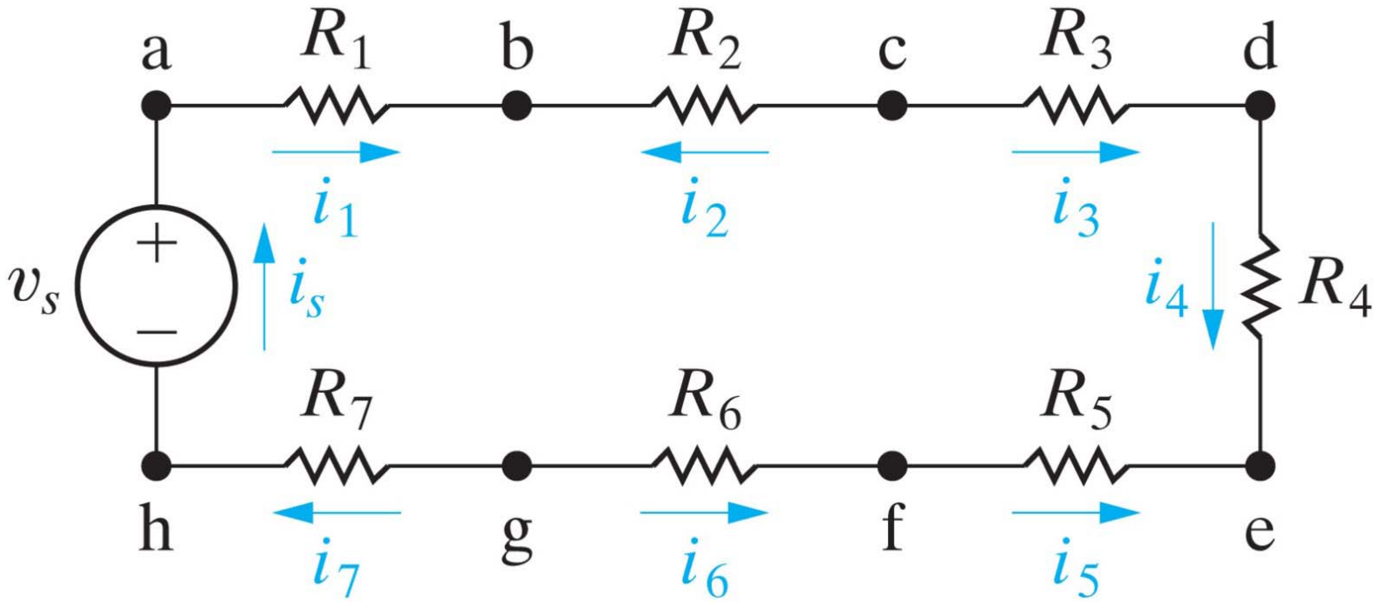
Chapter 3
Simple Resistive Circuits

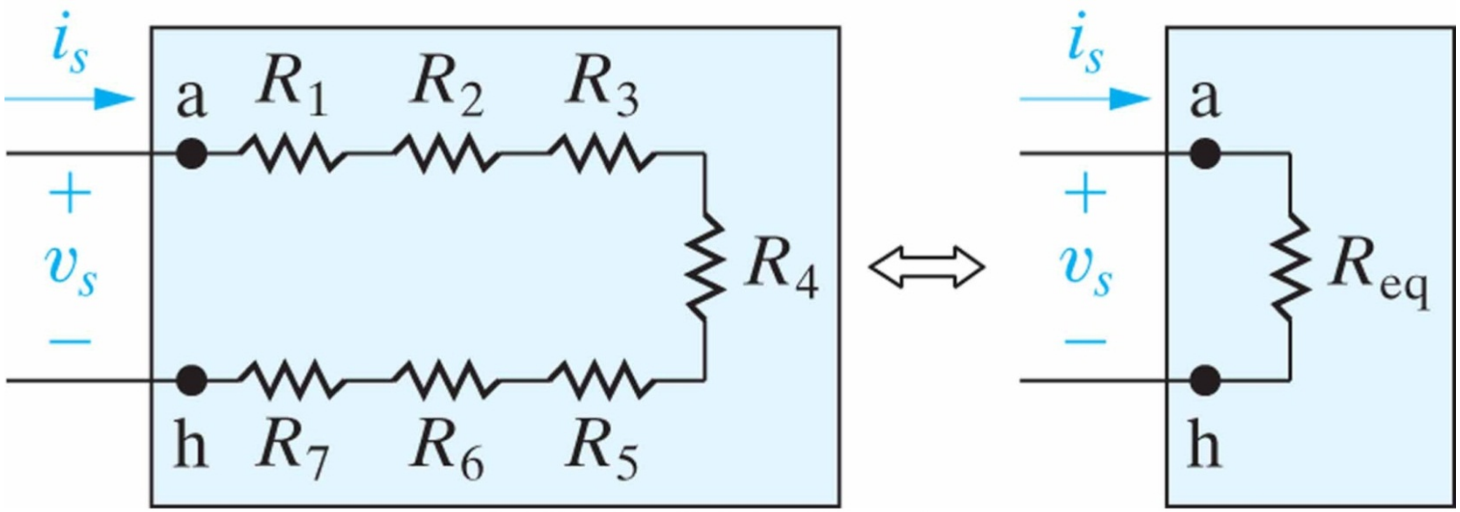
Section 3.1
Resistors in Series

Definition



- Two elements are said to be in series if they are connected at a single node.
- By KCL, all series-connected resistors carry the same current.

