Chapter 9
Sinusoidal Steady–State Analysis

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Overview

- We will generalize circuit analysis from constant to time-varying sources (Ch7-14).
- Sinusoidal sources are particularly important because: (1) Generation, transmission, consumption of electric energy occur under sinusoidal conditions. (2) It can be used to predict the behaviors of circuits with non-sinusoidal sources.
- Need to work in the realm of complex numbers.

Key points

- What is the phase of a sinusoidal function?
- What is the phasor of a sinusoidal function?
- What is the phase of an impedance? What are in-phase and quadrature?
- How to solve the sinusoidal steady-state response by using phasor and impedance?
- What is the reflected impedance of a circuit with transformer?

Section 9.1, 9.2 The Sinusoidal Source and Response

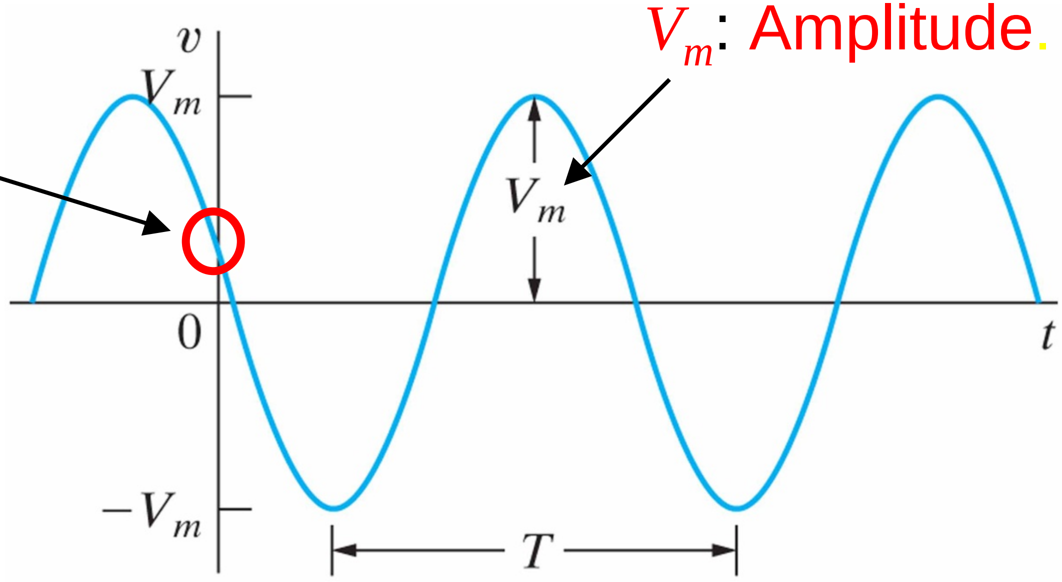
1. Definitions
2. Characteristics of sinusoidal response

Definition

- A source producing a voltage varying sinusoidally with time: $v(t) = V_m \cos(\omega t + \phi)$.

ϕ: Phase angle, determines the value at $t = 0$.

V_m: Amplitude.

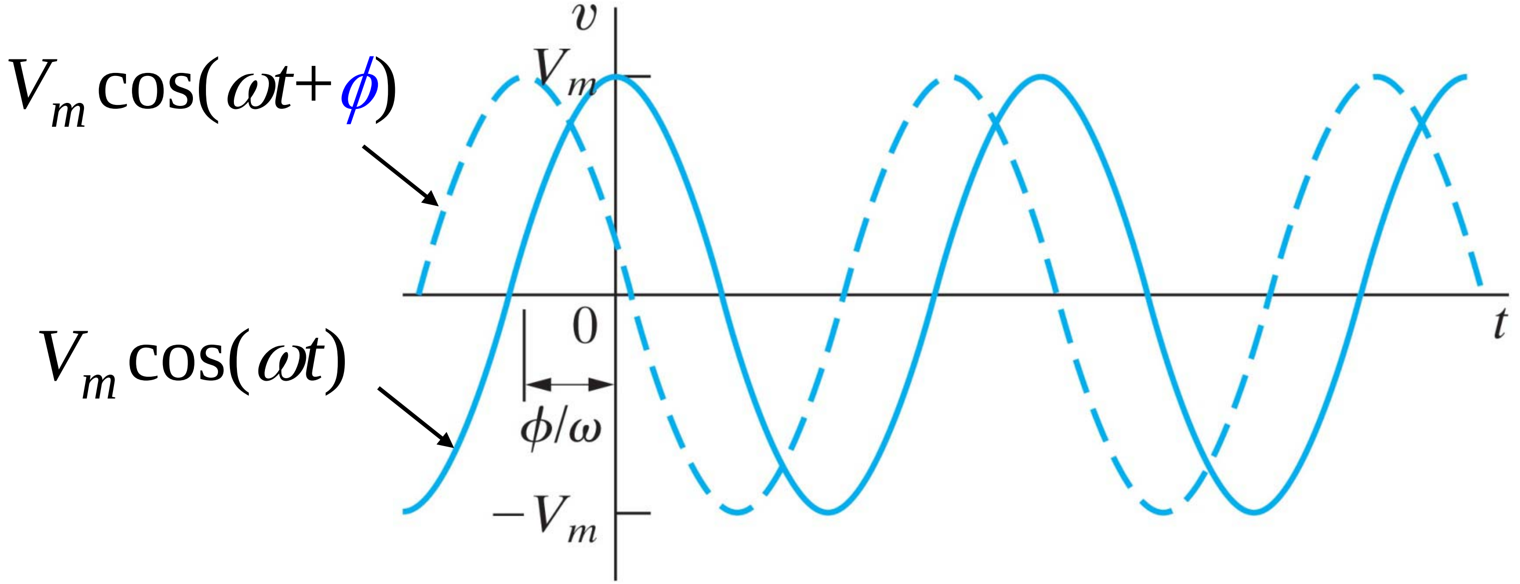


ω: Angular frequency, related to period T via $\omega = 2\pi/T$.

The argument ωt changes 2π radians (360°) in one period.

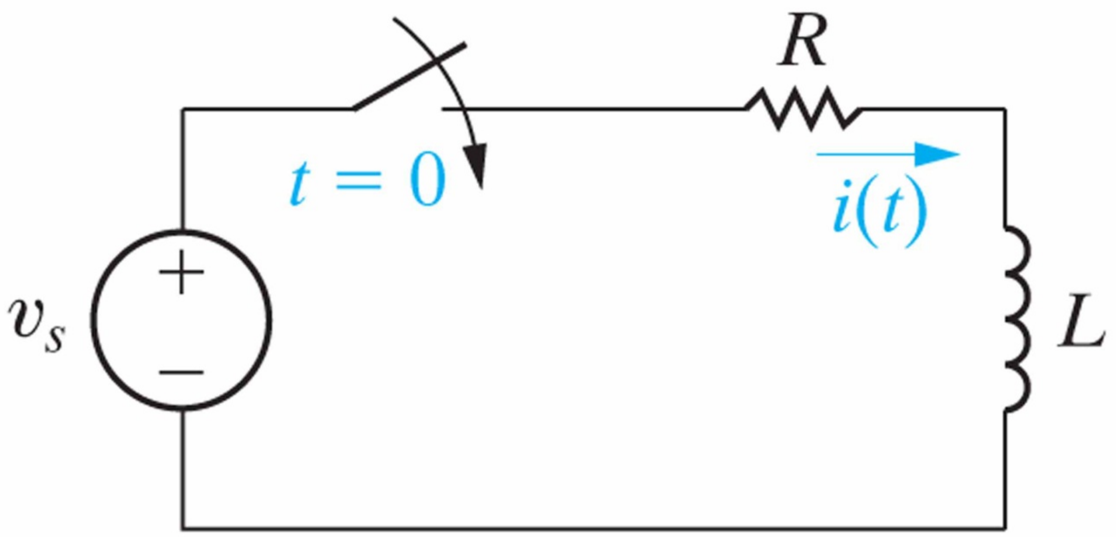
More on phase angle

- Change of phase angle shifts the curve along the time axis without changing the shape (amplitude, angular frequency).
- Positive phase ($\phi > 0$), $\Rightarrow$ the curve is shifted to the left by ϕ/ω in time, and vice versa.



Example: RL circuit (1)

- Consider an RL circuit with zero initial current $i(t=0^+)=0$ and driven by a sinusoidal voltage source $v_s(t)=V_m\cos(\omega t+\phi)$:



- By KVL: $L\dfrac{d}{dt}i+Ri=V_m\cos(\omega t+\phi)$.

Example: RL circuit (2)

- The complete solution to the ODE and initial condition is (verified by substitution):

$$i(t) = i_{tr}(t) + i_{ss}(t),$$

$$\begin{cases} i_{tr}(t) \equiv -\dfrac{V_m \cos(\phi - \theta)}{\sqrt{R^2 + \omega^2 L^2}} e^{-(R/L)t} \\ i_{ss}(t) \equiv \dfrac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta) \\ \theta = \tan^{-1}(\omega L / R). \end{cases}$$

Transient response, vanishes as $t \to \infty$.

Steady-state response, lasts even $t \to \infty$.

Characteristics of steady-state response

- $i_{ss}(t)$ of this example exhibits the following characteristics of steady-state response:

$$i_{ss}(t) \equiv \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \theta)$$

1. It remains sinusoidal of the same frequency as the driving source if the circuit is linear (with constant R, L, C values).
2. The amplitude differs from that of the source.
3. The phase angle differs from that of the source.

Purpose of Chapter 9

- Directly finding the steady-state response without solving the differential equation.
- According to the characteristics of steady-state response, the task is reduced to finding two real numbers, i.e. amplitude and phase angle, of the response. The waveform and frequency of the response are already known.
- Transient response matters in switching. It will be dealt with in Chapters 7, 8, 12, 13.

Section 9.3
The Phasor

1. Definitions
2. Solve steady-state response by phasor

Definition

- The phasor is a constant complex number that carries the amplitude and phase angle information of a sinusoidal function.
- The concept of phasor is rooted in Euler's identity, which relates the (complex) exponential function to the trigonometric functions: $e^{\pm j\theta} = \cos\theta \pm j\sin\theta.$

$$\Rightarrow \cos\theta = \mathrm{Re}\{e^{j\theta}\}, \quad \sin\theta = \mathrm{Im}\{e^{j\theta}\}.$$

Phasor representation

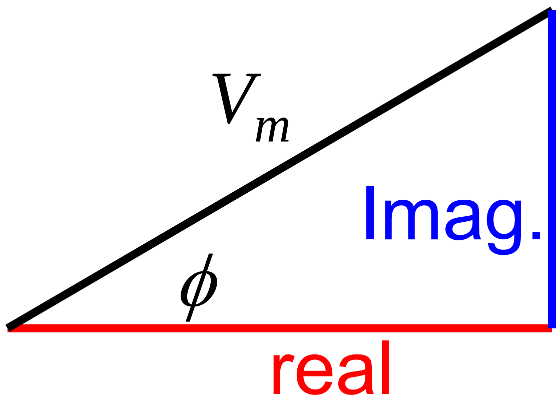
- A sinusoidal function can be represented by the real part of a phasor times the "complex carrier".

$$V_m \cos(\omega t + \phi) = V_m \operatorname{Re}\left\{ e^{j(\omega t + \phi)} \right\}$$
$$= \operatorname{Re}\left\{ \left(V_m e^{j\phi} \right) e^{j\omega t} \right\} = \operatorname{Re}\left\{ \mathbf{V} \times e^{j\omega t} \right\}$$

phasor carrier

- A phasor can be represented in two forms:

1. Polar form (good for ×, ÷):
$$\mathbf{V} \equiv V_m e^{j\phi} = V_m \angle \phi,$$
2. Rectangular form (good for +, -):
$$\mathbf{V} \equiv V_m \cos\phi + jV_m \sin\phi.$$



Phasor transformation

- A phasor can be regarded as the “phasor transform” of a sinusoidal function from the time domain to the frequency domain:

$$\mathbf{V} = P\{V_m \cos(\omega t + \phi)\} = V_m e^{j\phi}.$$

time domain freq. domain

- The “inverse phasor transform” of a phasor is a sinusoidal function in the time domain:

$$P^{-1}\{\mathbf{V}\} = \mathrm{Re}\{\mathbf{V}e^{j\omega t}\} = V_m \cos(\omega t + \phi).$$

Time derivative ↔ Multiplication of constant

Time domain:

$$\frac{d}{dt}V_m\cos(\omega t+\phi)=-\omega V_m\sin(\omega t+\phi)$$

$$=\omega V_m\cos(\omega t+\phi+90^\circ),$$

$$\frac{d^2}{dt^2}V_m\cos(\omega t+\phi)=-\omega^2V_m\cos(\omega t+\phi).$$

Frequency domain:

$$\mathrm{P}\left\{\frac{d}{dt}V_m\cos(\omega t+\phi)\right\}=\omega V_m e^{j(\phi+90^\circ)}$$

$$=\omega\left(V_m e^{j\phi}\right)e^{j90^\circ}=j\omega\mathbf{V},$$

$$\mathrm{P}\left\{\frac{d^2}{dt^2}V_m\cos(\omega t+\phi)\right\}=(j\omega)^2\mathbf{V}=-\omega^2\mathbf{V}.$$

How to calculate steady-state solution by phasor?

