

Chapter 1

Signals & Signal Processing

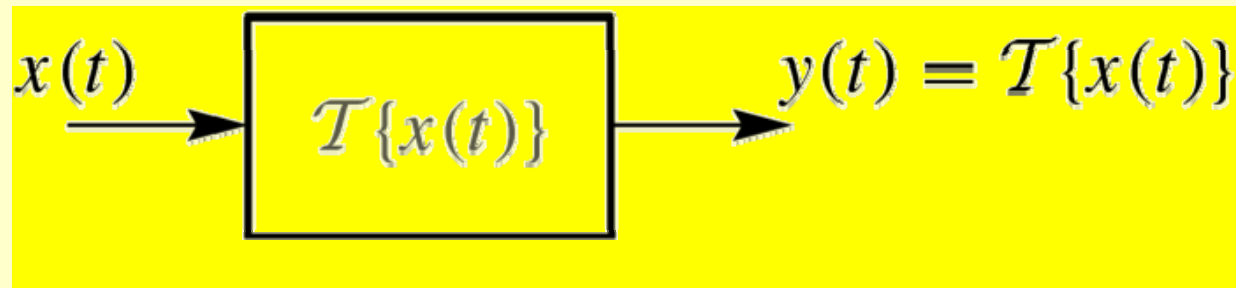
清大電機系林嘉文

cwlin@ee.nthu.edu.tw

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Signal & Signal Processing

- Signal: quantity that carries information
- Signal Processing is to study how to represent, convert, interpret, and process a signal and the information contained in the signal
- DSP: signal processing in the digital domain



Signals & Systems

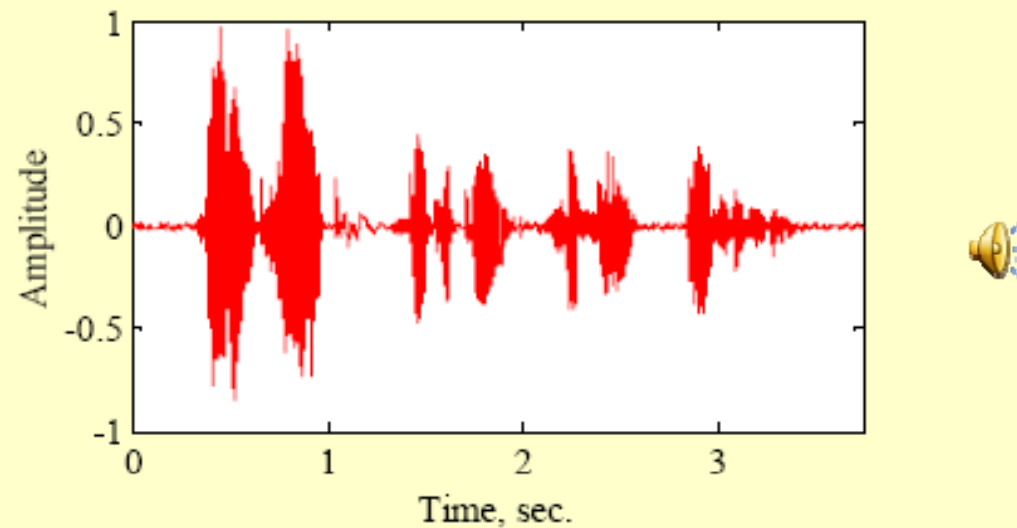
- Signals
 - “Something” that carries information
 - Speech, audio, image, video, biomedical signals, radar signals, seismic signals, etc.
- Systems
 - “Something” that can manipulate, change, record, or transmit signals
 - Examples: CD, VCD/DVD

“Discrete-Time” Signals vs. “Digital” Signals

- Discrete-Time signal
 - A “sampled” version of a continuous signal
 - What should be the sampling frequency which is enough for perfectly reconstructing the original continuous signal?
 - Nyquist rate (Shannon sampling theorem)
- Digital Signal
 - Sampling + Quantization
 - Quantization: use a number of finite bits (e.g., 8 bits) to represent a sampled value

Examples of Typical Signals

- **Speech and music signals** - Represent air pressure as a function of time at a point in space
- Waveform of the speech signal “I like digital signal processing” :

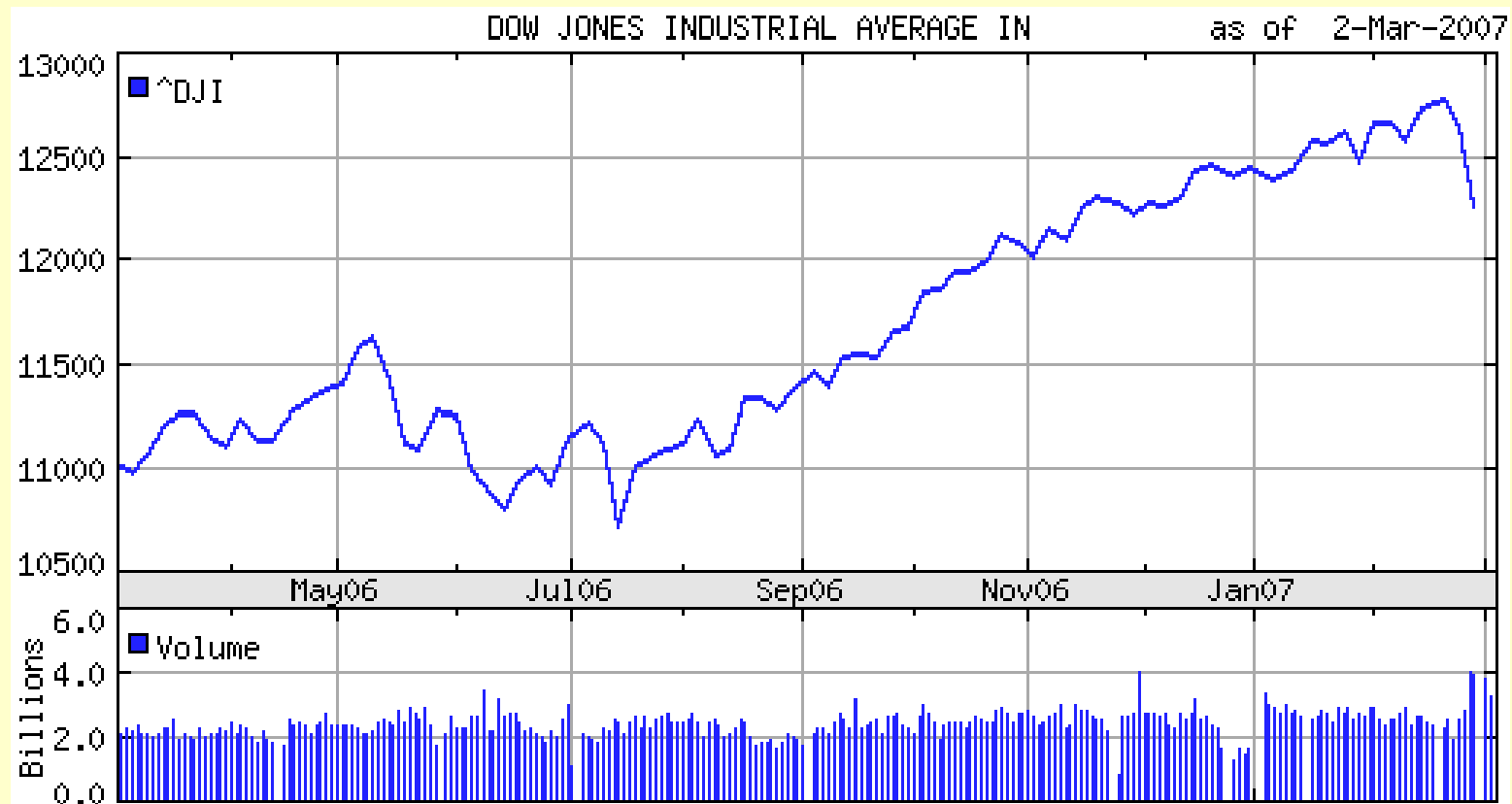


Digital Speech Signals

- Voice frequency range: 20Hz ~ 3.4 KHz
- Sampling rate: 8 KHz (8000 samples/sec)
- Quantization: 8 bits/sample
- Bit-rate: $8\text{K samples/sec} * 8 \text{ bits/sample} = 64 \text{ Kbps}$ (for uncompressed digital phone)
- In current Voice over IP (VOIP) technology, digital speech signals are usually compressed (compression ratio: 8~10)

Examples of Typical Signals

- **Dow Jones Industrial Average**



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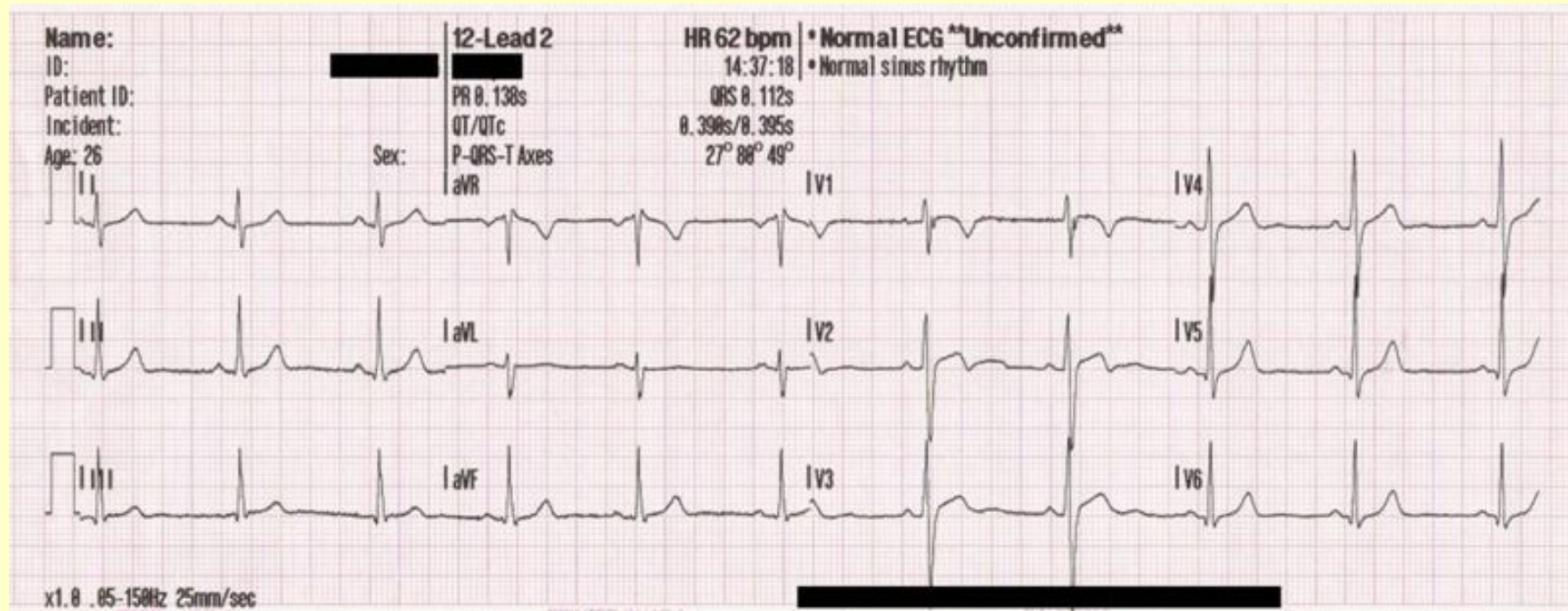
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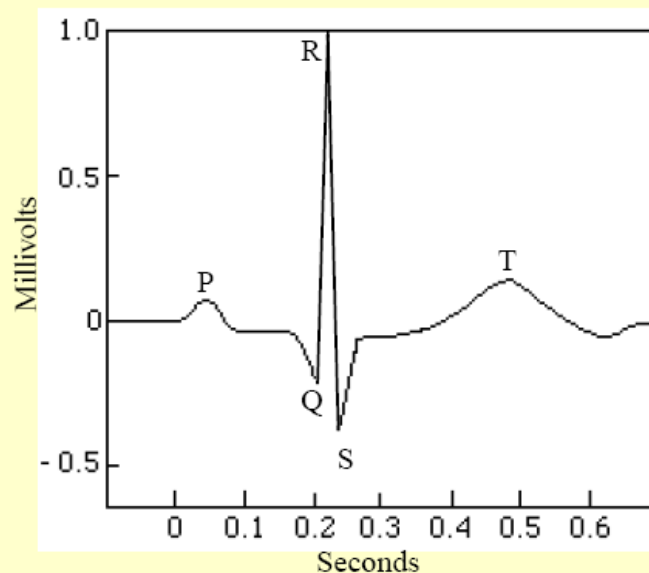
Examples of Typical Signals