Equivalent resistor

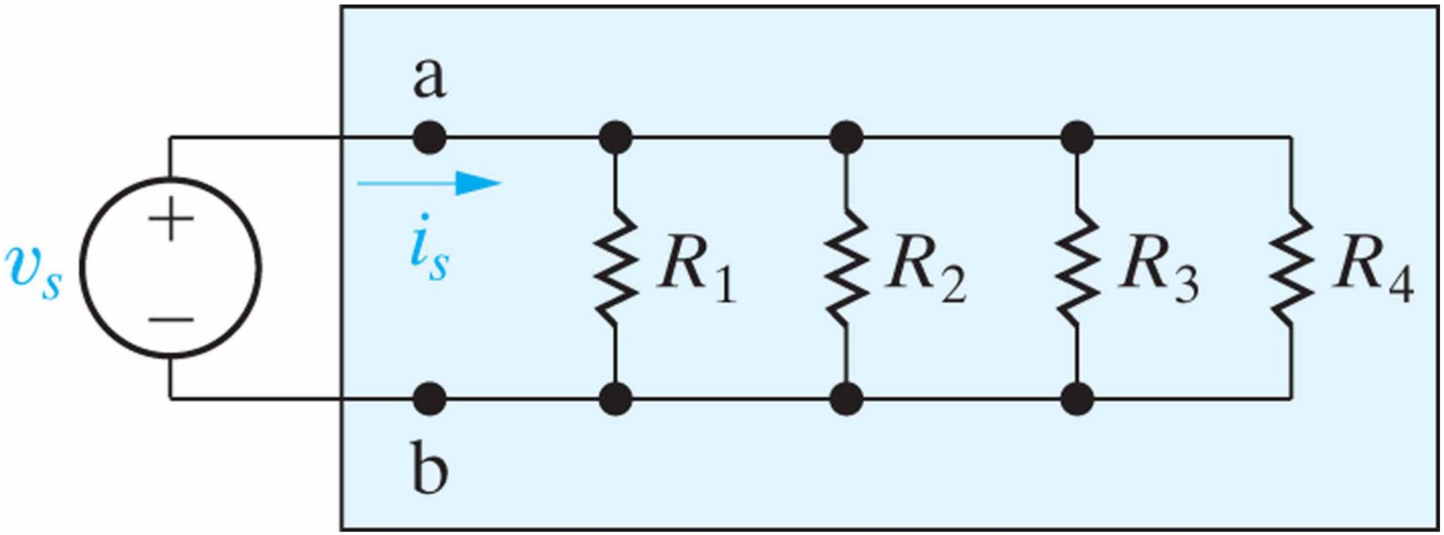


- By (1) KVL, (2) all series resistors share a common current i_s, and (3) Ohm's law, $\Rightarrow v_s = i_sR_1 + i_sR_2 + \ldots + i_sR_7 = i_s(R_1+R_2+\ldots+R_7) = i_sR_{eq}$,

$$\Rightarrow R_{eq} = \sum_{i=1}^{k} R_i = R_1 + R_2 + \ldots + R_k.$$

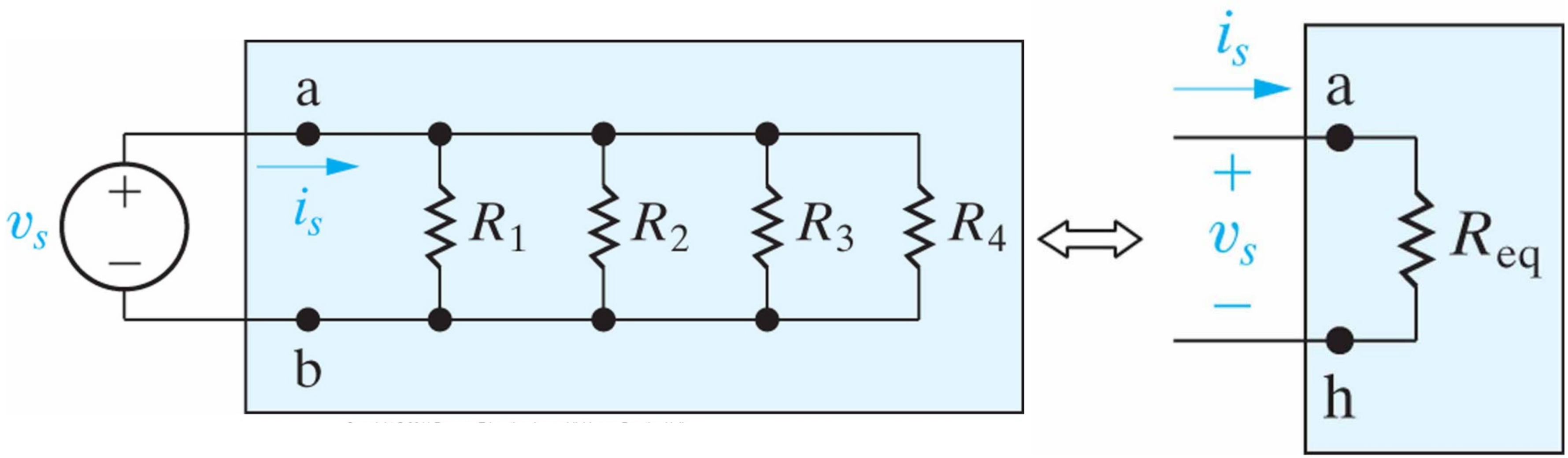
Section 3.2
Resistors in Parallel

Definition



- Two elements are in parallel if they are connected at a single node pair.
- By KVL, all parallel-connected elements have the same voltage across their terminals.

Equivalent resistor

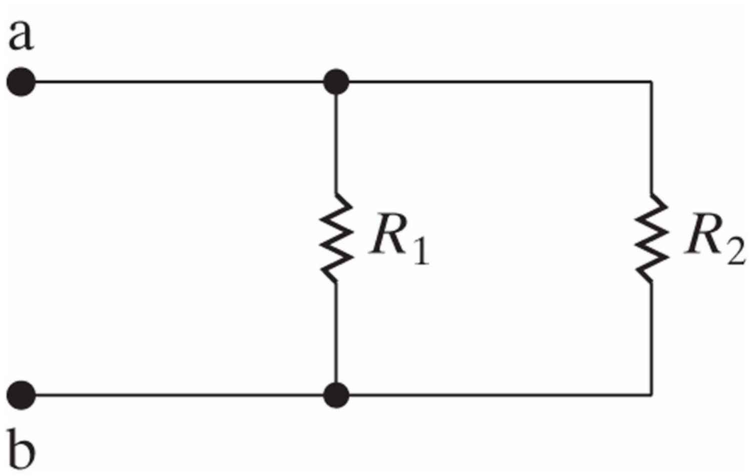


- By (1) KCL, (2) all parallel resistors share a common voltage v_s, and (3) Ohm's law, $i_s = v_s/R_1 + v_s/R_2 + \ldots + v_s/R_4 = v_s(1/R_1 + 1/R_2 + \ldots + 1/R_4) = v_s/R_{eq}$,

$$\Rightarrow \quad \frac{1}{R_{eq}} = \sum_{i=1}^{k} \frac{1}{R_i} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_k}.$$

Comments about resistors in parallel

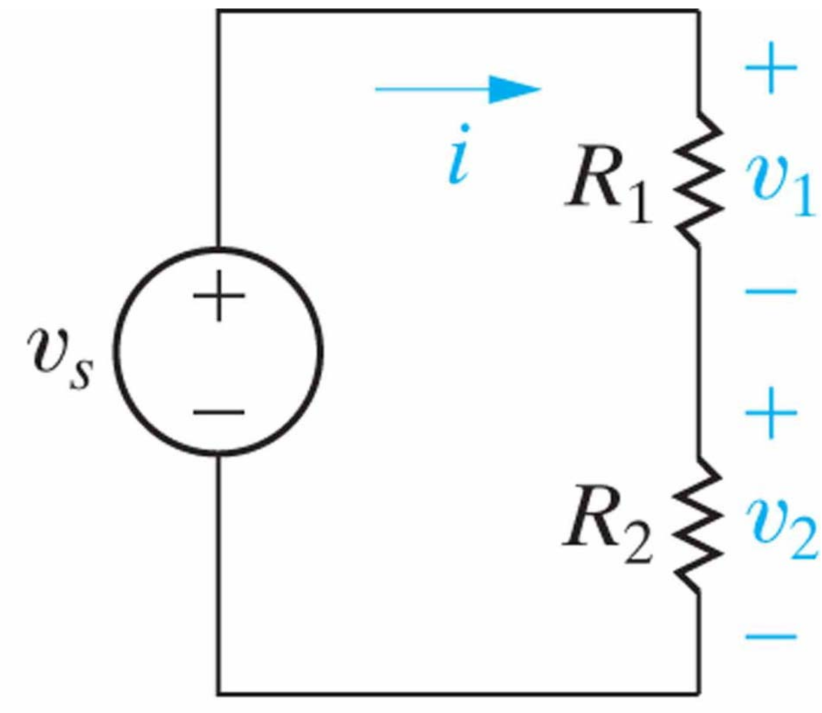
- R_{eq} is always smaller than any resistance in parallel connection.
- The smallest resistance dominates the equivalent value.