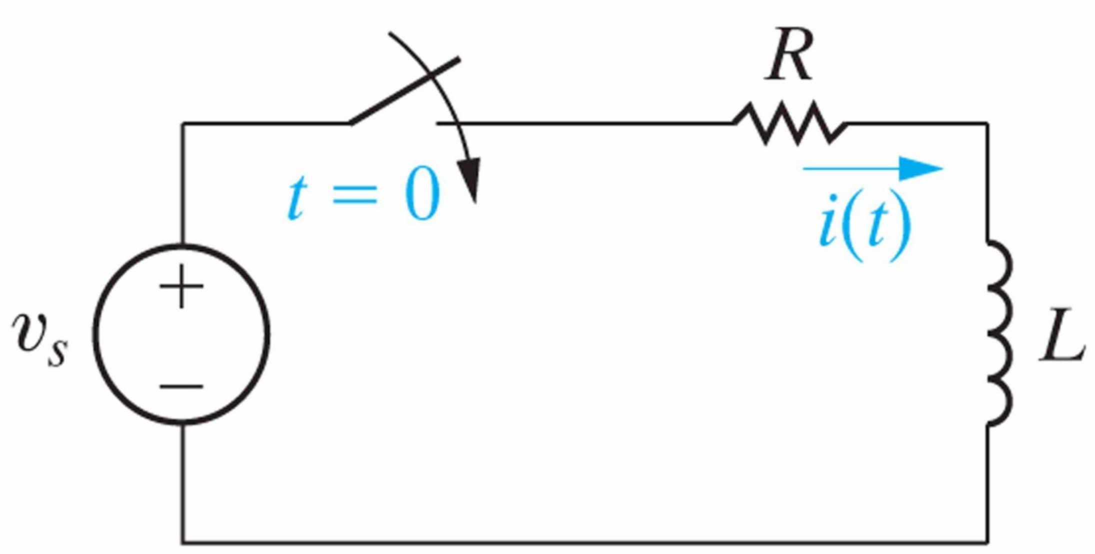
- Step 1: Assume that the solution is of the form:

$$\mathrm{Re}\left\{\left(Ae^{j\beta}\right)e^{j\omega t}\right\}$$

- Step 2: Substitute the proposed solution into the differential equation. The common time-varying factor $e^{j\omega t}$ of all terms will cancel out, resulting in two algebraic equations to solve for the two unknown constants $\{A, \beta\}$.

Example: RL circuit (1)

- Q: Given $v_s(t) = V_m \cos(\omega t + \phi)$, calculate $i_{ss}(t)$.



Assume

$$i_{ss}(t) = I_m \cos(\omega t + \beta).$$

$$\Rightarrow L\frac{d}{dt} i_{ss}(t) + R i_{ss}(t)$$

$$= V_m \cos(\omega t + \phi),$$

$$\Rightarrow L\frac{d}{dt}\left[I_m \cos(\omega t + \beta)\right] + R\left[I_m \cos(\omega t + \beta)\right] = V_m \cos(\omega t + \phi),$$

$$\Rightarrow -\omega L I_m \sin(\omega t + \beta) + R I_m \cos(\omega t + \beta) = V_m \cos(\omega t + \phi),$$

Example: RL circuit (2)

- By cosine convention:

$$\Rightarrow \omega L I_m \cos(\omega t + \beta + 90^\circ) + R I_m \cos(\omega t + \beta) = V_m \cos(\omega t + \phi),$$

$$\Rightarrow \mathrm{Re}\left\{\omega L I_m e^{j(\beta + 90^\circ)} e^{j\omega t}\right\} + \mathrm{Re}\left\{R I_m e^{j\beta} e^{j\omega t}\right\} = \mathrm{Re}\left\{V_m e^{j\phi} e^{j\omega t}\right\},$$

$$\Rightarrow \mathrm{Re}\left\{j\omega L \mathbf{I} e^{j\omega t}\right\} + \mathrm{Re}\left\{R \mathbf{I} e^{j\omega t}\right\} = \mathrm{Re}\left\{\mathbf{V} e^{j\omega t}\right\},$$

$$\Rightarrow \mathrm{Re}\left\{(j\omega L + R)\mathbf{I} e^{j\omega t}\right\} = \mathrm{Re}\left\{\mathbf{V} e^{j\omega t}\right\}.$$

- A necessary condition is:

$$(j\omega L + R)\mathbf{I} e^{j\omega t} = \mathbf{V} e^{j\omega t}, \Rightarrow (j\omega L + R)\mathbf{I} = \mathbf{V}.$$

Example: RL circuit (3)

- A more convenient way is directly transforming the ODE from time to frequency domain:

$$L\frac{d}{dt}i_{ss}(t)+Ri_{ss}(t)=V_m\cos(\omega t+\phi),$$

$$\Rightarrow L(j\omega)\mathbf{I}+R\mathbf{I}=\mathbf{V},\ (j\omega L+R)\mathbf{I}=\mathbf{V}.$$

- The solution can be obtained by one complex (i.e. two real) algebraic equation:

$$\mathbf{I}=\frac{\mathbf{V}}{j\omega L+R},\ \text{i.e. } I_m e^{j\beta}=\frac{V_m e^{j\phi}}{j\omega L+R}.$$

Section 9.4
Impedances of The Passive Circuit Elements

1. Generalize resistance to impedance
2. Impedances of R, L, C
3. In phase & quadrature

What is the impedance?

- For a resistor, the ratio of voltage $v(t)$ to the current $i(t)$ is a real constant R (Ohm's law):

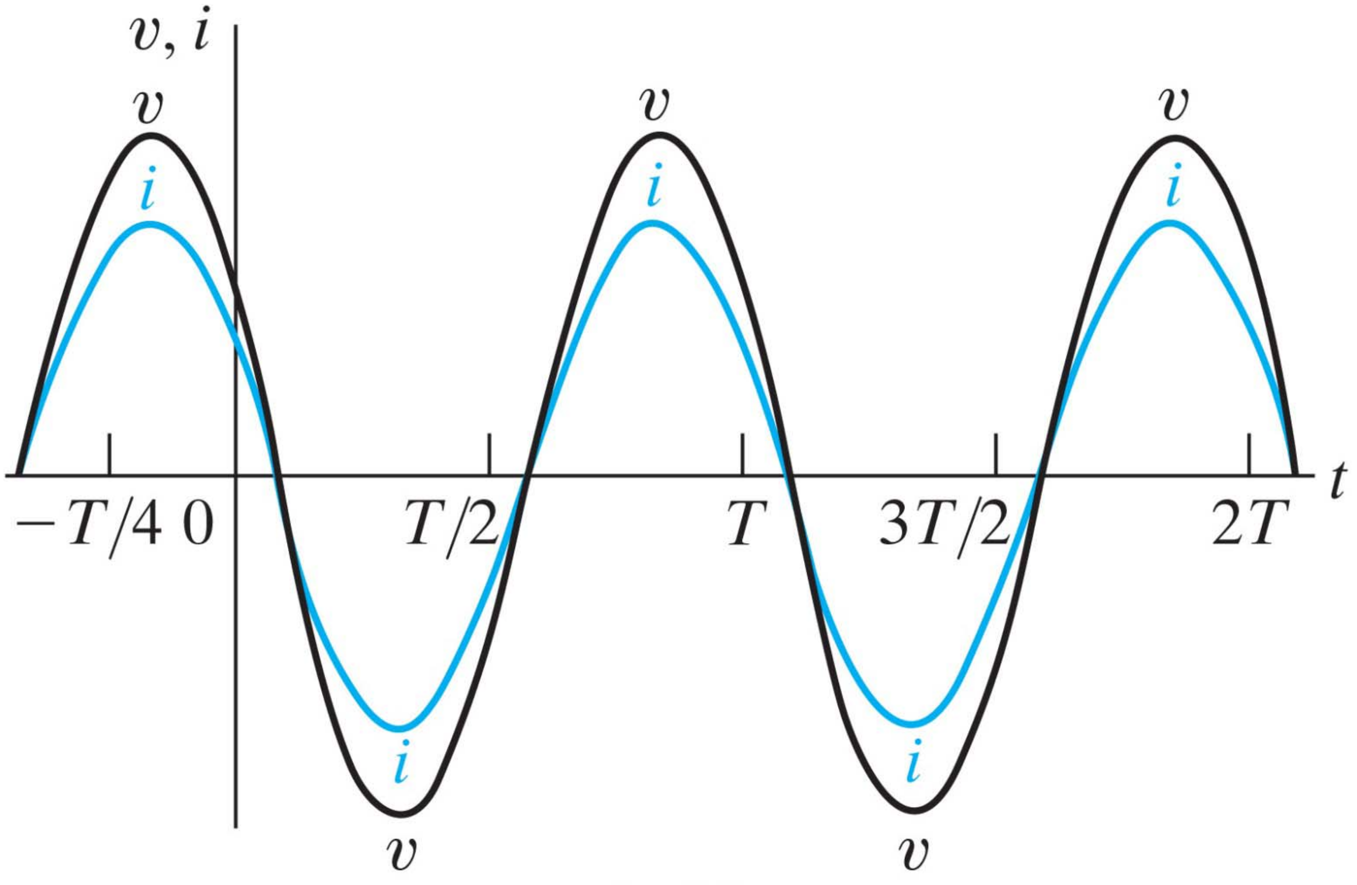
$$R = \frac{v(t)}{i(t)}. \quad \text{…resistance}$$

- For two terminals of a linear circuit driven by sinusoidal sources, the ratio of voltage phasor $\boldsymbol{V}$ to the current phasor $\boldsymbol{I}$ is a complex constant Z:

$$Z \equiv \frac{\mathbf{V}}{\mathbf{I}}. \quad \text{…impedance}$$

The i-v relation and impedance of a resistor

- $i(t)$ and $v(t)$ reach the peaks simultaneously (in phase), $\Rightarrow$ impedance $Z = R$ is real.



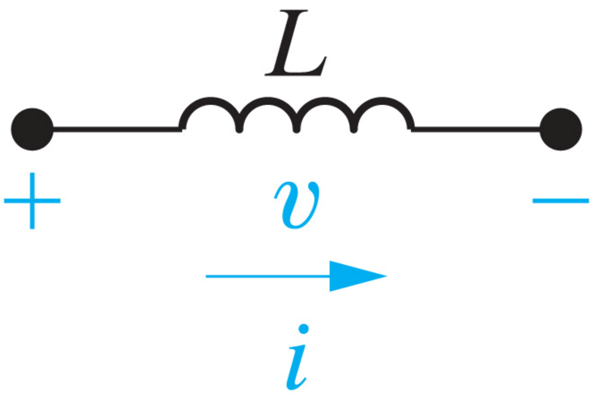
The *i*-*v* relation and impedance of an inductor (1)

- Assume $i(t) = I_m \cos(\omega t + \theta_i)$

$$\Rightarrow v(t) = L\frac{d}{dt}i(t)$$

$$= L\left[-\omega I_m \sin(\omega t + \theta_i)\right]$$

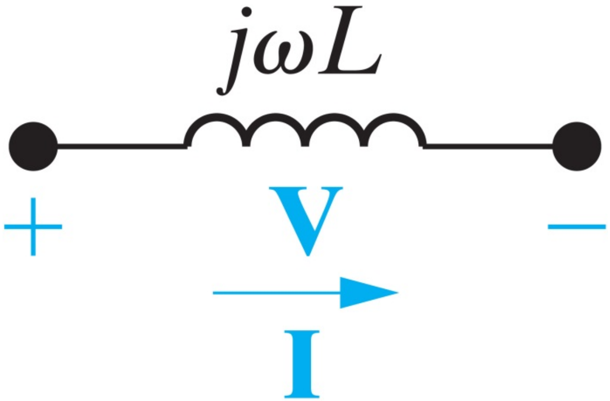
$$= \omega L I_m \cos(\omega t + \theta_i + 90^\circ).$$



