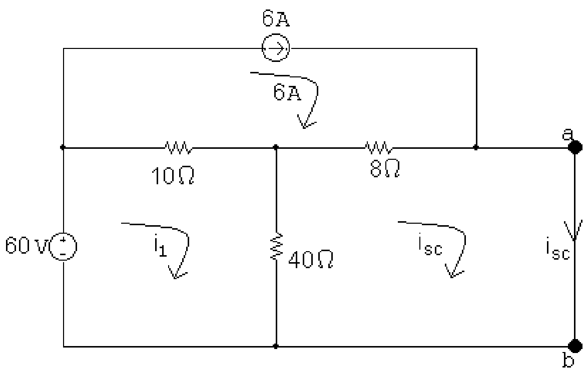
Homework3 solutions

(Due date: 2014/3/19)

This assignment covers Ch4 and Ch6.1-6.3 of the textbook. The full credit is 100 points. For each question, detailed derivation processes and accurate numbers are required to get full credit.

1) (10 points) Problem 4.66 of the textbook (p160), while the current source is changed from 4 A to 6 A.

Ans:

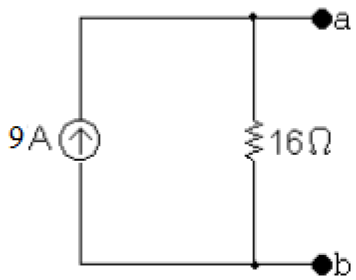


$50i_1 - 40i_{sc} = 60 + 60$

$-40i_1 + 48i_{sc} = 48$

Solving, $i_{sc} = 9$ A

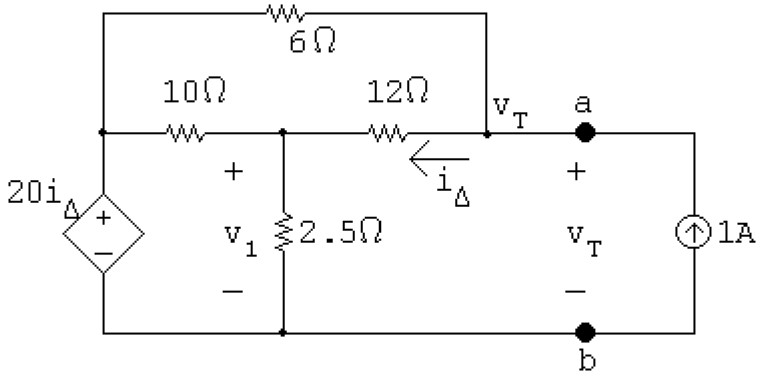
$$R_{Th} = 8 + \frac{(10)(40)}{50} = 16\ \Omega$$



2) (10 points) Problem 4.77 of the textbook (p161), while the current-controlled voltage source is changed from 10 i_Δ to 20 i_Δ.

Ans:

$V_{Th} = 0$, since circuit contains no independent sources.



$$\frac{v_1 - 20i_\Delta}{10} + \frac{v_1}{2.5} + \frac{v_1 - v_T}{12} = 0$$

$$\frac{v_T - v_1}{12} + \frac{v_T - 20i_\Delta}{6} - 1 = 0$$

$$i_\Delta = \frac{v_T - v_1}{12}$$

In standard form:

$$v_1\left(\frac{1}{10} + \frac{1}{2.5} + \frac{1}{12}\right) + v_T\left(-\frac{1}{12}\right) + i_\Delta(-2) = 0$$

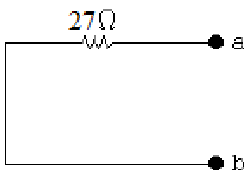
$$v_1\left(-\frac{1}{12}\right) + v_T\left(\frac{1}{12} + \frac{1}{6}\right) + i_\Delta\left(\frac{-20}{6}\right) = 1$$

$$v_1(1) + v_T(-1) + i_\Delta(12) = 0$$

Solving,

$v_1 = 9$ V; $v_T = 27$ V; $i_\Delta = 1.5$ A

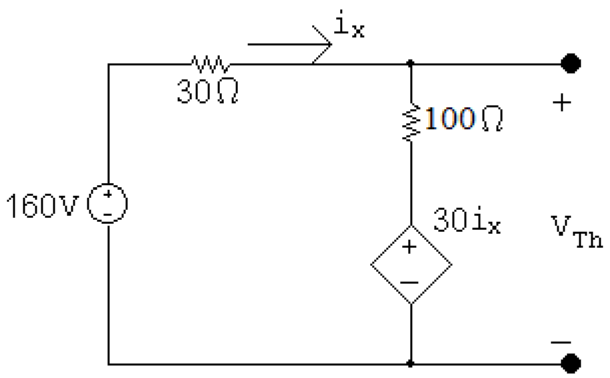
$$\therefore R_{Th} = \frac{v_T}{1\text{ A}} = 27\ \Omega$$



3) (20 points) Problem 4.83 of the textbook (p162), while the right resistor is changed from 20 Ω to 100 Ω and the power dissipated in the resistor R_0 is changed from 250 W to 225 W.

