Chapter 18
Two-Port Circuits

18.1 The Terminal Equations
18.2 The Two-Port Parameters
18.3 Analysis of the Terminated Two-Port Circuit
18.4 Interconnected Two-Port Circuits
Motivation

- Thévenin and Norton equivalent circuits are used in representing the contribution of a circuit to one specific pair of terminals.
- Usually, a signal is fed into one pair of terminals (input port), processed by the system, then extracted at a second pair of terminals (output port). It would be convenient to relate the $v/i$ at one port to the $v/i$ at the other port without knowing the element values and how they are connected inside the “black box”.
How to model the “black box”?

- We will see that a two-port circuit can be modeled by a $2 \times 2$ matrix to relate the $v/i$ variables, where the four matrix elements can be obtained by performing 2 experiments.
Restrictions of the model

- No energy stored within the circuit.
- No independent source.
- Each port is not a current source or sink, i.e.
  \[ i_1 = i_1', \quad i_2 = i_2'. \]
- No inter-port connection, i.e. between ac, ad, bc, bd.
Key points

- How to calculate the 6 possible $2 \times 2$ matrices of a two-port circuit?
- How to find the 4 simultaneous equations in solving a terminated two-port circuit?
- How to find the total $2 \times 2$ matrix of a circuit consisting of interconnected two-port circuits?
Section 18.1
The Terminal Equations
The most general description of a two-port circuit is carried out in the s-domain.

Any 2 out of the 4 variables \( \{V_1, I_1, V_2, I_2\} \) can be determined by the other 2 variables and 2 simultaneous equations.
Six possible sets of terminal equations (1)

\[
\begin{align*}
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} &= \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}; \ [Z] \text{ is the impedance matrix;} \\
\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} &= \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}; \ [Y] = [Z]^\dagger \text{ is the admittance matrix;} \\
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} &= \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}; \ [A] \text{ is a transmission matrix;} \\
\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} &= \begin{bmatrix} b_{11} & -b_{12} \\ b_{21} & -b_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}; \ [B] = [A]^\dagger \text{ is a transmission matrix;}
\end{align*}
\]
Six possible sets of terminal equations (2)

\[
\begin{aligned}
\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} &= \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \times \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}; [H] \text{ is a hybrid matrix;} \\
\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} &= \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \times \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}; [G] = [H]^\dagger \text{ is a hybrid matrix;}
\end{aligned}
\]

- Which set is chosen depends on which variables are given. E.g. If the source voltage and current \( \{V_1, I_1\} \) are given, choosing transmission matrix \([B]\) in the analysis.
Section 18.2
The Two-Port Parameters

1. Calculation of matrix $[Z]
2. Relations among 6 matrixes
Example 18.1: Finding $[Z]$ (1)

Q: Find the impedance matrix $[Z]$ for a given resistive circuit (not a “black box”):

$[\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [z_{11} \ z_{12}] \times [I_1 \\ I_2]$}

By definition, $z_{11} = (V_1/I_1)$ when $I_2 = 0$, i.e. the input impedance when port 2 is open. $\Rightarrow z_{11} = (20 \Omega)/(20 \Omega) = 10 \Omega$. 
Example 18.1: (2)

- By definition, \( z_{21} = \frac{V_2}{I_1} \) when \( I_2 = 0 \), i.e. the transfer impedance when port 2 is open.

- When port 2 is open:

\[
\begin{align*}
V_2 &= \frac{15 \, \Omega}{5 \, \Omega + 15 \, \Omega} V_1 = 0.75 V_1, \\
\frac{V_1}{I_1} &= z_{11} = 10 \, \Omega, \quad \Rightarrow I_1 = \frac{V_1}{10 \, \Omega}, \\
\Rightarrow z_{21} &= \frac{V_2}{I_1} = \frac{0.75V_1}{V_1/(10 \, \Omega)} = 7.5 \, \Omega.
\end{align*}
\]
Example 18.1: (3)

- By definition, $z_{22} = (V_2/I_2)$ when $I_1 = 0$, i.e. the output impedance when port 1 is open. \( \implies z_{22} = (15 \, \Omega)/(25 \, \Omega) = 9.375 \, \Omega \).

- $z_{12} = (V_1/I_2)$ when $I_1 = 0$, \( \implies \)
\[
\begin{align*}
V_1 &= \frac{20 \, \Omega}{20 \, \Omega + 5 \, \Omega} V_2 = 0.8V_2, \\
V_2 &= z_{22} = 9.375 \, \Omega, \implies I_2 = \frac{V_2}{9.375 \, \Omega}, \\
\Rightarrow z_{12} &= \frac{V_1}{I_2} = \frac{0.8V_2}{V_2/(9.375 \, \Omega)} = 7.5 \, \Omega.
\end{align*}
\]
Comments

- When the circuit is well known, calculation of $[Z]$ by circuit analysis methods shows the physical meaning of each matrix element.

