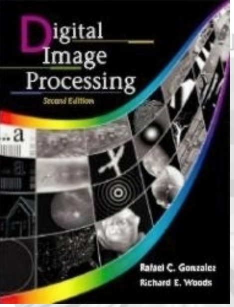


Chapter 11
Representation & Description

- Image segmented into regions, how to represent and describe these regions?

1) In terms of its external characteristics (boundary)
2) In terms of its internal characteristics (pixels in the region)

11.1 Representation – Chain code

- Chain codes are used to represent a boundary as a connected sequence of straight line segments of specified length and direction.
- The representation is based on 4- or 8- connectivity.
- Chain code is generated by following a boundary in clockwise direction and assigning a direction to the segments connecting every pair of pixels.
- Disadvantages of chain codes:

1) The chain code is quite long
2) Any small disturbance along the boundary due to noise cause change in the code that may not related to the shape of the boundary.

11.1 Representation – Chain code

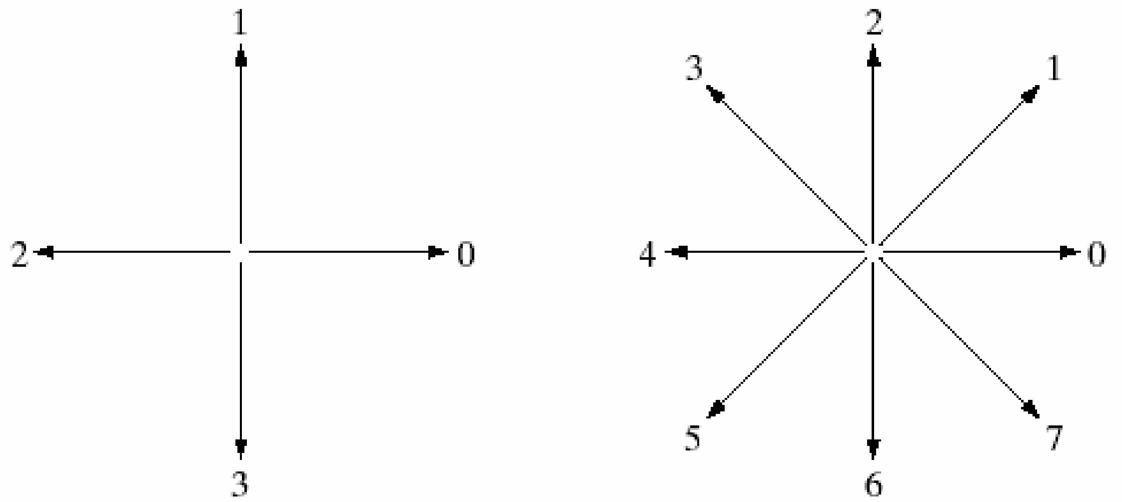
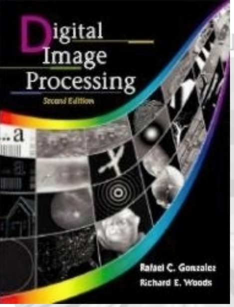


FIGURE 11.1 Direction numbers for (a) 4-directional chain code, and (b) 8-directional chain code.