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Section 3.3
The Voltage-Divider & Current-Divider Circuits

The voltage-divider circuit



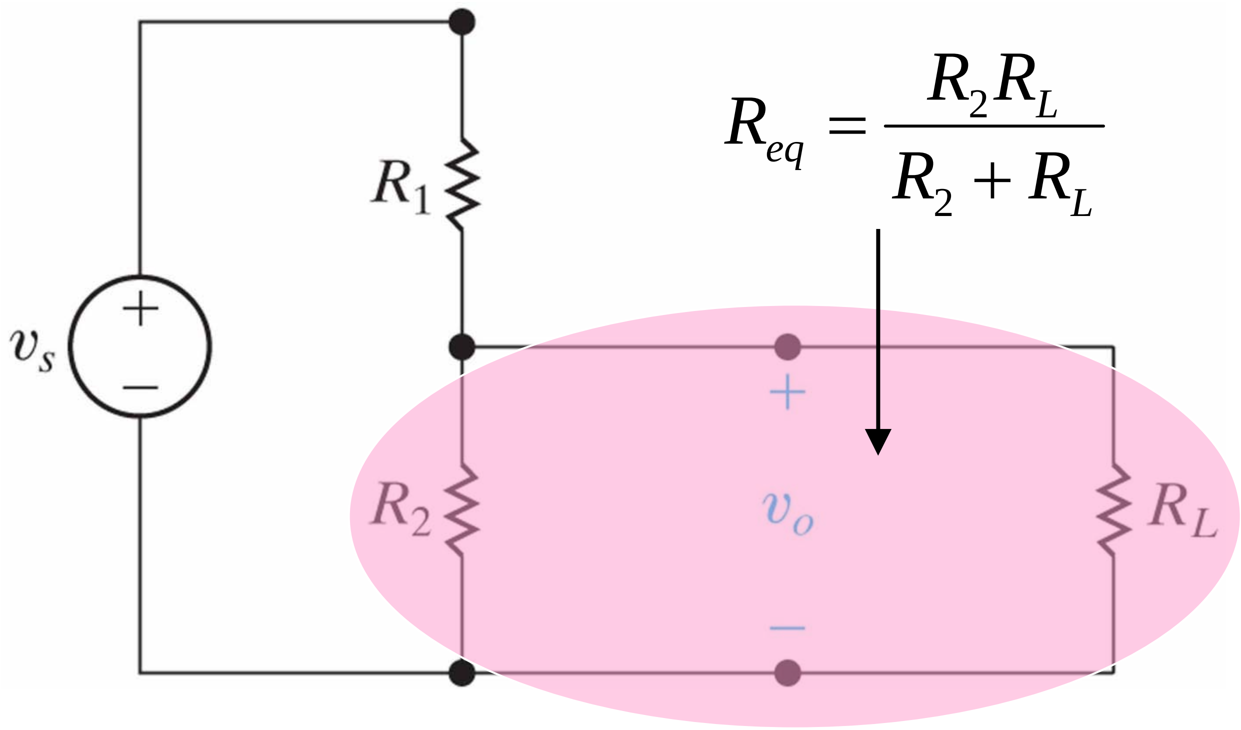
By KVL, $v_s = iR_1 + iR_2$,

$\Rightarrow i = v_s/(R_1+R_2)$,

$$\Rightarrow \begin{cases} v_1 = iR_1 = \dfrac{R_1}{R_1 + R_2} v_s, \\ v_2 = iR_1 = \dfrac{R_2}{R_1 + R_2} v_s. \end{cases}$$

- v_1, v_2 are fractions of v_s depending only on the ratio of resistance.

Practical concern: Load resistance



$$R_{eq} = \frac{R_2 R_L}{R_2 + R_L}$$

$$v_o = \frac{R_{eq}}{R_1 + R_{eq}} v_s$$

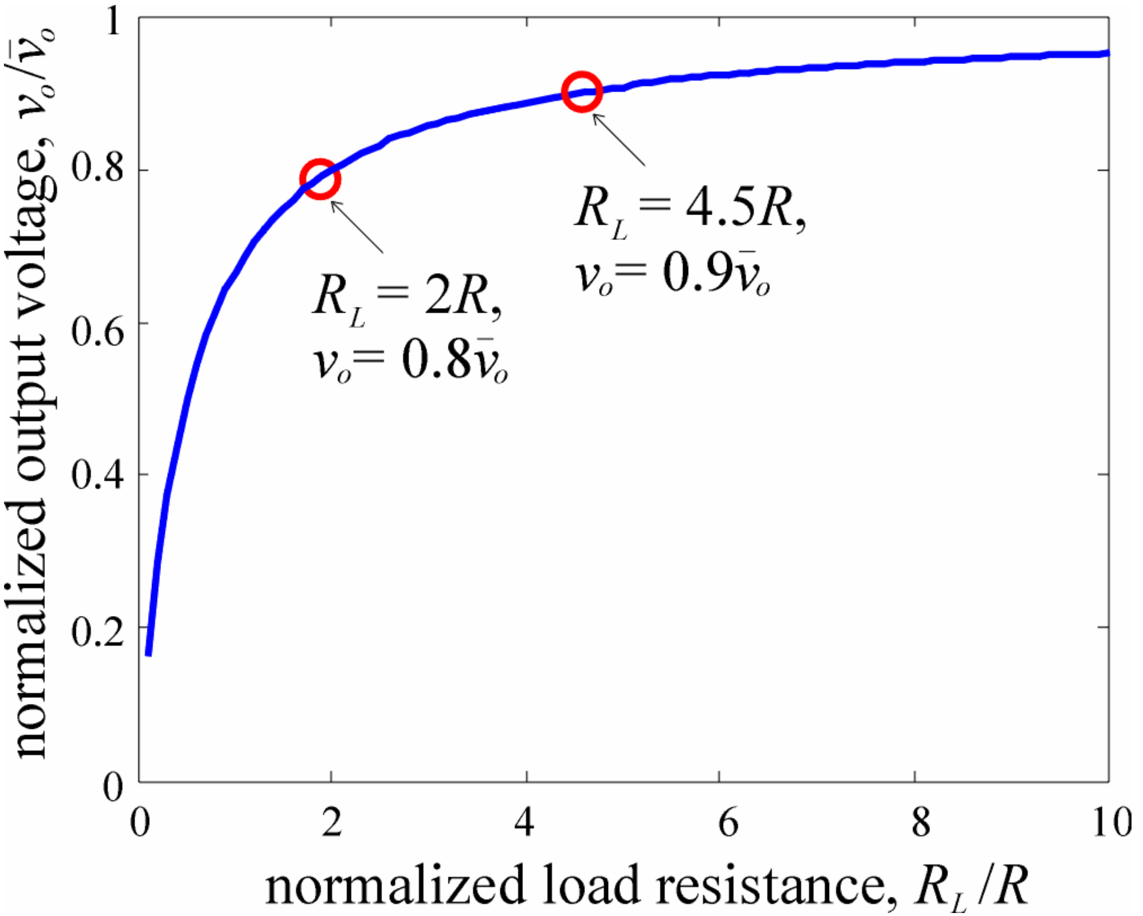
$$= \frac{R_2}{(1+a)R_1 + R_2} v_s,$$

$$\text{where } a = \frac{R_2}{R_L}.$$

- The effect of R_1 is amplified by a factor of $a = R_2/R_L$, reducing the output voltage v_o.
- Large load resistance $R_L >> R_2$ is preferred.