- When the circuit is a “black box”, we can perform 2 test experiments to get $[Z]$: (1) Open port 2, apply a current $I_1$ to port 1, measure the input voltage $V_1$ and output voltage $V_2$. (2) Open port 1, apply a current $I_2$ to port 2, measure the terminal voltages $V_1$ and $V_2$. 
Relations among the 6 matrixes

- If we know one matrix, we can derive all the others analytically (Table 18.1).
- \([Y] = [Z]^{-1}, [B] = [A]^{-1}, [G] = [H]^{-1}\), elements between mutually inverse matrixes can be easily related.
- E.g.

\[
\begin{bmatrix}
  z_{11} & z_{12} \\
  z_{21} & z_{22}
\end{bmatrix}
= \begin{bmatrix}
  y_{11} & y_{12} \\
  y_{21} & y_{22}
\end{bmatrix}^{-1}
= \frac{1}{\Delta y}
\begin{bmatrix}
  y_{22} & -y_{12} \\
  -y_{21} & y_{11}
\end{bmatrix},
\]

where \(\Delta y \equiv \det[Y] = y_{11} y_{22} - y_{12} y_{21}\).
Represent $[Z]$ by elements of $[A]$ (1)

- $[Z]$ and $[A]$ are not mutually inverse, relation between their elements are less explicit.

- By definitions of $[Z]$ and $[A],$

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
z_{11} & z_{12} \\
z_{21} & z_{22}
\end{bmatrix}
\times
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix},
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
a_{11} & -a_{12} \\
a_{21} & -a_{22}
\end{bmatrix}
\times
\begin{bmatrix}
V_2 \\
I_2
\end{bmatrix},
\]

the independent variables of $[Z]$ and $[A]$ are $\{I_1, I_2\}$ and $\{V_2, I_2\}$, respectively.

- Key of matrix transformation: Representing the distinct independent variable $V_2$ by $\{I_1, I_2\}$. 
Represent $[Z]$ by elements of $[A]$ (2)

- By definitions of $[A]$ and $[Z]$,

\[
\begin{aligned}
V_1 &= a_{11}V_2 - a_{12}I_2 \cdots (1) \\
I_1 &= a_{21}V_2 - a_{22}I_2 \cdots (2)
\end{aligned}
\]

\[
(2) \implies V_2 = \frac{1}{a_{21}}I_1 + \frac{a_{22}}{a_{21}}I_2 = z_{21}I_1 + z_{22}I_2 \cdots (3),
\]

\[
(1),(3) \implies V_1 = a_{11}\left(\frac{1}{a_{21}}I_1 + \frac{a_{22}}{a_{21}}I_2\right) - a_{12}I_2
\]

\[
= \frac{a_{11}}{a_{21}}I_1 + \left(\frac{a_{11}a_{22}}{a_{21}} - a_{12}\right)I_2 = z_{11}I_1 + z_{12}I_2 \cdots (4)
\]

\[
\Rightarrow \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \frac{1}{a_{21}} \begin{bmatrix} a_{11} & \Delta a \\ 1 & a_{22} \end{bmatrix}, \text{ where } \Delta a \equiv \det[A].
\]
Section 18.3
Analysis of the Terminated Two-Port Circuit

1. Analysis in terms of $[Z]$
2. Analysis in terms of $[T] \neq [Z]$
A two-port circuit is typically driven at port 1 and loaded at port 2, which can be modeled as:

The goal is to solve \( \{V_1, I_1, V_2, I_2\} \) as functions of given parameters \( V_g, Z_g, Z_L \), and matrix elements of the two-port circuit.
Analysis in terms of $[Z]$

- Four equations are needed to solve the four unknowns $\{V_1, I_1, V_2, I_2\}$.

\[
\begin{align*}
V_1 &= z_{11}I_1 + z_{12}I_2 \quad \cdots (1) \\
V_2 &= z_{21}I_1 + z_{22}I_2 \quad \cdots \text{two-port equations} \\
V_1 &= V_g - I_1Z_g \quad \cdots (3) \\
V_2 &= -I_2Z_L \quad \cdots \text{constraint equations due to terminations}
\end{align*}
\]

\[
\Rightarrow \begin{bmatrix}
-1 & 0 & z_{11} & z_{12} \\
0 & -1 & z_{21} & z_{22} \\
1 & 0 & Z_g & 0 \\
0 & 1 & 0 & Z_L
\end{bmatrix}
\times
\begin{bmatrix}
V_1 \\
V_2 \\
I_1 \\
I_2
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
V_g \\
0
\end{bmatrix},
\]