11.1 Representation – chain code

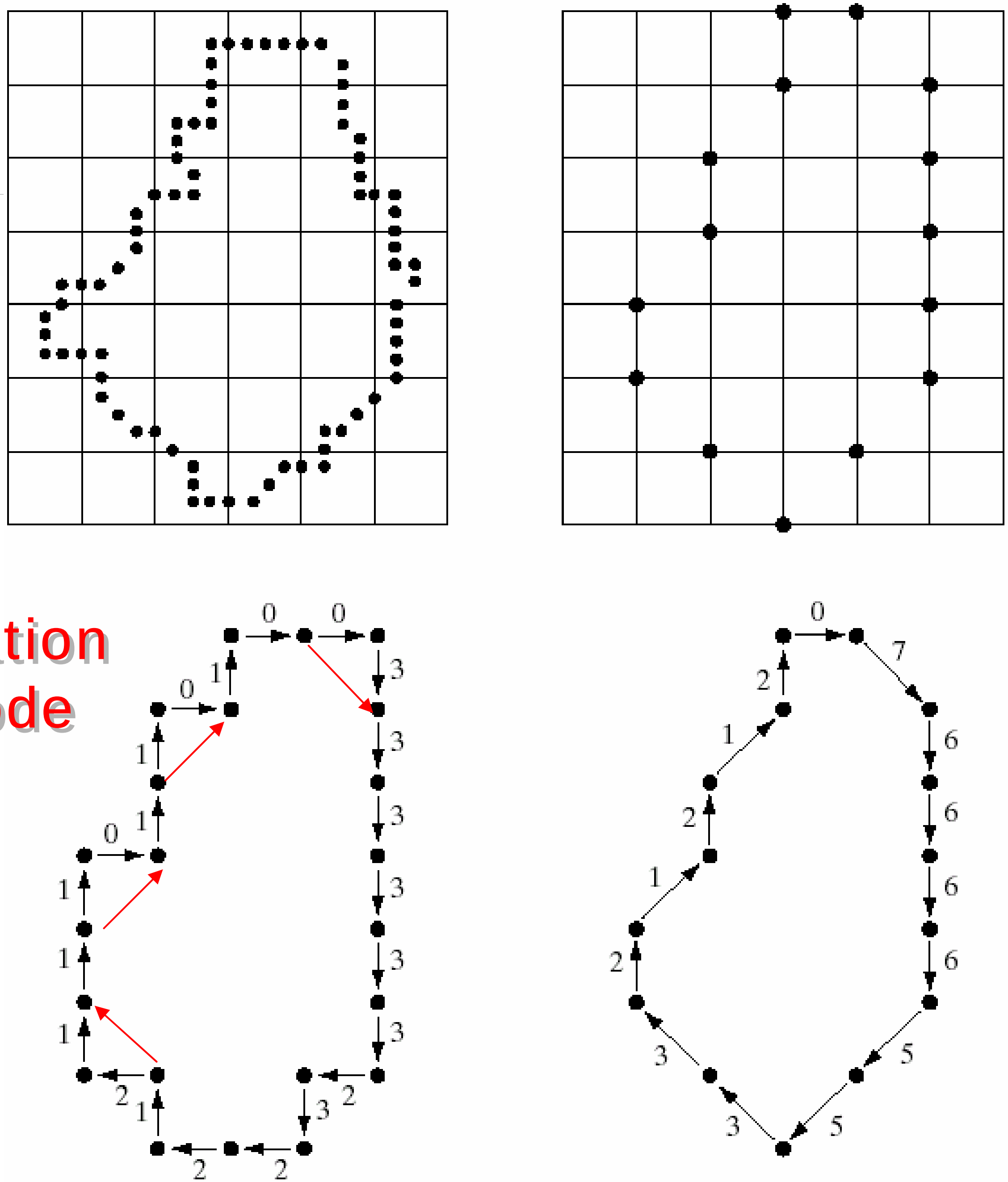


FIGURE 11.2 (a) Digital boundary with resampling grid superimposed. (b) Result of resampling. (c) 4-directional chain code. (d) 8-directional chain code.

11.1 Representation – Chain code

- The chain code of a boundary depends on the ***starting point***.
- ***Normalize*** the chain code by using the first difference of the chain code.
- ***Example***: the chain code is 10103322,

  the first difference is 3133030 or 33133030, the 1st "3" is obtained by connecting the last and the first element of the chain.
- ***Size normalization*** can be obtained by alternating the size of the sampling grid.