- **Electrocardiography (ECG) Signal** - Represents the electrical activity of heart



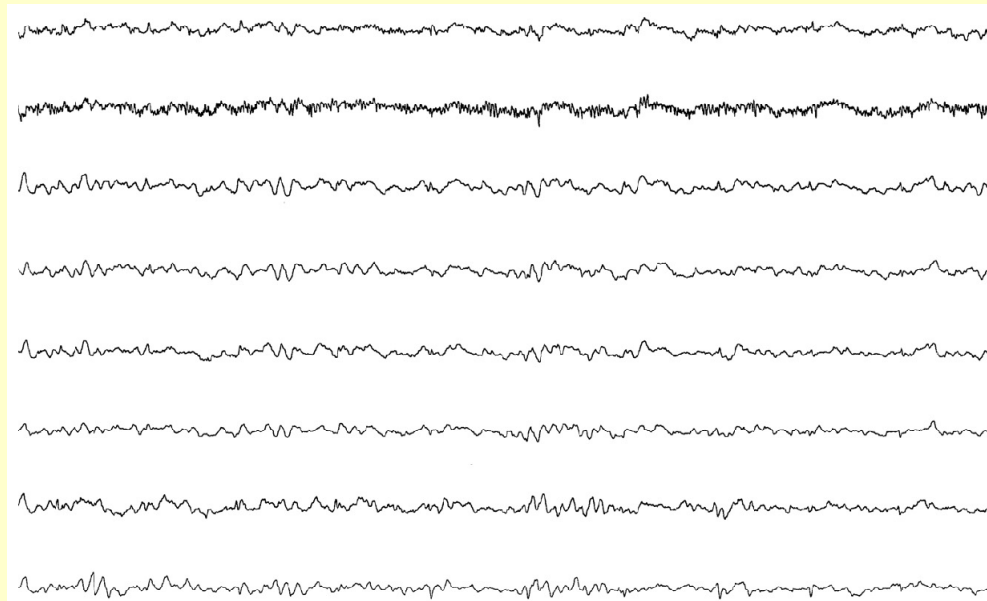
ECG Signal

- The ECG trace is a periodic waveform
- One period of the waveform shown below represents one cycle of the blood transfer process from the heart to the arteries



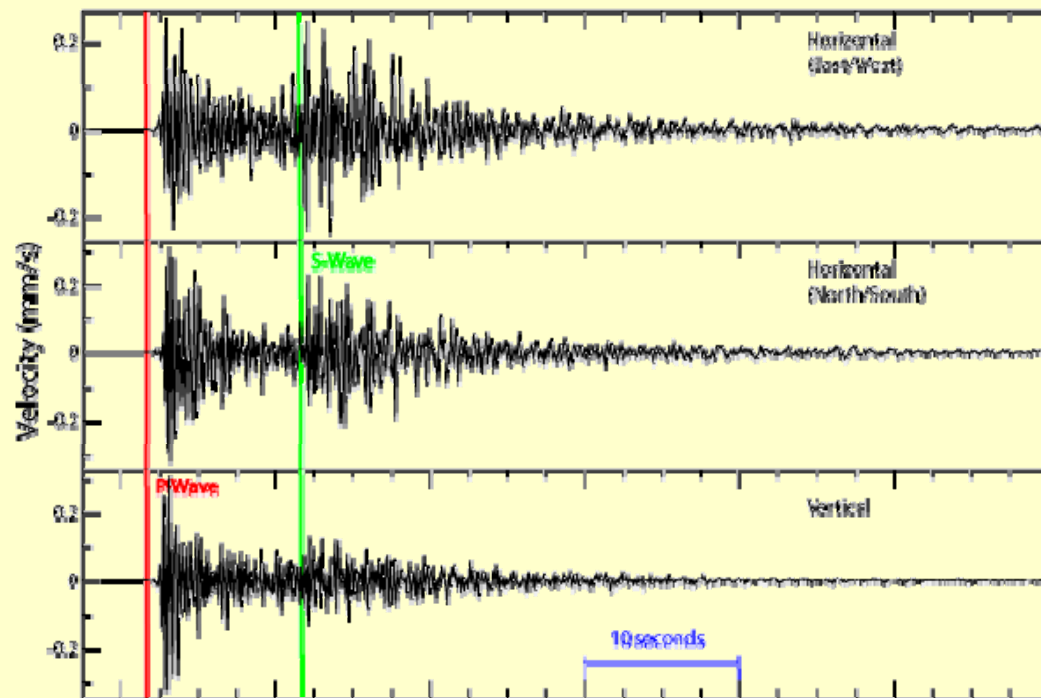
Examples of Typical Signals

- **Electroencephalogram (EEG) Signals -**
Represent the electrical activity caused by the random firings of billions of neurons in the brain



Examples of Typical Signals

- **Seismic Signals - Caused by the movement** of rocks resulting from an earthquake, a volcanic eruption, or an underground explosion



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Examples of Typical Signals

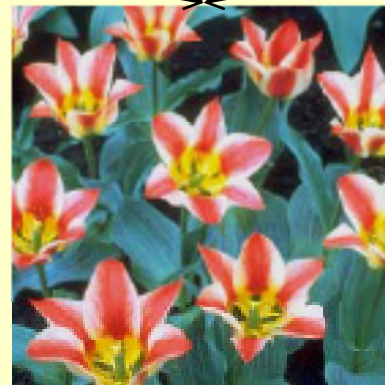
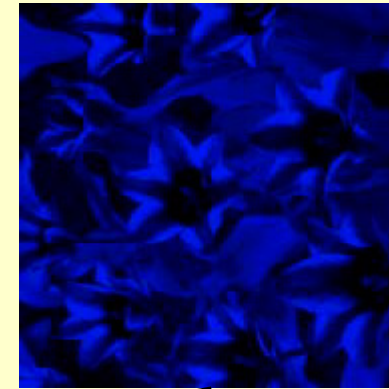
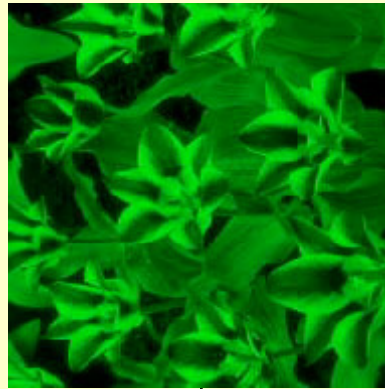
- Black-and-white picture - Represents light intensity as a function of two spatial coordinates



$I(x,y)$

Examples of Typical Signals

- **Color Image** – Consists of Red, Green, and Blue (RGB) components



Examples of Typical Signals

- Surface Search Radar Image



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Digital Image

- An one mega-pixel image (1024x1024)
- Quantization: 24 bits/pixel for the RGB full-color space, and 12 bits/pixel for a reduced color space (YCbCr)
- Bit-rate: $1024 \times 1024 \text{ samples/sec} * 12 \text{ bits/pixel} = 12 \text{ Mbits} = 1.5 \text{ Mbytes}$ (for uncompressed digital phone)
- How many uncompressed images can be stored in a 2G SD flash-memory card?
- What is the compression ratio of JPEG used in your digital camera?

Digital Image (Cont.)

- In your image processing course, you were taught how to do
 - Edge detection (high-pass filtering)
 - Image blurring or noise reduction (low-pass filtering)
 - Object segmentation (spatial coherence classification)
 - Image compression (retaining most significant info)
- The above are all about mathematical manipulations
 - Could you give mathematical formulations for the above manipulations?
 - Could you characterize the frequency behaviors of the above operations?
 - Could you design an image processing tool to meet a given spec?

