Queueing Analysis of Loss Systems with Variable Optical Delay Lines

Duan-Shin Lee, Cheng-Shang Chang, Jay Cheng, and Horng-Sheng Yan
Institute of Communications Engineering
National Tsing Hua University
Hsinchu 300, Taiwan, R.O.C.
Email: lds@cs.nthu.edu.tw
{cschang, jcheng}@ee.nthu.edu.tw
g944362@oz.nthu.edu.tw

***Abstract*—Recently, a new optical device called variable optical delay line (VODL) has been proposed in the literature. As suggested by its name, the delay of a VODL can be dynamically set within a certain range. Once set, a VODL behaves like a traditional fiber delay line and can admit packets requiring the same delay as that set by the VODL. As in the queueing context, a VODL can thus be viewed as a server that serves packets with the service times equal to the required delays.**

In this paper, we consider loss systems with parallel VODLs subject to various classes of packet arrivals. Such loss systems are different from the classical loss systems as a VODL, even when occupied, can still admit new packets with the same delay. For the case with an infinite number of VODLs, we show that the number of VODLs occupied by different classes of packets still has a product form solution. However, the analysis for the case with a finite number of VODLs is much more difficult. For this, we propose an approximation method based on state truncation. We show that the packet loss probabilities derived from our approximation are very close to those generated from simulations. In order to minimize the packet loss probabilities in such loss systems, we also consider the problem of assigning dedicated VODLS to various classes of packets. We show under the light traffic condition, the complete sharing policy, i.e., the policy that does not assign any dedicated VODLs, is optimal. For the general traffic condition, we propose a greedy search algorithm to find a suboptimal assignment of dedicated VODLS. Simulation results show that our greedy algorithm yields very good assignments when comparing with the optimal ones.

***Index Terms*—optical buffers, variable optical delay lines, optimal assigment, complete sharing**

I. Introduction

The increasing varieties of Internet services have pushed the need for high-speed packet switched network. Optical fiber is an ideal transmission medium because it promises lossless transmission over a very broad frequency range. Although the optical transmission technology is very powerful in transmission capability, it is relatively limited in signal processing ability. Therefore, the current packet switched communication networks transmit information in optical form in optical links. However, optical information in the form of packets are received, converted into electronic form, stored, processed and converted back to optical form for transmission in routers or switches. This optical-electric-optical conversion seriously slows down the speed of the optical networks.

To avoid optical-electric-optical conversion and to achieve all-optical packet switched networks, one needs to store packets in optical form. The device that stores packets in optical form is called an optical buffer. There has been a surge of research interest in optical buffers recently. In the late nineties and early 2000s, designs of optical buffers and experimental studies were reported in the research literature [8], [9], [23], [11], [14], [12], [13]. A series of theoretical research on the design of optical buffers has been reported in [2], [3], [10], [4], [6], [7], [22]. These studies focus on clever designs of optical buffers using switched delay lines that exactly emulate optical buffers of various service disciplines. In addition, these designs achieve logarithmic or sublinear complexity. Rigorous mathematical proofs are presented to support the claims on sublinear complexity and exact emulation. Some optical buffers built with fiber delay lines allow asynchronous and variable length packets. Queueing analyses have been proposed to study the effect of granularity to such buffers [1], [16], [18], [17].

Recently, a new optical device called variable optical delay line (VODL) has been proposed in [5], [15], [24]. It is achieved by varying the dispersion curve of the medium based on the use of semiconductor quantum dots and the electromagnetically induced transparency mechanism [19]. When we set the adjustable optical memory to certain storage time, it acts like traditional optical delay lines and thus can accept packets with the same delay requirement. One can not modify the storage time when there is an optical signal in the variable optical memory. This is because it may cause an unexpected release time. When all optical signals are released, it can be adjusted to another storage time. As in the queueing context, a VODL can thus be viewed as a server that serves packets with the service times equal to the required delays (a schematic diagram of a VODL is shown in Figure 1).

In this paper, we consider loss systems with parallel VODLs subject to various classes of packet arrivals. Instead of focusing on exact emulation or sublinear complexity, we focus on queueing analysis and optimal operation of the loss systems such that the packet loss probabilities are minimized. In our setting, VODLs can be tuned to different delay values when

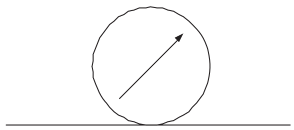

Fig. 1. A variable optical delay line

they are idle. Unlike the classical queueing context, a busy VODL can accept further packets before the packets in it have left. However, for each time slot, a VODL can accept at most one packet and the requested delay of the accepted packet must be identical to that of packets stored in the VODL.

For such loss systems, we show that they can be modeled by Markov chains. When the number of VODLs is infinite, the corresponding Markov chain can be solved easily. In particular, we show that the number of VODLs occupied by different classes of packets still has a product form solution. However, the analysis for the case with a finite number of VODLs is much more difficult. For this, we propose an approximation method based on state truncation. We show that the packet loss probabilities derived from our approximation are very close to those generated from simulations. In order to minimize the packet loss probabilities in such loss systems, we also consider the problem of assigning dedicated VODLS to various classes of packets. We show under the light traffic condition, the complete sharing policy, i.e., the policy that does not assign any dedicated VODLs, is optimal. For the general traffic condition, we propose a greedy search algorithm to find a suboptimal assignment of dedicated VODLS. Simulation results show that our greedy algorithm yields very good assignments when comparing with the optimal ones.