Example: output voltage vs. load resistance

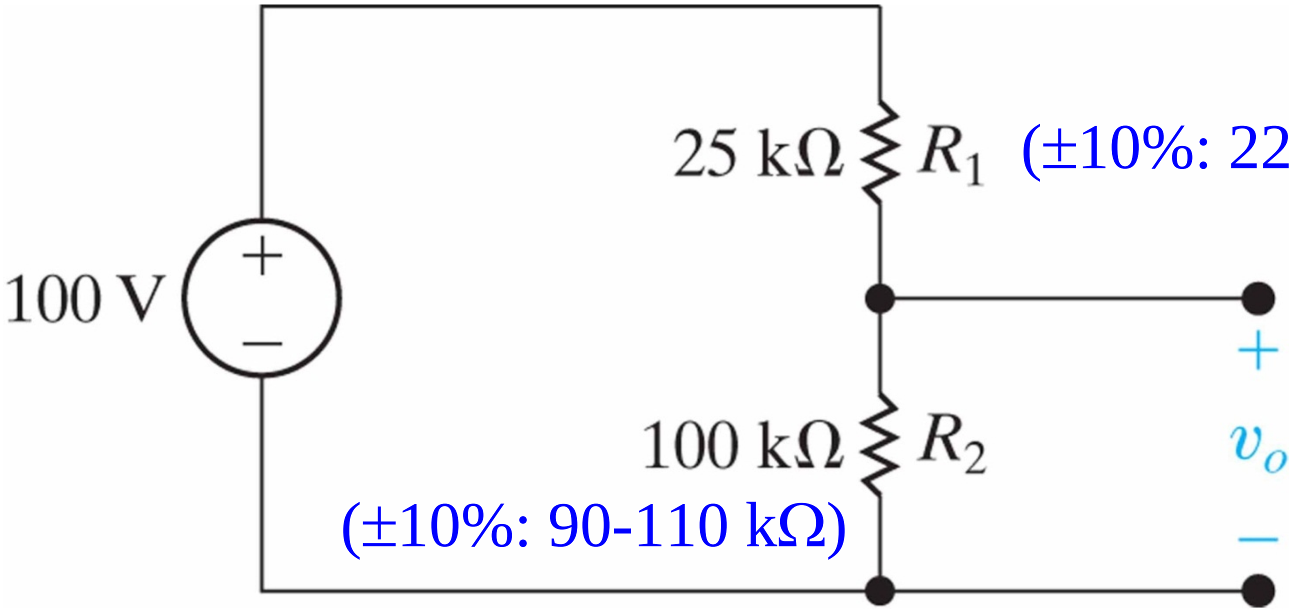
$$\text{Let } R_1 = R_2 = R, \Rightarrow \text{open-circuit output voltage } \bar{v}_o = 0.5 v_s.$$



Reference: Matlab™ codes

```
clear                          % empty the variables in the working space
close all                      % close all existing figures

R2 = 1;                        % resistance R2 normalized to resistance R1

RL = linspace(0.1,10,100);     % load resistance normalized to R1

vo_noload = R2/(1+R2);         % output voltage normalized to vs without load
vo = 1./(2+1./RL);             % output voltage normalized to vs with load

plot(RL,vo/vo_noload)          % plot a curve
ylim([0 1])                    % y-axis is shown between 0 and 1
xlabel(['normalized load resistance, R_L/R_1'])     % label the x-axis
ylabel(['normalized output voltage, v_o/v''_o'])    % label the y-axis
```

Practical concern: Tolerance of resistance



(±10%: 22.5-27.5 kΩ)

(±10%: 90-110 kΩ)

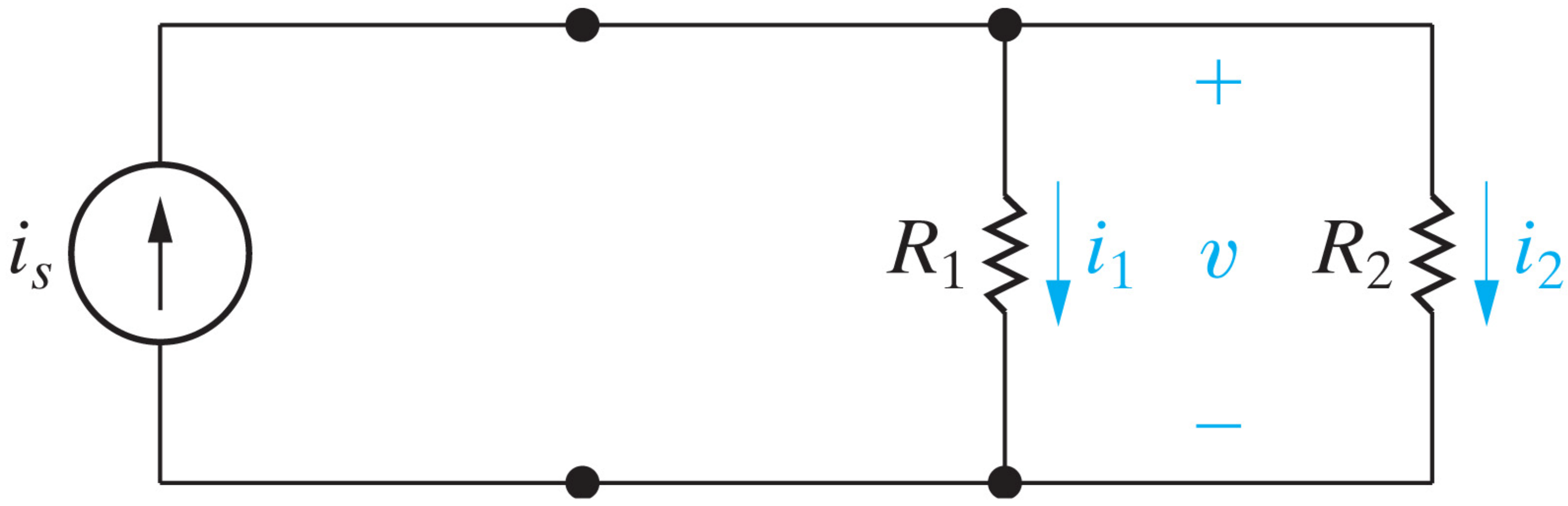
ideal voltage:

$$\bar{v}_o = \frac{100}{25+100}100$$
$$= 80\ \text{V}$$

$$v_{o,\max} = \frac{110}{22.5+110}100 = 83.02\ \text{V} = (1+3.78\%)\bar{v}_0;$$