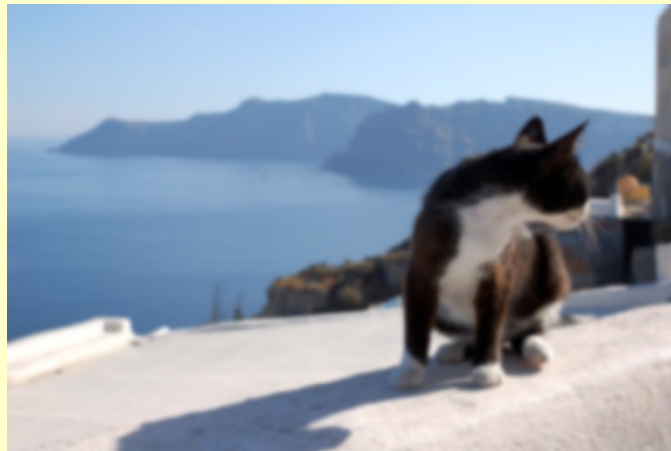
Example of Digital Image Processing



Original Image



Edge Detection



Blurring

Examples of Typical Signals

- Video signals - Consists of a sequence of images, called frames, and is a function of 3 variables: 2 spatial coordinates and time



Frame 1



Frame 3



Frame 5



Video



Classifications of Signals (1/4)

- **Types of signal** depend on the nature of the independent variables and the value of the function defining the signal
 - for example, the independent variables can be continuous or discrete
 - likewise, the signal can be a continuous or discrete function of the independent variables
 - for an 1-D signal, the independent variable is usually labeled as time
- A signal can be either a real-valued function or a complex-valued function
- A signal generated by a single source is called a **scalar signal**, where as a signal generated by multiple sources is called a **vector signal** or a **multichannel signal**

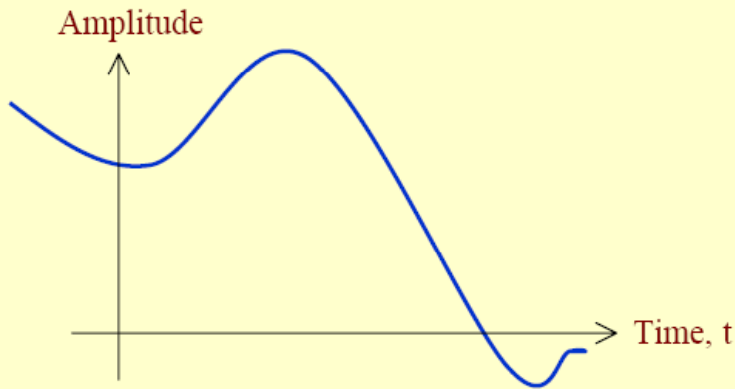
Classifications of Signals (2/4)

- A **continuous-time signal** is defined at every instant of time
- A **discrete-time signal** is defined at discrete instants of time, and hence, it is a sequence of numbers
- A continuous-time signal with a continuous amplitude is usually called an **analog signal** (e.g., speech)
- A discrete-time signal with discrete-valued amplitudes represented by a finite number of digits is referred to as a **digital signal**
- A discrete-time signal with continuous-valued amplitudes is called a **sampled-data signal**
- A continuous-time signal with discrete-value amplitudes is usually called a **quantized boxcar signal**

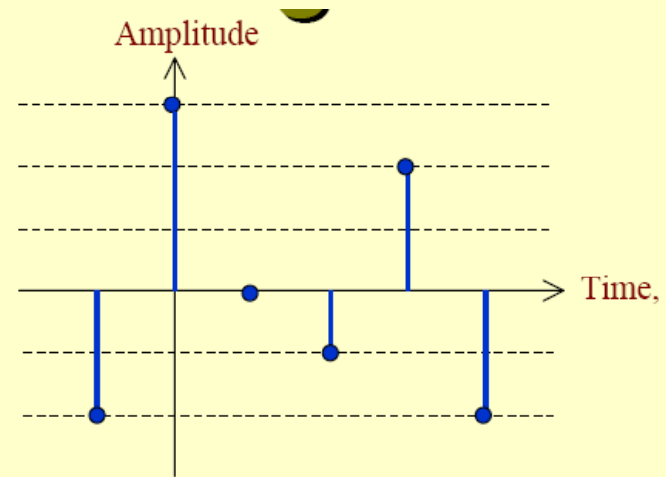
Classifications of Signals (3/4)

- A signal that can be uniquely determined by a well-defined process, such as a mathematical expression or rule, or table look-up, is called a **deterministic signal**
- A signal that is generated in a random fashion and cannot be predicted ahead of time is called a **random signal**

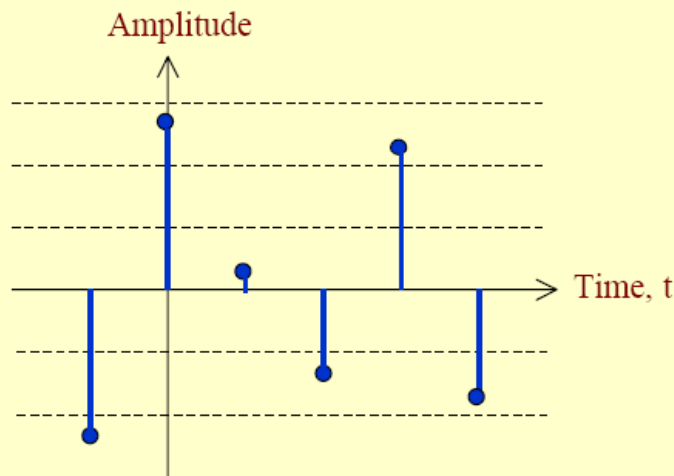
Classification of Signals (4/4)



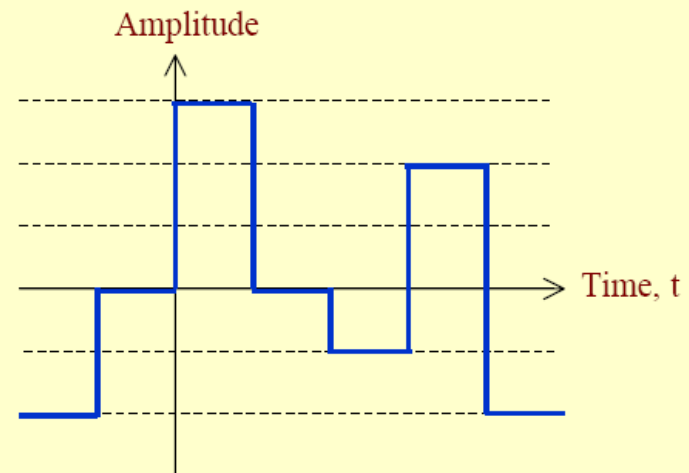
A continuous-time signal



A digital signal



A sampled - data signal



A quantized boxcar signal

Typical Signal Processing Operations

- Most signal processing operations in the case of analog signals are carried out in the time-domain
- In the case of discrete-time signals, both time-domain or frequency-domain operations are usually employed
- Continuous-time Fourier transform (CTFT) is used to transform a signal into the frequency domain

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t} dt$$

Elementary Time-Domain Operations

- Three most basic time-domain signal operations: scaling, delay, and addition

- Integration

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

- Differentiation

$$w(t) = \frac{dx(t)}{dt}$$

- More complex operations are implemented by combining two or more elementary operations

Filtering (1/3)

- Filtering is one of the most widely used complex signal processing operations
- A filter passes certain frequency components and blocks other frequency components
- Passband vs. stopband of a filter
- The filtering operation of a linear analog filter is described by the convolution integral

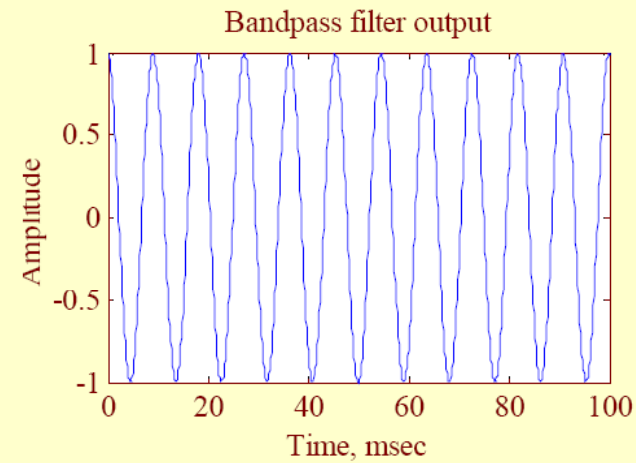
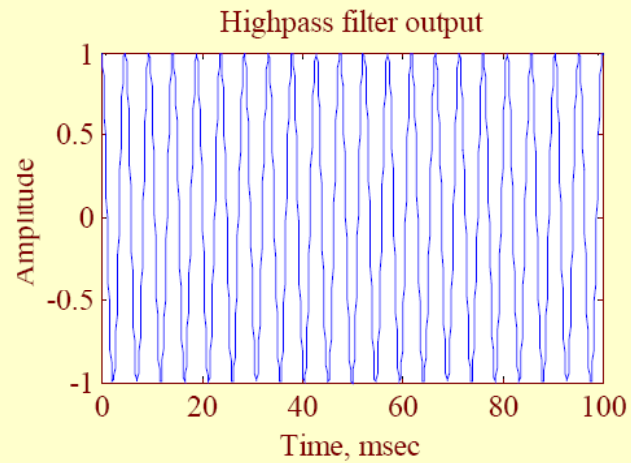
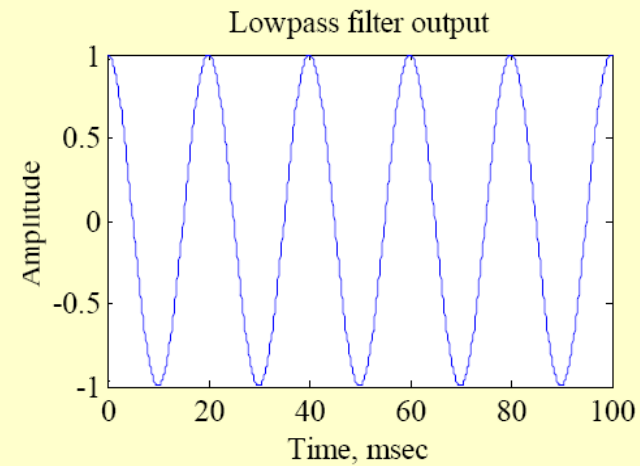
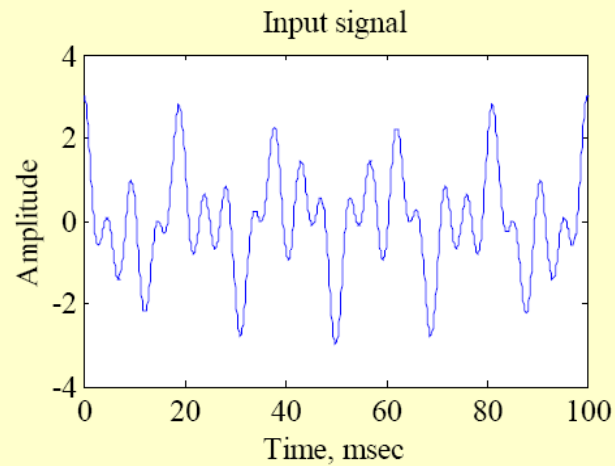
$$y(t) = \int_{-\infty}^{\infty} h(t - \tau)x(\tau)d\tau$$

where $x(t)$ is the input signal, $y(t)$ is the output of the filter, and $h(t)$ is the impulse response of the filter