- By phasor transformation:

$$\mathbf{V} = L \cdot j\omega \mathbf{I}$$

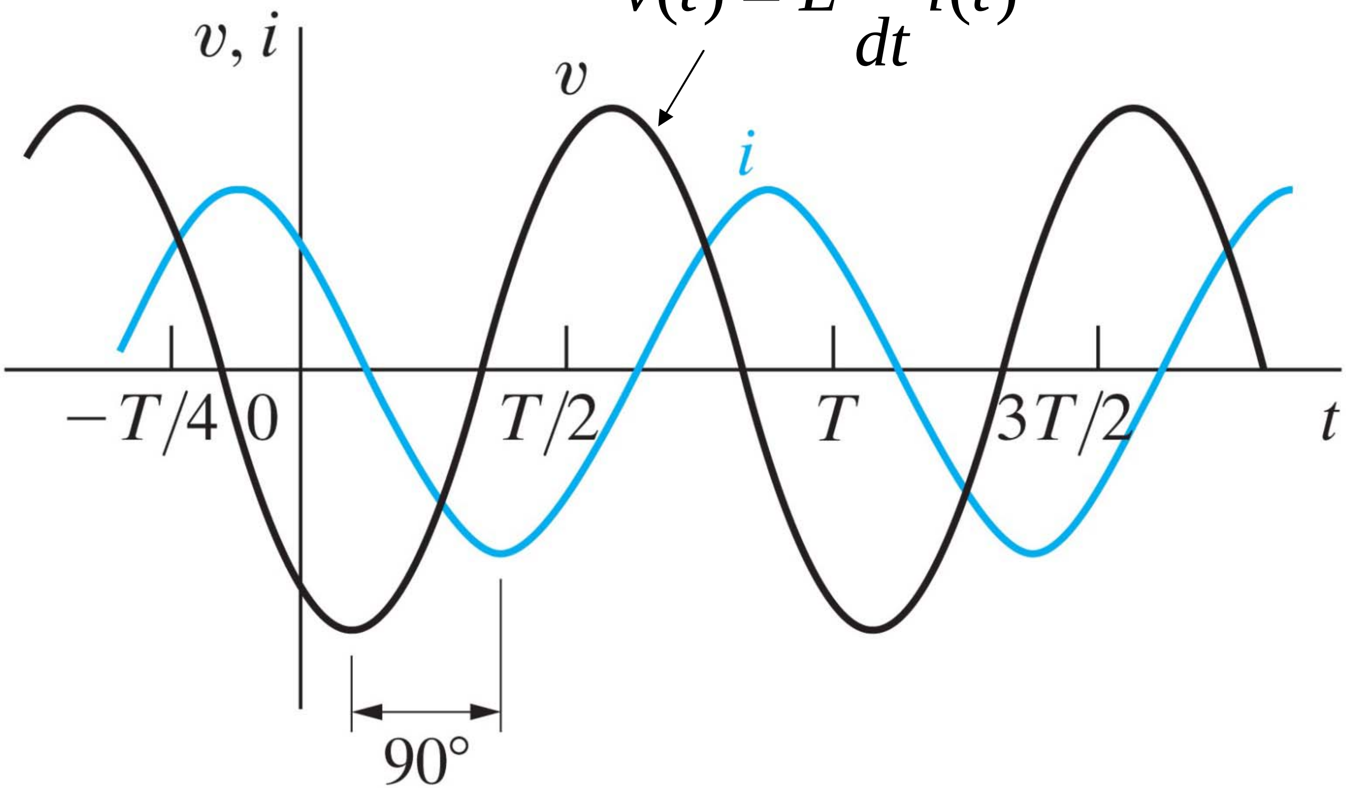
$$\Rightarrow Z = \frac{\mathbf{V}}{\mathbf{I}} = j\omega L.$$



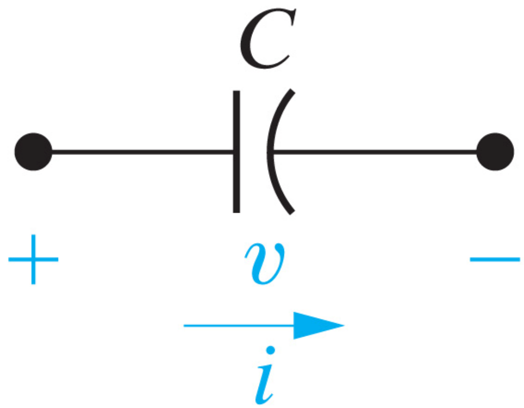
The i-v relation and impedance of an inductor (2)

- $v(t)$ leads $i(t)$ by $T/4$ (+90° phase, i.e. quadrature)
  $\Rightarrow$ impedance $Z = j\omega L$ is purely positive imaginary.

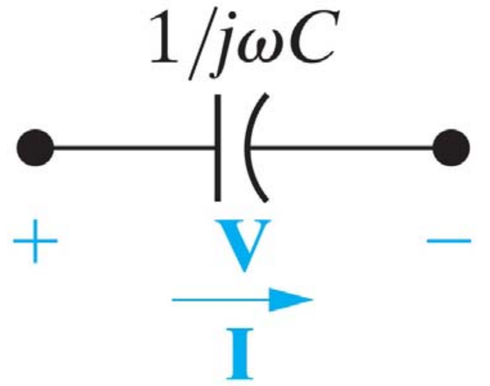
$$v(t) = L\frac{d}{dt}i(t)$$



The i-v relation and impedance of an capacitor (1)



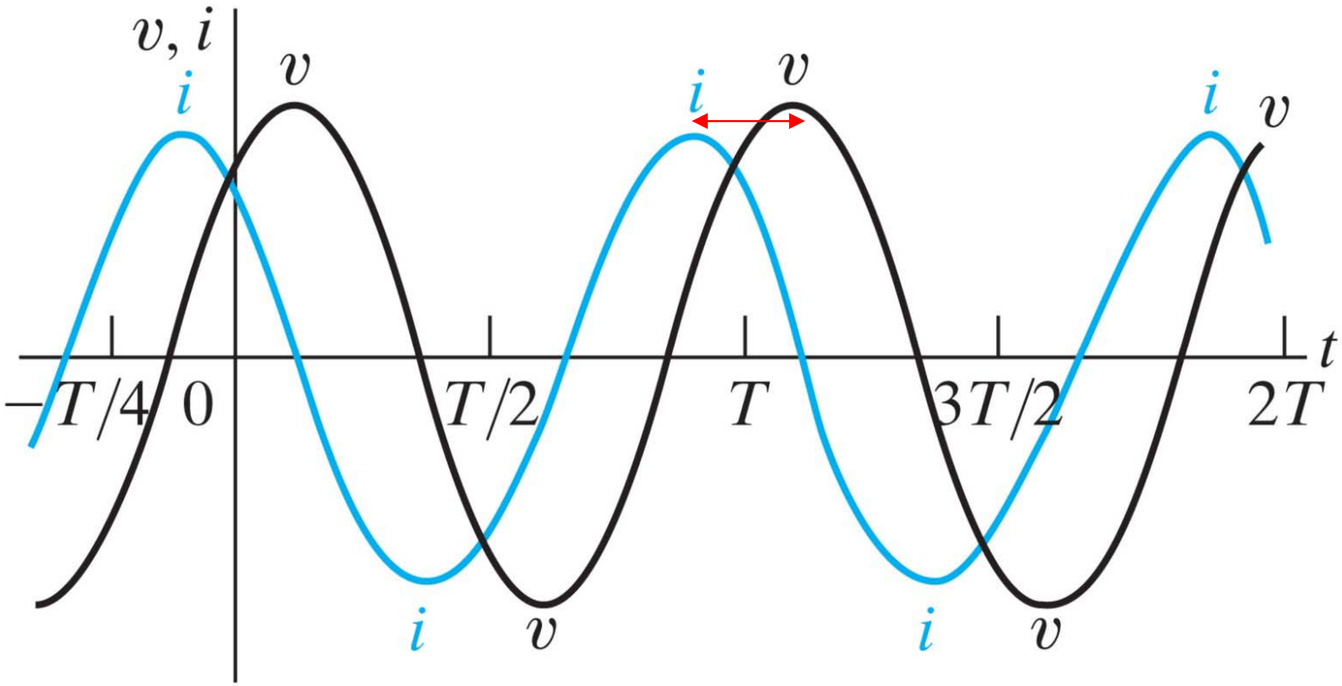
$$i(t) = C\frac{d}{dt}v(t).$$



$$\mathbf{I} = C \cdot j\omega\mathbf{V}, \Rightarrow Z = \frac{\mathbf{V}}{\mathbf{I}} = \frac{1}{j\omega C}.$$

The *i*-*v* relation and impedance of a capacitor (2)

- $v(t)$ lags $i(t)$ by $T/4$ (-90° phase, i.e. quadrature) $\Rightarrow$ impedance $Z = 1/(j\omega C)$ is purely negative imaginary.



More on impedance

- Impedance Z is a complex number in units of Ohms.
- Impedance of a "mutual" inductance M is $j\omega M$.
- $\mathrm{Re}(Z) = R,\ \mathrm{Im}(Z) = X$ are called resistance and reactance, respectively.
- Although impedance is complex, it's not a phasor. In other words, it cannot be transformed into a sinusoidal function in the time domain.

Section 9.5-9.9
Circuit Analysis Techniques in the Frequency Domain

Summary

- All the DC circuit analysis techniques:

1. KVL, KCL;
2. Series, parallel, Δ-Y simplifications;
3. Source transformations;
4. Thévenin, Norton equivalent circuits;
5. NVM, MCM;

are still applicable to sinusoidal steady-state analysis if the voltages, currents, and passive elements are replaced by the corresponding phasors and impedances.

KVL, KCL

- KVL: $$v_1(t)+v_2(t)+\ldots+v_n(t)=\sum_{q=1}^{n} v_q(t)=0,$$

$$\Rightarrow \sum_{q=1}^{n} V_{mq}\cos(\omega t+\theta_q)=\sum_{q=1}^{n}\mathrm{Re}\left[V_{mq}e^{j(\omega t+\theta_q)}\right]$$

$$=\left\{\sum_{q=1}^{n}\mathrm{Re}\left[V_{mq}e^{j\theta_q}\right]\right\}e^{j\omega t}=0,$$

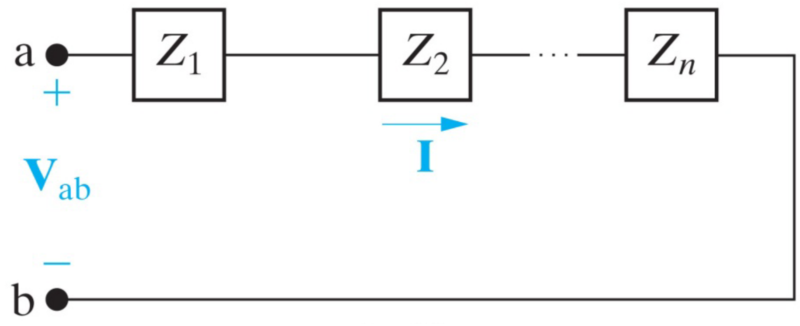
$$\mathbf{V}_1+\mathbf{V}_2+\ldots+\mathbf{V}_n=0.$$