This paper is organized as follows. In Section II, we model and analyze the loss system with parallel VODLs by a Markov chain. We then show that the Markov chain has a product form solution in Section II-C when the number of VODLs is infinite. Based on state truncation, we then derive an approximation in Section II-D for the case with a finite number of VODLs. An upper bound for the packet loss probability is also derived in Section II-E. In Section III, we consider the problem of optimal assignment of dedicated VODLS to minimize the loss probability. When the traffic is light, we show that the complete sharing policy is optimal in Section III-A. For the general traffic condition, a greedy search algorithm is proposed in Section III-B. Various numerical results are obtained to verify our approximations in Section IV. The paper is concluded in Section V, where we address possible extensions.

II. Modelling and analysis

A. *queueing model*

In this section, we present our queueing model for loss systems with variable optical delay lines (VODLs). Here we consider the discrete-time setting and make the following assumptions:

(A1) Packets are of the same size.

(A2) Time is slotted and synchronized so that a packet can be transmitted within a time slot.

In the literature of optical switching, fiber delay lines are often used for delaying packets to a fixed integer number of time slots. To avoid conflict, packets might be required to have different delays. As such, one needs to have a network element like a VODL that is capable of delaying packets to a range of delays. Here we assume that the delay of each VODL can be dynamically set within the range $[1, d]$. A VODL is said to be *occupied* (resp. idle) if there is at least one packet (resp. no packet) in it. When a packet arrives requesting a certain delay within the range of the delay of an *idle* VODL, the delay of the VODL can be set to match the required delay of the packet and the packet is then "served" by the VODL with the service time equal to the delay set by the VODL. In the classical queueing setting, a server, once occupied, can no longer admit new customers. However, in our setting, an *occupied* VODL, due to its physical property, can still admit new packets with the same delay as that set by the VODL. This makes the analysis of VODLs a new paradigm of queueing theory.

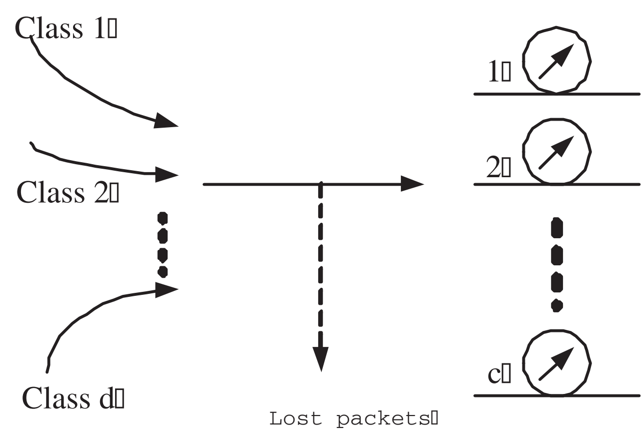

Fig. 2. A loss system with c variable optical delay lines and d classes of packets

To carry out our analysis, we assume that there are c VODLs in total, and that there are d classes of packets (as shown in Figure 2). A class i packet is required to be delayed by i time slots, $i = 1, 2, \ldots, d$. Since an occupied VODL can keep on accepting new packets, VODLs occupied by packets requesting long delay can be held up for a long time. Thus, packets requesting long delay are more likely to be served and packets requesting short delay are more likely to be lost. To minimize packet loss probability, we assign dedicated VODLs to each traffic class. Specifically for class i packets, we dedicate N_i VODLs statically. These VODLs will be called *dedicated* VODLs. The remaining $c - \sum_{i=1}^{d} N_i$ VODLs can be dynamically set to accommodate packets of any class. These VODLs will be called *shared* VODLs. Let

$$b = c - \sum_{i=1}^{d} N_i \tag{1}$$

be the number of shared VODLs.

Now we specify the policy of placing packets into VODLs. Let $a_i(t)$ be the number of class i packets arriving at time t.

Step 1. As new class i packets arrive, they first try to enter their dedicated VODLs.

Step 2. If there are not enough dedicated VODLs, the rest $(a_i(t) - N_i)^+$ arrivals attempt to enter the shared VODLs that are already occupied by class i packets that arrive before time t. Here we use the symbol $(x)^+$ to denote $\max(x, 0)$.

Step 3. If there are still class i new arrivals that cannot be assigned to VODLs after the above two steps, they can be assigned to idle shared VODLs according to a priority order that is randomly chosen in every time slot.

Step 4. If there are not enough idle shared VODLs left, then packets are lost.

Specifically, let the priority of class i at time t be $p(i, t)$. Numerically smaller priority values correspond to higher priorities. Also, let $X_i(t)$ be the number of class i packets placed into *shared* VODLs at time t, and $Y_i(t)$ be the number of shared VODLs occupied by earlier class i packets that arrive before t. Note that if a class i packet is placed into a VODL at time t, that VODL will remain occupied from time t to time $t + i - 1$. As we always try those VODLs occupied by class i packets first, we have

$$Y_i(t) = \max[X_i(t-1), \ldots, X_i(t-i+1)]. \tag{2}$$

To ease our presentation, we let $Y_1(t) = 0$ for all t (as class 1 packets with delay 1 only occupy VODLs for one time slot). We note that (2) is in fact the governing equation that makes the loss system with VODLs different from the traditional loss system. In the traditional loss system, the number of "busy" servers is represented by the *sum* of all the customers in the system. Here in (2), it is represented by the *maximum* of packets placed in various time slots. Clearly, this implies that the loss system with VODLs will yield a much lower packet loss probability than that in the traditional loss system. We will further verify this by computer simulations in Section IV.