Filtering (2/3)

- Frequency-selective filters can be classified into the following types according to their passbands and stopbands: **low-pass**, **high-pass**, **bandpass**, and **bandstop** filters
- **Notch filter**: blocks a single frequency component
- **Multiband filter**: has more than one passband and more than one stopband
- **Comb filter**: blocks frequencies that are integral multiples of a low frequency

Filtering (3/3)



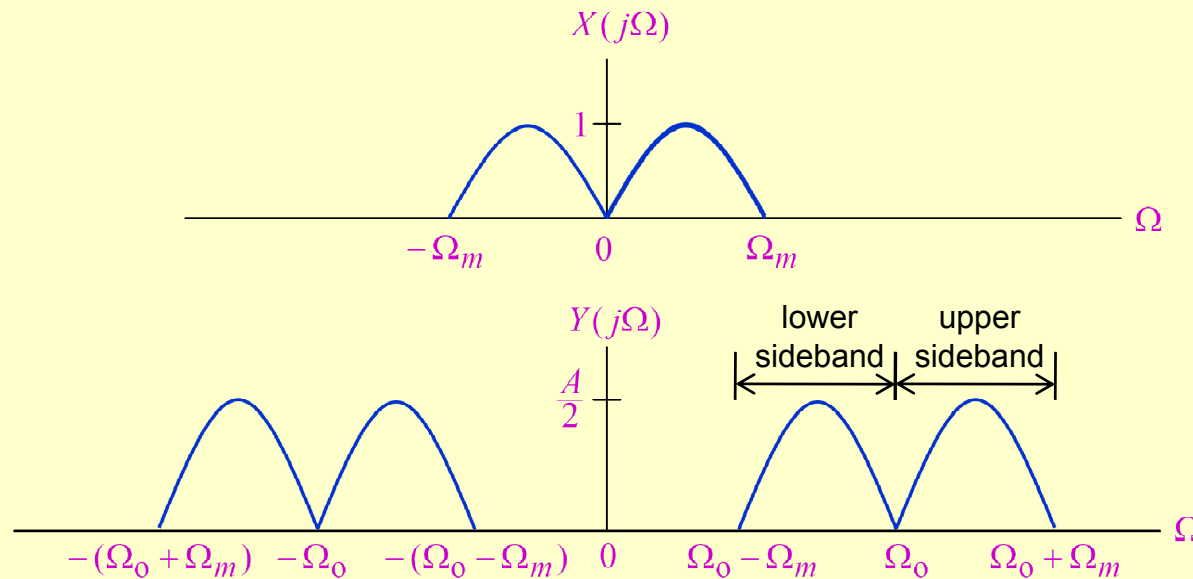
Modulation

- For efficient transmission of a low-frequency signal over a channel, it is necessary to transform the signal to a high-frequency signal by means of a modulation operation
- Four major types of modulation of analog signals:
 - Amplitude modulation
 - Frequency modulation
 - Phase modulation
 - Pulse amplitude modulation

Amplitude Modulation (1/3)

- The amplitude of a high-frequency sinusoidal signal $A\cos(\Omega_0 t)$, called the carrier signal, is varied by a low-frequency signal $x(t)$, called the modulating signal by

$$y(t) = Ax(t)\cos(\Omega_0 t)$$



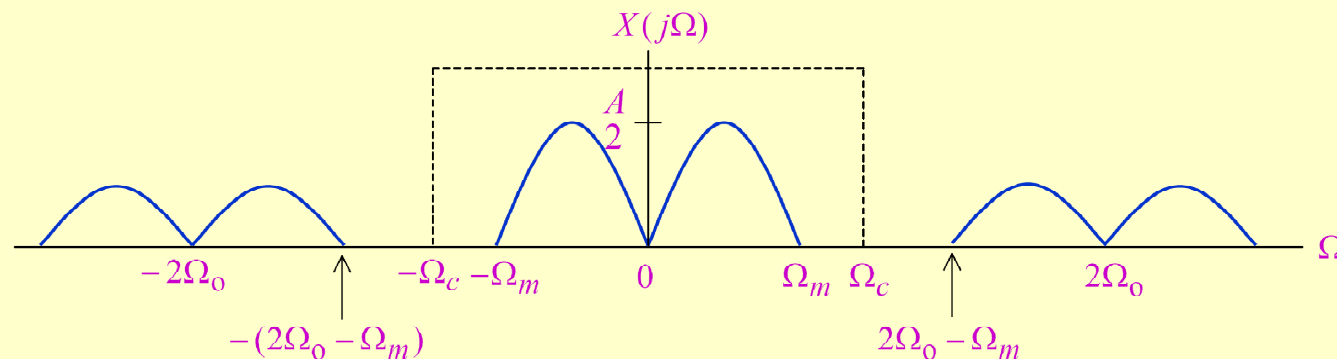
Double-SideBand Suppressed Carrier (DSB-SC) modulation

Amplitude Modulation (2/3)

- To demodulate, $y(t)$ is first multiplied with a sinusoidal signal of the same frequency as the carrier:

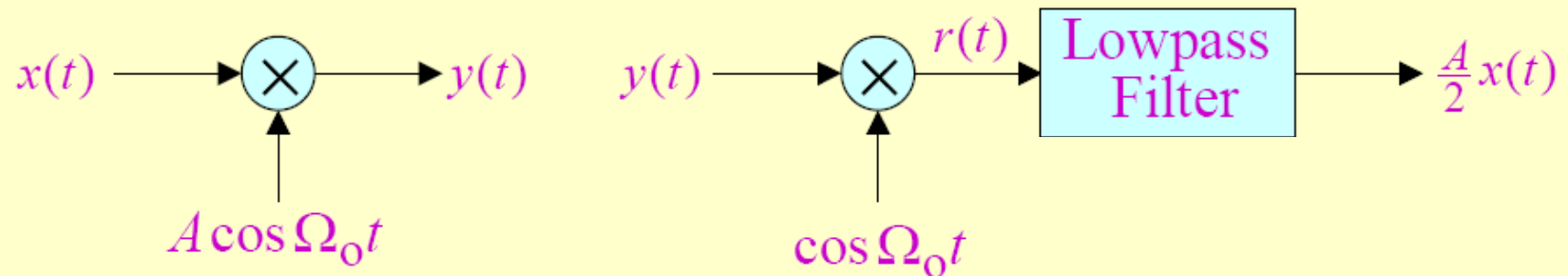
$$\begin{aligned} r(t) &= y(t) \cos \Omega_o t = Ax(t) \cos^2 \Omega_o t \\ &= \frac{A}{2} x(t) + \frac{A}{2} x(t) \cos(2\Omega_o t) \end{aligned}$$

- Thus $x(t)$ can be recovered from $r(t)$ by passing it through a low-pass filter with a cutoff frequency at Ω_c satisfying the relation $\Omega_m < \Omega_c < 2\Omega_o - \Omega_m$

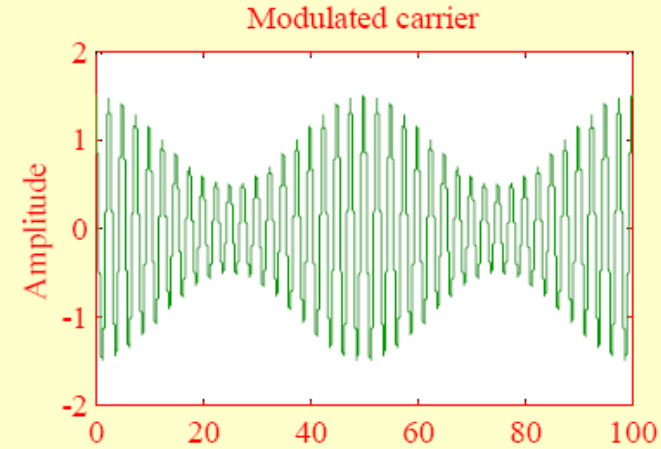
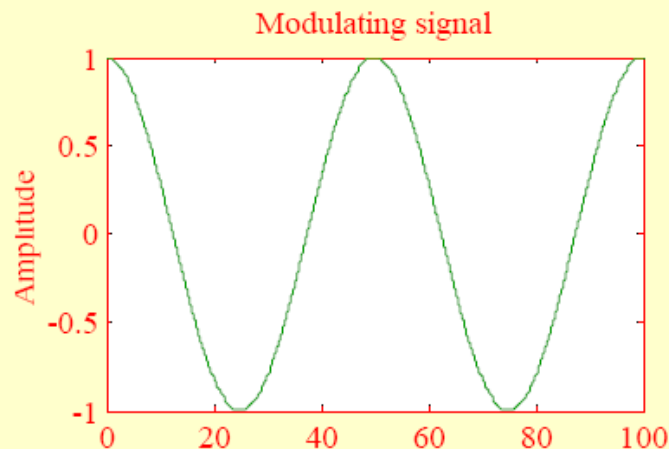


Amplitude Modulation (3/3)

- Modulation & Demodulation of AM:



- A modulating signal (20 Hz) and the amplitude-modulated carrier (400 Hz) obtained using the DSB modulation



Hilbert Transform

- The impulse response of Hilbert transform is defined as

$$h_{HT}(t) = \frac{1}{\pi t}$$

- The continuous-time Fourier transform (CTFT) $X_{HT}(j\Omega)$ of $h_{HT}(t)$ is

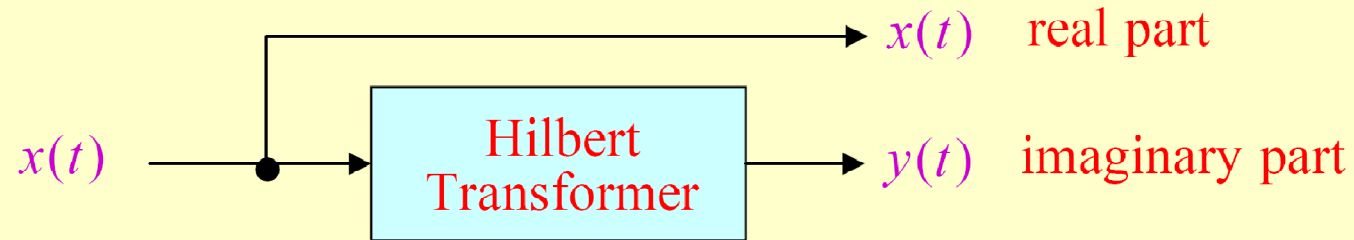
$$H_{HT}(j\Omega) = \begin{cases} -j, & \Omega > 0 \\ j, & \Omega < 0 \end{cases}$$