- KCL: $$i_1(t)+i_2(t)+\ldots+i_n(t)=0,\Rightarrow$$

$$\mathbf{I}_1+\mathbf{I}_2+\ldots+\mathbf{I}_n=0.$$

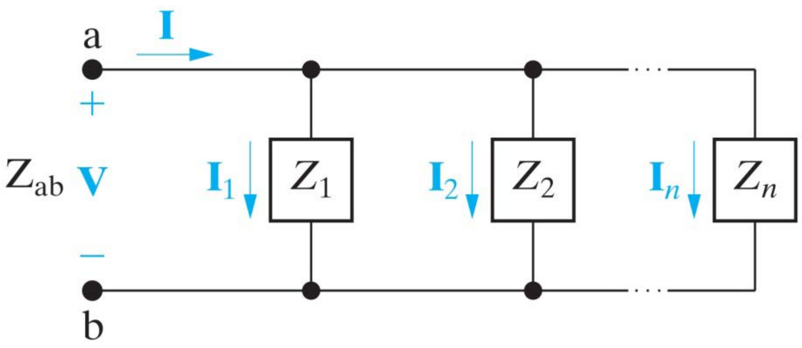
Equivalent impedance formulas

- Impedances in series



$$Z_{ab} = \sum_j Z_j$$

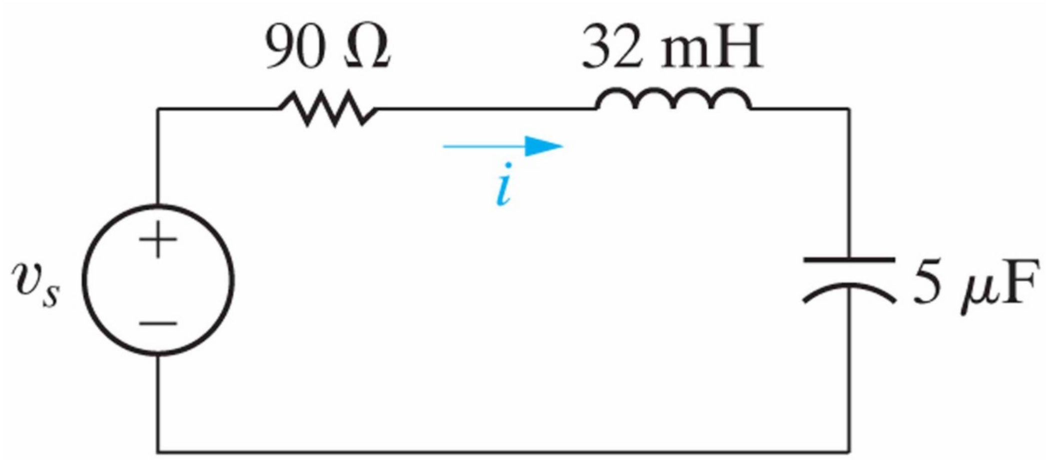
- Impedances in parallel



$$\frac{1}{Z_{ab}} = \sum_j \frac{1}{Z_j}$$

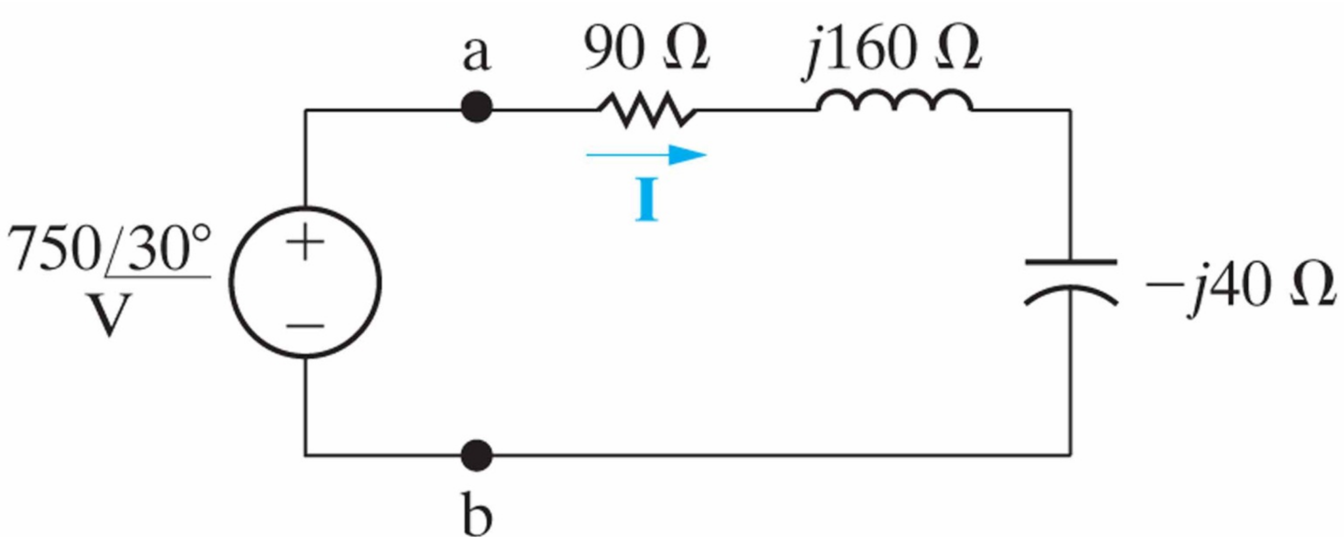
Example 9.6: Series RLC circuit (1)

- Q: Given $v_s(t)$=750 cos(5000t+30°), $\Rightarrow$ $i(t)$=?



$$\begin{cases} Z_L = j\omega L = j(5000)(32\times 10^{-3}) = j160\,\Omega, \\ Z_C = \dfrac{1}{j\omega C} = -j\dfrac{1}{(5000)(5\times 10^{-6})} = -j40\,\Omega, \\ \mathbf{V}_s = 750\angle 30^\circ \text{ V}, \end{cases}$$

Example 9.6: Series RLC circuit (2)



$$Z_{ab} = 90 + j160 - j40 = 90 + j120$$

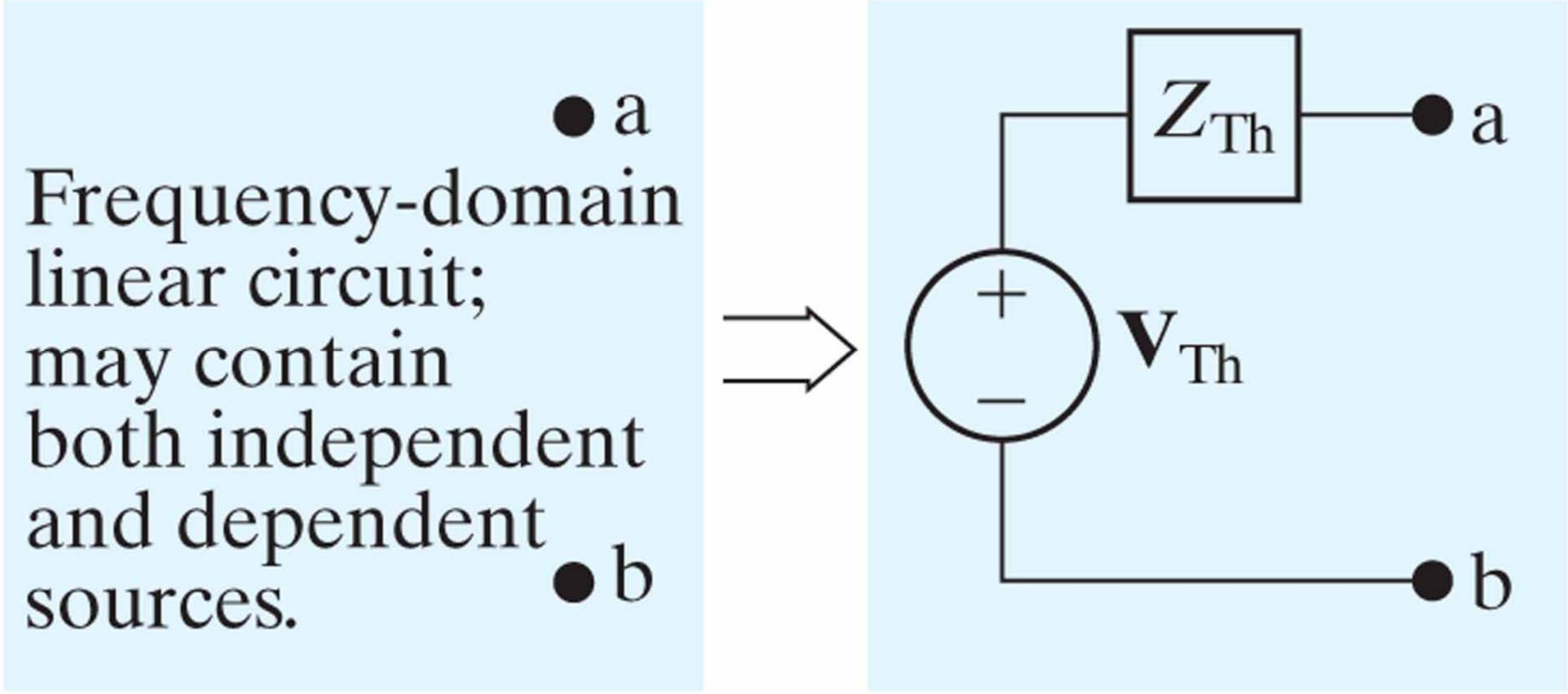
$$= \sqrt{90^2 + 120^2} \angle \tan^{-1}(120/90) = 150\angle 53.13^\circ\ \Omega,$$

$$\Rightarrow \mathbf{I} = \frac{\mathbf{V}_s}{Z_{ab}} = \frac{750\angle 30^\circ\ \text{V}}{150\angle 53.13^\circ\ \Omega} = 5\angle -23.13^\circ\ \text{A},$$

$$\Rightarrow i(t) = 5\cos(5000t - 23.13^\circ)\ \text{A}.$$

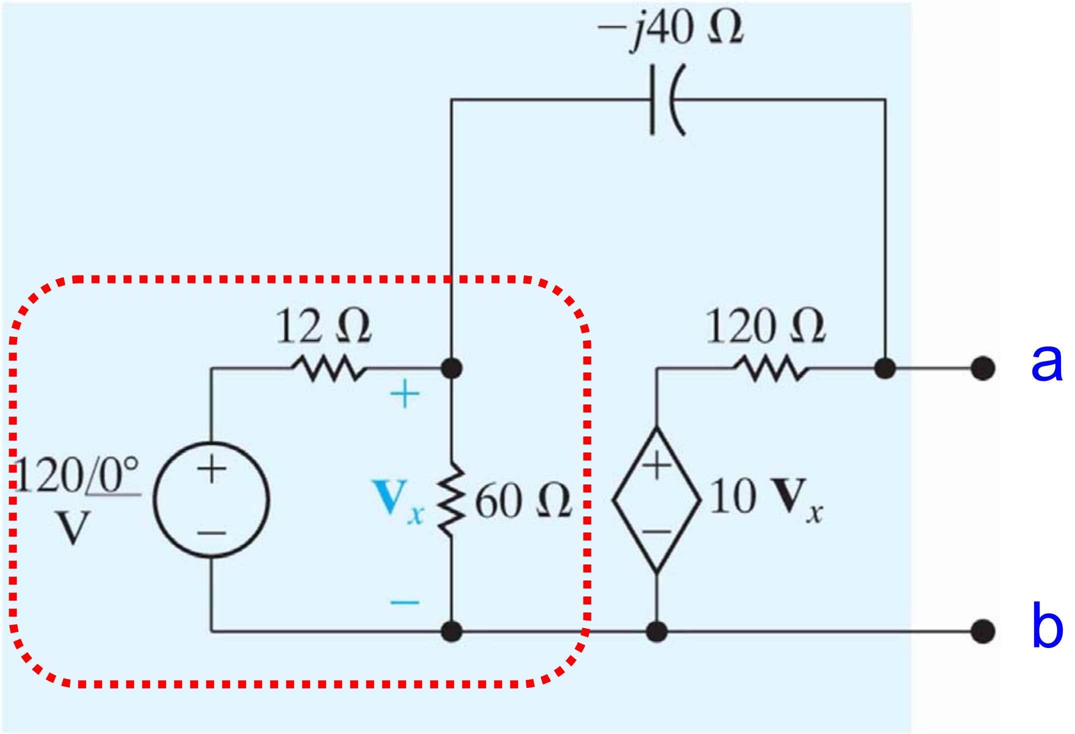
Thévenin equivalent circuit

- Terminal voltage phasor and current phasor are the same by using either configuration.



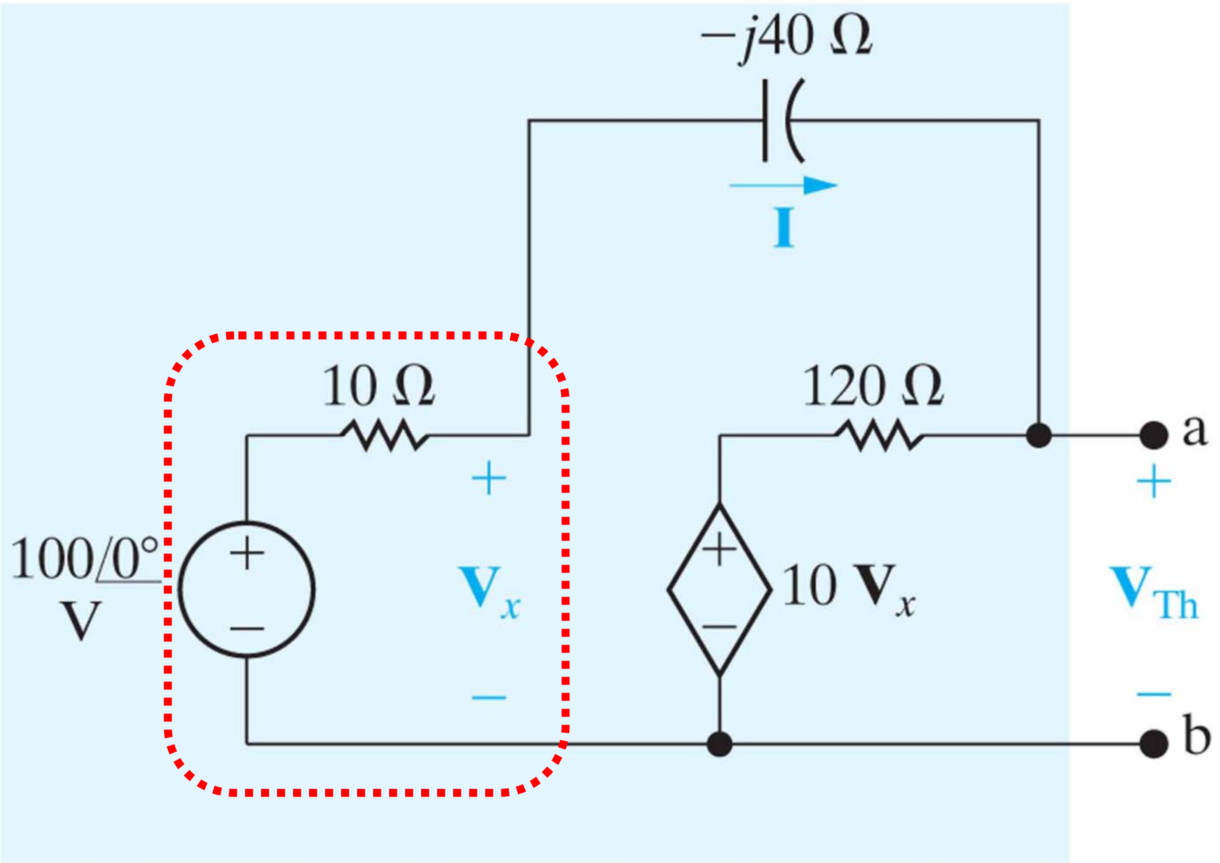
Example 9.10 (1)

- Q: Find the Thévenin circuit for terminals a, b.



- Apply source transformation to {120V, 12Ω, 60Ω} twice to get a simplified circuit.