According to the priority assignment, the number of shared VODLs that can be used by class i packets, denoted by $N_i(t)$, is

$$N_i(t) = b - \sum_{p(j,t)<p(i,t)} \max[X_j(t), Y_j(t)] - \sum_{p(j,t)>p(i,t)} Y_j(t), \tag{3}$$

where the first (resp. second) summation is the number of shared VODLs used by those classes of packets with priority higher (resp. lower) than class i packets at time t. Thus, we have

$$X_i(t) = \min[(a_i(t) - N_i)^+, N_i(t)], \tag{4}$$

and the number of lost class i packets at time t is

$$(a_i(t) - N_i - N_i(t))^+. \tag{5}$$

To compute the total number of lost packets at time t, one may simply add up all the lost packets for each class in (5). However, the computation of the total number of lost packets at time t using that approach depends on the priority order. Here we derive a much simpler expression that is independent of the priority order (once $Y_i(t)$'s are given). First, we note that the number of class i packets that need to be assigned to idle shared VODLs at time t (after the second step) is

$$(a_i(t) - N_i - Y_i(t))^+.$$

Thus, the total number of packets that need to be assigned to idle shared VODLs at time t is

$$\sum_{i=1}^{d} (a_i(t) - N_i - Y_i(t))^+.$$

On the other hand, the total number of idle shared VODLs at time t is

$$b - \sum_{i=1}^{d} Y_i(t).$$

From these, the total number of lost packets at time t is

$$\Big(\sum_{i=1}^{d} (a_i(t) - N_i - Y_i(t))^+ - (b - \sum_{i=1}^{d} Y_i(t))\Big)^+.$$

Since

$$(a_i(t) - N_i - Y_i(t))^+ + Y_i(t) = \max[a_i(t) - N_i, Y_i(t)],$$

the total number of lost packets at time t, denoted by $L(t)$, is

$$L(t) = \Big((\sum_{i=1}^{d} \max[a_i(t) - N_i, Y_i(t)]) - b\Big)^+. \tag{6}$$

B. The Markov chain

To carry out a probabilistic analysis, we need to make an assumption on the arrival processes:

(A3) $\{a_i(t), t \geq 1\}$ is a sequence of independent and identically distributed (i.i.d.) random variables with the common distribution

$$F_i(z) = \Pr(a_i(1) \leq z).$$

The sequence is also independent of everything else.

Under (A3), the packet loss probability, denoted by $\ell(N_1, N_2, \ldots, N_d)$, is then given by the ratio of the expected number of lost packets to the expected number of packet arrivals in a time slot. Specifically, we have from (6) that

$$\begin{aligned} &\ell(N_1, N_2, \ldots, N_d) \\ &= \mathsf{E}\Big[\sum_{i=1}^{d} \max[a_i(t) - N_i, Y_i(t)] - b\Big]^+ \Big/ \sum_{i=1}^{d} \mathsf{E}[a_i(t)]. \end{aligned} \tag{7}$$

For $i = 2, \ldots, d$, let

$$\mathbf{X}_i(t) = (X_i(t-1), \ldots, X_i(t-i+1)).$$

Also let

$$\mathbf{X}(t) = (\mathbf{X}_2(t), \ldots, \mathbf{X_d}(t)).$$

In view of (4) and (3), we know that $\mathbf{X}(t+1)$ is a deterministic function of $\mathbf{X}(t)$ and $a_i(t), i = 1, 2, \ldots, d$, once the priority order is given. From (A3), $a_i(t), i = 1, 2, \ldots, d$, are independent of $\mathbf{X}(t)$. Thus, $\mathbf{X}(t)$ is a discrete-time Markov chain and the packet loss probability in (7) can be obtained by solving the steady state probabilities of the Markov chain $\mathbf{X}(t)$. However, the size of the state space of $\mathbf{X}(t)$ is astronomical even for small d and c. Thus, we have to seek for an approximation method.

C. The case with an infinite number of VODLs

Suppose that the number of shared VODLs b is infinite. In that case, there are no lost packets and we will show that the Markov chain $\mathbf{X}(t)$ has a product form joint distribution. As b is infinite, we note from (4) and (3) that

$$X_i(t) = (a_i(t) - N_i)^+, \quad i = 1, 2, \ldots, d. \tag{8}$$

Let $\mathbf{x}_i = (x_{i,1}, \ldots, x_{i,i-1})$ be a nonnegative $(i-1)$-vector, and $\mathbf{x} = (\mathbf{x_1}, \ldots, \mathbf{x}_d)$. Then we have from (A3) that

$$\begin{aligned}\Pr(\mathbf{X}_i(t) \le \mathbf{x_i}) &= \prod_{j=1}^{i-1} \Pr(a_i(t-j) - N_i \le x_{i,j}) \\ &= \prod_{j=1}^{i-1} F_i(N_i + x_{i,j}).\end{aligned} \tag{9}$$

Moreover, it follows from (A3) that $\mathbf{X}_i(t)$, $i = 1, 2, \ldots, d$, are independent. As such,

$$\begin{aligned}\Pr(\mathbf{X}(t) \le \mathbf{x}) &= \prod_{i=1}^{d} \Pr(\mathbf{X}_i(t) \le \mathbf{x_i}) \\ &= \prod_{i=1}^{d}\prod_{j=1}^{i-1} F_i(N_i + x_{i,j}).\end{aligned} \tag{10}$$

In view of (9) and (2), we then have for $i = 2, \ldots, d$,

$$\begin{aligned}\Pr(Y_i(t) \le k) &= \Pr(\mathbf{X}_i(t) \le (k, k, \ldots, k)) \\ &= F_i(N_i + k)^{i-1}.\end{aligned} \tag{11}$$

Let $\pi_i(n) = \Pr(Y_i(t) = n)$, $n \ge 0$, when b is infinite. It then follows that

$$\pi_i(0) = \Pr(Y_i(t) = 0) = F_i(N_i)^{i-1}, \tag{12}$$

$$\begin{aligned}\pi_i(n) &= \Pr(Y_i(t) = n) \\ &= F_i(N_i + n)^{i-1} - F_i(N_i + n - 1)^{i-1}. \quad n \ge 1.\end{aligned} \tag{13}$$

Note from (A3) and (2) that $Y_i(t)$'s are independent. As such,

$$\begin{aligned}&\Pr(Y_2(t) = n_2, \ldots, Y_d(t) = n_d) \\ &= \prod_{i=2}^{d} \Pr(Y_i(t) = n_i) \\ &= \prod_{i=2}^{d} \pi_i(n_i).\end{aligned} \tag{14}$$

This shows that the number of VODLs occupied by different classes of packets still has a product form solution.