$$v_{o,\min} = \frac{90}{27.5+90}100 = 76.60\ \text{V} = (1-4.25\%)\bar{v}_o.$$

Current-divider circuit



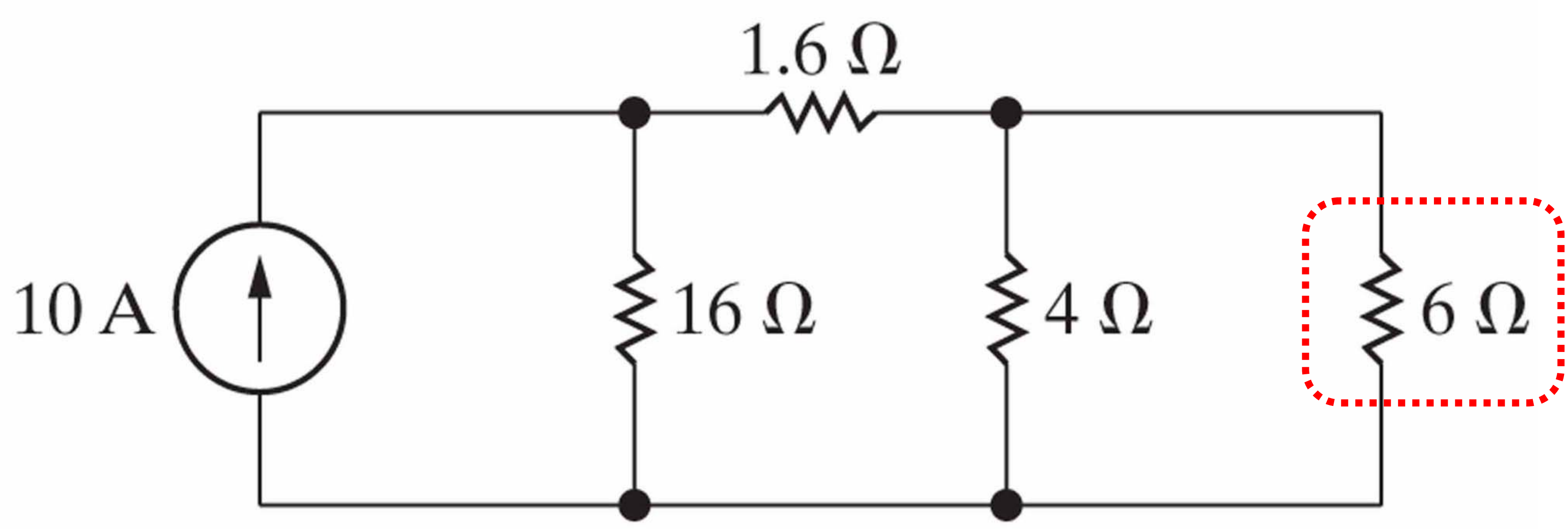
By Ohm's law & resistors in parallel,

$v = i_1 R_1 = i_2 R_2 = i_s R_{eq} = i_s[R_1 R_2/(R_1+R_2)]$,

$$\Rightarrow i_1 = \frac{R_2}{R_1 + R_2} i_s, \quad i_2 = \frac{R_1}{R_1 + R_2} i_s.$$

Example 3.3 (1)

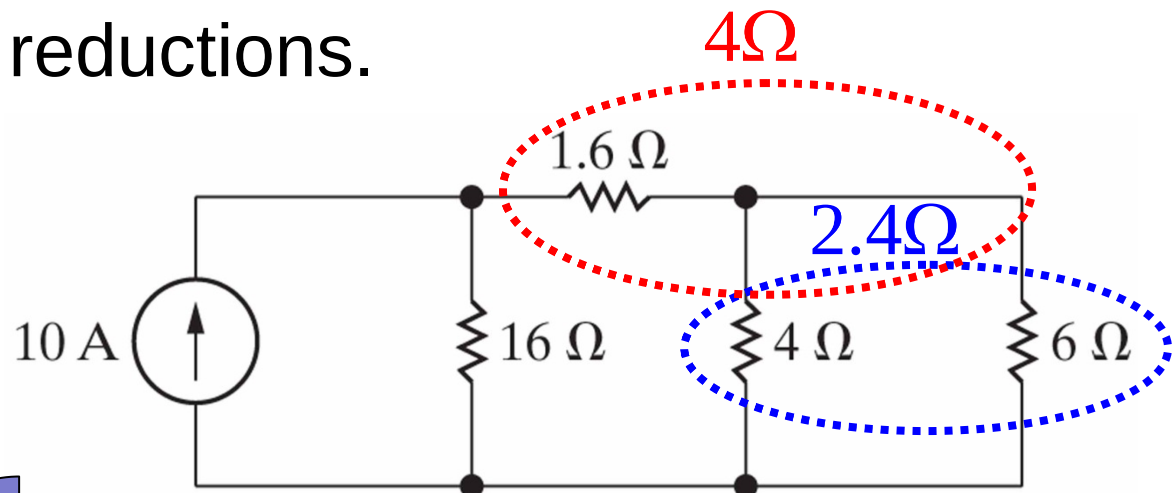
- Q: Find the power dissipated at the 6Ω-resistor.

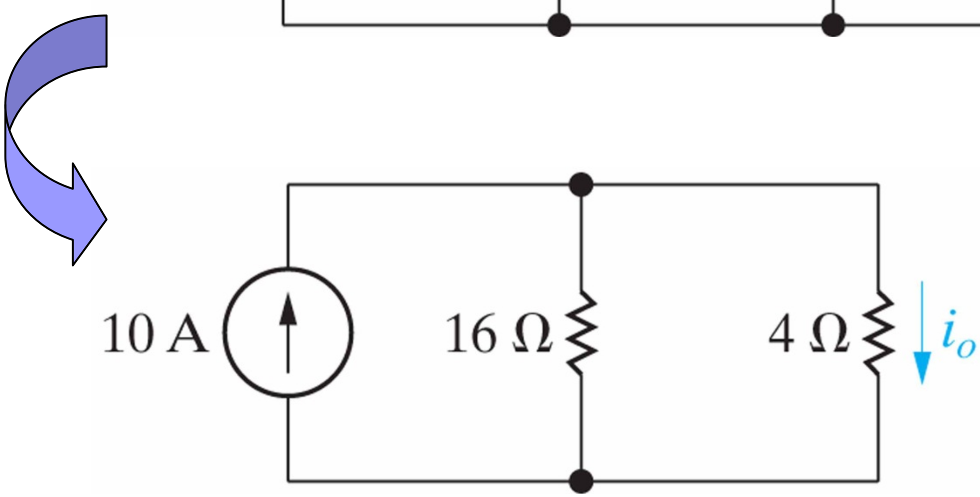


- Strategy: Find the current $i_{6\Omega}$, then use $p = i^2R$ to calculate the power.

Example 3.3 (2)

Step 1: Simplifying the circuit with series-parallel reductions.





