Homework2 solutions

(Due date: 2014/3/12)

This assignment covers Ch3 and Ch4.1-4.9 of the textbook. The full credit is 100 points. For each question, detailed derivation processes and accurate numbers are required to get full credit.

1) (10 points) Problem 3.8 of the textbook (p100), while the right resistor is changed from 6 Ω to 9 Ω.

Ans:

1a)

$p_{4\Omega} = i_s^2 4 = \left(\frac{120}{11.2}\right)^2 4 \cong 459.1837\ \Omega$ $\quad p_{18\Omega} = i_1^2 18 = \left(\frac{120}{11.2} \times \frac{2}{5}\right)^2 18 \cong 330.6122\ \Omega$

$p_{3\Omega} = i_2^2 3 = \left(\frac{120}{11.2} \times \frac{3}{5}\right)^2 3 \cong 123.9796\ \Omega$ $\quad p_{9\Omega} = i_2^2 9 = \left(\frac{120}{11.2} \times \frac{3}{5}\right)^2 9 \cong 371.9388\ \Omega$

1b)

$$p_{120\text{V}}(\text{delivered}) = 120 i_s = 120 \times \frac{120}{11.2} \cong 1285.714\ \text{W}$$

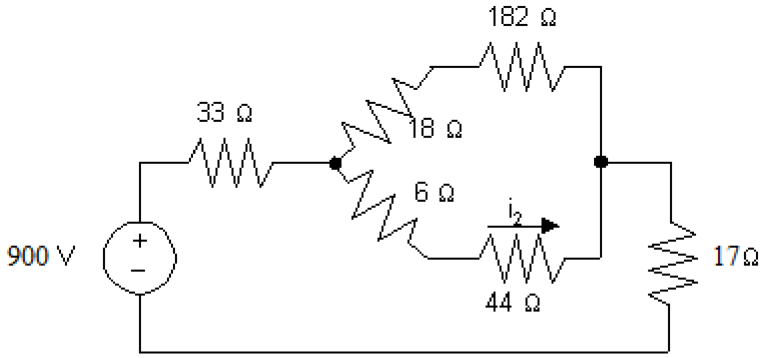
1c)

$$p_{\text{diss}} = 459.1837 + 330.6122 + 123.9796 + 371.9388 \cong 1285.714\ \text{W}$$

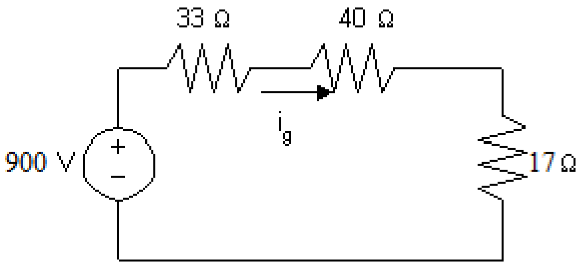
2) (10 points) Problem 3.60 of the textbook (p107), while the voltage source is changed from 500 V to 900 V and the right resistor is changed from 27 Ω to 17 Ω.

Ans:

Replace the 30—60—10 Ω delta with a wye equivalent to get



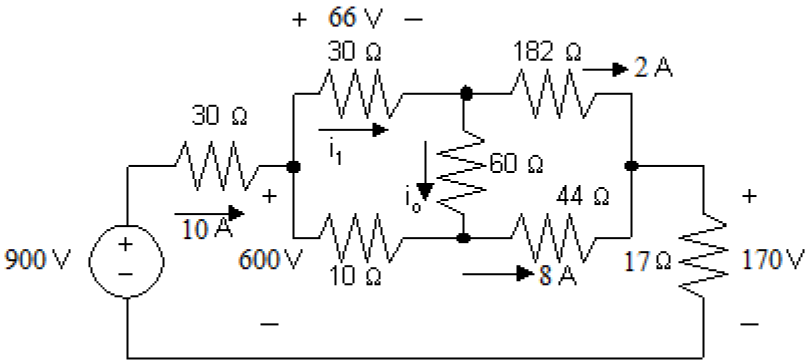
Using series/parallel reductions the circuit reduces to



$$i_g = \frac{900}{90} = 10 \text{ A}$$

$$i_2 = \frac{200}{250}(10) = 8 \text{ A}$$

Returning to the original circuit we have



$$i_1 = \frac{66}{30} = 2.2 \text{ A}$$

$$i_o = 2.2 - 2 = 0.2 \text{ A}$$

$$\text{v} = 60 i_o = 12 \text{ V}$$

$$P_{\text{supplied}} = 900 \times 10 = 9000 \text{ W}$$

3) (15 points) Problem 3.71 of the textbook (p109).