D. The case with a finite number of VODLs

When b is not infinite, $Y_i(t)$'s are not independent and we do not have the product form in (14). When b is finite, we know that

$$\sum_{i=2}^{d} Y_i(t) \le b. \tag{15}$$

Thus, the stochastic process $(Y_2(t), \ldots, Y_d(t))$ is constrained. In view of this constraint, we propose the following state truncation approximation for the joint distribution of $Y_i(t)$'s:

$$\Pr(Y_2(t) = n_2, \ldots, Y_d(t) = n_d) = G \prod_{i=2}^{d} \pi_i(n_i), \tag{16}$$

where $\pi_i(n)$'s are defined in (12) and (13), and

$$G = \Big(\sum_{\{(n_2,\ldots,n_d) \in S\}} \prod_{i=2}^{d} \pi_i(n_i) \Big)^{-1} \tag{17}$$

is the normalization constant with respect to the truncated state space

$$\mathcal{S} = \left\{ (n_2, \ldots, n_d) \,\Big|\, \sum_{i=2}^{d} n_i \le b, n_j \ge 0, j = 2, \ldots, d \right\}. \tag{18}$$

Such an approximation would be *exact* if the unrestricted process $(Y_2(t), \ldots, Y_d(t))$ formed a time-reversible Markov process (see e.g., [20][p. 443]). However, $(Y_2(t), \ldots, Y_d(t))$, as defined in (2), is not even a Markov process.

With the approximation for the joint distribution of $Y_i(t)$'s in (16), the packet loss probability in (7) can be obtained by calculating the conditional expected number of lost packets. Specifically, for state $\boldsymbol{n} = (n_2, \ldots, n_d) \in \mathcal{S}$, let $\phi(\boldsymbol{n})$ denote the conditional expected number of lost packets given that $(Y_2(t), \ldots, Y_d(t))$ is in state $\boldsymbol{n}$. Since $a_i(t)$'s are independent of $Y_i(t)$'s, we have from (6) that

$$\begin{aligned}\phi(\boldsymbol{n}) &= \mathsf{E}[L(t) | (Y_2(t), \ldots, Y_d(t)) = (n_2, \ldots, n_d)] \\ &= \sum_{k_1,\ldots,k_d=0}^{\infty} \left[\sum_{j=1}^{d} \max[(k_j - N_j)^+, n_j] - b \right]^+ \\ &\quad \times \prod_{i=1}^{d} (F(k_i) - F(k_i - 1)),\end{aligned} \tag{19}$$

where $n_1 = 0$ and $F(k_i) - F(k_i - 1)$ is simply the probability that there are k_i class i arrivals at time t. Finally, the approximated packet loss probability, denoted by $\Phi(N_1, \ldots, N_d)$, is given by

$$\Phi(N_1, \ldots, N_d) = \frac{G \displaystyle\sum_{(n_2,\ldots,n_d) \in S} \prod_{i=2}^{d} \pi_i(n_i) \phi(\boldsymbol{n})}{\sum_{i=1}^{d} \mathsf{E}[a_i]}, \tag{20}$$

where G is the normalization constant in (17).

E. An upper bound

In this section, we derive an upper bound for the packet loss probability. Note that the number of class i packets that can be placed in the shared VODLs at time t is bounded above $(a_i(t) - N_i)^+$, i.e.,

$$X_i(t) \leq (a_i(t) - N_i)^+, \tag{21}$$

and the inequality becomes an equality for the unrestricted process, i.e., b is infinite. As such, we have from (2) that

$$\begin{aligned} &Y_i(t) = \max[X_i(t-1), \ldots, X_i(t-i+1)] \\ &\leq \max[(a_i(t-1) - N_i)^+, \ldots, (a_i(t-i+1) - N_i)^+] \\ &= Y_i^u(t), \end{aligned} \tag{22}$$

where $Y_i^u(t)$ is the unrestricted process of $Y_i(t)$ in Section II-C and it has the steady state distribution specified in (11). Using this in (7) yields

$$\begin{aligned} &\ell(N_1, N_2, \ldots, N_d) \\ &\leq \mathsf{E}\Big[\sum_{i=1}^{d} \max[a_i(t) - N_i, Y_i^u(t)] - b\Big]^+ \Big/ \sum_{i=1}^{d} \mathsf{E}[a_i(t)]. \end{aligned} \tag{23}$$

Following the same argument as that for (20), we can compute the expectation in (23) and derive the following upper bound:

$$\ell(N_1, N_2, \ldots, N_d) \leq \frac{\sum_{n_2,\ldots,n_d} \prod_{i=2}^{d} \pi_i(n_i)\phi(\boldsymbol{n})}{\sum_{i=1}^{d} \mathsf{E}[a_i]}, \tag{24}$$

where $\phi(\boldsymbol{n})$ is defined in (19) and $\pi_i(n)$'s are defined in (12) and (13).

Note that the difference between the upper bound in (24) and the approximation in (20) is that the sum in (24) is unconstrained. This implies that the approximation in (20) should be very good when b is very large or when the traffic is very light.