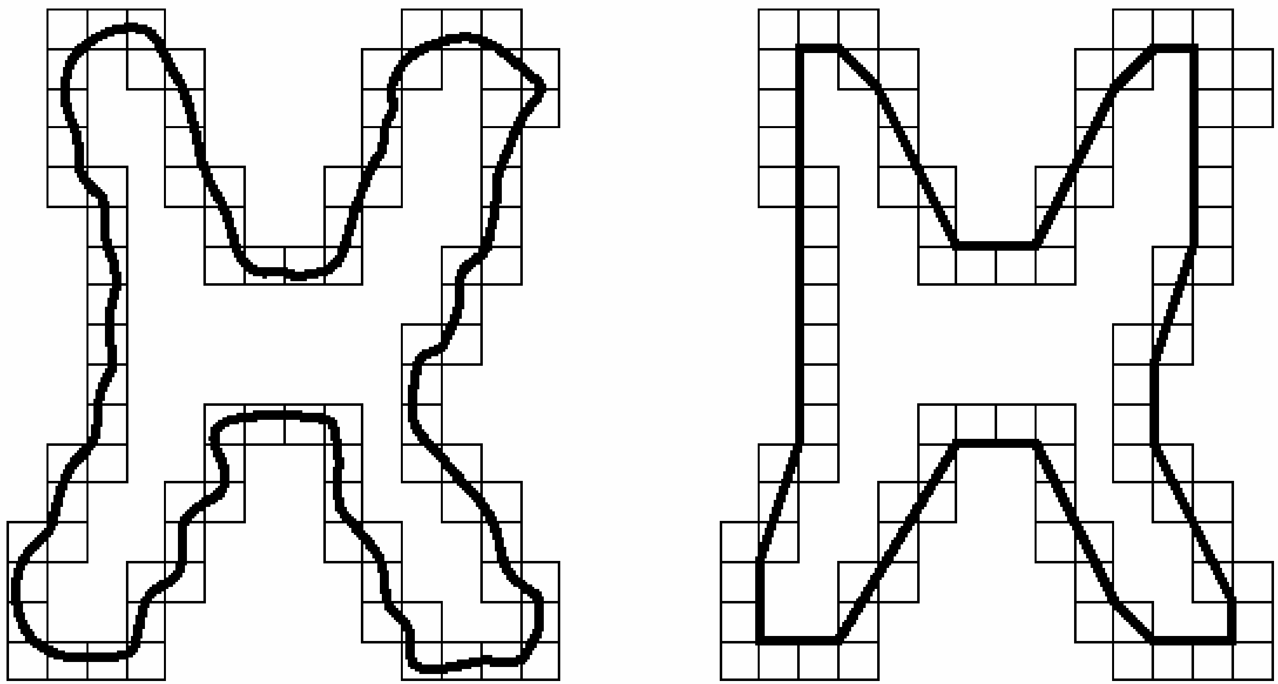
11.1 Representation - Polygon approximation

- Minimum perimeter polygons
  - Enclose the boundary by a set of concatenated cells (Fig. 11.3).
  - The enclosure has two walls corresponding to the inside and outside boundaries of the strip of cell.
  - Think of the object boundary as a rubber band contained within the wall.
  - The rubber band shrinks and produces a polygon of minimum perimeter that fit the geometry established by the cell strip.

11.1 Representation - Polygon approximation

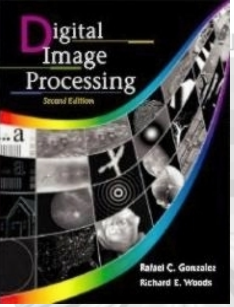
a b

FIGURE 11.3 (a) Object boundary enclosed by cells. (b) Minimum perimeter polygon.