$i_o = [16/(16+4)]10 = 8$ A,

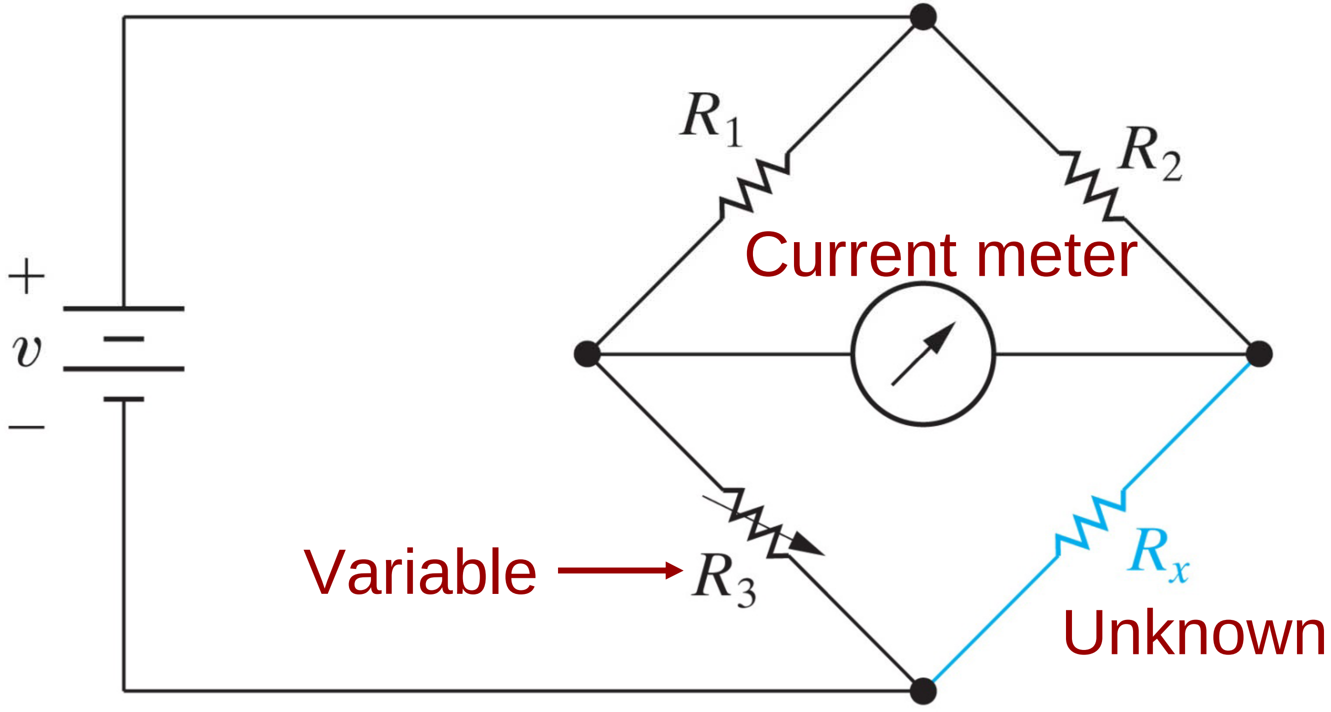
$i_{6\Omega} = [4/(6+4)]8 = 3.2$ A,

$\Rightarrow\ p = (3.2)^2\,6 = 61.44$ W.

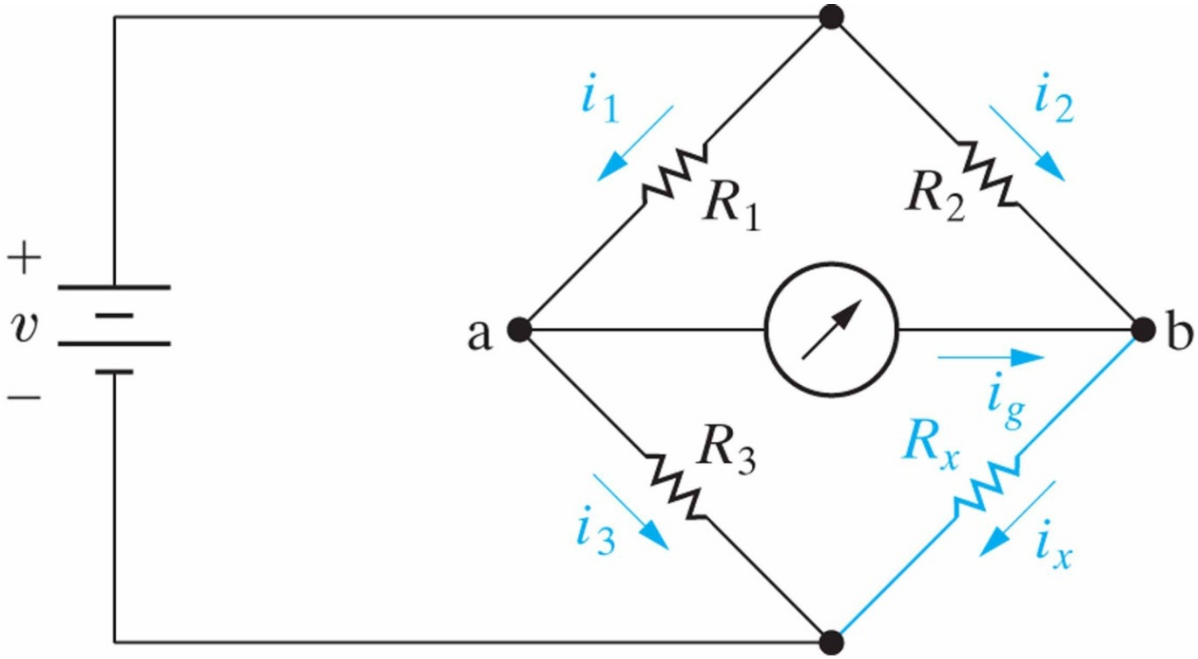
Section 3.6
The Wheatstone Bridge

The Wheatstone bridge

- Goal: Measuring a resistor's value.
- Apparatus: Fixed-value resistors 2×, variable resistor 1×, current meter 1×.



The working principle



Tune R_3 until

$i_{ab} = 0, \Rightarrow v_{ab} = 0,$

terminals ab

become both

open and short!