III. Optimal assignment of dedicated VODLs

In this section, we study the problem of optimal assignment of the numbers of dedicated VODLs, $N_1, N_2, \ldots, N_d$, so that the packet loss probability is minimized.

A. Complete sharing in light traffic

As argued in Section II-E, the packet loss probability approaches to the upper bound in (24) when the traffic is light. In such a case, we will show that the complete sharing policy, i.e., choosing $N_1 = N_2 = \ldots = N_d = 0$, is the optimal policy that minimizes the loss probability. To see this, note from (23), (1), and (22) that

$$\begin{aligned} &(\sum_{i=1}^{d} \max[a_i(t) - N_i, Y_i^u(t)]) - b \\ &= (\sum_{i=1}^{d} \max[a_i(t) - N_i, Y_i^u(t)] + N_i) - c \\ &= (\sum_{i=1}^{d} \max[N_i, \max_{0 \leq j \leq i-1}[a_i(t-j)]]) - c, \end{aligned} \tag{25}$$

where c is the total number of VODLs. Clearly, the expression in (25) is increasing in N_i. Thus, the minimum of the expression in (25) is achieved when $N_1 = N_2 = \ldots = N_d = 0$.

B. A Greedy Search Algorithm

For the general traffic condition, we use the approximation for the packet loss probability in (20) as the objective function. Let $\mathcal{S}'$ be the set that contains all the legitimate vector $(N_1, N_2, \ldots, N_d)$, *i.e.,*

$$\mathcal{S}' = \left\{(n_1, n_2, \ldots, n_d) \,\Big|\, \sum_{i=1}^{d} n_i \leq c, n_j \geq 0, j = 1, 2, \ldots, d\right\}. \tag{26}$$

To find the value of $(N_1, N_2, \ldots, N_d)$ that minimizes the packet loss probability in (20), one needs to solve the following nonlinear integer programming problem

$$\min_{(N_1,\ldots,N_d) \in \mathcal{S}'} \Phi(N_1, \ldots, N_d).$$

In general, this is an NP-complete problem. We now present a greedy search algorithm that gives us a suboptimal solution. We construct a search graph which consists of nodes corresponding to points in $\mathcal{S}'$ and directed links. To construct the graph we start with node $(0, 0, \ldots, 0)$, corresponding to the complete sharing policy. Each node in the graph has d descendant nodes. Consider node $(n_1, n_2, \ldots, n_d)$ for example. The d descendant nodes of this node are

$$\begin{aligned} &(n_1 + 1, n_2, \ldots, n_d) \\ &(n_1, n_2 + 1, n_3, \ldots, n_d) \\ &\vdots \\ &(n_1, \ldots, n_{d-1}, n_d + 1). \end{aligned}$$

We connect node $(n_1, \ldots, n_d)$ with all its descendant nodes by directed links which point from node $(n_1, n_2, \ldots, n_d)$ to its descendant nodes. We recursively construct for each node in the graph its descendant nodes until the node satisfies $n_1 + n_2 + \ldots + n_d = c$. The search graph corresponding to $c = 4$ and $d = 2$ is shown in Figure 3.

The greedy search algorithm starts from the node $(0, \ldots, 0)$. We compute the approximated packet loss probabilities of this node and all its descendant nodes. If there is a descendant node whose approximated packet loss probability is less than that of the node $(0, \ldots, 0)$, we move to the descendant node that has the smallest approximated packet loss probability. We repeat this process until we reach a node that has a smaller approximated packet loss probability than that of all its descendant nodes. This is the solution of the greedy search algorithm.

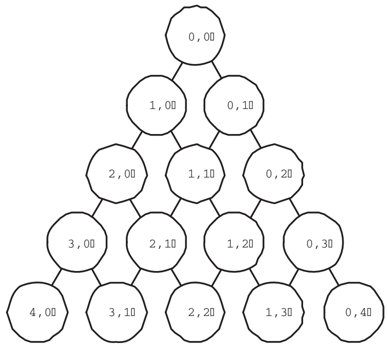

Fig. 3. The search graph for $c = 4$ and $d = 2$

IV. Simulation and Numerical Results

In this section we present our simulation and numerical results. First we examine the accuracy of the approximation by state truncation in (20). For any c, d and d-vector $(N_1, N_2, \ldots, N_d)$, we obtain packet loss probability $\ell(N_1, \ldots, N_d)$ by simulation. The length of our simulation is one million time slots. We use batch mean method to estimate 99-percent confidence intervals. The number of batches is ten. Therefore, there are 10^5 time slots in each batch. In our simulation results, all simulated packet loss probabilities fall inside their confidence intervals. In Figure 4 we show the packet loss probability as a function of the total arrival rate. In this example, $c = 10$ and $d = 5$ and $N_i = 0$ for all i. The arrivals of each class of packets are generated by i.i.d. Poisson random variables with equal rates. From this figure, we see that the approximation fits the simulation result very well. We also compare the performance with the traditional loss system where servers cannot admit new customers once they are occupied. The loss probability in such a loss system can be obtained by the famous Erlang loss formula [21][p. 170], i.e.,

$$B(c, \rho) = \frac{\rho^c / c!}{\sum_{j=0}^{c} \rho^j / j!}, \tag{27}$$

where

$$\rho \stackrel{\text{def}}{=} \sum_{i=1}^{d} i \lambda_i.$$

From Figure 4 we see that the loss systems with VODLs can be three orders of magnitude better than the traditional loss systems in packet loss probability.

In Figure 5 we keep the same c and d. The Poisson arrival rates are increasing. Specifically, if λ_i is the arrival rate of class i, it satisfies

$$\lambda_i = \lambda_{i+1}/2. \tag{28}$$

Similarly, we show the packet loss probability for decreasing arrival rates

$$\lambda_i = 2\lambda_{i+1} \tag{29}$$

in Figure 6. In these three figures, it seems that our approximations fit the simulation results very well.