$\{V_1, I_1, V_2, I_2\}$ are derived by inverse matrix method.
Thévenin equivalent circuit with respect to port 2

- Once \( \{V_1, I_1, V_2, I_2\} \) are solved, \( \{V_{Th}, Z_{Th}\} \) can be determined by \( Z_L \) and \( \{V_2, I_2\} \):

\[
\begin{align*}
V_2 &= \frac{Z_L}{Z_{Th} + Z_L} V_{Th} \cdots (1) \\
I_2 &= \frac{V_2 - V_{Th}}{Z_{Th}} \cdots (2)
\end{align*}
\]

\[
\Rightarrow \begin{bmatrix} Z_L & -V_2 \\ 1 & I_2 \end{bmatrix} \begin{bmatrix} V_{Th} \\ Z_{Th} \end{bmatrix} = \begin{bmatrix} V_2 Z_L \\ V_2 \end{bmatrix}, \quad \begin{bmatrix} V_{Th} \\ Z_{Th} \end{bmatrix} = \begin{bmatrix} Z_L & -V_2 \\ 1 & I_2 \end{bmatrix}^{-1} \begin{bmatrix} V_2 Z_L \\ V_2 \end{bmatrix}.
\]

The terminal behavior of the circuit can be described by manipulations of \( \{V_1, I_1, V_2, I_2\} \):

- **Input impedance:**
  \[
  Z_{in} \equiv \frac{V_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{z_{22} + Z_L};
  \]

- **Output current:**
  \[
  I_2 = \frac{-z_{21}V_g}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}};
  \]

- **Current gain:**
  \[
  \frac{I_2}{I_1} = -\frac{z_{21}}{z_{22} + Z_L};
  \]

- **Voltage gains:**
  \[
  \begin{align*}
  \frac{V_2}{V_1} & = \frac{z_{21}Z_L}{z_{11}Z_L + \Delta z}; \\
  \frac{V_2}{V_g} & = \frac{z_{21}Z_L}{(z_{11} + Z_g)(z_{22} + Z_L) - z_{12}z_{21}};
  \end{align*}
  \]
Terminal behavior (2)

- Thévenin voltage: \[ V_{Th} = \frac{z_{21}}{z_{11} + Z_g} V_g; \]

- Thévenin impedance: \[ Z_{Th} = z_{22} - \frac{z_{12}z_{21}}{z_{11} + Z_g}; \]
Analysis in term of a two-port matrix $[T] \neq [Z]$

- If the two-port circuit is modeled by $[T] \neq [Z]$, $T = \{Y, A, B, H, G\}$, the terminal behavior can be determined by two methods:
  - Use the 2 two-port equations of $[T]$ to get a new $4 \times 4$ matrix in solving $\{V_1, I_1, V_2, I_2\}$ (Table 18.2);
  - Transform $[T]$ into $[Z]$ by Table 18.1, borrow the formulas derived by analysis in terms of $[Z]$. 
Example 18.4: Analysis in terms of $[B]$ (1)

Q: Find (1) output voltage $V_2$, (2,3) average powers delivered to the load $P_2$ and input port $P_1$, for a terminated two-port circuit with known $[B]$.

\[
[B] = \begin{bmatrix}
-20 & 3 \text{k} \Omega \\
-2 \text{ mS} & 0.2 \\
\end{bmatrix}
\]
Example 18.4 (2)

- Use the voltage gain formula of Table 18.2:

\[
\frac{V_2}{V_g} = \frac{\Delta b Z_L}{b_{12} + b_{11}Z_g + b_{22}Z_L + b_{21}Z_gZ_L};
\]

\[
\Delta b = b_{11}b_{22} - b_{12}b_{21} = (-20)(-0.2) - (-3 \text{kΩ})(-2 \text{ mS}) = 4 - 6 = -2,
\]

\[
\Rightarrow \frac{V_2}{V_g} = \frac{(-2)(5 \text{kΩ})}{(-3 \text{kΩ}) + (-20)(0.5 \text{kΩ}) + (-0.2)(5 \text{kΩ}) + ...} = \frac{10}{19},
\]

\[
\Rightarrow V_2 = \frac{10}{19} \cdot 500 \angle 0^\circ = 263.16 \angle 0^\circ \text{ V}.
\]
Example 18.4 (3)