- The input signal $x(t)$ can be divided into two components:

$$X(j\Omega) = X_p(j\Omega) + X_n(j\Omega)$$

where $X_p(j\Omega)$ is the portion of $X(j\Omega)$ occupying the **positive frequency** range and $X_n(j\Omega)$ is the portion occupying the **negative frequency** range

Hilbert Transform (2/2)



$$g(t) = x(t) + j y(t)$$

- The CTFT of $y(t)$ becomes

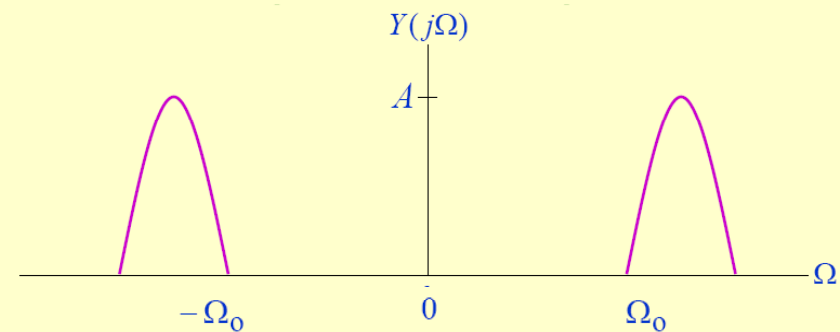
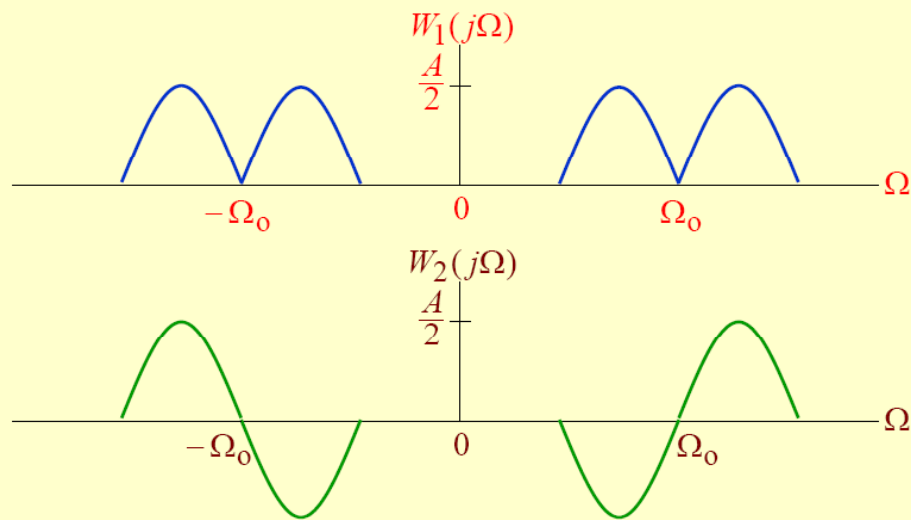
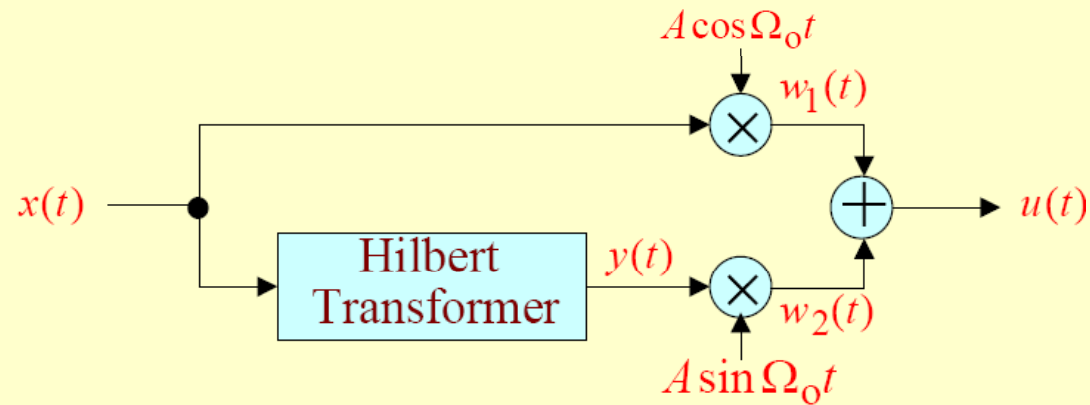
$$\begin{aligned} Y(j\Omega) &= H_{HT}(j\Omega)X(j\Omega) \\ &= -j X_p(j\Omega) + j X_n(j\Omega) \end{aligned}$$

- Consider $g(t) = x(t) + j y(t)$. The CTFT of $g(t)$ is

$$G(j\Omega) = X(j\Omega) + j Y(j\Omega) = 2X_p(j\Omega)$$

only the **positive-frequency** component is retained

Single-SideBand (SSB) Modulation



Quadrature Amplitude Modulation (1/3)

- QAM uses DSB modulation to modulate two different signals so that they both occupy the same bandwidth

$$y(t) = Ax_1(t)\cos(\Omega_0 t) + Ax_2(t)\sin(\Omega_0 t)$$

- The two carrier signals have the same carrier frequency Ω_0 but have a phase difference of 90°
- QAM takes up as much bandwidth as the SSB method, and only half of DSB

$$Y(j\Omega) = \frac{A}{2} \{X_1(j(\Omega - \Omega_0)) + X_1(j(\Omega + \Omega_0))\} \\ + \frac{A}{2j} \{X_2(j(\Omega - \Omega_0)) - X_2(j(\Omega + \Omega_0))\}$$

Quadrature Amplitude Modulation (2/3)

- To recover $x_1(t)$ and $x_2(t)$, $y(t)$ is multiplied by both the in-phase and the quadrature components of the carrier separately:

$$r_1(t) = y(t) \cos(\Omega_0 t)$$

$$r_2(t) = y(t) \sin(\Omega_0 t)$$

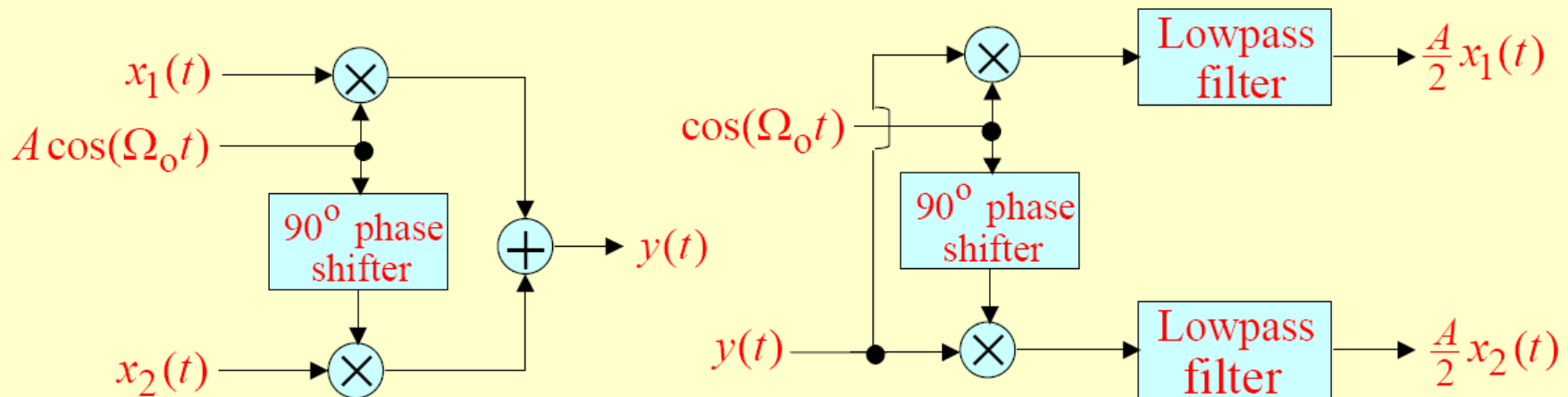
$$r_1(t) = \frac{A}{2} x_1(t) + \frac{A}{2} x_1(t) \cos(2\Omega_0 t) + \frac{A}{2} x_2(t) \sin(2\Omega_0 t)$$

$$r_2(t) = \frac{A}{2} x_2(t) + \frac{A}{2} x_1(t) \sin(2\Omega_0 t) - \frac{A}{2} x_2(t) \cos(2\Omega_0 t)$$

- Lowpass filtering of $r_1(t)$ and $r_2(t)$ by filters with a cutoff at Ω_m yields $x_1(t)$ and $x_2(t)$

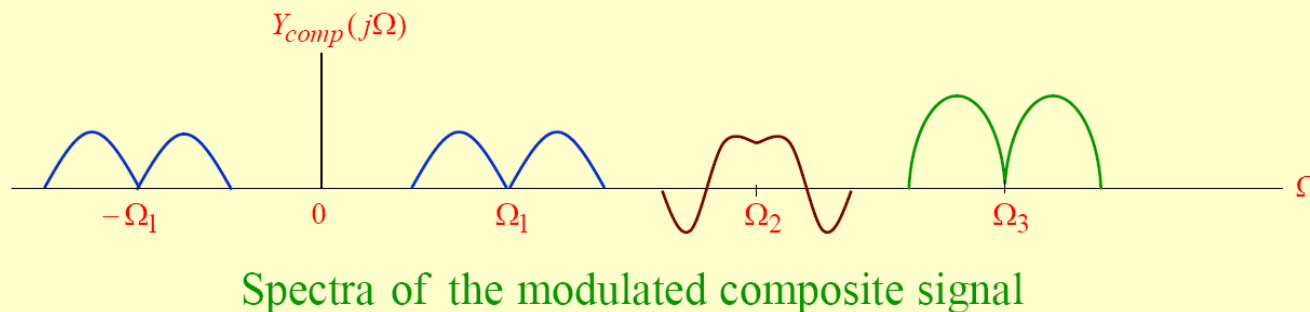
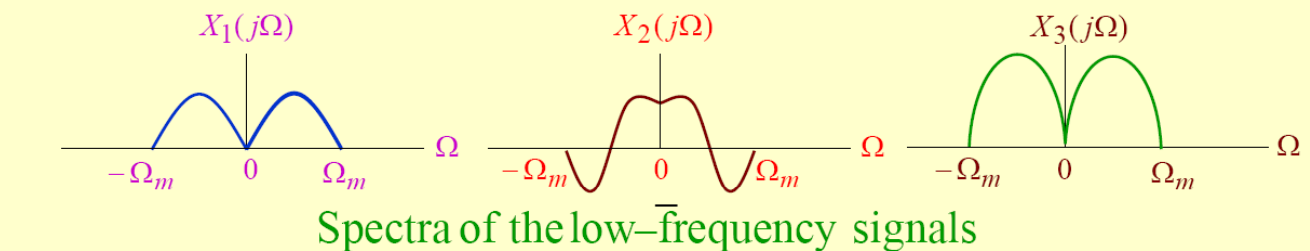
Quadrature Amplitude Modulation (3/3)

- QAM Modulation & Demodulation :



Multiplexing & Demultiplexing

- Purpose: For an efficient utilization of a wideband channel, many narrow-bandwidth low-frequency signals are combined for a composite wideband signal that is transmitted as a single signal
- Illustration of Frequency-Division Multiplexing (FDM):



Why DSP?