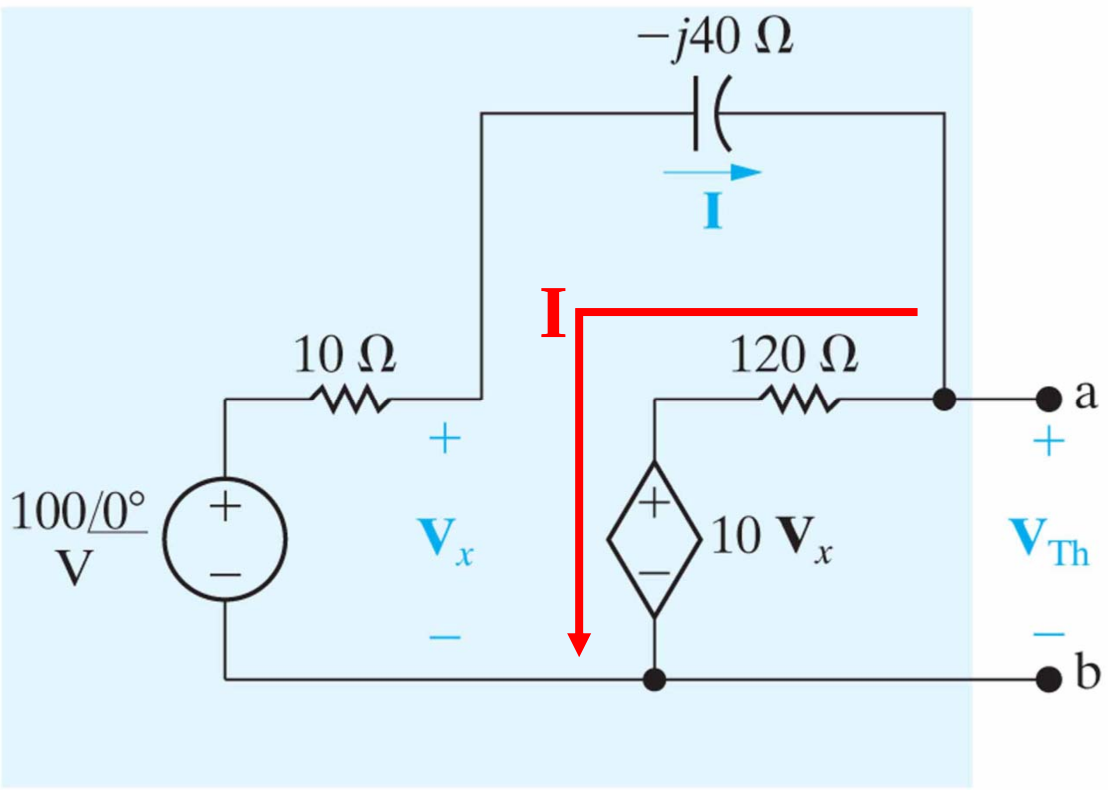
Example 9.10 (2)



$$100 = (10 - j40 + 120)\mathbf{I} + 10\mathbf{V}_x, \Rightarrow (130 - j40)\mathbf{I} + 10\mathbf{V}_x = 100 \cdots (1)$$

$$\mathbf{V}_x = 100 - 10\mathbf{I} \cdots (2)$$

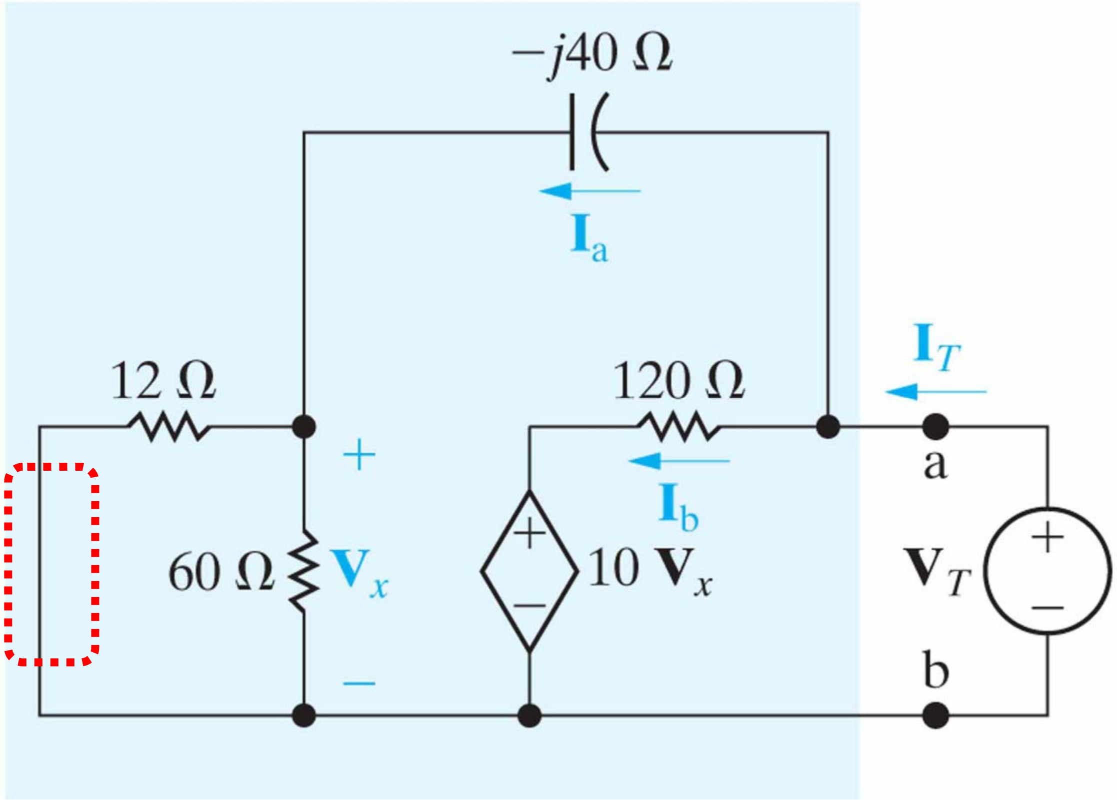
Example 9.10 (3)



$$\mathbf{I} = \frac{-900}{30 - j40} = 18\angle -126.87^\circ \text{ A},$$

$$\mathbf{V}_{Th} = 10(100 - 10\mathbf{I}) + 120\mathbf{I} = 835.22\angle -20.17^\circ \text{ V}.$$

Example 9.10 (4)



$$\mathbf{I}_a = \frac{\mathbf{V}_T}{-j40 + (12 // 60)}$$

$$= \frac{\mathbf{V}_T}{10 - j40},$$

$$\mathbf{V}_x = (12 // 60)\mathbf{I}_a = 10\mathbf{I}_a,$$

$$\mathbf{I}_b = \frac{\mathbf{V}_T - 10\mathbf{V}_x}{120},$$

$$\mathbf{I}_T = \mathbf{I}_a + \mathbf{I}_b = \mathbf{I}_a + \frac{\mathbf{V}_T - 100\mathbf{I}_a}{120} = \frac{\mathbf{I}_a}{6} + \frac{\mathbf{V}_T}{120} = \frac{1}{6}\frac{\mathbf{V}_T}{10 - j40} + \frac{\mathbf{V}_T}{120},$$

$$Z_{Th} = \mathbf{V}_T / \mathbf{I}_T = 91.2 - j38.4\,\Omega.$$

Section 9.10, 9.11
The Transformer

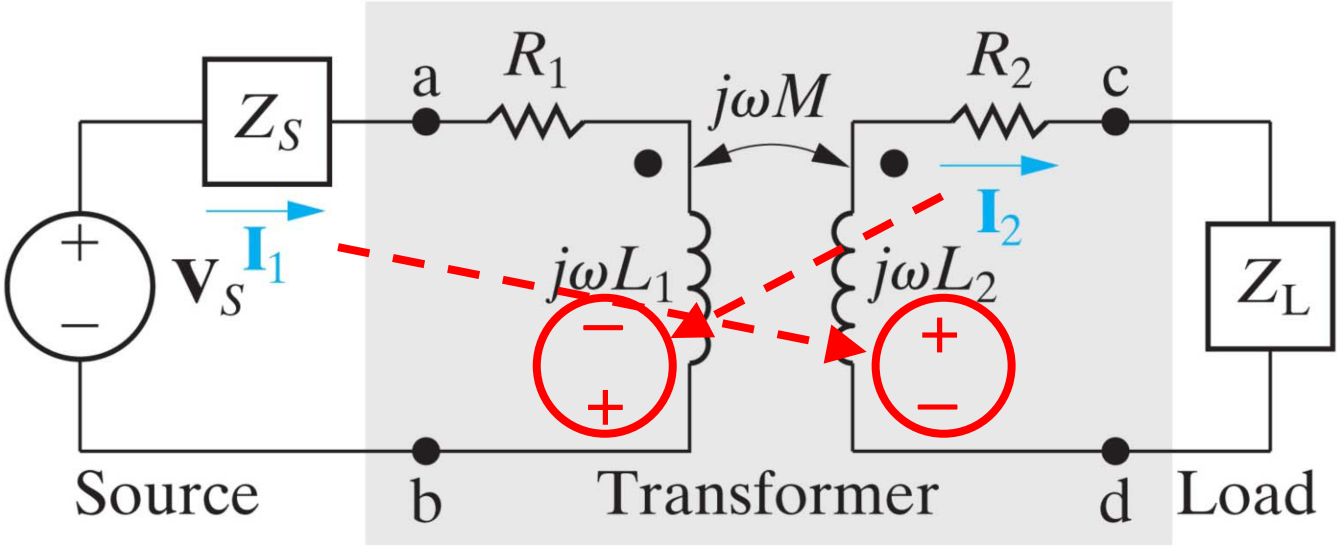
1. Linear transformer, reflected impedance
2. Ideal transformer

Summary

- A device based on magnetic coupling.
- Linear transformer is used in communication circuits to (1) match impedances, and (2) eliminate dc signals.
- Ideal transformer is used in power circuits to establish ac voltage levels.
- MCM is used in transformer analysis, for the currents in various coils cannot be written by inspection as functions of the node voltages.

Analysis of linear transformer (1)

- Consider two coils wound around a single core (magnetic coupling):

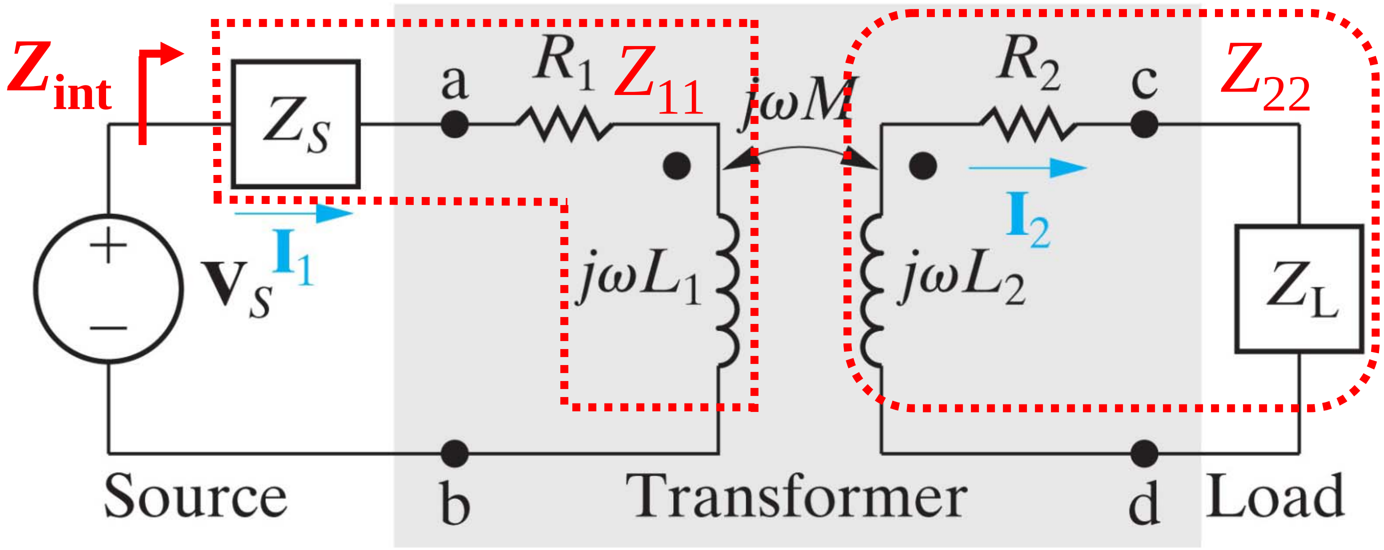


Mesh current equations:

$$\begin{cases} \mathbf{V}_s = \underbrace{(Z_s + R_1 + j\omega L_1)}_{Z_{11}}\mathbf{I}_1 - j\omega M\mathbf{I}_2; \\ 0 = -j\omega M\mathbf{I}_1 + \underbrace{(R_2 + j\omega L_2 + Z_L)}_{Z_{22}}\mathbf{I}_2. \end{cases}$$