Ans:

From Eq 3.69

$$\frac{i_1}{i_3} = \frac{R_2 R_3}{D}$$

But $D = (R_1 + 2R_{\mathrm{a}})(R_2 + 2R_{\mathrm{b}}) + 2R_{\mathrm{b}}R_2$

Where $R_{\mathrm{a}} = \sigma R_1$; $R_2 = (1+2\sigma)^2 R_1$ and $R_{\mathrm{b}} = \frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}$

Therefore D can be written as

$$D = (R_1 + 2\sigma R_1)\left[(1+2\sigma)^2 R_1 + \frac{2(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}\right] + 2(1+2\sigma)^2 R_1 \left[\frac{(1+2\sigma)^2 \sigma R_1}{4(1+\sigma)^2}\right]$$

$$= (1+2\sigma)^3 {R_1}^2 \left[1 + \frac{\sigma}{2(1+\sigma)^2} + \frac{(1+2\sigma)\sigma}{2(1+\sigma)^2}\right]$$

$$= \frac{(1+2\sigma)^3 {R_1}^2}{2(1+\sigma)^2}\{2(1+\sigma)^2 + \sigma + (1+2\sigma)\sigma\}$$

$$= \frac{(1+2\sigma)^3 {R_1}^2}{2(1+\sigma)^2}\{1 + 3\sigma + 2\sigma^2\}$$

$$D = \frac{(1+2\sigma)^4 {R_1}^2}{1+\sigma}$$

$$\therefore \frac{i_1}{i_3} = \frac{R_2 R_3 (1+\sigma)}{(1+2\sigma)^4 {R_1}^2} = \frac{(1+2\sigma)^2 R_1 R_3 (1+\sigma)}{(1+2\sigma)^4 {R_1}^2} = \frac{(1+\sigma) R_3}{(1+2\sigma)^2 R_1}$$

When this result is substituted into Eq 3.69 we get

$$R_3 = \frac{(1+\sigma)^2 {R_3}^2 R_1}{(1+2\sigma)^4 {R_1}^2}$$

Solving for R_3 gives

$$R_3 = \frac{(1+2\sigma)^4 R_1}{(1+\sigma)^2}$$

4) (15 points) Problem 4.27 of the textbook (p155), while the voltage source is changed

from 24 V to 18 V and the voltage-controlled voltage source is changed from $5v_\Delta$ to 3 v_Δ. Also calculate v_0 when the 33-Ω resistor is eliminated.

Ans:

Place $5v_\Delta$ inside a supernode and use the lower node as a reference. Then

$$\frac{v_\Delta - 18}{10} + \frac{v_\Delta}{2} + \frac{v_\Delta - 3v_\Delta}{20} + \frac{v_\Delta - 3v_\Delta}{40} = 0$$

$18v_\Delta = 72; \quad v_\Delta = 4$ V

$v_o = v_\Delta - 3v_\Delta = -8$ V

When the 33-Ω resistor is eliminated,

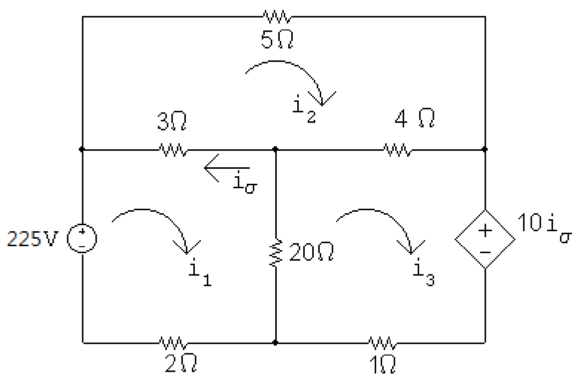
$$\frac{v_\Delta - 18}{10} + \frac{v_\Delta}{2} + i_{5v_\Delta} = 0$$

$$\frac{-2v_\Delta}{20} + \frac{-2v_\Delta}{40} = i_{5v_\Delta}$$

$v_o = -8$ V

5) (20 points) Problem 4.38 of the textbook (p156), while the voltage source is changed from 135 V to 225 V. Also find the power extracted or dissipated by the current controlled voltage source.

