Homework 5

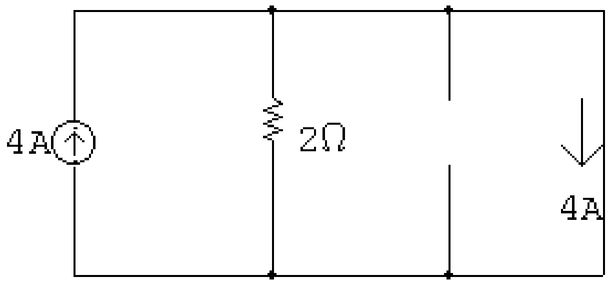
(Due date: 2014/4/30)

This assignment covers Ch7 and Ch8 of the textbook. The full credit is 100 points. For each question, detailed derivation processes and accurate numbers are required to get full credit.

1) (10 points) Problem 7.17 of the textbook (p271), while the dependent source current is changed from $4v_o$ to $2.5v_o$.

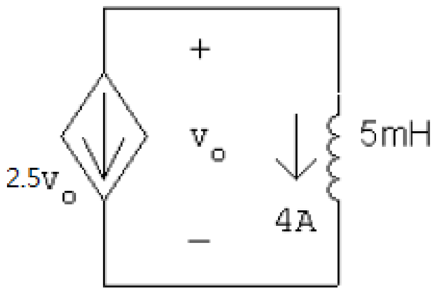
Ans:

$t < 0$

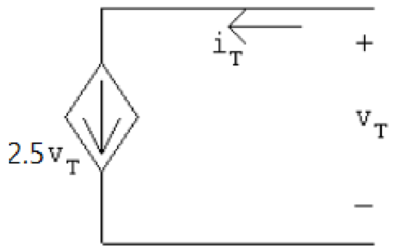


$i_L(0^-) = i_L(0^+) = 4\text{ A}$

$t > 0$

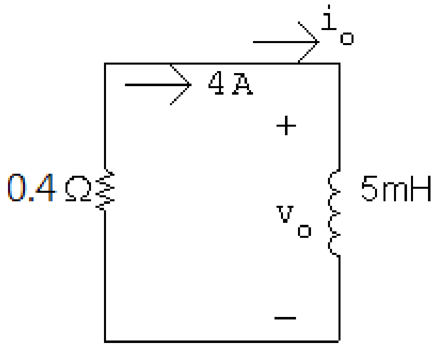


Find Thévenin resistance seen by inductor :



$i_T = 2.5v_T; \quad \frac{v_T}{i_T} = R_{Th} = \frac{1}{2.5} = 0.4\ \boldsymbol{\Omega}$

$\tau = \frac{L}{R} = \frac{5 \times 10^{-3}}{0.4} = 12.5\text{ ms}, \qquad \frac{1}{\tau} = 80$



$i_o = 4e^{-80t}\text{ A}, \qquad t \geq 0$

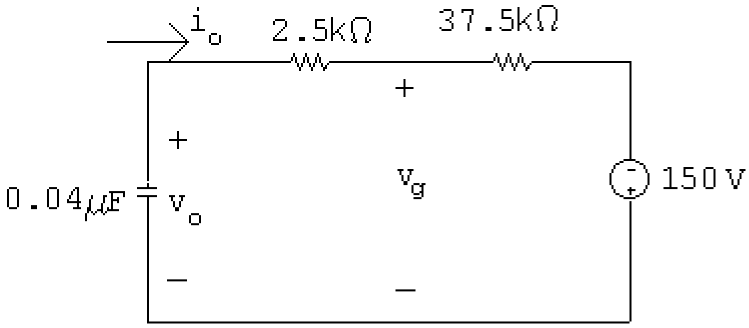
$v_o = L\frac{di_o}{dt} = (5 \times 10^{-3})(-320e^{-80t}) = -1.6e^{-80t}\text{ V}, \qquad t \geq 0^+$

2) (20 points) Problem 7.52 of the textbook (p276), while the middle resistor is changed from 12.5 kΩ to 2.5 kΩ.