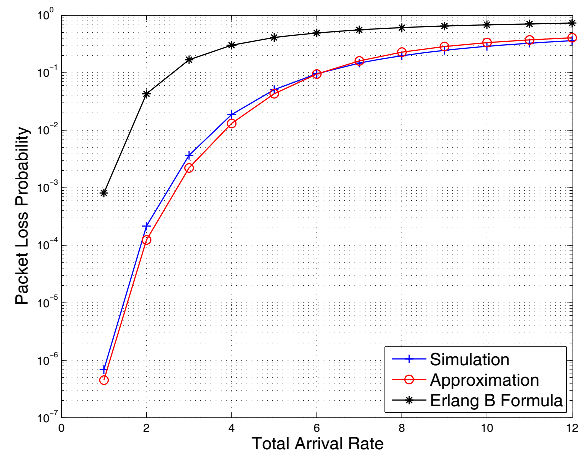

Fig. 4. Packet loss probability vs. traffic arrival rate (with equal arrival rates among all traffic classes)

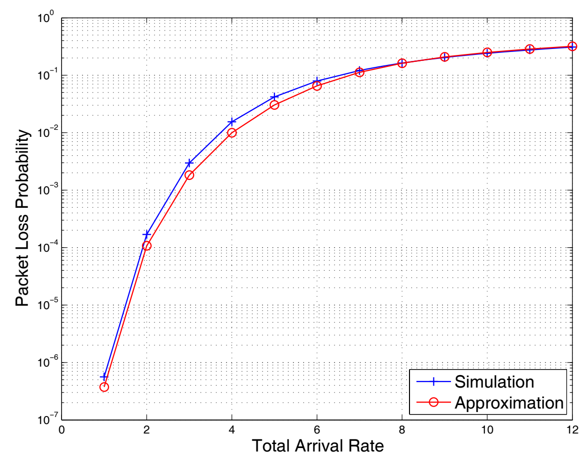

Fig. 5. Packet loss probability vs. traffic arrival rate (with increasing arrival rates)

We then conduct a larger experiment. We consider two values for c, two values for d and three traffic loadings corresponding to low, medium and heavy traffic conditions. The total arrival rates of these three cases are 1, 6 and 12 when $c = 10$. When $c = 5$, the arrival rates are half of the above values. For each traffic loading, we consider equal arrival rates, increasing arrival rates (see (28)) and decreasing arrival rates (see (29)) for traffic classes. The two values for c are 5 and 10, and the two values for d are 3 and 5, respectively. In addition, we consider three different $(N_1, N_2, \ldots, N_d)$ vectors. The first two vectors are $(0, \ldots, 0)$ corresponding to the complete sharing policy and $(1, 1, 0, \ldots, 0)$ which assigns dedicated VODLs to packets requesting short delay. For the third vector we introduce a class of assignment method based on proportionality. Specifically, we allocate roughly $r \cdot c$ dedicated VODLs and roughly $(1 - r) \cdot c$ shared VODLs. For

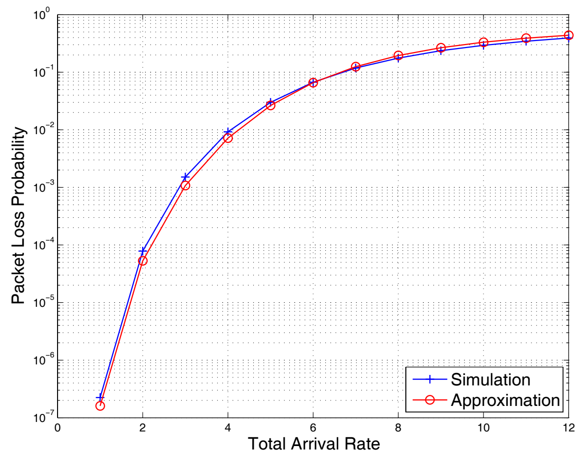

Fig. 6. Packet loss probability vs. traffic arrival rate (with decreasing arrival rates)

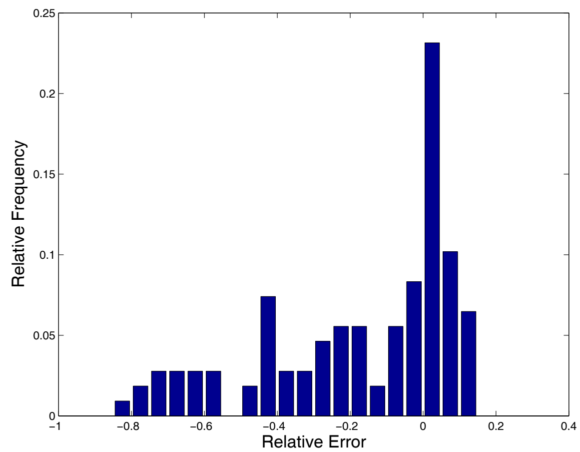

Fig. 7. The relative frequency of the relative errors

class i traffic, we allocate

$$N_i = \left\lfloor \frac{rc\lambda_i}{\sum_{j=1}^{d} \lambda_j} \right\rfloor, \quad i = 2, 3, \dots, d, \tag{30}$$

and for class 1 we allocate

$$N_1 = \lfloor rc \rfloor - \sum_{i=2}^{d} N_i. \tag{31}$$

For the third vector, we assign dedicated VODLs according to (30) and (31) with $r = 1/2$. In this experiment, there are totally 108 cases. We define relative error to be

$$\frac{\Phi(N_1, N_2, \dots, N_d) - \ell^s(N_1, N_2, \dots, N_d)}{\ell^s(N_1, N_2, \dots, N_d)},$$

where $\ell^s(N_1, N_2, \dots, N_d)$ is the packet loss probability obtained by simulation using the d-vector $(N_1, N_2, \dots, N_d)$. In Figure 7 we show the relative frequency of the relative errors occurred in the 108 cases. We can see that in most cases the approximation method works fairly well.