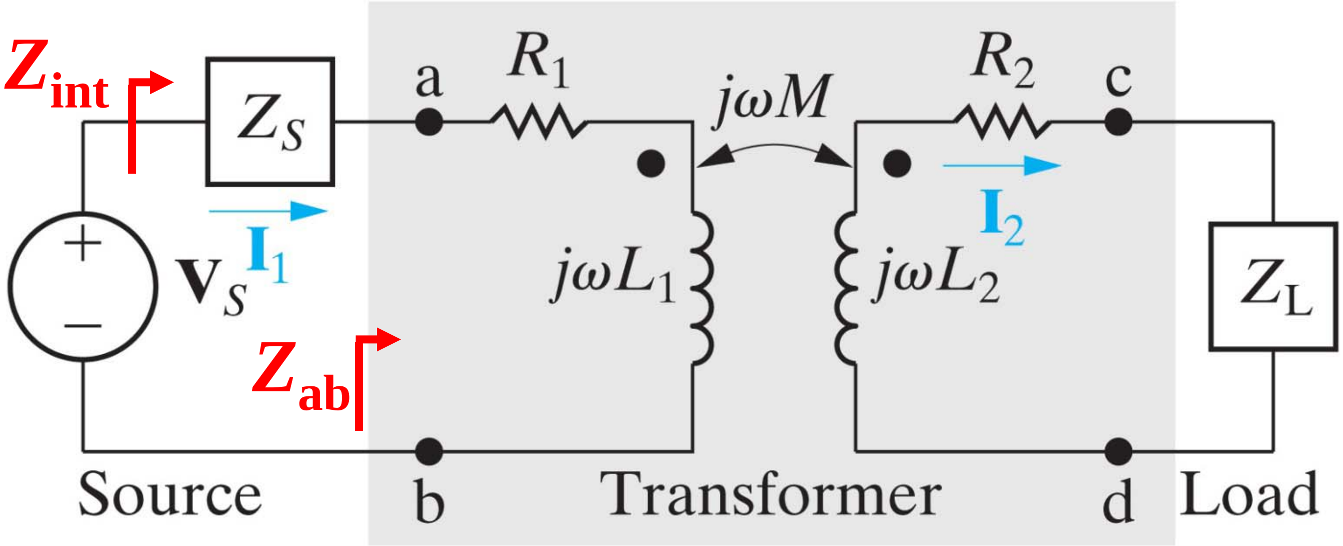
Analysis of linear transformer (2)

$$\Rightarrow \mathbf{I}_1 = \frac{Z_{22}}{Z_{11}Z_{22} + \omega^2 M^2}\mathbf{V}_s, \quad \mathbf{I}_2 = \frac{j\omega M}{Z_{22}}\mathbf{I}_1 = \frac{j\omega M}{Z_{11}Z_{22} + \omega^2 M^2}\mathbf{V}_s.$$



$$\Rightarrow Z_{\text{int}} = \frac{\mathbf{V}_s}{\mathbf{I}_1} = \frac{Z_{11}Z_{22} + \omega^2 M^2}{Z_{22}} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}.$$

Input impedance of the primary coil

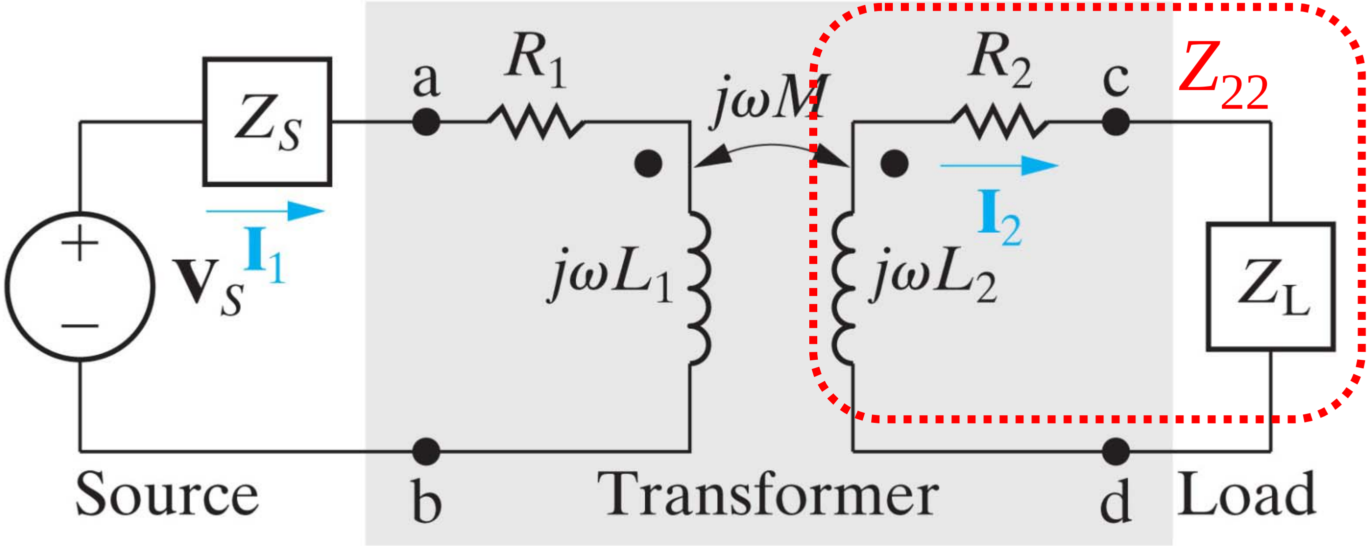


$$Z_{ab} = Z_{int} - Z_s = \left(R_1 + j\omega L_1\right) + Z_r, \quad Z_r = \frac{\omega^2 M^2}{Z_{22}} = \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}.$$

- Z_r is the equivalent impedance of the secondary coil and load due to the mutual inductance.
- $Z_{ab} = Z_S$ is needed to prevent power reflection.

Reflected impedance

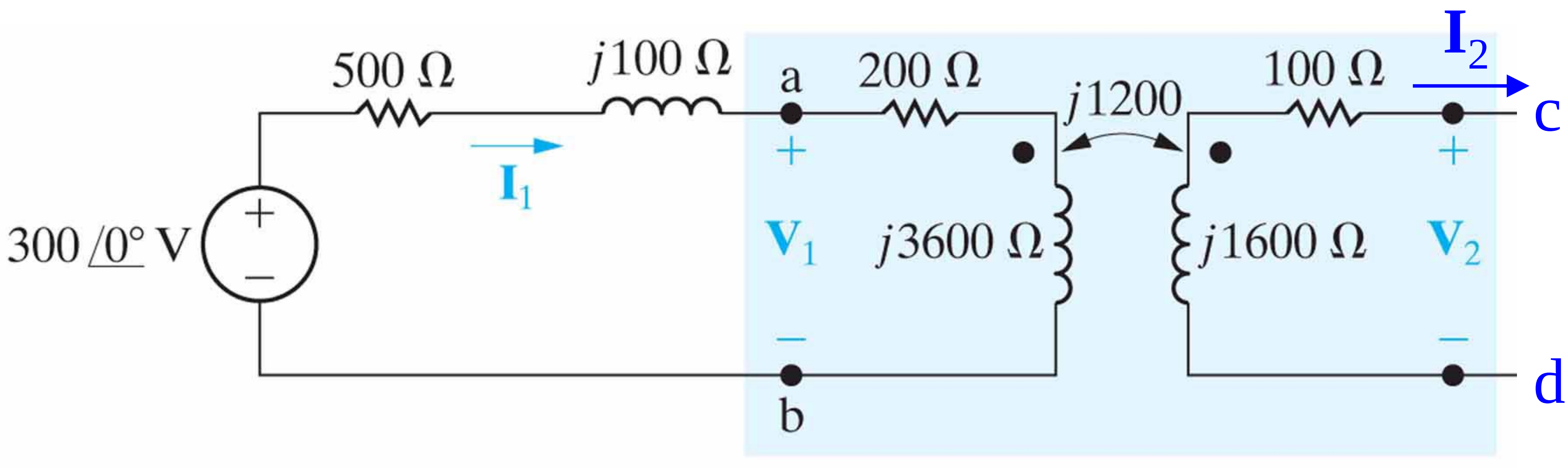
$$Z_r = \frac{\omega^2 M^2}{Z_{22}} = \frac{\omega^2 M^2}{Z_{22} Z_{22}^*} Z_{22}^* = \left(\frac{\omega M}{|Z_{22}|} \right)^2 Z_{22}^*.$$



- Linear transformer reflects $(Z_{22})^*$ into the primary coil by a scalar multiplier $(\omega M/|Z_{22}|)^2$.

Example 9.13 (1)

- Q: Find the Thévenin circuit for terminals c, d.

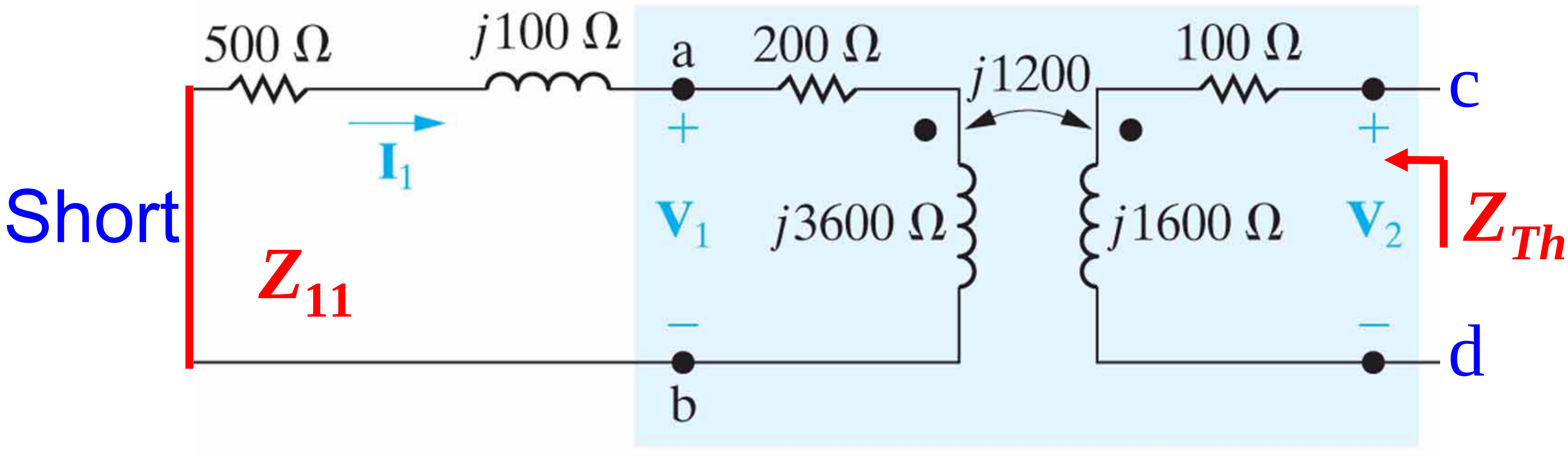


$\mathbf{V}_{Th} = \mathbf{V}_{cd}$. Since $\mathbf{I}_2 = 0$, $\Rightarrow \mathbf{V}_{cd} = \mathbf{I}_1 \times j\omega M$, where

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{Z_{11}} = \frac{300\angle 0^\circ}{(500 + j100) + (200 + j3600)} = 79.67\angle -79.29^\circ \text{ A.}$$

$$\Rightarrow \mathbf{V}_{Th} = (79.67\angle -79.29^\circ) \times (j1200) = 95.6\angle 10.71^\circ \text{ V.}$$

Example 9.13 (2)



- $Z_{Th} = (100 + j1600) + Z_r$, where Z_r is the reflected impedance of Z_{11} due to the transformer:

$$Z_{11} = (500 + j100) + (200 + j3600) = (700 + j3700)\,\Omega.$$

$$Z_r = \left(\frac{\omega M}{|Z_{11}|}\right)^2 Z_{11}^* = \left(\frac{1200}{|700 + j3700|}\right)^2 (700 - j3700),$$

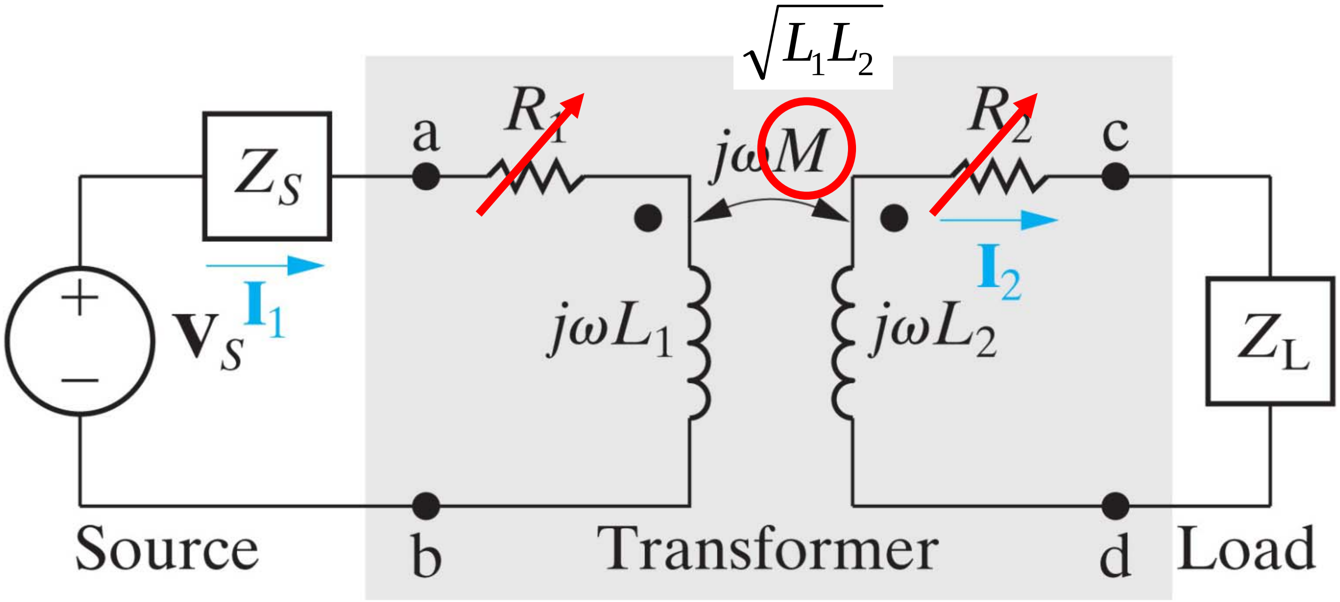
$$Z_{Th} = (100 + j1600) + Z_r = (171.09 + j1224.26)\,\Omega.$$

Characteristics of ideal transformer

- An ideal transformer consists of two magnetically coupled coils with N_1 and N_2 turns, respectively. It exhibits three properties:

1. Magnetic field is perfectly confined within the magnetic core, $\Rightarrow$ magnetic coupling coefficient is $k=1$, $\Rightarrow M = \sqrt{L_1 L_2}$.
2. The self-inductance of each coil $\left(L_i \propto N_i^2\right)$ is large, i.e. $L_1 = L_2 \rightarrow \infty$.
3. The coil loss is negligible: $R_1 = R_2 \rightarrow 0$.