Ans:

2a)

$v_0(0^-) = v_0(0^+) = 120\text{ V}$



$v_0(\infty) = -150\text{ V};\quad \tau = 1.6\text{ ms};\quad \frac{1}{\tau} = 625$

$v_0 = -150 + (120 - (-150))e^{-625t}$

$v_0 = -150 + 270e^{-625t}\text{ V}, t \geq 0$

2b)

$i_0 = -0.04 \times 10^{-6}(-625)[\,270e^{-625t}] = 6.75e^{-625t}\text{ mA}, t \geq 0^+$

2c)

$v_g = v_0 - 2.5 \times 10^3 i_0 = -150 + 253.125e^{-625t}\text{ V}$

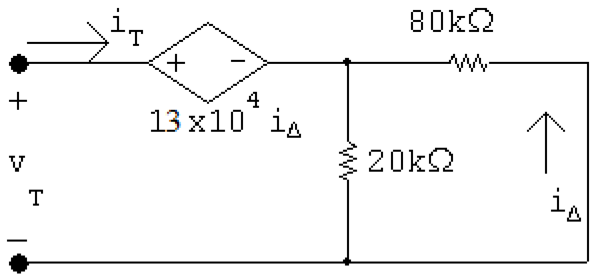
2d)

$v_g(0^+) = -150 + 253.125 = 103.125\text{ V}$

3) (10 points) Problem 7.84 of the textbook (p280), while the dependent source voltage is changed from $12{\times}10^4\, i_\Delta$ to $13{\times}10^4\, i_\Delta$.

Ans:

t > 0:



$v_T = 13\times 10^4 i_\Delta + 16\times 10^3 i_T$

$i_\Delta = -\frac{20}{100} i_T = -0.2 i_T$

$\therefore v_T = -26\times 10^3 i_T + 16\times 10^3 i_T$

$R_{\mathrm{Th}} = \frac{v_T}{i_T} = -10\ \mathrm{k}\Omega$

$\tau = RC = (-10\times 10^3)(2.5\times 10^{-6}) = -0.025, \qquad \frac{1}{\tau} = -40$

$v_c = 20e^{40t}\mathrm{V}; \qquad 20e^{40t} = 20000$

$40t = \ln(1000), \qquad \therefore t =$ 172.7 ms

4) (20 points) Problem 8.16 of the textbook (p321), while the voltage source is changed from -15 V to -10 V.

Ans:

4a)

$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(2.5)(100)} = 4\times 10^6$

$\omega_o = 2000\ \mathrm{rad/s}$

$\frac{1}{2RC} = 2000; \quad R = \frac{1}{4000C} =$ 2500 Ω

4b)

$v(t) = D_1 t e^{-2000t} + D_2 e^{-2000t}$

$v(0) = -10\text{ V} = D_2$

$i_C(0) = 6 + \frac{10}{2.5} = 10\text{ mA}$

$\frac{dv}{dt}(0) = \frac{i_C(0)}{C} = \frac{10 \times 10^{-3}}{100 \times 10^{-9}} = 1 \times 10^5$

$D_1 - 2000(-10) = 1 \times 10^5 \quad \text{so} \;\; D_1 = 80000\frac{\text{V}}{\text{s}}$

$\therefore v(t) = (80000t - 10)e^{-2000t}\text{ V}, \quad t \geq 0$

4c)

$i_C(t) = 0 \;\text{ when}\; \frac{dv}{dt}(t) = 0$

$\frac{dv}{dt} = (100{,}000 - 160 \times 10^6 t)e^{-2000t}$

$\frac{dv}{dt} = 0 \;\text{ when }\; 160 \times 10^6 t_1 = 100000; \quad \therefore t_1 = 625\ \mu\text{s}$

$v(625\ \mu\text{s}) = (50 - 10)e^{-1.25} = 11.4602\text{ V}$

4d)

$w(0) = \frac{1}{2}(100 \times 10^{-9})(10)^2 + \frac{1}{2}(2.5)(0.006)^2 = 50\ \mu\text{J}$

$w(625\ \mu\text{s}) = \frac{1}{2}(100 \times 10^{-9})(11.46)^2 + \frac{1}{2}(2.5)\left(\frac{11.46}{2500}\right)^2 = 32.83\ \mu\text{J}$

$\%\text{ remaining} = \frac{32.83}{50}(100\%) = 65.66\%$

5) (10 points) Problem 8.29 of the textbook (p322), while the resistor is changed from 150 Ω to 100 Ω.