Ans:

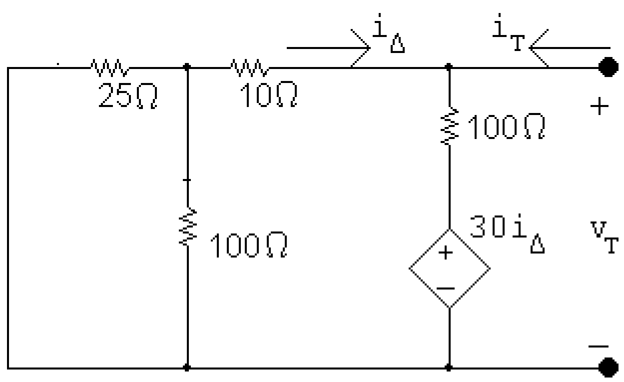
We begin by finding the Thévenin equivalent with respect to R_o. After making a couple of source transformations the circuit simplifies to



$$i_x = \frac{160 - 30i_x}{130}; \quad i_x = 1\text{ A}$$

$$V_{Th} = 100i_x + 30i_x = 130i_x = 130\text{ V}$$

Using the test-source method to find the Thévenin resistance gives

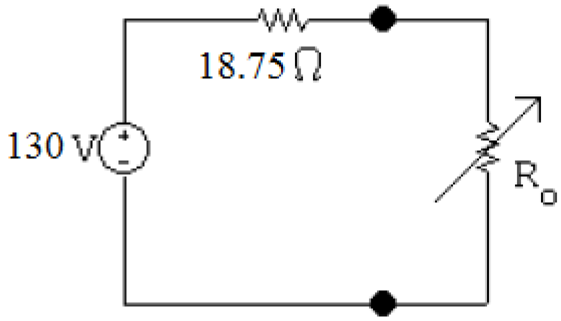


$$i_T = \frac{v_T}{30} + \frac{v_T - 30(-\frac{v_T}{30})}{100}$$

$$\frac{i_{\mathrm{T}}}{v_{\mathrm{T}}} = \frac{1}{30} + \frac{1}{50} = \frac{8}{150} = \frac{4}{75}$$

$$R_{\mathrm{Th}} = \frac{v_{\mathrm{T}}}{i_{\mathrm{T}}} = \frac{75}{4} = 18.75\ \Omega$$

Thus our problem is reduced to analyzing the circuit shown below.



$$p = \left(\frac{130}{18.75 + R_{\mathrm{o}}}\right)^2 R_{\mathrm{o}} = 225$$

$$\frac{(130)^2}{{R_{\mathrm{o}}}^2 + 37.5R_{\mathrm{o}} + 18.75^2} R_{\mathrm{o}} = 225$$

$$\frac{(130)^2}{225} R_{\mathrm{o}} = {R_{\mathrm{o}}}^2 + 37.5R_{\mathrm{o}} + 18.75^2$$

$${R_{\mathrm{o}}}^2 + \left(37.5 - \frac{(130)^2}{225}\right) R_{\mathrm{o}} + 18.75^2 = 0$$

Solving,

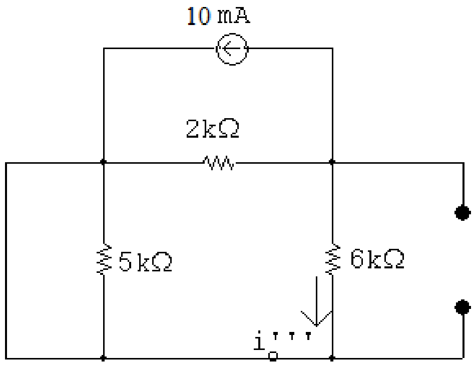
$R_{\mathrm{o}} =$ 20.25 Ω & 17.36 Ω

4) (10 points) Problem 4.95 of the textbook (p163), while the top current source is changed from 5 mA to 10 mA.

Ans:

4a)

By hypothesis $i_o' + i_o'' = 3.5$ mA



$i_o''' = -10\frac{2}{8} = -2.5; \quad i_o = 3.5 - 2.5 = 1\text{ mA}$

4b)

With all three sources in the circuit write a single node voltage equation.

$\frac{v_b}{6} + \frac{v_b - 8}{2} + 10 - 10 = 0$

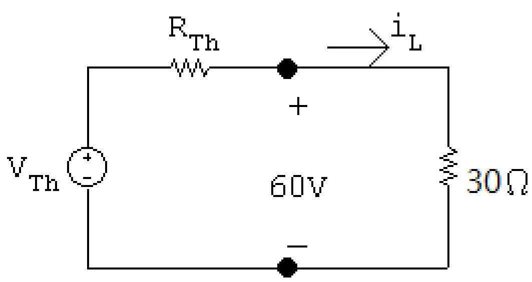
$\therefore v_b = 6\text{ V}$

$i_o = \frac{v_b}{6} = 1\text{ mA}$

5) (10 points) Problem 4.102 of the textbook (p165), while the resistor is changed from 20 Ω to 30 Ω.