Current ratio

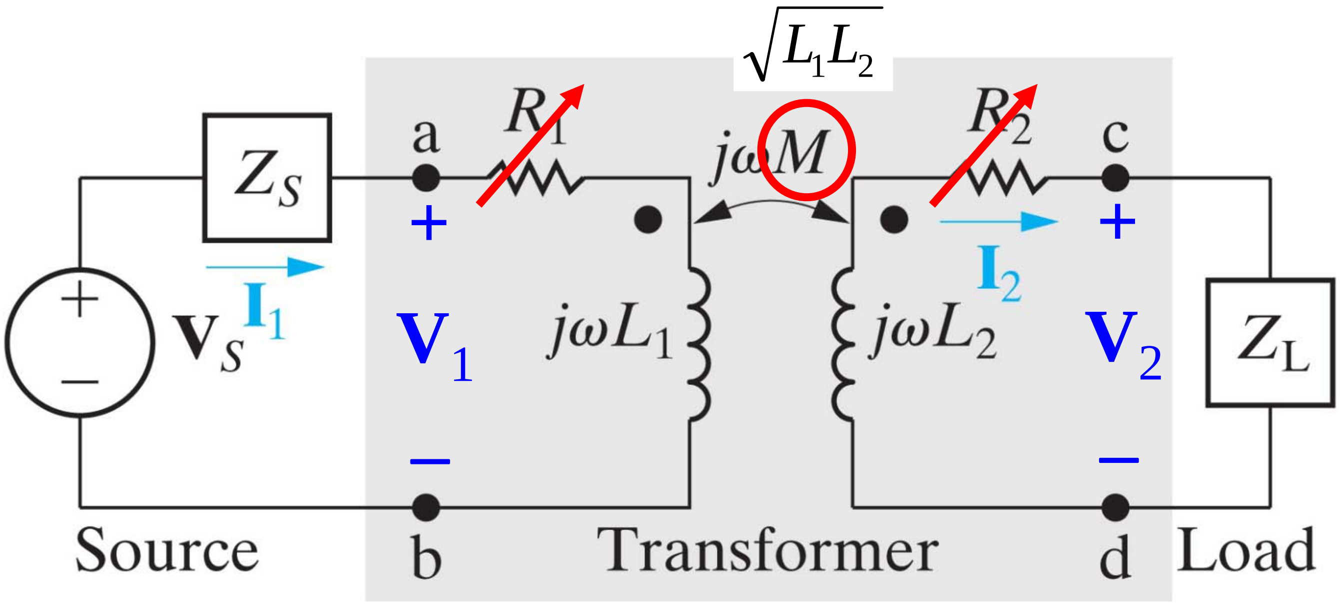


- By solving the two mesh equations of a general linear transformer:

if $\omega L_2 >> |Z_L|$

$$\Rightarrow \frac{\mathbf{I}_1}{\mathbf{I}_2} = \frac{Z_{22}}{j\omega M} = \frac{j\omega L_2 + Z_L}{j\omega\sqrt{L_1L_2}} \rightarrow \sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1}.$$

Voltage ratio

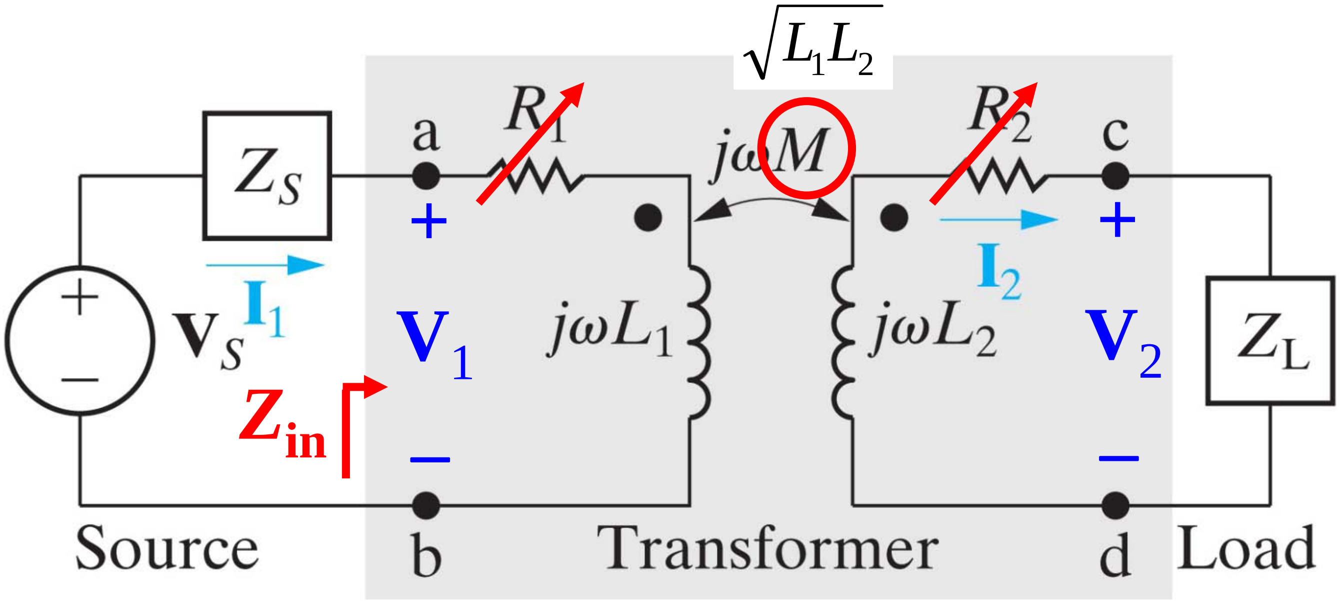


- Substitute $\mathbf{I}_2 = \dfrac{j\omega M}{Z_{22}} \mathbf{I}_1$ into $\begin{cases} \mathbf{V}_1 = j\omega L_1 \mathbf{I}_1 - j\omega M \mathbf{I}_2, \\ \mathbf{V}_2 = Z_L \mathbf{I}_2. \end{cases}$

$j\omega L_2 + Z_L$

$$\Rightarrow \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{Z_{22}(j\omega L_1) + \omega^2 M^2}{j\omega M Z_L} = \frac{L_1}{M} = \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2}.$$

Input impedance



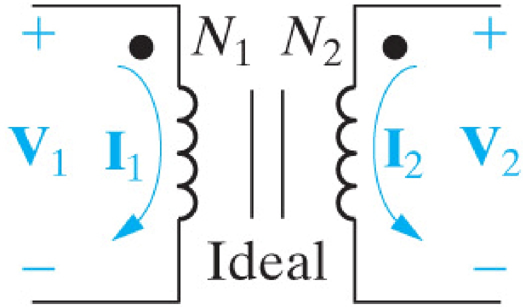
- By the current and voltage ratios,

in-phase

$$\frac{Z_{ab}}{Z_L}=\frac{\mathbf{V}_1/\mathbf{I}_1}{\mathbf{V}_2/\mathbf{I}_2}=\frac{\mathbf{V}_1}{\mathbf{V}_2}\frac{\mathbf{I}_2}{\mathbf{I}_1}=\left(\frac{N_1}{N_2}\right)^2, \Rightarrow Z_{in}=Z_{ab}=\left(\frac{N_1}{N_2}\right)^2 Z_L.$$

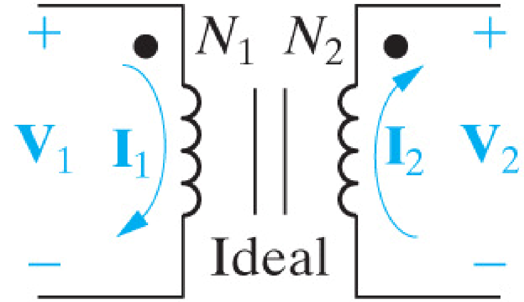
- For lossy transformer, $\Rightarrow Z_{ab}\rightarrow R_1+\left(\frac{N_1}{N_2}\right)^2(R_2+Z_L).$

Polarity of the voltage and current ratios



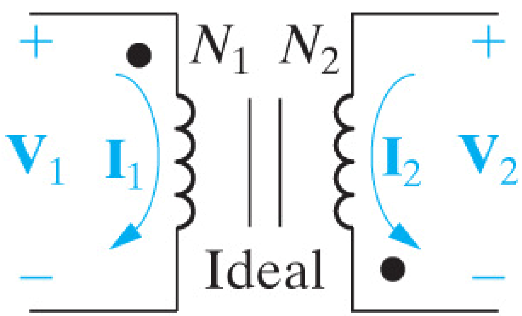
$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$



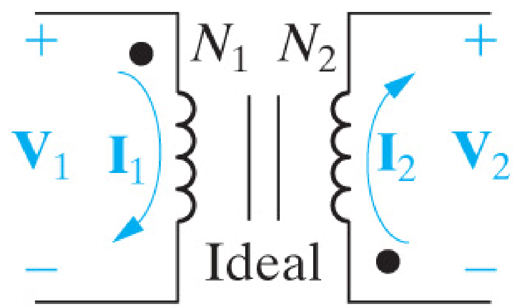
$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$