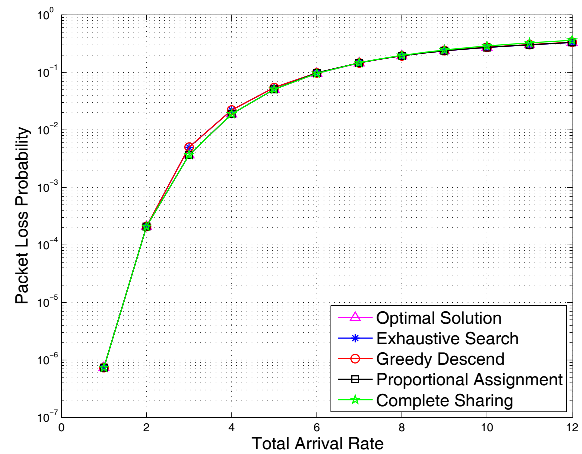

Fig. 8. The packet loss probability of the solution obtained by the greedy search algorithm, the exhaustive algorithm, the complete sharing scheme and a hybrid proportional assignment scheme

Next we study the greedy search algorithm. We are interested in whether its solution is close to an optimal solution. We consider the case where $d = 5$ and $c = 10$ and assume that the arrival rates of all traffic classes are equal. The greedy search algorithm starts from node $(0, 0, \dots, 0)$ and descends according to the packet loss probabilities calculated by the approximation. The greedy search algorithm stops at a node, say node $(\tilde{N}_1, \tilde{N}_2, \dots, \tilde{N}_d)$. The packet loss probability $\ell^s(\tilde{N}_1, \tilde{N}_2, \dots, \tilde{N}_d)$ is shown in Figure 8 and is labeled with "Greedy Descend". By exhaustive search, we find the vector $(\hat{N}_1, \hat{N}_2, \dots, \hat{N}_d)$ such that $\Phi(\hat{N}_1, \hat{N}_2, \dots, \hat{N}_d)$ is minimized. We plot $\ell^s(\hat{N}_1, \hat{N}_2, \dots, \hat{N}_d)$ in Figure 8 with label "Exhaustive Search". Finally, we find the vector $(N_1^\star, N_2^\star, \dots, N_d^\star)$ such that $\ell^s(N_1^\star, N_2^\star, \dots, N_d^\star)$ is minimized. This result is labeled with "Optimal Solution" in Figure 8. From Figure 8, we find that the greedy search algorithm produces solutions that are extremely close to optimal solutions. Figure 8 also suggests that the packet loss probability is not very sensitive to the selection of the number of dedicated VODLs.

To further examine the effectiveness of the greedy search algorithm, we form an ascending list of the packet loss probability $\ell^s(N_1, N_2, \dots, N_d)$ for all $(N_1, \dots, N_d)$ in $\mathcal{S}'$. The optimality ranking for the greedy search algorithm is defined as the place of $\ell^s(\tilde{N}_1, \dots, \tilde{N}_d)$ in the list. Similarly, the optimality ranking of the exhaustive search method is defined as the place of $\ell^s(\hat{N}_1, \dots, \hat{N}_d)$ in the list. The optimality ranking is shown in Figure 9. For Figure 9 we mention that the state space $\mathcal{S}'$ has 3003 points. We define the optimality difference for the greedy search algorithm as

$$\frac{\ell^s(\tilde{N}_1, \tilde{N}_2, \dots, \tilde{N}_d) - \ell^s(N_1^\star, N_2^\star, \dots, N_d^\star)}{\ell^s(N_1^\star, N_2^\star, \dots, N_d^\star)}.$$

The optimality difference for the exhaustive algorithm is defined similarly. In Figure 10 We show the optimality difference. These studies indicate that although the approximation based on state truncation produces errors, the greedy search algorithm still finds suboptimal solutions that are close enough

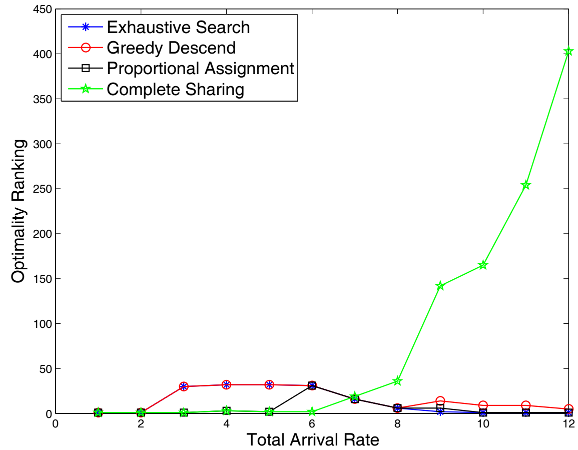

Fig. 9. The optimality ranking of the greedy search algorithm, the exhaustive search algorithm, the complete sharing scheme and a hybrid proportional assignment scheme

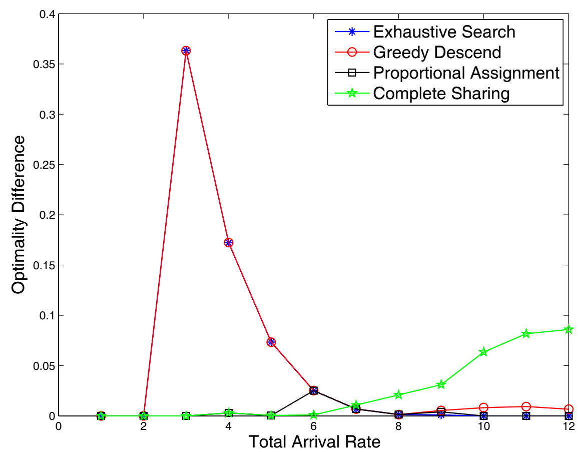

Fig. 10. The optimality difference of the greedy search algorithm, the exhaustive search algorithm, the complete sharing scheme and a hybrid proportional assignment scheme

to the optimal solutions. We have also studied the greedy search algorithm for other values of c and d, and various settings of packet arrival rates. The results are similar to what are shown in Figure 8, Figure 9 and Figure 10.