- Mathematical **abstractions** lead to generalization and discovery of new processing techniques
- Computer implementations are **flexible**
- Applications provide a **physical** context

Advantages of DSP (1/2)

- Absence of drift in the filter characteristics
 - Processing characteristics are fixed, e.g. by binary coefficients stored in memories
 - Independent of the external environment and of parameters such as temperature and device aging
- Improved quality level
 - Quality of processing limited only by economic considerations
 - Desired quality level achieved by increasing the number of bits in data/coefficient representation (SNR improvement: 6 dB/bit)

Advantages of DSP (2/2)

- Reproducibility
 - Component tolerances do not affect system performance with correct operation
 - No adjustments necessary during fabrication
 - No realignment needed over lifetime of equipment
- Ease adjustment of processor characteristics
 - Easy to develop and implement adaptive filters, programmable filters and complementary filters
- Time-sharing of processor (multiplexing & modularity)
- No loading effect
- Realization of certain characteristics not possible or difficult with analog implementations

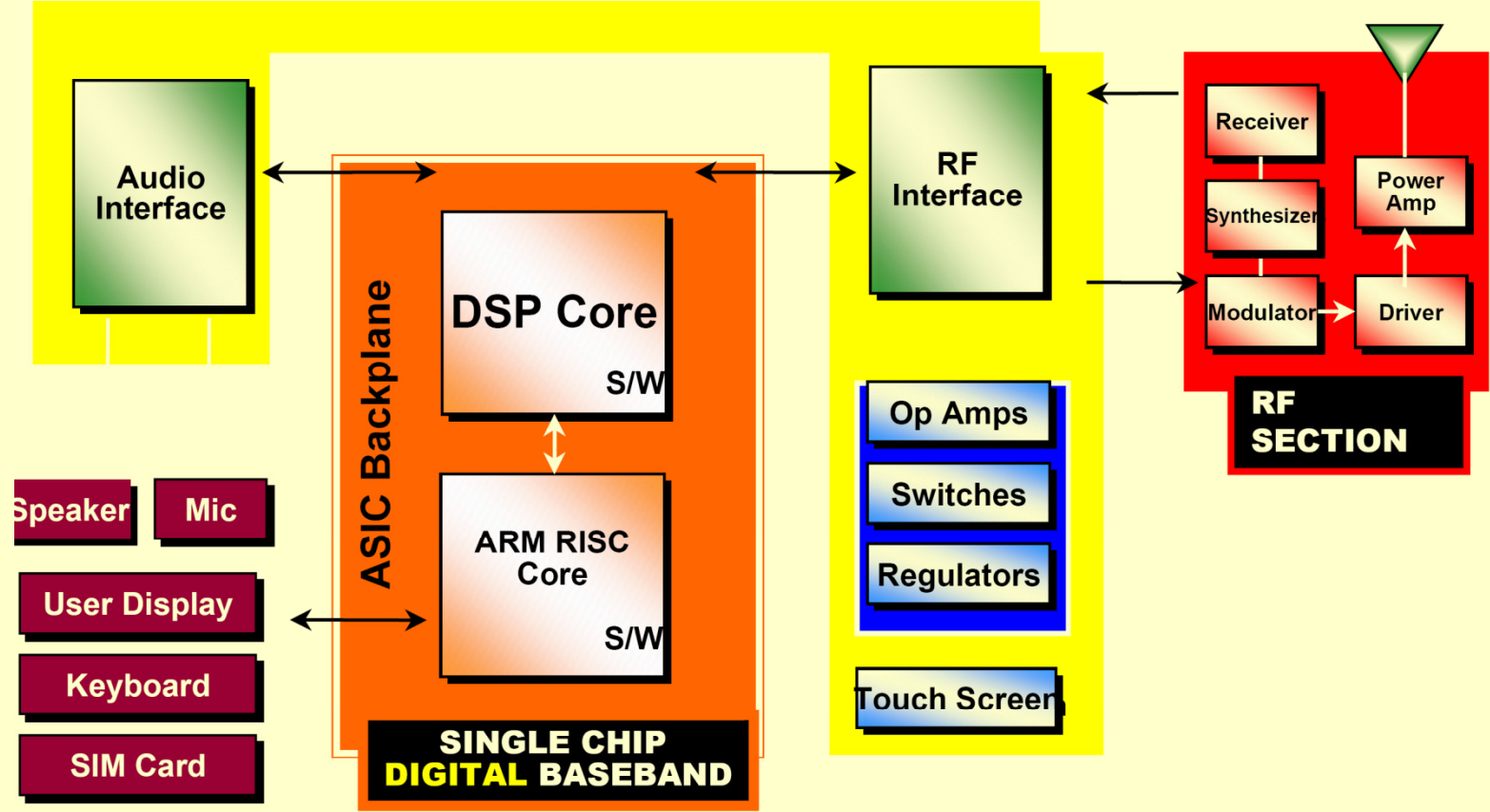
Limitations of DSP

- Limited Frequency Range of Operation
 - Frequency range technologically limited to values corresponding to maximum computing capacities (e.g., A/D converter) that can be developed and exploited
- Digital systems are active devices, thereby consuming more power and being less reliable
- Additional Complexity in the Processing of Analog Signals
 - A/D and D/A converters must be introduced adding complexity to overall system
- Inaccuracy due to finite precision arithmetic

Application Examples of DSP

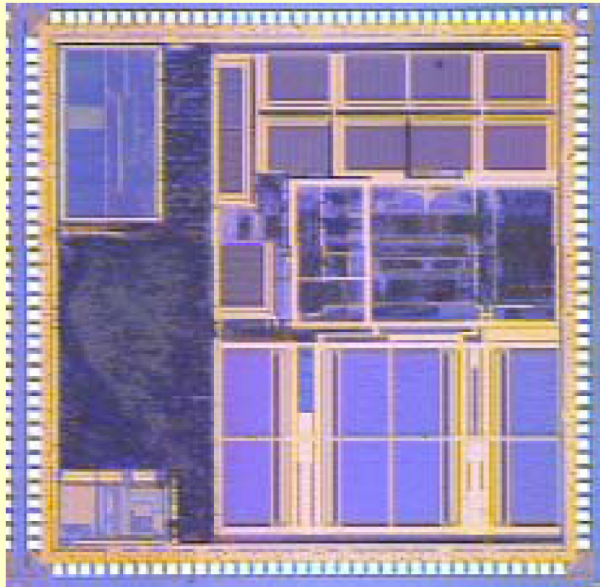
- Cellular Phone
- Discrete Multitone Transmission (ADSL)
- Digital Camera
- Digital Sound Synthesis
- Signal Coding & Compression
- Signal Enhancement

Cellular Phone Block Diagram



Courtesy : Texas Instruments

Cellular Phone Baseband SOC



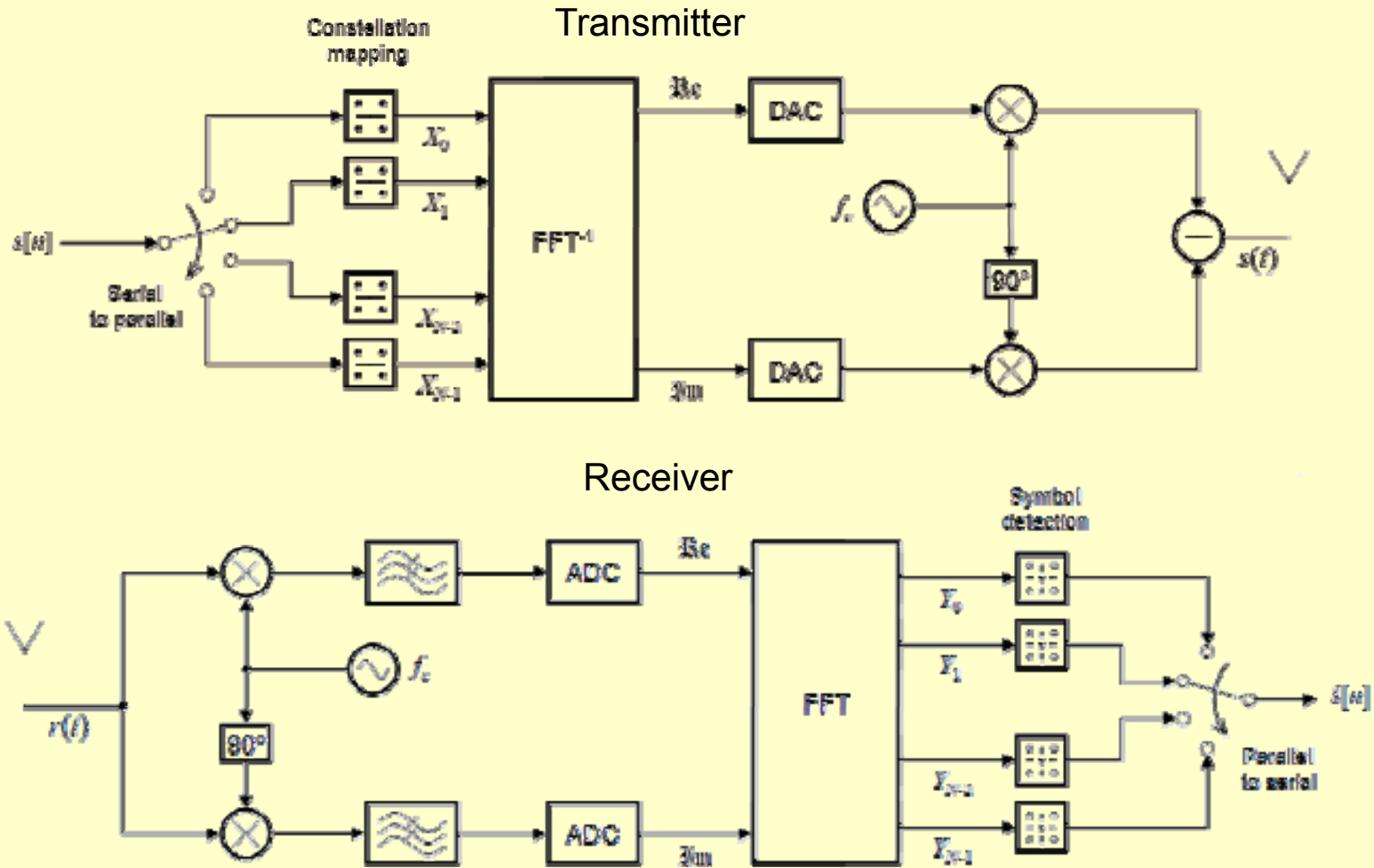
- **100-200 MHz DSP + MCU**
- **ASIC Logic**
- **Dense Memory**
- **Analog**