Ans:



$25i_1 - 3i_2 - 20i_3 + 0i_\sigma = 225$

$-3i_1 + 12i_2 - 4i_3 + 0i_\sigma = 0$

$-20i_1 - 4i_2 + 25i_3 + 10i_\sigma = 0$

$1i_1 - 1i_2 + 0i_3 + 1i_\sigma = 0$

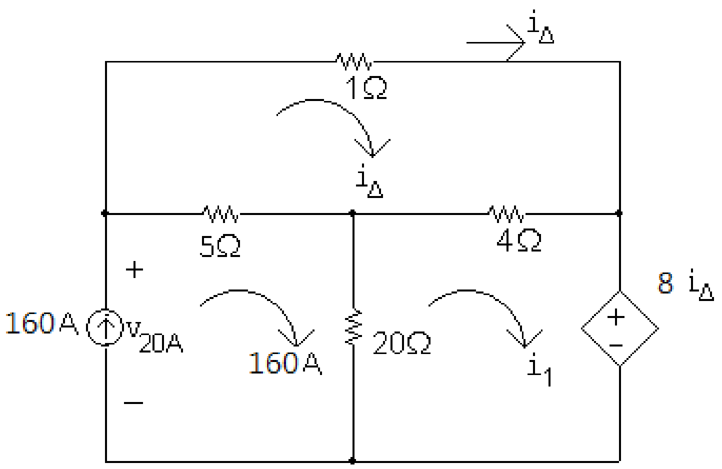
Solving, $i_1 = 108$ A , $i_2 = 65$ A , $i_3 = 114$ A , $i_\sigma = -43$ A

$P_{20\Omega} = (114 - 108)^2(20) = 720$ W

$P_{10i_\sigma} = 10i_\sigma i_3 = -49.02$ kW ,extracted.

6) (10 points) Problem 4.45 of the textbook (p157), while the current source is changed from 20 A to 160 A and the current-controlled voltage source is changed from 6.5 i_Δ to 8 i_Δ.

Ans:



$10i_\Delta - 4i_1 = 800$

$-4i_\Delta + 24i_1 + 8i_\Delta = 3200$

Solving, $i_1 = 112.5$ A , $i_\Delta = 125$ A

$v_{20A} = 1i_\Delta + 8i_\Delta = 9(125) = 1125$ V

$p_{20A} = -160v_{20A} = -160(1125) = -180$ kW

$p_{8i_\Delta} = 8i_\Delta i_1 = (8)(125)(112.5) = 112.5$ kW

Therefore, the independent source is developing 180 kW, all other elements are absorbing power, and the total power developed is thus 180 kW.

CHECK:

$p_{1\Omega} = (125)^2(1) = 15.625\ \text{kW}$

$p_{5\Omega} = (160 - 125)^2(5) = 6.125\ \text{kW}$

$p_{4\Omega} = (125 - 112.5)^2(4) = 0.625\ \text{kW}$

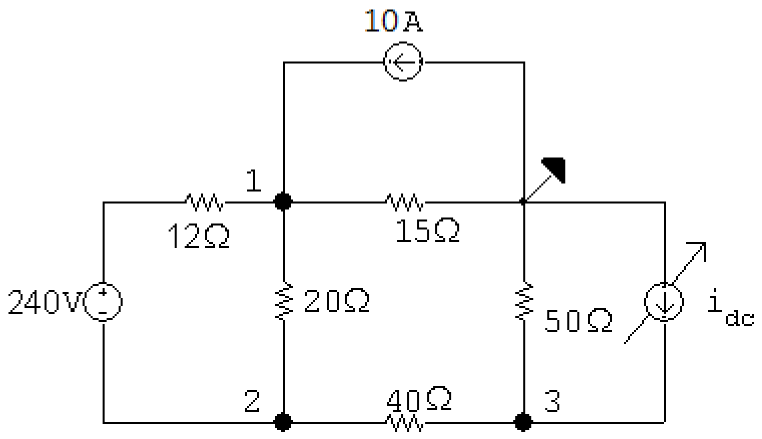
$p_{20\Omega} = (160 - 112.5)^2(20) = 45.125\ \text{kW}$