Open: $i_1 = i_3,\ i_2 = i_x \ \ldots(1)$

Short: $i_1 R_1 = i_2 R_2,\ i_3 R_3 = i_x R_x \ \ldots(2)$

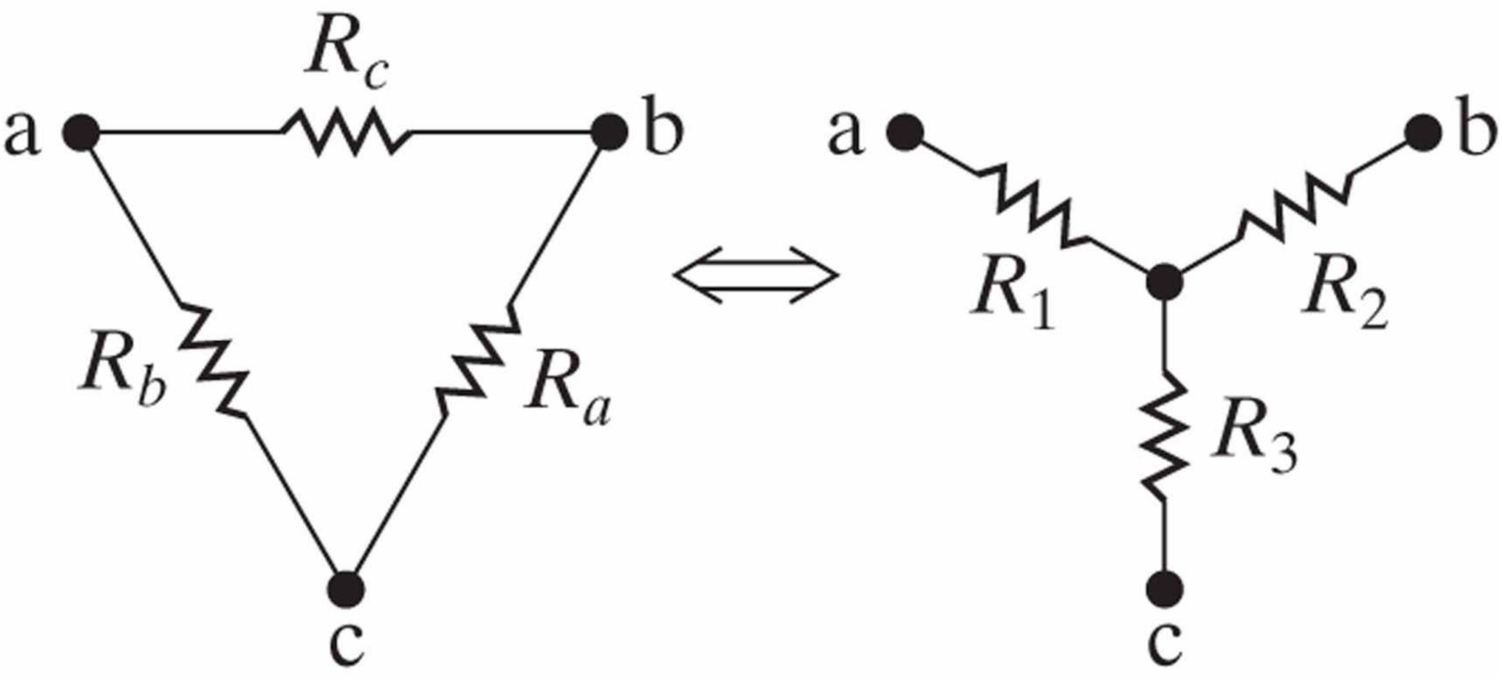
(1) into (2): $i_1 R_3 = i_2 R_x \ \ldots(3)$

$$\frac{(2)}{(3)} = \frac{R_1}{R_3} = \frac{R_2}{R_x},$$

$$\Rightarrow R_x = \frac{R_2}{R_1} R_3$$

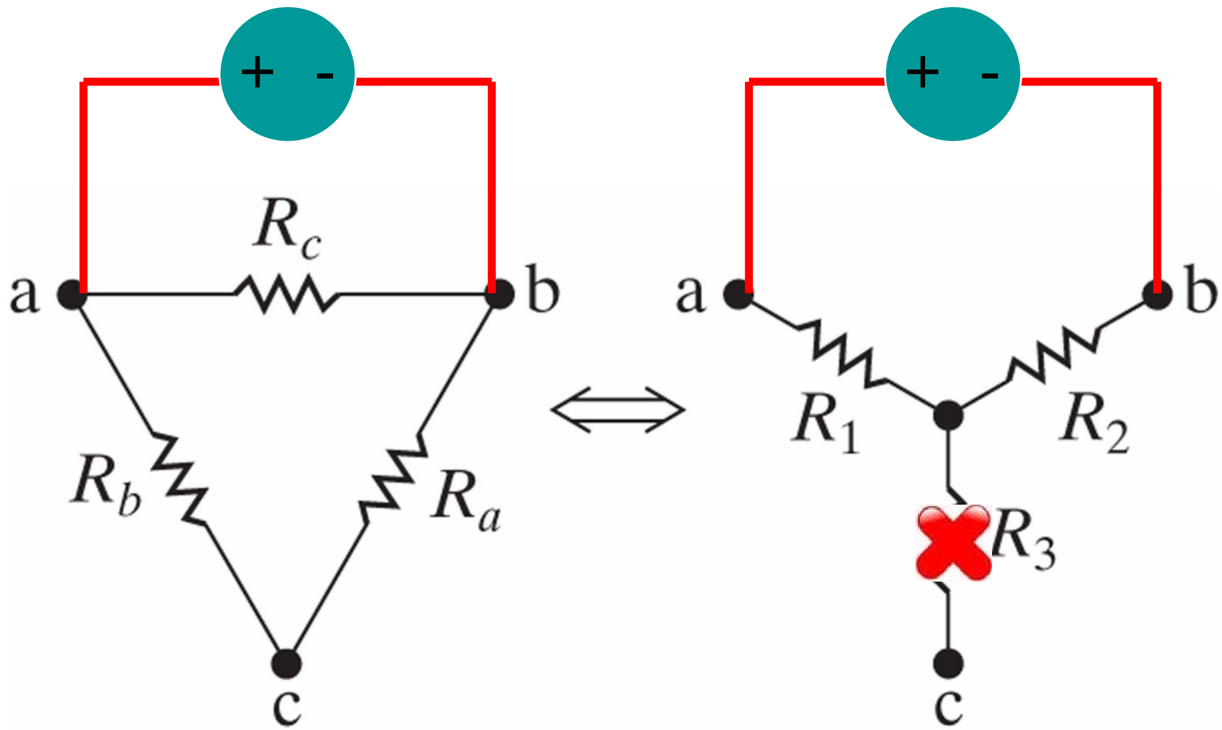
Section 3.7
Δ-to-Y (Π-to-T) Equivalent Circuits

Definition of Δ-to-Y (Π-to-T) transformation



- Two circuits of Δ and Y configurations are equivalent if the terminal behavior of the two configurations are the same. $\Rightarrow R_{ab,\Delta} = R_{ab,\mathrm{Y}}$; $R_{bc,\Delta} = R_{bc,\mathrm{Y}}$; $R_{ca,\Delta} = R_{ca,\mathrm{Y}}$

Terminal resistances



$$R_{ab} = R_c // (R_a + R_b) = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c} = R_1 + R_2 \cdots (1)$$

$$R_{bc} = R_a // (R_b + R_b) = R_2 + R_3 \cdots (2)$$

$$R_{ca} = R_b // (R_c + R_a) = R_3 + R_1 \cdots (3)$$

Transformation formulas

- Solving simultaneous equations (1-3),

$$\begin{cases} R_1 = \dfrac{R_b R_c}{R_a + R_b + R_c}, \\ R_2 = \dfrac{R_c R_a}{R_a + R_b + R_c}, \\ R_3 = \dfrac{R_a R_b}{R_a + R_b + R_c}. \end{cases}$$