Courtesy : Texas Instruments

Discrete MultiTone Modulation (DMT)

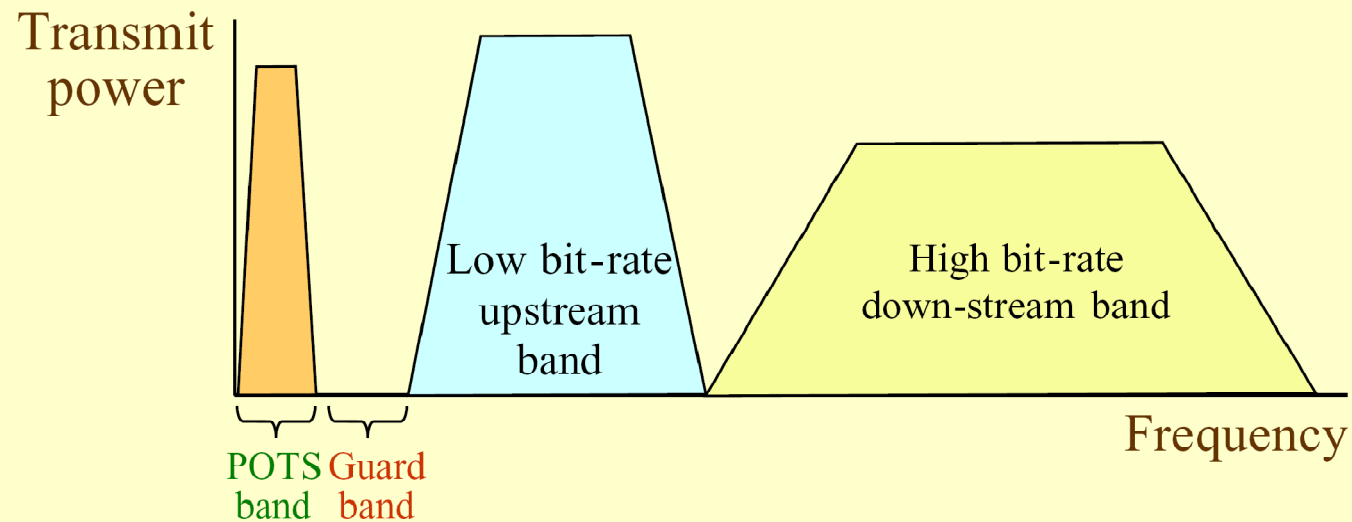
- Core technology in the implementation of the asymmetric digital subscriber line (ADSL) and very-high-rate DSL (VDSL)
- ADSL:
 - Downstream bit-rate: up to 9 Mb/s
 - Upstream bit-rate: up to 1 Mb/s
- VDSL:
 - Downstream bit-rate: 13 to 26 Mb/s
 - Upstream bit-rate: 2 to 3 Mb/s
 - Distance: less than 1 km
- Orthogonal Frequency-Division Multiplexing (OFDM) for wireless communications

DMT



ADSL Band Allocation

- Band-allocations for an ADSL system



Digital Camera

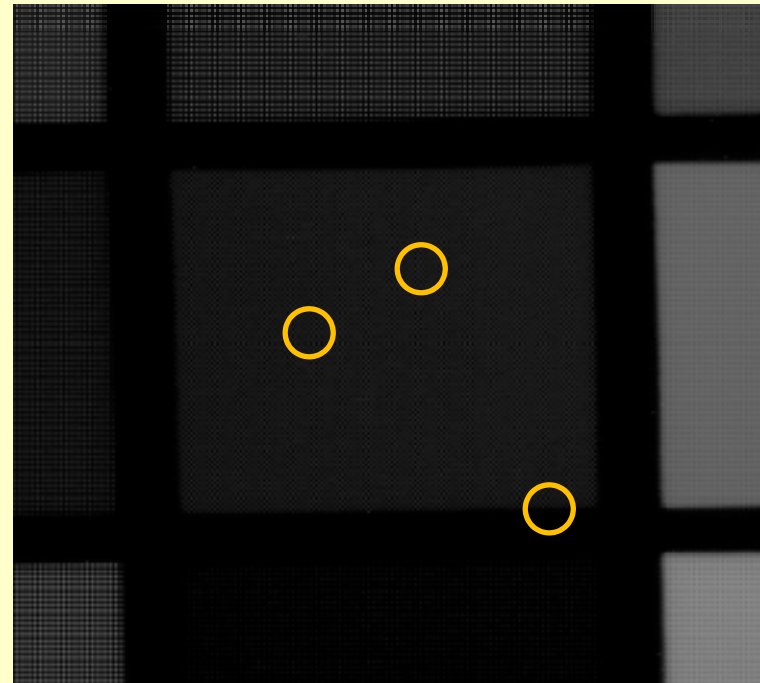
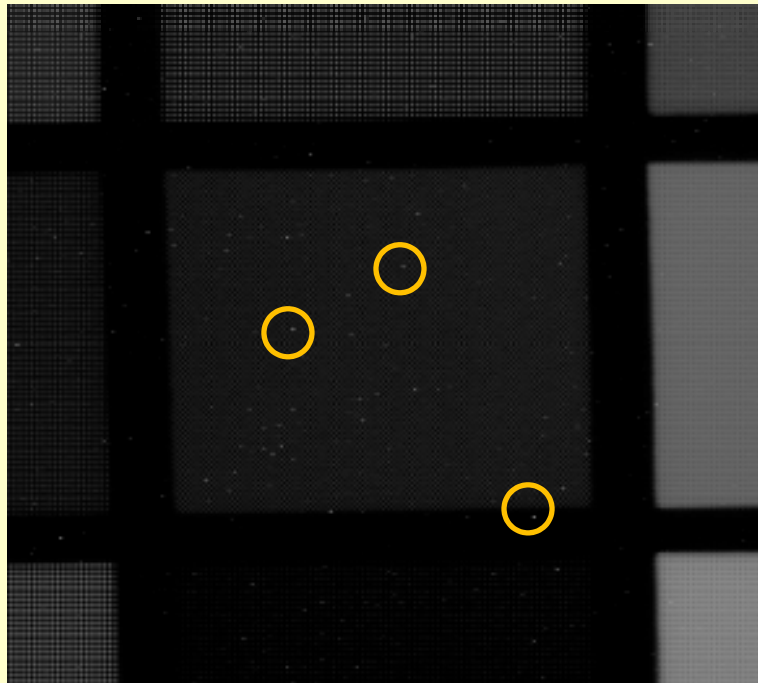
- CMOS Imaging Sensor
 - Increasingly being used in digital cameras
 - Single chip integration of sensor and other image processing algorithms needed to generate final image
 - Can be manufactured at low cost
 - Less expensive cameras use single sensor with individual pixels in the sensor covered with either a red, a green, or a blue optical filter

Digital Camera

- Image Processing Algorithms
 - Bad pixel detection and masking
 - Color interpolation
 - Color balancing
 - Contrast enhancement
 - False color detection and masking
 - Image and video compression

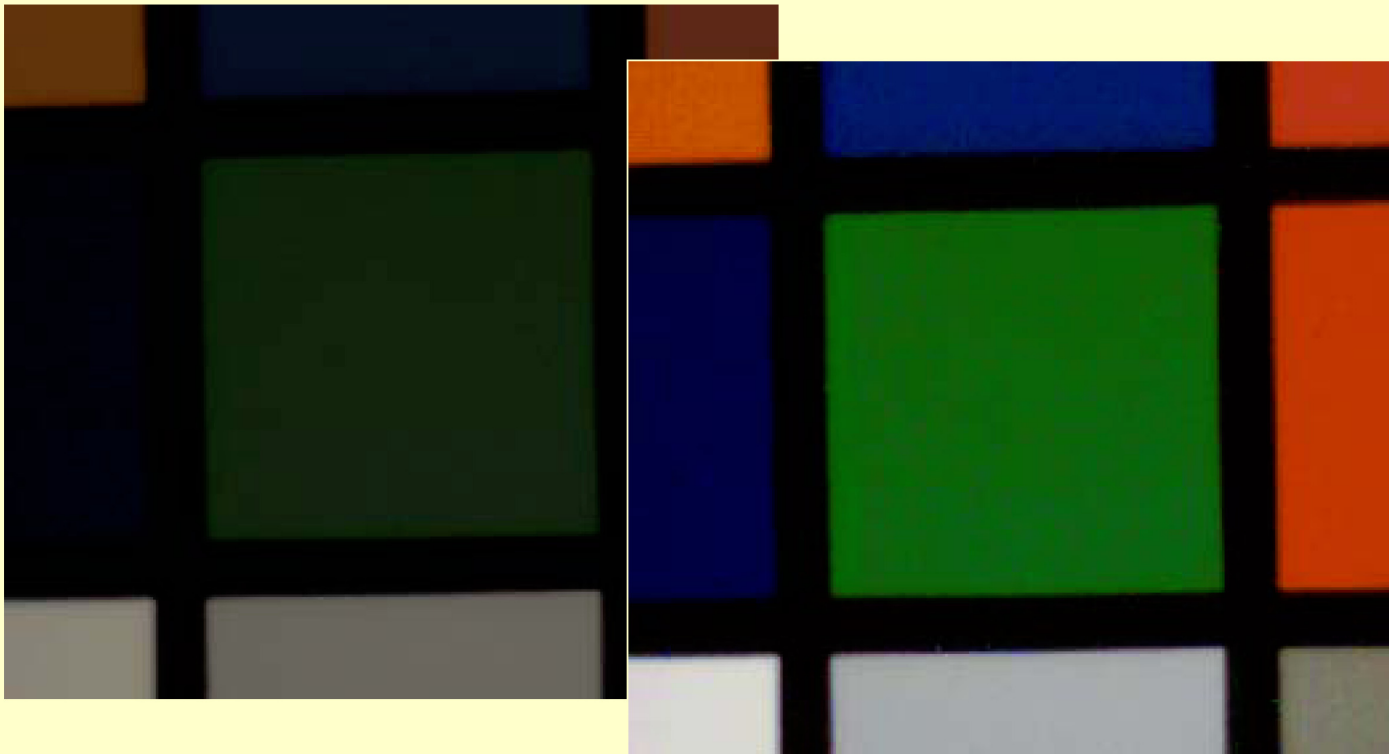
Digital Camera

- Bad pixel detection and masking



Digital Camera


- Color Interpolation and Balancing




Digital Sound Synthesis (1/5)