$$\sum p_{abs} = 112.5 + 15.625 + 6.125 + 0.625 + 45.125 = 180\ \ \text{kW}$$

7) (10 points) Problem 4.58 of the textbook (p158), while the top current source is changed from 4 A to 10 A.

Ans:

Choose the reference node so that a node voltage is identical to the voltage across the 10 A source; thus:



Since the 10 A source is developing 0 W, v_1 must be 0 V.

Since v_1 is known, we can sum the currents away from node 1 to find v_2 ; thus:

$$\frac{0-(240+v_2)}{12} + \frac{0-v_2}{20} + \frac{0}{15} - 10 = 0$$

$\therefore v_2 = -225\text{V}$

Now that we know v_2 , we sum the currents away from node 2 to find v_3 ; thus:

$$\frac{v_2 + 240 - 0}{12} + \frac{v_2 - 0}{20} + \frac{v_2 - v_3}{40} = 0$$

$\therefore v_3 = -625\ \text{V}$

Now that we know v_3, we sum the currents away from node 3 to find i_{dc}; thus:

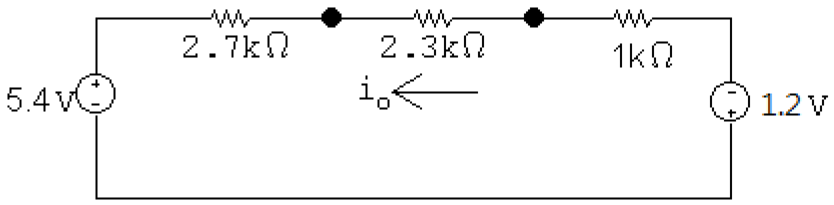
$$\frac{v_3}{50} + \frac{v_3 - v_2}{40} = i_{dc}$$

$\therefore i_{dc} = -22.5\text{A}$

8) (10 points) Problem 4.59 of the textbook (p159), while the right current source is changed from 0.6 mA to 1.2 mA.

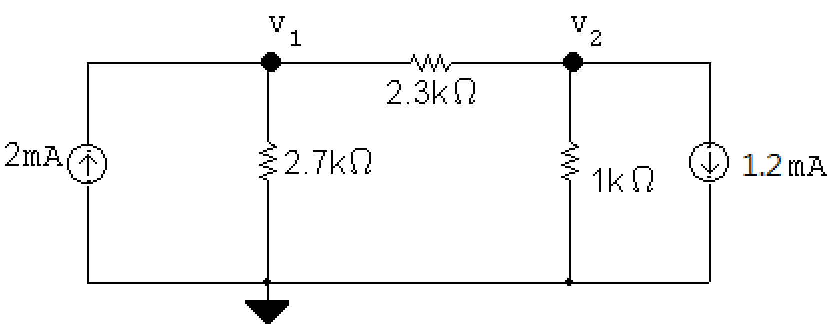
Ans:

8a) Apply source transformations to both current sources to get



$$i_o = \frac{-(5.4 + 1.2)}{2700 + 2300 + 1000} = -1.1\ \text{mA}$$

8b)



The node voltage equation:

$$-2 \times 10^{-3} + \frac{v_1}{2700} + \frac{v_1 - v_2}{2300} = 0$$

$$\frac{v_2}{1000} + \frac{v_2 - v_1}{2300} + 1.2 \times 10^{-3} = 0$$

Solving, $v_1 = 2.43$ V , $v_2 = -0.1$ V

$$\therefore\ i_o = \frac{v_2 - v_1}{2300} = \frac{-2.53}{2300} = -1.1 \text{ mA}$$