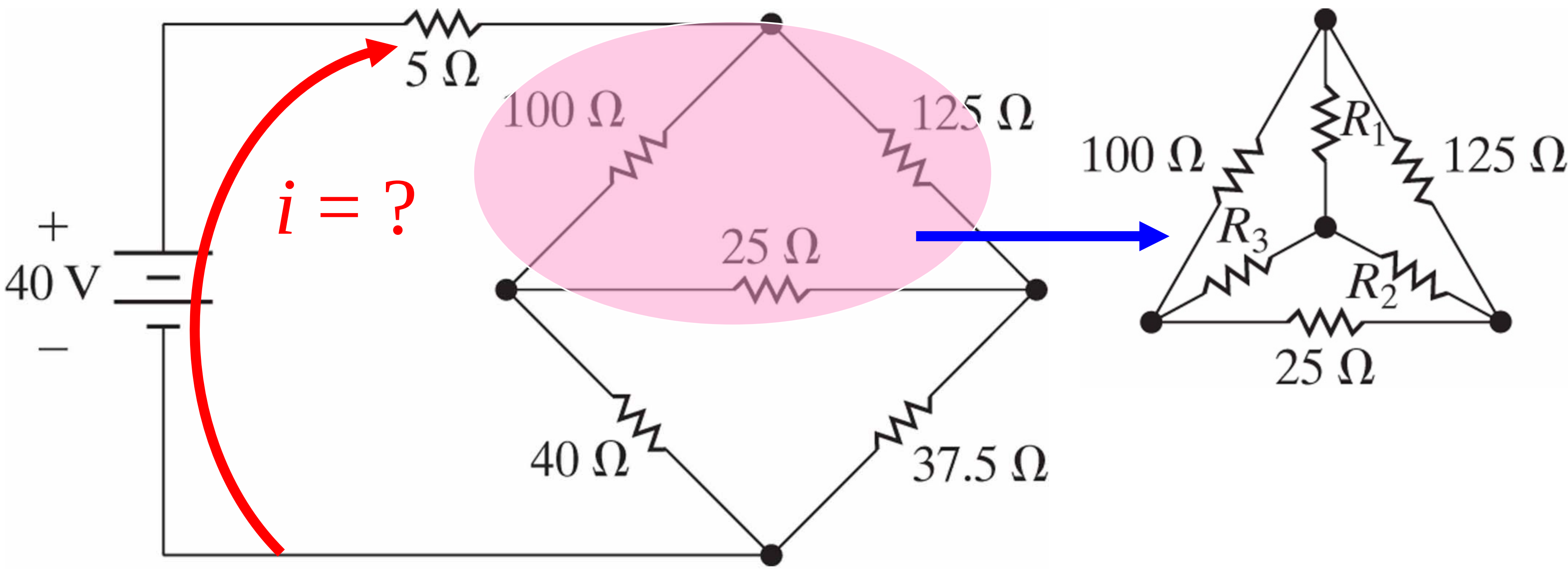
(Δ-to-Y)

$$\begin{cases} R_a = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}, \\ R_b = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}, \\ R_c = \dfrac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}. \end{cases}$$

(Y-to-Δ)

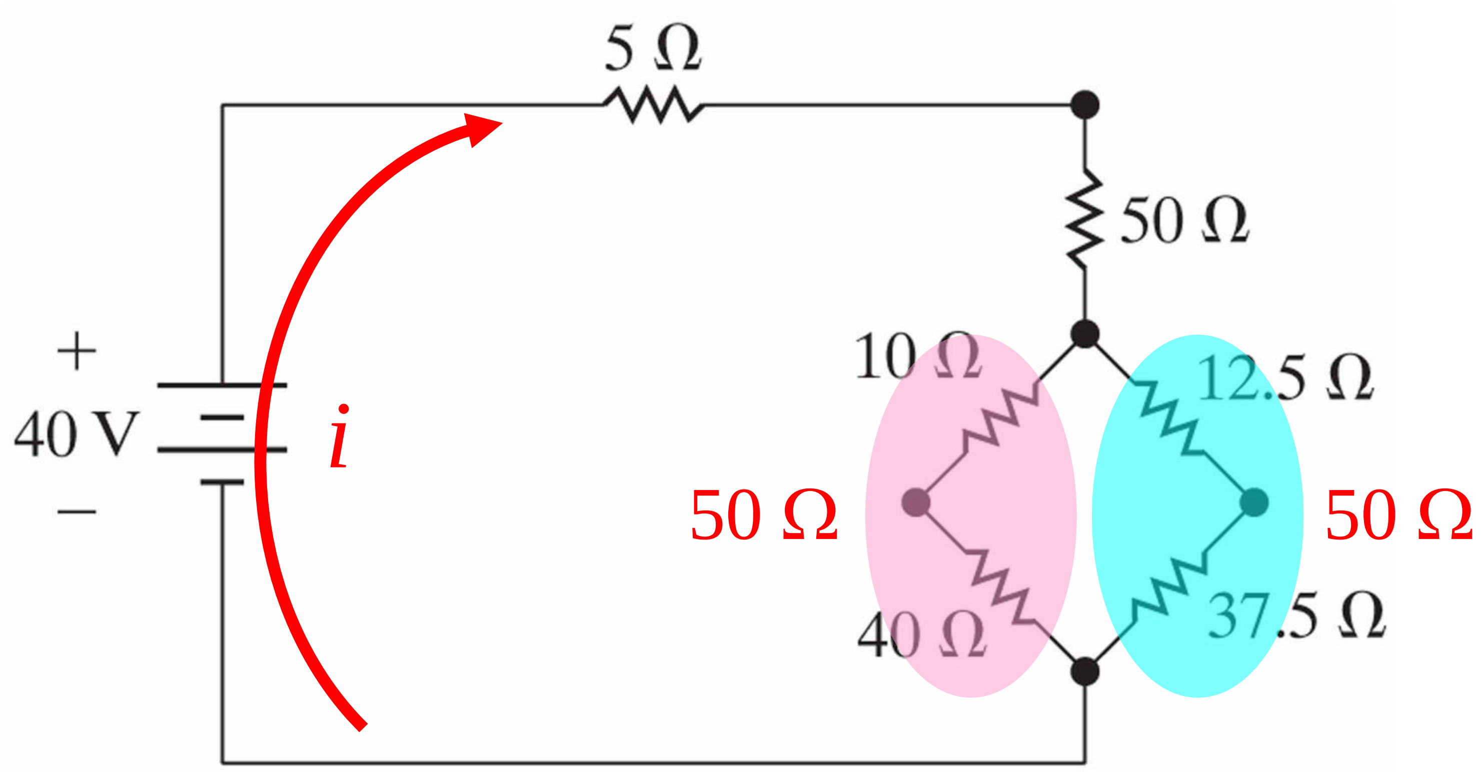
Example 3.7 (1)

- Q: Find the source current i.



$$R_1 = \frac{100 \times 125}{25 + 100 + 125} = 50\ \Omega,\ R_2 = 12.5\ \Omega,\ R_3 = 10\ \Omega.$$

Example 3.7 (2)



$$R_{eq} = (5\ \Omega) + (50\ \Omega) + (50 // 50\ \Omega) = 80\,\Omega,$$

$$i = (40\text{ V})/(80\,\Omega) = 0.5\text{ A}.$$