- Four methods for the synthesis of musical sound:
 - Wavetable synthesis
 - Spectral modeling synthesis
 - Nonlinear synthesis
 - Synthesis by physical modeling


Digital Sound Synthesis (2/5)

- Wavetable Synthesis
 - Recorded or synthesized musical events stored in internal memory and played back on demand
 - Playback tools consists of various techniques for sound variation during reproduction such as **pitch shifting, looping, enveloping and filtering**
 - Example : Giga sampler 




Digital Sound Synthesis (3/5)

- Spectral Modeling Synthesis
 - Produces sounds from frequency domain models
 - Signal represented as a superposition of basis functions with time-varying amplitudes
 - Practical implementation usually consist of a combination of **additive synthesis, subtractive synthesis, and granular synthesis**
 - Example: Kawai K500 Demo 

Digital Sound Synthesis (4/5)

- Nonlinear Synthesis
 - Frequency modulation method: Time dependent phase terms in the sinusoidal basis Functions
 - An inexpensive method frequently used in synthesizers and in sound cards for PC
 - Example: Variation modulation index complex algorithm (Pulsar) 

Digital Sound Synthesis (5/5)

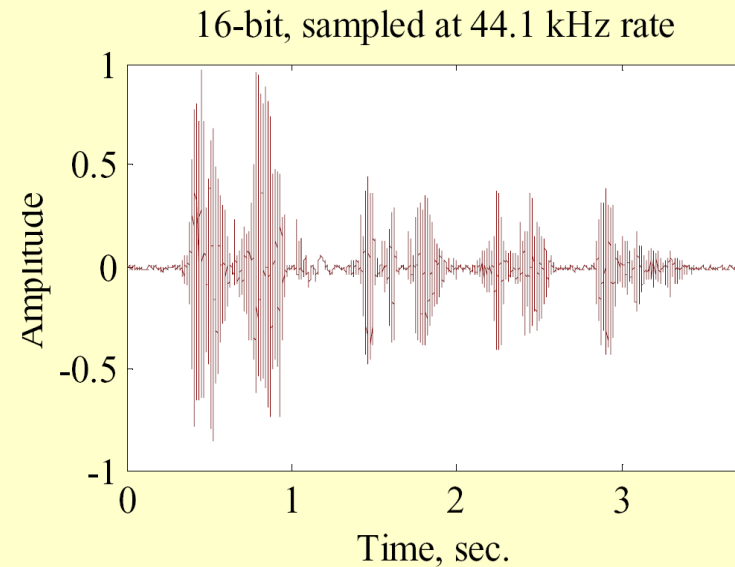
- Physical Modeling
 - Models the sound production method
 - Physical description of the main vibrating structures by partial differential equations
 - Physical description of the main vibrating structures by partial differential equation
 - Examples: (CCRMA, Stanford)
 - Guitar with nylon strings 
 - Marimba (木琴) 
 - Tenor saxophone 

Signal Coding & Compression

- Concerned with efficient digital representation of audio or visual signal for storage and transmission to provide maximum quality to the listener or viewer



Signal Compression Example (1/3)

- Original Speech 🗣️
Data size: 330,780 bytes



- Compressed Speech(GSM 6.10) 🗣️
Sampled at 22.050 kHz, Data size 16,896 bytes
- Compressed speech (Lernout & Hauspie CELP 4.8kbit/s) 🗣️
Sampled at 8 kHz, Data size 2,302 bytes

Signal Compression Example (2/3)

- Original Music 
Audio Format: PCM 16.000 kHz, 16 Bits
(Data size 66206 bytes)
- Compressed Music 
Audio Format: GSM 6.10, 22.05 kHz
(Data size 9295 bytes)

Courtesy: Dr. A. Spanias

Signal Compression Example (3/3)



Original Lena Image
File Size = 256K bytes





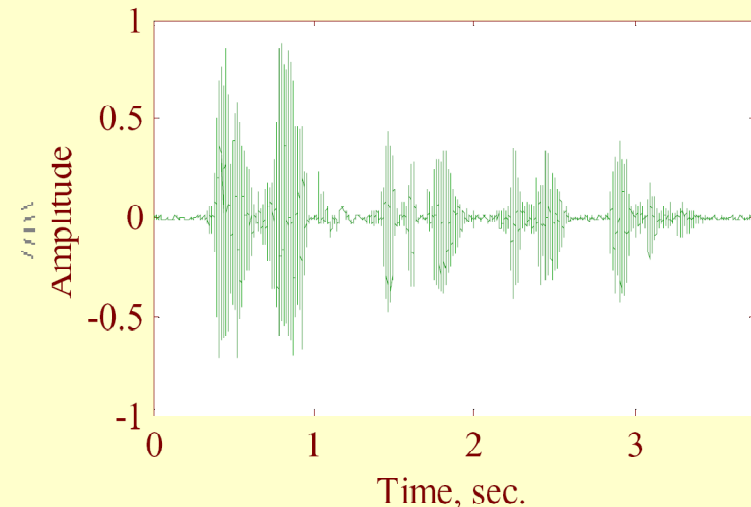
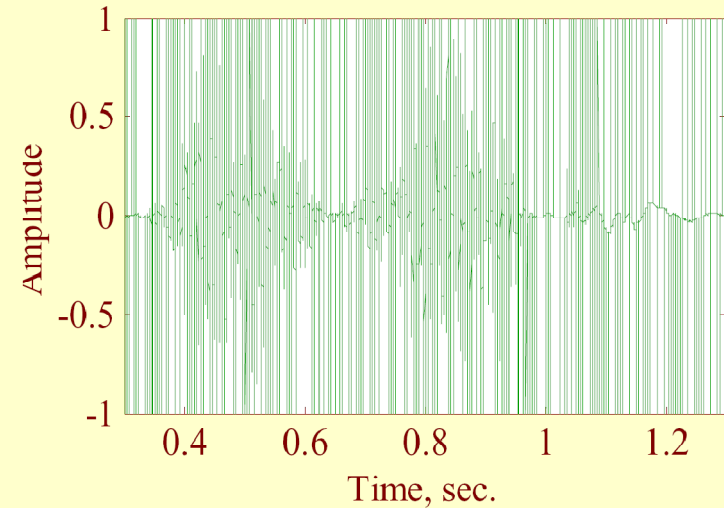
Compressed Lena Image
File Size = 13K bytes

Applications: Signal Enhancement

- Purpose: To emphasize specific signal features to provide maximum quality to the listener or viewer
- For speech signals, algorithms include removal of background noise or interference
- For image or video signals, algorithms include contrast enhancement, sharpening and noise removal

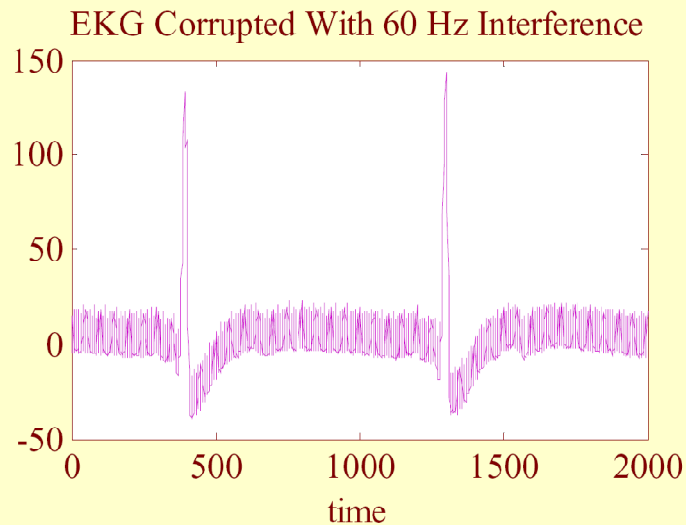
Signal Enhancement Examples (1/4)

- Noisy speech signal 
- Noise removed speech 

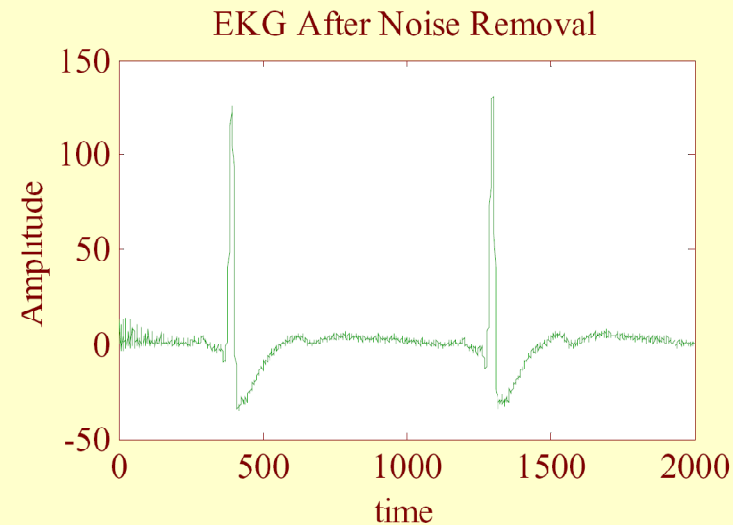


Signal Enhancement Examples (2/4)

EKG corrupted with
60 Hz interference



EKG after filtering with
a notch filter



Signal Enhancement Examples (3/4)

- Original image and its contrast enhanced version



Original



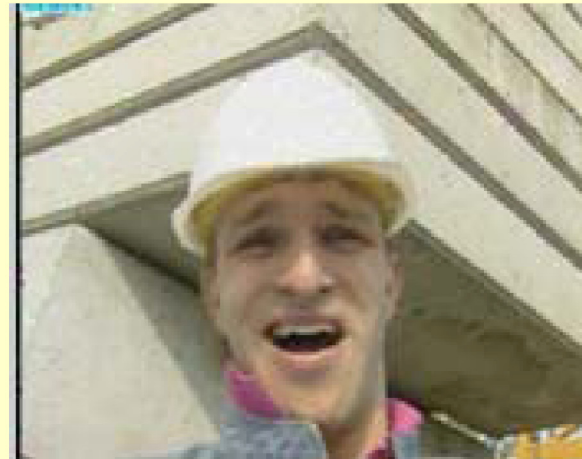
Enhanced

Signal Enhancement Examples (4/4)

- Noisy image & denoised image



20% pixels corrupted with
additive impulse noise



Noise-removed version