- The average power of the load is formulated by

\[
P_2 = \frac{1}{2} \frac{|V_2|^2}{R_L} = \frac{1}{2} \left| \frac{263.16 \angle 0^\circ \text{ V}}{5 \text{ k}\Omega} \right|^2 = 6.93 \text{ W}.
\]

- The average power delivered to port 1 is formulated by

\[
P_1 = \frac{1}{2} |I_1|^2 \text{ Re}(Z_{in}).
\]

\[
Z_{in} \equiv \frac{V_1}{I_1} = \frac{b_{22} Z_L + b_{12}}{b_{21} Z_L + b_{11}} = \frac{(-0.2)(5 \text{ k}\Omega) - (3 \text{ k}\Omega)}{(-2 \text{ mS})(5 \text{ k}\Omega) - 20}\]

\[
I_1 = \frac{V_g}{Z_g + Z_{in}} = \frac{500 \angle 0^\circ \text{ V}}{(500 \text{ \Omega}) + (133.33 \text{ \Omega})} = 0.789 \angle 0^\circ \text{ A},
\]

\[
\Rightarrow P_1 = \frac{1}{2} (0.789)^2 (133.33) = 41.55 \text{ W}.
\]
Section 18.4
Interconnected Two-Port Circuits
Why interconnected?

- Design of a large system is simplified by first designing subsections (usually modeled by two-port circuits), then interconnecting these units to complete the system.
Five types of interconnections of two-port circuits

a. Cascade: Better use \([A]\).
b. Series: \([Z]\)
c. Parallel: \([Y]\)
d. Series-parallel: \([H]\).
e. Parallel-series: \([G]\).
Analysis of cascade connection (1)

- Goal: Derive the overall matrix \([A]\) of two cascaded two-port circuits with known transmission matrixes \([A']\) and \([A'']\).
Analysis of cascade connection (2)

\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
A'
\end{bmatrix} \times \begin{bmatrix}
V_2' \\
I_2'
\end{bmatrix} = \begin{bmatrix}
A'
\end{bmatrix} \times \begin{bmatrix}
V_1' \\
-I_1'
\end{bmatrix} \quad \ldots (1)
\]

\[
\begin{bmatrix}
V_1' \\
I_1'
\end{bmatrix} = \begin{bmatrix}
A''
\end{bmatrix} \times \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix} = \begin{bmatrix}
a''_{11} & -a''_{12} & a''_{21} & -a''_{22}
\end{bmatrix} \times \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix},
\]

\[
\Rightarrow \begin{bmatrix}
V_1' \\
-I_1'
\end{bmatrix} = \begin{bmatrix}
a''_{11} & -a''_{12} & a''_{21} & -a''_{22}
\end{bmatrix} \times \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix} = \begin{bmatrix}
A''
\end{bmatrix} \times \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix} \quad \ldots (2)
\]

By (1), (2),
\[
\begin{bmatrix}
V_1 \\
I_1
\end{bmatrix} = \begin{bmatrix}
A'
\end{bmatrix} \times \begin{bmatrix}
A''
\end{bmatrix} \times \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix} = \begin{bmatrix}
A
\end{bmatrix} \times \begin{bmatrix}
V_2 \\
I_2
\end{bmatrix},
\]

\[
\Rightarrow \begin{bmatrix}
A
\end{bmatrix} = \begin{bmatrix}
A'
\end{bmatrix} \times \begin{bmatrix}
A''
\end{bmatrix},
\]

\[
\begin{bmatrix}
a_{11} & -a_{12} \\
a_{21} & -a_{22}
\end{bmatrix} = \begin{bmatrix}
a'_{11}a''_{11} + a'_{12}a''_{21} & -(a'_{11}a''_{12} + a'_{12}a''_{22}) \\
a'_{21}a''_{11} + a'_{22}a''_{21} & -(a'_{21}a''_{12} + a'_{22}a''_{22})
\end{bmatrix}.
\]
Key points

- How to calculate the 6 possible $2 \times 2$ matrices of a two-port circuit?
- How to find the 4 simultaneous equations in solving a terminated two-port circuit?
- How to find the total $2 \times 2$ matrix of a circuit consisting of interconnected two-port circuits?
Practical Perspective
Audio Amplifier
Application of two-port circuits

Q: Whether it would be safe to use a given audio amplifier to connect a music player modeled by \( \{ V_g = 2 \text{ V (rms)}, Z_g = 100 \Omega \} \) to a speaker modeled by a load resistor \( Z_L = 32 \Omega \) with a power rating of 100 W?
Find the \([H]\) by 2 test experiments (1)

