Homework4 solutions

(Due date: 2014/4/16)

This assignment covers Ch9 and Ch10 of the textbook. The full credit is 100 points. For each question, detailed derivation processes and accurate numbers are required to get full credit.

1) (10 points) Problem 9.14 of the textbook (p371), while the maximum amplitude of the steady-state current in the inductor is changed from 25 A to 20 A.

Ans:

1a)

400 Hz

1b)

$\theta_v = 0^o$

$\mathbf{I} = \frac{100\angle 0^o}{jwL} = \frac{100}{wL}\angle -90^o, \quad \theta_i = -90^o$

1c)

$\frac{100}{wL} = 20, \qquad wL = 5\ \Omega$

1d)

$L = \frac{5}{800\pi} \approx 2\ \text{mH}$

1e)

$Z_L = jwL = j5\ \Omega$

2) (10 points) Problem 9.35 of the textbook (p374), while the impedance of the left inductor is changed from j5 Ω to j2 Ω.

Ans:

V_2 is the voltage across the $-j10\Omega$ impedance.

$$\frac{V_1 - V_g}{20} + \frac{V_1}{j2} + \frac{V_1 - V_2}{Z} = 0$$

$$\frac{(40 + j30) - (100 - j50)}{20} + \frac{40 + j30}{j2} + \frac{(40 + j30) - V_2}{Z} = 0$$

$$\therefore V_2 = 40 + j30 + (12 - j16)Z$$

$$\frac{V_2 - V_1}{Z} + \frac{V_2}{-j10} - (20 + j30) + \frac{V_2 - (100 - j50)}{3 + j1} = 0$$

$$\frac{V_2 - (40 + j30)}{Z} + \frac{V_2}{-j10} - (20 + j30) + \frac{V_2 - (100 - j50)}{3 + j1} = 0$$

Substituting the expression for V_2 found at the start and simplifying yields:

$Z = 0.5 + j4\ \Omega$

3) (20 points) Problem 9.45 of the textbook (p375), while the impedance of capacitor is changed from –j48 Ωto –j36 Ω and the resistor R_0 is changed from 36 Ω to 24 Ω.

Ans:

Step 1 to Step 2:

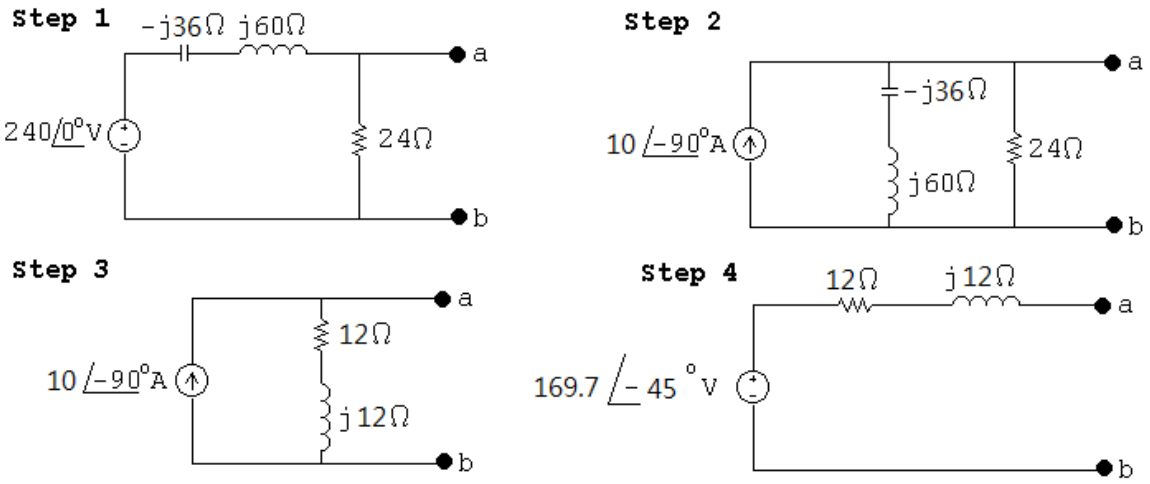
$$\frac{240\angle 0^o}{j24} = -j10 = 10\angle -90^o\ \text{A}$$

Step 2 to Step 3:

$j24||24 = 12 + j12\ \Omega$

Step 3 to Step 4:

$(10\angle -90^o)(12 + j12) = 120 - j120 = 169.7\angle -45^o\ \text{V}$



4) (10 points) Problem 9.59 of the textbook (p377), while the current source is changed from 10+j10 to 10+j20.

Ans:

$$\frac{V_o}{-j8}+\frac{V_o-2.4I_\Delta}{j4}+\frac{V_o}{5}-(10+j20)=0$$

The constant equation is:

$$I_\Delta=\frac{V_o}{-j8}, \text{Sovling that, } V_o=-40+j120=126.5\angle 108.43°V$$

5) (10 points) Problem 9.78 of the textbook (p379), while the left resistor is changed from 5 Ω to 10 Ω.

