Homework 7

(Due date: 2014/6/11)

This assignment covers Ch14 and Ch18 of the textbook. The full credit is 100 points. For each question, detailed derivation processes and accurate numbers are required to get full credit.

1) (10 points) Problem 14.8 of the textbook (p574), while the inductor in (e) is changed from 10 mH to 5 mH.

Ans:

1a)

$Z_L = j\omega L = j0L = 0$ so it is a short circuit.

At $\omega = 0$, $V_o = V_i$

1b)

$Z_L = j\omega L = j\infty L = \infty$ so it is a open circuit.

At $\omega = \infty$, $V_o = 0$

1c)

This is a low pass filter, with a gain of 1 at low frequencies and a gain of 0 at high frequencies.

1d)

$$H(s) = \frac{V_o}{V_i} = \frac{R}{R + sL} = \frac{\frac{R}{L}}{s + \frac{R}{L}}$$

1e)

$$\omega_c = \frac{R}{L} = \frac{330}{0.005} = 66\ \text{krad/s}$$

2) (10 points) Problem 14.15 of the textbook (p575), while the resistor is changed from 150 Ω to 200 Ω.

Ans:

2a)

For $\omega = 0$, the inductor behaves as a short circuit, so $V_o = 0$.

For $\omega = \infty$, the inductor behaves as an open circuit, so $V_o = V_i$.

2b)

$$H(s) = \frac{sL}{R + sL} = \frac{s}{s + R/L} = \frac{s}{s + 20000}$$

2c)

$$\omega_c = \frac{R}{L} = 20000\ \text{rad/s}$$

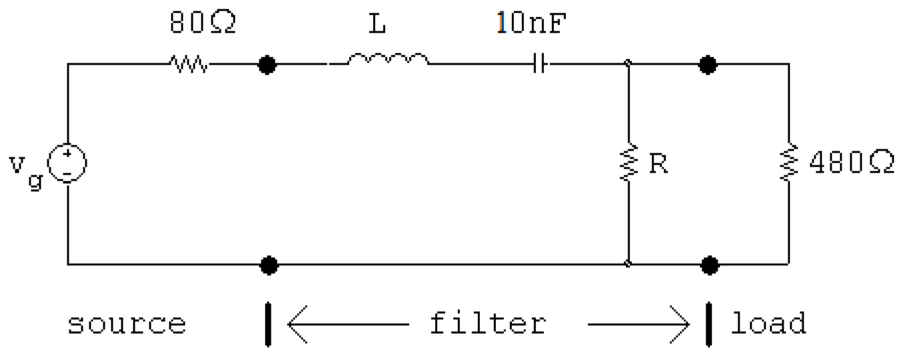
2d)

$$|H(jR/L)| = \left|\frac{jR/L}{\frac{jR}{L} + R/L}\right| = \left|\frac{j}{j + 1}\right| = \frac{1}{\sqrt{2}}$$

3) (10 points) Problem 14.30 of the textbook (p576), while the center frequency ω_0 is changed from 50 krad/s to 100 krad/s and the capacitor is changed from 20 nF to 10 nF.

Ans:

3a)



3b)

$$L = \frac{1}{\omega_o^2 C} = \frac{1}{(100 \times 10^3)^2 (10 \times 10^{-9})} = 10 \text{ mH}$$

$$R = \frac{\omega_o L}{Q} = \frac{(100 \times 10^3)(10 \times 10^{-3})}{6.25} = 160\ \Omega$$

3c)

$$R_e = 160 || 480 = 120\ \Omega$$

$$R_e + R_i = 120 + 80 = 200\ \Omega$$

$$Q_{\text{system}} = \frac{\omega_o L}{R_e + R_i} = \frac{(100 \times 10^3)(10 \times 10^{-3})}{200} = 5$$

3d)

$$\beta_{\text{system}} = \frac{\omega_o}{Q_{\text{system}}} = \frac{100 \times 10^3}{5} 20 \text{ krad/s}$$

$$\beta_{\text{system}}(\text{Hz}) = \frac{20000}{2\pi} = 3183.1 \text{ Hz}$$

4) (20 points) Problem 14.31 of the textbook (p577).