- Definition of hybrid matrix \([H]\):
  \[
  \begin{bmatrix}
  V_1 \\
  I_2 \\
  \end{bmatrix} = \begin{bmatrix}
  h_{11} & h_{12} \\
  h_{21} & h_{22} \\
  \end{bmatrix} \times \begin{bmatrix}
  I_1 \\
  V_2 \\
  \end{bmatrix},
  \]

- Test 1:
  \[I_1 = 2.5 \text{ mA (rms)}\]
  \[V_1 = 1.25 \text{ V (rms)}\]
  \[V_2 = 0 \text{ (short)}\]
  \[I_2 = 3.75 \text{ A (rms)}\]

\[V_1 = h_{11} I_1, \Rightarrow h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0} = \frac{1.25 \text{ V}}{2.5 \text{ mA}} = 500 \text{ \Omega}.\]

\[I_2 = h_{21} I_1, \Rightarrow h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0} = \frac{3.75 \text{ A}}{2.5 \text{ mA}} = 1500.\]
Find the $[H]$ by 2 test experiments (2)

- Definition of hybrid matrix $[H]$: 
  \[
  \begin{bmatrix}
  V_1 \\ I_2
  \end{bmatrix} =
  \begin{bmatrix}
  h_{11} & h_{12} \\ h_{21} & h_{22}
  \end{bmatrix}
  \begin{bmatrix}
  I_1 \\ V_2
  \end{bmatrix};
  \]

- Test 2:
  
  $I_1 = 0$ (open)
  
  $V_1 = 50$ mV (rms)
  
  $V_2 = 50$ V (rms)
  
  $I_2 = 2.5$ A (rms)

  \[
  V_1 = h_{12}V_2, \quad h_{12} = \frac{V_1}{V_2}igg|_{I_1=0} = \frac{50 \text{ mV}}{50 \text{ V}} = 10^{-3},
  \]
  
  Voltage gain

  \[
  I_2 = h_{22}V_2, \quad h_{22} = \frac{I_2}{V_2}igg|_{I_1=0} = \frac{2.5 \text{ A}}{50 \text{ V}} = (20 \Omega)^{-1},
  \]
  
  Output admittance
Find the power dissipation on the load

- For a terminated two-port circuit:

\[
P_L = \text{Re}\{-V_2 I_2^*\} = \text{Re}\{-(-I_2 Z_L)I_2^*\} = |I_2|^2 \text{Re}\{Z_L\},
\]

where \(I_2\) is the rms output current phasor.
Method 1: Use terminated 2-port eqs for $[H]$

- By looking at Table 18.2:

\[
I_2 = \frac{h_{21}V_g}{(1 + h_{22}Z_L)(h_{11} + Z_g) - h_{12}h_{21}Z_L} = 1.98 \text{ A (rms)},
\]

where

\[
\begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix} = \begin{bmatrix}
500 \Omega & 10^{-3} \\
1500 & (20 \Omega)^{-1}
\end{bmatrix};
\]

$V_g = 2 \text{ V (rms)}, Z_g = 100 \Omega, Z_L = 32 \Omega.$

$\Rightarrow P_L = |I_2|^2 \text{ Re}\{Z_L\} = (1.98)^2(32) = 126 \text{ W} > 100 \text{ W}.$

Not safe!
Method 2: Use system of terminated eqs of \([Z]\)

- Transform \([H]\) to \([Z]\) (Table 18.1):

\[
\begin{bmatrix}
 z_{11} & z_{12} \\
 z_{21} & z_{22}
\end{bmatrix} = \frac{1}{h_{22}} \begin{bmatrix}
 \Delta h & h_{12} \\
 -h_{21} & 1
\end{bmatrix} \begin{bmatrix}
 470 & 0.02 \\
 -30,000 & 20
\end{bmatrix} \Omega.
\]

- By system of terminated equations:

\[
\begin{bmatrix}
 V_1 \\
 V_2 \\
 I_1 \\
 I_2
\end{bmatrix} = \begin{bmatrix}
 -1 & 0 & z_{11} & z_{12} \\
 0 & -1 & z_{21} & z_{22} \\
 1 & 0 & Z_g & 0 \\
 0 & 1 & 0 & Z_L
\end{bmatrix}^{-1} \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 V_g
\end{bmatrix} = \begin{bmatrix}
 1.66 \text{ V} \\
 -63.5 \text{ V} \\
 3.4 \text{ mA} \\
 1.98 \text{ A}
\end{bmatrix}.
\]

\[
\Rightarrow P_L = |I_2|^2 \text{Re}\{Z_L\} = (1.98)^2 (32) = 126 \text{ W} > 100 \text{ W}.
\]