$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

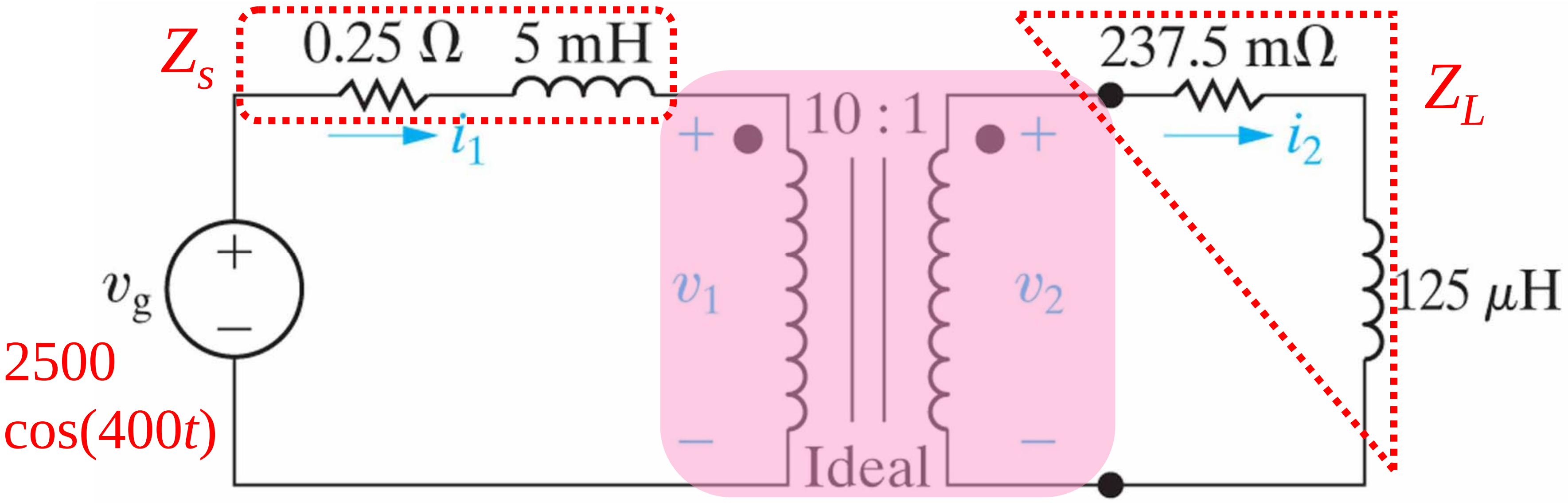


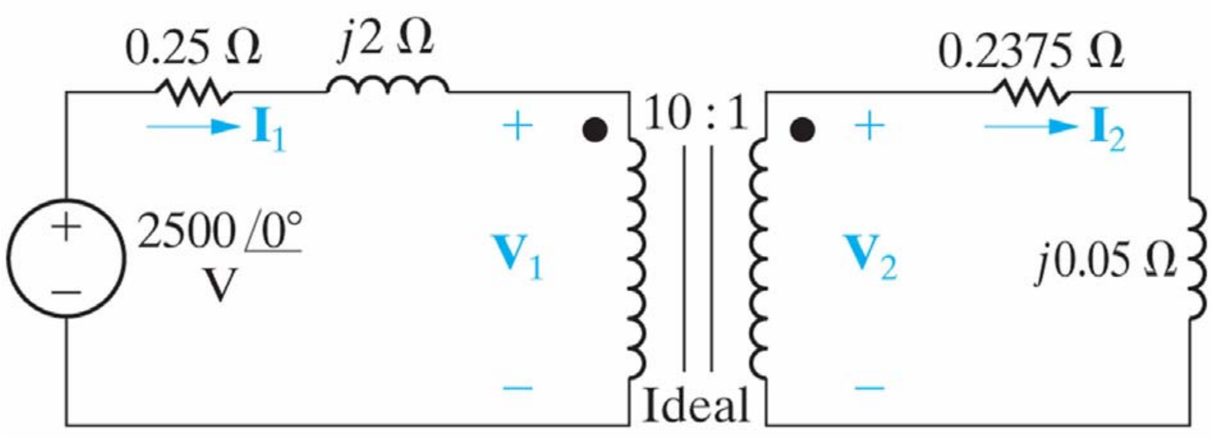
$$\frac{\mathbf{V}_1}{N_1} = -\frac{\mathbf{V}_2}{N_2},$$

$$N_1\mathbf{I}_1 = -N_2\mathbf{I}_2$$

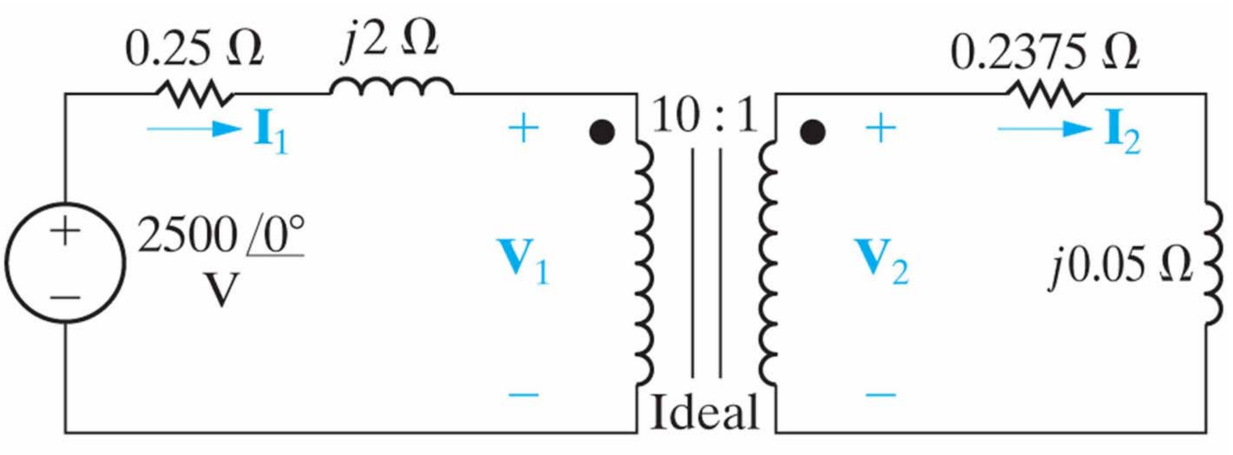
Example 9.14 (1)

- Q: Find v_1, i_1, v_2, i_2.





Example 9.14 (2)



$$2500\angle 0^\circ = (0.25 + j2)\mathbf{I}_1 + \mathbf{V}_1 \cdots (1)$$

$$\left\{ \begin{array}{l} \mathbf{V}_2 = (0.2375 + j0.05)\mathbf{I}_2, \\ \mathbf{V}_1 = 10\mathbf{V}_2,\ \mathbf{I}_2 = 10\mathbf{I}_1, \end{array} \right\} \Rightarrow \mathbf{V}_1 = (23.75 + j5)\mathbf{I}_1 \cdots (2)$$

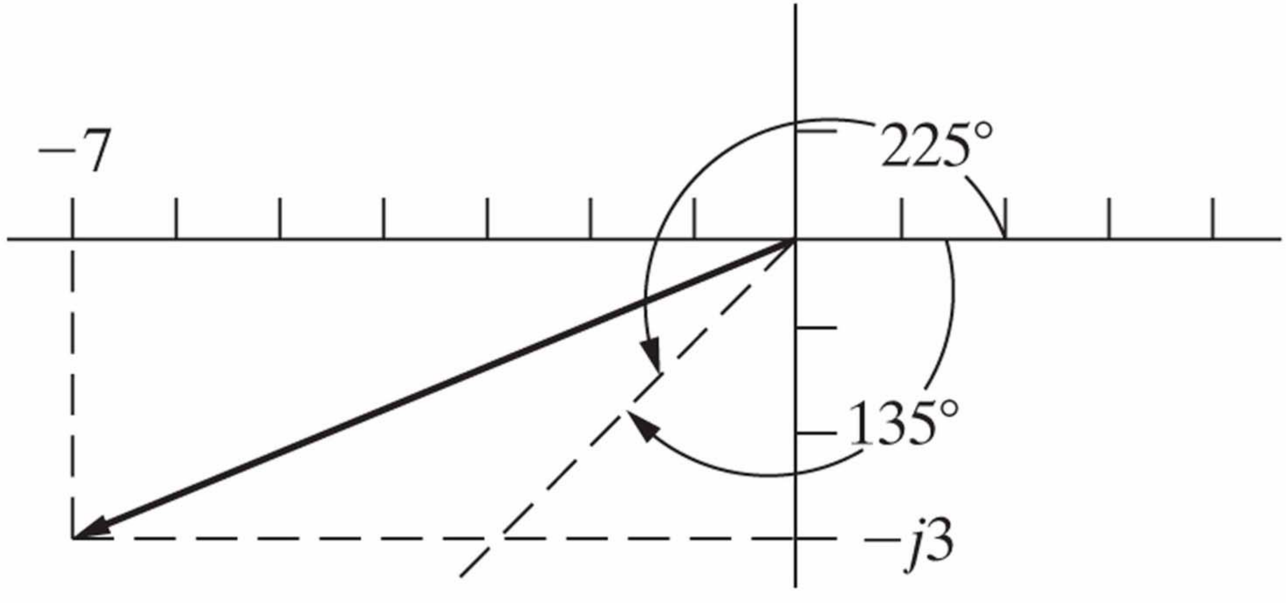
$$(2) \to (1):\ \mathbf{I}_1 = \frac{2500\angle 0^\circ}{24 + j7} = 100\angle -16.26^\circ,\ i_1 = 100\cos(400t - 16.26^\circ).$$

$$\text{By } (2): \mathbf{V}_1 = (23.75 + j5)(100\angle -16.26^\circ) = 2427\angle -4.37^\circ, v_1 = \ldots$$

Section 9.12
Phasor Diagrams

Definition

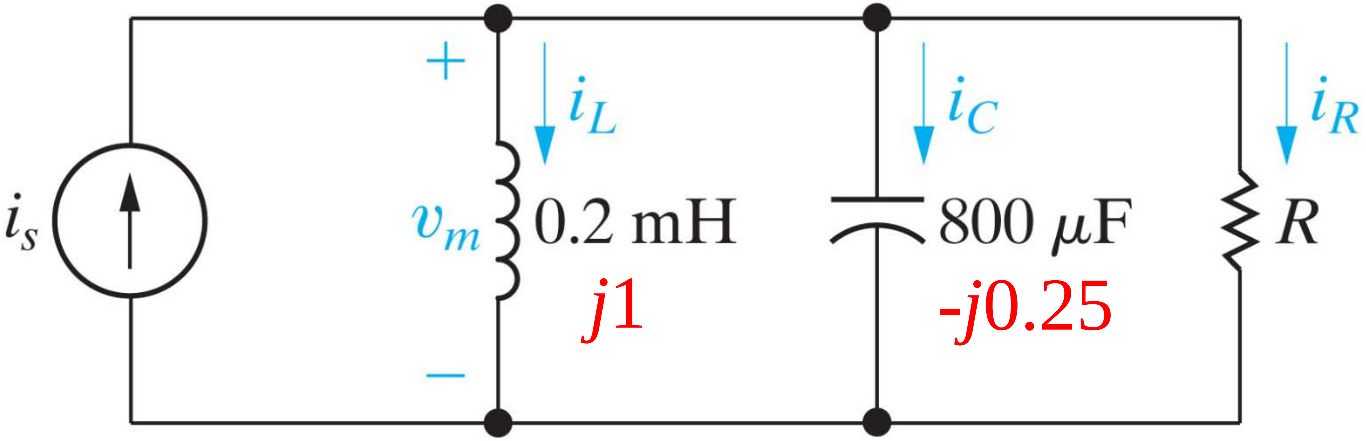
- Graphical representation of $-7-j3 = 7.62\angle{-156.8°}$ on the complex-number plane.



- Without calculation, we can anticipate a magnitude >7, and a phase in the 3rd quadrant.

Example 9.15 (1)

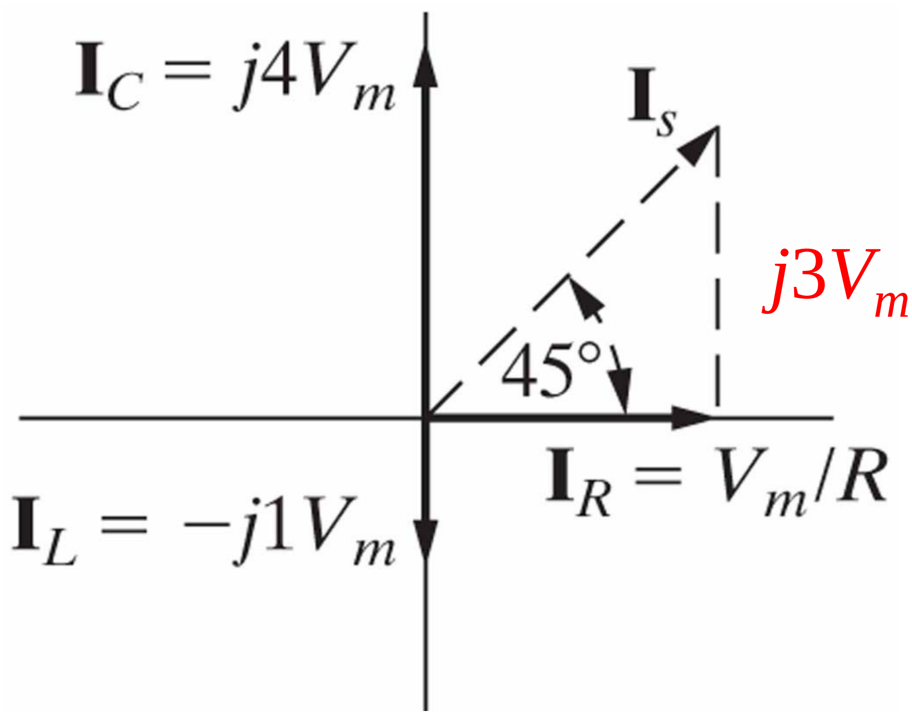
- Q: Use a phasor diagram to find the value of ***R*** that will cause i_R to lag the source current i_s by 45° when $\omega = 5$ krad/s.



$$\mathbf{I}_L = \frac{\mathbf{V}_m}{j1} = V_m\angle -90^\circ,\ \mathbf{I}_C = \frac{\mathbf{V}_m}{-j0.25} = 4V_m\angle 90^\circ,\ \mathbf{I}_R = \frac{\mathbf{V}_m}{R} = V_m\angle 0^\circ.$$

Example 9.15 (2)

- By KCL, $\mathbf{I}_s = \mathbf{I}_L + \mathbf{I}_C + \mathbf{I}_R$. Addition of the 3 current phasors can be visualized by vector summation on a phase diagram:



To make $\angle \mathbf{I}_s = 45°$,

$\mathbf{I}_R = 3V_m$,

$\Rightarrow R = 1/3\ \Omega$.

Key points

- What is the phase of a sinusoidal function?
- What is the phasor of a sinusoidal function?
- What is the phase of an impedance? What are in-phase and quadrature?
- How to solve the sinusoidal steady-state response by using phasor and impedance?
- What is the reflected impedance of a circuit with transformer?