From Figure 8, Figure 9 and Figure 10, we see that the greedy search algorithm is useful in finding a very good solution. However one may wonder if one can find a good solution without executing the greedy search algorithm. From our experience with the vast cases that we have studied, it seems that a hybrid policy that combines the complete sharing scheme and the proportional assignment scheme in (30) and (31) is a good policy. Specifically, the hybrid scheme adopts the complete sharing scheme if the total arrival rate is less than or equal to $c/2$. If the total arrival rate is more than $c/2$ but less than c, the hybrid scheme adopts the proportional assignment scheme with $r = 1/2$ in (30) and (31). If the total arrival rate is more than or equal to c, the hybrid scheme adopts the proportional scheme with $r = 1$. The intuition of the proportional assignment scheme is as follows. Suppose that arrivals are deterministic. That is, traffic class i has λ_i new packet arrivals in every time slot. Since a VODL can accept exactly one packet in a time slot, a scheme that assigns dedicated VODLs proportional to the numbers of arrivals can equalize the individual packet loss probabilities of traffic classes. In reality, arrivals are not deterministic. Therefore, we set aside roughly $(1-r)c$ VODLs as shared VODLs to handle the statistical variance of the arrivals. The performance of this dedicated VODL assignment scheme is shown Figure 8, Figure 9 and Figure 10 with label "Proportional Assignment". For comparison purpose, the performance of the complete sharing scheme is also shown in these three figures. From these figures, the complete sharing scheme performs well only under the light traffic condition. However, the proportional assignment scheme performs quite well in the medium and heavy traffic conditions as well.

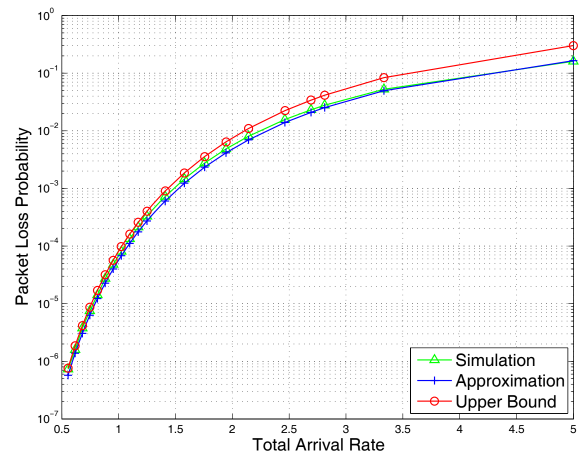

Fig. 11. comparison of the approximation method and the upper bound in light traffic

We study the accuracy of the upper bound in (24) in light traffic. As the complete sharing policy is optimal in light traffic, we do not assign any dedicated VODLs in this experiment. In Figure 11 we show the packet loss probability obtained by simulation, the approximation in (20) and the upper bound in (24). Since the packet loss probability is quite small, we lengthen the simulation time to 5×10^8 time slots. Moreover we assume that the number of arrivals for each class is a geometrically distribution random variable. The arrival rates for different traffic classes are equal in this study. Eq. (24) can be easily computed for geometrically distributed arrivals. From Figure 11, we see that both the approximation and the upper bound are very close to the simulation under light traffic condition.

V. Conclusion

Motivated by the recent development of optical memory in the literature, in this paper, we performed queueing analysis of loss systems with parallel VODLs subject to various classes of packet arrivals. Such loss systems are different from the classical loss systems as a VODL, even occupied, can still admit new packets with the same delay. For the case with an infinite number of VODLs, we showed that the number of VODLs occupied by different classes of packets has a product form solution. When the number of VODLs is finite , we proposed an approximation method based on state truncation. By computer simulation, we showed that the packet loss probabilities derived from our approximation were very good. In order to minimize the packet loss probabilities in such loss systems, we also considered the problem of assigning dedicated VODLS to various classes of packets. We showed that the complete sharing policy is optimal under the light traffic condition. For the general traffic condition, a greedy search algorithm was proposed to find a suboptimal assignment of dedicated VODLS. The effectiveness of the greedy search algorithm was also verified by computer simulations.

At last, we address some problems for future research.

(i) Continuous-time setting: in this paper, we only considered the discrete-time setting. As synchronization of optical packet might be difficult, it would be of interest to extend our analysis to the continuous-time setting, where packets arrive asynchronously. However, the state space for the corresponding Markov chain in the continuous-time setting appears to be much larger than that in the discrete-time setting.

(ii) Dispatching policies: when dispatching a packet to an occupied VODL with the same delay, we did not consider the remaining service time for that VODL to become idle. To use VODLs more efficiently, it is intuitive to dispatch a packet to the VODL with the longest remaining service time among all the occupied VODLs with the same delay. The gain of doing that is not clear (at least it cannot be seen from our approximation).

(iii) Fast simulation in light traffic: as pointed out in our simulation, the simulation length to obtain the packet loss probability is much longer in light traffic. To accelerate the speed of the simulation in light traffic, one may consider using the method of importance sampling. However, how to choose the right change of measure may require further study.

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