Ans:

4a)

$$\frac{V_o}{V_i} = \frac{Z}{Z+R} \text{ where } Z = \frac{1}{Y}$$

And $Y = sC + \frac{1}{sL} + \frac{1}{R_L} = \frac{LCR_Ls^2+sL+R_L}{R_LLs}$

$$H(s) = \frac{R_LLs}{RLCR_Ls^2 + (R+R_L)Ls + RR_L} = \frac{\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}}$$

$$= \frac{\left(\frac{R_L}{R+R_L}\right)\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)s}{s^2 + \left[\left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)\right]s + \frac{1}{LC}} = \frac{K\beta s}{s^2+\beta s+\omega_o^2},$$

$$K = \frac{R_L}{R+R_L}, \qquad \beta = \frac{1}{(R||R_L)C}$$

4b)

$$\beta = \left(\frac{R+R_L}{R_L}\right)\left(\frac{1}{RC}\right)$$

4c)

$$\beta_U = \frac{1}{RC}$$

$$\therefore \beta_L = \left(\frac{R+R_L}{R_L}\right)\beta_U = \left(1+\frac{R}{R_L}\right)\beta_U$$

4d)

$$Q = \frac{\omega_o}{\beta} = \frac{\omega_o RC}{\left(\frac{R+R_L}{R_L}\right)}$$

4e)

$$Q_U = \omega_o RC$$

$$\therefore Q_L = \left(\frac{R+R_L}{R_L}\right)Q_U = \frac{1}{\left(1+\frac{R}{R_L}\right)}Q_U$$

4f)

$$H(j\omega) = \frac{Kj\omega\beta}{\omega_o^2 - \omega^2 + j\omega\beta}$$

$$H(j\omega_o) = K$$

Let ω_c represent a corner frequency. Then

$$|H(j\omega_c)| = \frac{K}{\sqrt{2}} = \frac{K\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c{}^2)^2 + \omega_c{}^2\beta^2}}$$

$$\therefore \frac{1}{\sqrt{2}} = \frac{\omega_c\beta}{\sqrt{(\omega_o^2 - \omega_c{}^2)^2 + \omega_c{}^2\beta^2}}$$

Squaring both sides leads to

$(\omega_o^2 - \omega_c{}^2)^2 = \omega_c{}^2\beta^2$ or $\omega_o^2 - \omega_c{}^2 = \pm\omega_c\beta$

$$\therefore \omega_c{}^2 \pm \omega_c\beta - \omega_o^2 = 0$$

Or

$$\omega_c = \mp\frac{\beta}{2} \pm \sqrt{\frac{\beta^2}{4} + \omega_o^2}$$

The two positive roots are

$\omega_{c1} = -\frac{\beta}{2} + \sqrt{\frac{\beta^2}{4} + \omega_o^2}$ and $\omega_{c2} = \frac{\beta}{2} - \sqrt{\frac{\beta^2}{4} + \omega_o^2}$

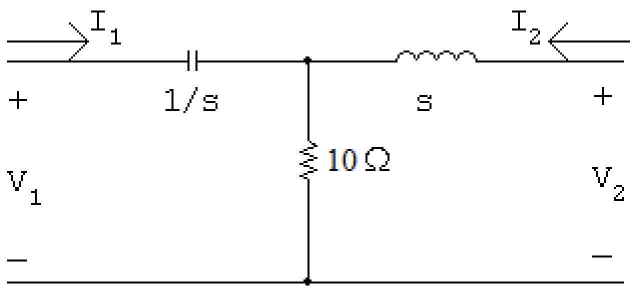
Where

$\beta = \left(1 + \frac{R}{R_L}\right)\frac{1}{RC}$ and $\omega_o^2 = \frac{1}{LC}$

5) (10 points) Problem 18.18 of the textbook (p722), while the resistor is changed from 4 Ω to 10 Ω and the goal is changed from expressions for the *a* parameters to those of the *g* parameters.

Ans:

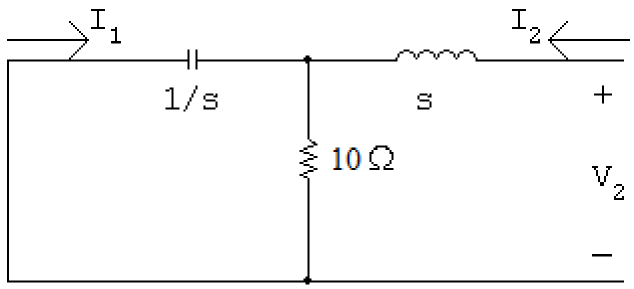
For I_2=0:



$$V_2 = \frac{V_1}{10 + \frac{1}{s}} \times 10 = \frac{10sV_1}{10s+1}, \qquad g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = \frac{10s}{10s+1}$$

$$I_1 = \frac{V_1}{10 + \frac{1}{s}} = \frac{sV_1}{10s+1}, \qquad g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{s}{10s+1}$$

For V_1=0:



$$I_1 = \frac{-I_2 \times 10}{10 + \frac{1}{s}}, \qquad g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0} = \frac{-10s}{10s+1}$$

$$V_2 = \frac{(10s^2+s+10)I_2}{1+10s}, \qquad g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0} = \frac{(10s^2+s+10)}{1+10s}$$

6) (10 points) Problem 18.19 of the textbook (p722).