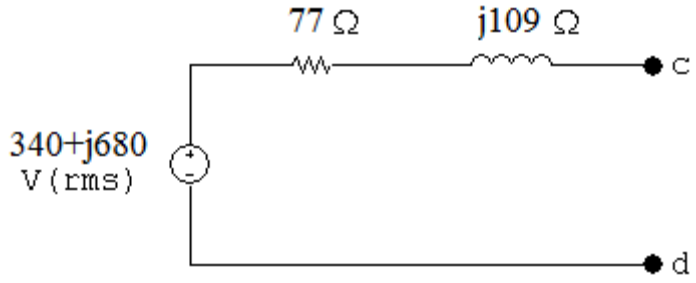
Ans:

Remove the voltage source to find the equivalent impedance:

$$Z_{Th}=45+j125+\left(\frac{20}{|10+j5|}\right)^2(10-j5)=77+j109\ \Omega$$

Using voltage division:

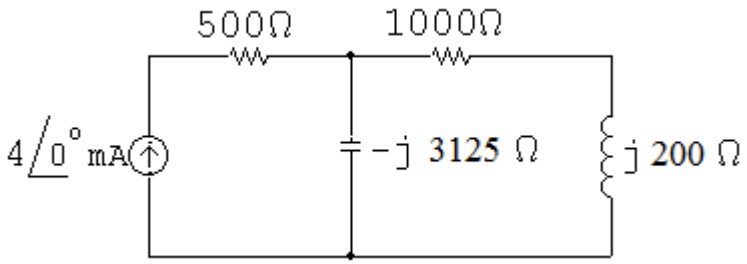
$$V_{Th}=V_{cd}=j20I_1=j20\left(\frac{425}{10+j5}\right)=340+j680\ V$$



6) (10 points) Problem 10.5 of the textbook (p409), while the current source is changed from 4 cos 5000*t* mA to 4 cos 2000*t* mA.

Ans:

$\mathrm{I_g} = 4\angle 0°$ mA; $\quad \dfrac{1}{j\omega C} = -j3125\ \Omega$; $\quad j\omega L = j200\ \Omega$



$Z_{\mathrm{eq}} = 500 + [-j3125 \parallel (1000 + j200)] = 1522 - \mathrm{j}135.72\ \Omega$

$P_{\mathrm{g}} = -\dfrac{1}{2}|I|^2 \mathrm{Re}\{Z_{\mathrm{eq}}\} = -\dfrac{1}{2}(0.004)^2(1522) = -0.0122 = -12.2$ mW

The source delivers 12.2 mW of power to the circuit.

7) (10 points) Problem 10.27 of the textbook (p412), while the voltage drop is changed from 250 V to 200 V (rms).

Ans:

7a)

$S_1 = 16 + j28$ kVA; $\quad S_2 = 6 - j8$ kVA; $\quad S_3 = 8 + j0$ kVA

$S_T = S_1 + S_2 + S_3 = 30 + j20$ kVA

$200\mathbf{I}^* = (30 + j20) \times 10^3; \quad \therefore \mathbf{I} = 150 - \text{j}100$ A

$$Z = \frac{200}{150 - \text{j}100} = 0.9234 + j0.6154\ \Omega = 1.1094\angle 33.69°$$

7b)

$\text{pf} = \cos(33.69°) = 0.8321$

8) (20 points) Problem 10.56 of the textbook (p416).

Ans:

8a)

$$\text{V}_{\text{Th}} = \frac{760\angle 0°}{28 + j96}(j50) = 380\angle 16.26°\ \text{V}$$

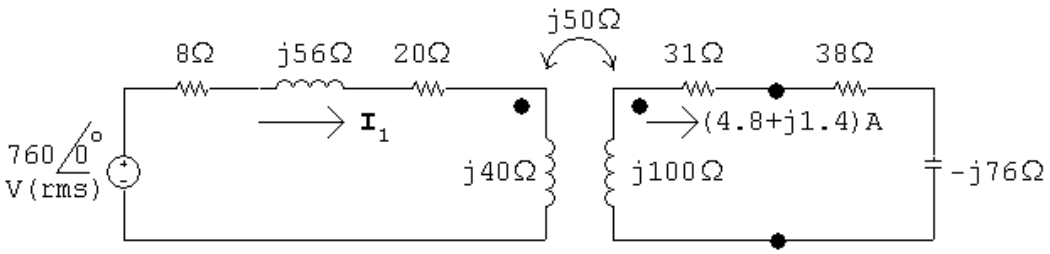
$$\text{Z}_{\text{Th}} = 31 + j100 + \left(\frac{50}{100}\right)^2 (28 - j96) = 38 + j76\ \Omega$$

$\therefore\ \text{Z}_{\text{L}} = 38 - 76\ \Omega$

$$\mathbf{I}_{\text{L}} = \frac{380\angle 16.26°}{76} = 4.8 + j1.4 = 5\angle 16.26°\ \text{A(rms)}$$

$\text{P}_{\text{L}} = |\mathbf{I}_{\text{L}}|^2(38) = 950$ W

8b)



$760\angle 0° = \mathbf{I}_1(28 + j96) - j50(4.8 + j1.4)$

$$\therefore\ \mathbf{I}_1 = \frac{690 + j240}{100\angle 73.74°} = 7.31\angle -54.56° = 4.24 - j5.95\ \text{A}$$

$S_{\text{g}}(\text{delivered}) = 760(4.24 + j5.95) = 3219.36 + j4523.52 \text{ VA}$

$P_{\text{loss}} = |\mathbf{I}_1|^2(8) = 426.96 \text{ W}$

$P_{\text{in}}(\text{transformer}) = 3219.36 - 426.96 = 2792.4 \text{ W}$

$\%$ delivered to $\text{Z}_\text{L} = \dfrac{950}{2792.4} \times 100\% = 34.02\%$