Ans:

$\omega_o^2 = \frac{1}{LC} = \frac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8; \quad \omega_o = 10^4\ \text{rad/s}$

$\alpha = \frac{1}{2RC} = \frac{1}{2(100)(0.2 \times 10^{-6})} = 25000\, \text{rad/s} \quad \therefore \text{overdamped}$

$s_{1,2} = -25000 \pm \sqrt{(-25000)^2 - 10^8} = -25000 \pm 22912.9\ \text{rad/s}$

$s_1 = -2087.1\ \text{rad/s}; \quad s_2 = -47912.9\ \text{rad/s};$

$I_f = 60$ mA

$i_L = 60 \times 10^{-3} + A_1' e^{-2087.1t} + A_2' e^{-47912.9t}$

$\therefore -45 \times 10^{-3} = 60 \times 10^{-3} + A_1' + A_2'; \quad A_1' + A_2' = -105 \times 10^{-3}$

$\frac{di_L}{dt} = -2087.1A_1' - 47912.9A_2' = \frac{15}{0.05} = 300$

Solving, $A_1' = -0.10324 \quad A_2' = -0.00176$

$i_L = 60 \times 10^{-3} - 0.10324e^{-2087.1t} - 0.00176A_2'e^{-47912.9t}$ (A)

6) (10 points) Problem 8.44 of the textbook (p323), while the initial capacitor voltage is changed from 15 V to 10 V.

Ans:

6a)

$\omega_o^2 = \frac{1}{LC} = \frac{10^9}{(125)(0.32)} = 25 \times 10^6$

$\alpha = \frac{R}{2L} = \omega_o = 5000\ \text{rad/s}$

$\therefore R = (5000)(2)L = 1250\ \Omega$

6b)

$i(0) = i_L(0) = 4$ mA

$v_L(0) = 10 - (0.004)(1250) = 5$ V

$\frac{di}{dt}(0) = \frac{5}{0.125} = 40\ \text{A/s}$

6c)

$v_C = D_1 t e^{-5000t} + D_2 e^{-5000t}$

$v_C(0) = D_2 = 10 \text{ V}$

$\frac{dv_C}{dt}(0) = D_1 - 5000D_2 = \frac{i_C(0)}{C} = \frac{-i_L(0)}{C} = -12500$

$\therefore D_1 = 37500 \text{ V/s}$

$v_C = 37500te^{-5000t} + 10e^{-5000t} \text{ V}, \ t \geq 0$

7) (10 points) Problem 8.50 of the textbook (p324), while the voltage source is changed from 60 V to 30 V.

Ans:

$\alpha = \frac{R}{2L} = 2000 \text{ rad/s}$

$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(6.25 \times 10^{-6})} = 256 \times 10^4$

$s_{1,2} = -2000 \pm \sqrt{4 \times 10^6 - 256 \times 10^4} = -2000 \pm 1200 \text{ rad/s}$

$v_o = V_f + A_1' e^{-800t} + +A_2' e^{-3200t}$

$v_o(0) = 0 = V_f + A_1' + A_2'$

$v_o(\infty) = 30 \text{ V}; \quad \therefore A_1' + A_2' = -30$

$\frac{dv_o(0)}{dt} = 0 = -800A_1' - 3200A_2'$

$\therefore A_1' = -40 \text{ V}; \quad A_2' = 10 \text{ V}$

$v_o = 30 - 40e^{-800t} + 10e^{-3200t} \text{ V}, \ t \geq 0$

8) (10 points) Problem 8.57 (a) of the textbook (p325).

Ans:

$$v_c = V_f + [B_1' \cos w_d t + +B_2' \sin w_d t]e^{-\alpha t}$$

$$\frac{dv_c}{dt} = [(w_d B_2' - \alpha B_1') \cos w_d t - (\alpha B_2' + w_d B_1') \sin w_d t]e^{-\alpha t}$$

Since the initial stored energy is zero,

$v_c(0^+) = 0$ and $\dfrac{dv_c(0^+)}{dt} = 0$

It follows that $B_1' = -V_f$ and $B_2' = \alpha B_1'/w_d$

When these values are substituted into the expression for $[dv_c/dt]$, we get

$$\frac{dv_c}{dt} = \left(\frac{\alpha^2}{w_d} + w_d\right) V_f e^{-\alpha t} \sin w_d t$$

But $V_f = V$ and $\dfrac{\alpha^2}{w_d} + w_d = \dfrac{\alpha^2 + {w_d}^2}{w_d} = \dfrac{w_o^2}{w_d}$

Therefore $\dfrac{dv_c}{dt} = \left(\dfrac{w_o^2}{w_d}\right) V e^{-\alpha t} \sin w_d t$