Ans:

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = \frac{(1+1/s)(1)}{2+1/s} + s = \frac{s+1}{2s+1} + s = \frac{2s^2+2s+1}{2s+1}$$

$z_{22} = z_{11}$ (the circuit is reciprocal and symmetrical)

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

$$V_2 = I_1 \frac{1}{2+\frac{1}{s}}(1) + sI_1; \quad \frac{V_2}{I_1} = \frac{s}{2s+1} + s = \frac{2s^2+2s}{2s+1}$$

$$z_{21} = \frac{2s^2+2s}{2s+1} = \frac{2s(s+1)}{2s+1}$$

$z_{12} = z_{21}$ (the circuit is reciprocal and symmetrical)

7) (15 points) Problem 18.29 of the textbook (p722), while the right resistor is changed from 100 Ω to 60 Ω.

$$\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_gZ_L}$$

$$\Delta b = b_{11}b_{22} - b_{12}b_{21} = (25)(-40) - (1000)(-1.25) = 250$$

$$\therefore \frac{V_2}{V_g} = \frac{250(60)}{1000 + 25(20) - 40(60) - 1.25(1200)} = -6.25$$

$$V_2 = -6.25(120\angle 0°) = 750\angle 180° \text{ V(rms)}$$

$$I_2 = \frac{-V_2}{60} = \frac{-750\angle 180°}{60} = 12.5 \text{ A(rms)}$$

$$\frac{I_2}{I_1} = \frac{-\Delta b}{b_{11} + b_{21}Z_L} = \frac{-250}{25 - 1.25(60)} = 5$$

$$\therefore \quad I_1 = \frac{I_2}{5} = \frac{12.5}{5} = 2.5 \text{ A(rms)}$$

$$\therefore P_g = (120)(2.5) = 300 \text{ W}; \quad P_o = 12.5^2(60) = 9375 \text{ W}$$

$$\therefore \frac{P_o}{P_g} = \frac{9375}{300} = 31.25$$

8) (15 points) Problem 18.34 of the textbook (p723), while the value of V_1 in Measurement 2 is changed from 20 mV to 25 mV.

Ans:

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$I_2 = h_{21}I_1 + h_{22}V_2$

From the first measurement:

$$h_{11} = \frac{V_1}{I_1} = \frac{4}{5 \times 10^{-3}} = 800\ \Omega$$

$$h_{21} = \frac{I_2}{I_1} = \frac{-200}{5} = -40$$

From the second measurement:

$$h_{22}V_2 = 40I_1, \qquad h_{22} = 40 \times \frac{20 \times 10^{-6}}{40} = 20\ \mu\text{S}$$

$$25 \times 10^{-3} = 800(20 \times 10^{-6}) + 40h_{12}, \qquad h_{12} = 2.25 \times 10^{-4}$$

Summary:

$h_{11} = 800\ \Omega;\ h_{12} = 2.25 \times 10^{-4};\ h_{21} = -40;\ h_{22} = 20\ \mu\text{S}$

From the circuit,

$Z_g = 250\ \Omega;\ V_g = 5.25\ \text{mV}$

$$Z_{\text{Th}} = \frac{Z_g + h_{11}}{h_{22}Z_g + \Delta h};\ \ \Delta h = 800(20 \times 10^{-6}) + 40(2.25 \times 10^{-4}) = 0.025$$

$$Z_{\text{Th}} = \frac{250 + 800}{20 \times 10^{-6}(250) + 0.025} = 35\ \text{k}\Omega$$

$$V_{\text{Th}} = \frac{-h_{21}V_g}{h_{22}Z_g + \Delta h} = \frac{40(5.25 \times 10^{-3})}{20 \times 10^{-6}(250) + 0.025} = 7\ \text{V}$$

$$i = \frac{7}{70000} = 1 \times 10^{-4}\ \text{A}$$

$$P_{\max} = (1 \times 10^{-4})^2 \times 35000 = 0.00035 = 350\ \mu\text{W}$$