Ans:

5a)



$v_{oc} = V_{Th} = 75\text{ V}; \quad i_L = 60/30 = 2\text{ A};$

$i_L = 75 - 60/R_{Th} = 15/R_{Th}; \quad R_{Th} = 7.5\ \Omega$

5b)

$$i_{\mathrm{L}} = \frac{v_{\mathrm{o}}}{R_{\mathrm{L}}} = \frac{V_{\mathrm{Th}} - v_{\mathrm{o}}}{R_{\mathrm{Th}}}$$

$$R_{\mathrm{Th}} = \frac{V_{\mathrm{Th}} - v_{\mathrm{o}}}{v_{\mathrm{o}}/R_{\mathrm{L}}} = \left(\frac{V_{\mathrm{Th}}}{v_{\mathrm{o}}} - 1\right) R_{\mathrm{L}}$$

6) (10 points) Problem 6.15 of the textbook (p228), while the initial voltage on the capacitor is changed from -60.6 V to -100 V.

Ans:

6a)

$$v = 5 \times 10^6 \int_0^{250\times10^{-6}} 100 \times 10^{-3} \mathrm{e}^{-1000t} dt - 100$$

$$= 500 \times 10^3 \frac{\mathrm{e}^{-1000t}}{-1000}\Big|_0^{250\times10^{-6}} - 100$$

$$= 500(1 - \mathrm{e}^{-0.25}) - 100 = 10.6 \text{ V}$$

$$w = \frac{1}{2} C v^2 = \frac{1}{2}(0.2)(10^{-6})(10.6)^2 = 11.236 \ \mu\text{J}$$

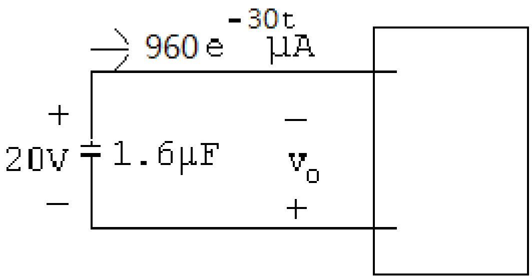
6b)

$$v = 500 - 100 = 400 \text{ V}$$

$$w = \frac{1}{2}(0.2) \times 10^{-6}(400)^2 = 16 \text{ mJ} = 16000 \ \mu\text{J}$$

7) (20 points) Problem 6.31 of the textbook (p230), while the resulting current i(t) is changed from $800e^{-25t}$ to $960e^{-30t}$.

Ans:



7a)

$$v_o = \frac{10^6}{1.6}\int_0^t 960 \times 10^{-6} e^{-30x}dx - 20$$

$$= 600\frac{e^{-30x}}{-30}\Big|_0^t = -20e^{-30t}\ \text{V},\ \ t \geq 0$$

7b)

$$v_1 = \frac{10^6}{2}(960 \times 10^{-6})\frac{e^{-30x}}{-30}\Big|_0^t + 5 = -16e^{-30t} + 21\ \text{V},\ \ t \geq 0$$

7c)

$$v_2 = \frac{10^6}{8}(960 \times 10^{-6})\frac{e^{-30x}}{-30}\Big|_0^t - 25 = -4e^{-30t} - 21\ \text{V},\ \ t \geq 0$$

7d)

$$p = -vi = -(-20e^{-30t})(960 \times 10^{-6})e^{-30t} = 19.2 \times 10^{-3}e^{-60t}$$

$$w = \int_0^\infty 19.2 \times 10^{-3}e^{-60t}dt = 19.2 \times 10^{-3}\frac{e^{-60x}}{-60}\Big|_0^\infty = 320\ \mu\text{J}$$

7e)

$$w = \frac{1}{2}(2 \times 10^{-6})(5)^2 + \frac{1}{2}(8 \times 10^{-6})(25)^2 = 2525\ \mu\text{J}$$

7f)

$$w_{\text{trapped}} = w_{\text{initial}} - w_{\text{delivered}} = 2525 - 320 = 2205\ \mu\text{J}$$

7g)

$$w_{\text{trapped}} = \frac{1}{2}(2\times10^{-6})(21)^2 + \frac{1}{2}(8\times10^{-6})(-21)^2 = 2205\ \mu\text{J}$$

8) (10 points) Problem 6.34 of the textbook (p230).

Ans:

$$v_2(\text{t}) = L\frac{di_o}{dt}$$

$$= 20\times10^{-3}\times50\times(-8000\text{e}^{-8000\text{t}})\times(\cos6000\text{t}+\sin6000\text{t})\times10^{-3}$$

$$+20\times10^{-3}\times50\times\text{e}^{-8000\text{t}}\times(6000\times(-\sin6000\text{t})+12000\cos6000\text{t})\times10^{-3}$$

$$= \text{e}^{-8000\text{t}}(4\cos6000\text{t}-22\sin6000\text{t})$$

$$v_2(0^+) = 4\ \text{V}$$

$$\text{V}_{320\Omega}(0^+) = 320\times i_o(0^+) = 320\times50\times10^{-3} = 16\ \text{V}$$

$$\therefore v_1(0^+) = 16+4 = 20\ \text{V}$$