11.1 Representation - Polygon approximation

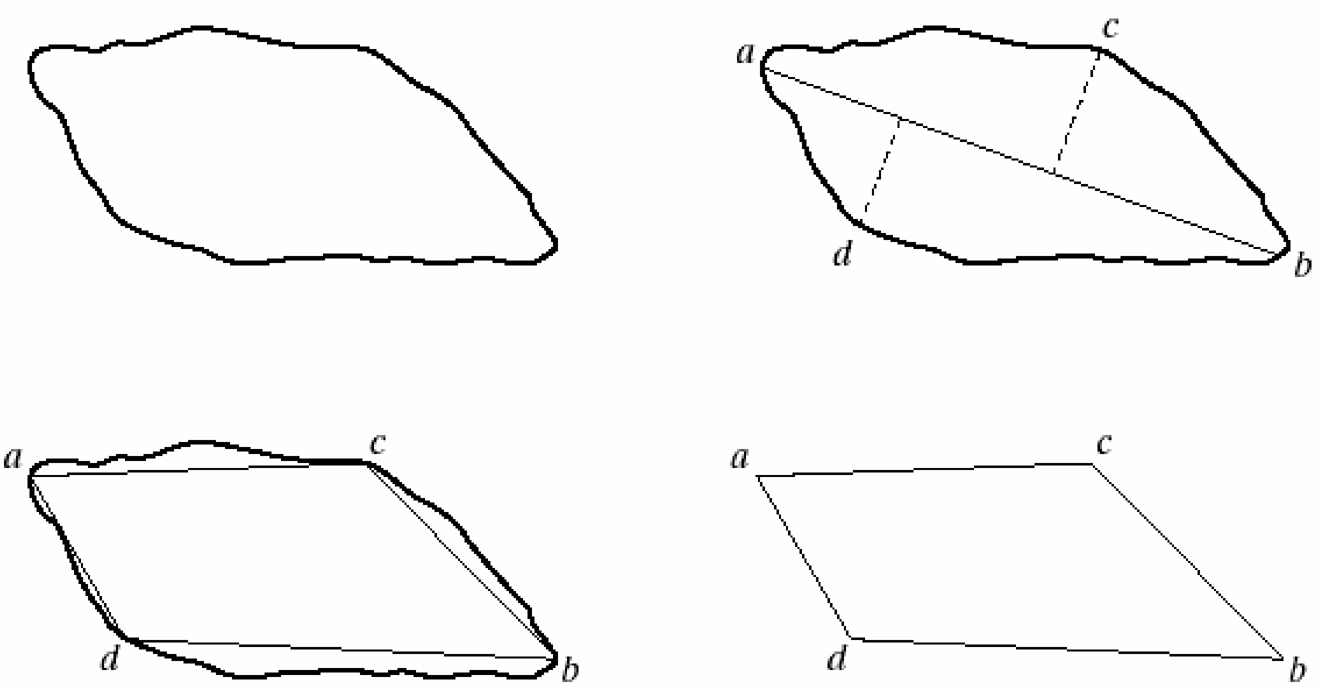
- Merging technique
  - Merge points along the boundary until the least square error line fit of the points merged so far exceeds a preset threshold.
  - Difficulties: the vertices do not always correspond to inflections (corners) in the original boundary.



11.1 Representation Polygon approximation

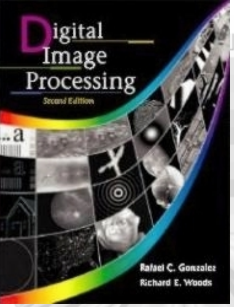
- ***Splitting techniques***:
- Subdivide a segment successively into two parts until a specified criterion is satisfied.
- The maximum perpendicular distance from a boundary segment to the line joining its two end points not exceed a preset threshold.
- If it does, the farthest point from the line become a vertex, thus subdivide the segment into two sub-segments,
- This approach has the advantage in seeking prominent inflection points

11.1 Representation - Polygon approximation



a b
c d

FIGURE 11.4
(a) Original boundary.
(b) Boundary divided into segments based on extreme points. (c) Joining of vertices.
(d) Resulting polygon.



11.1 Representation - Polygon approximation

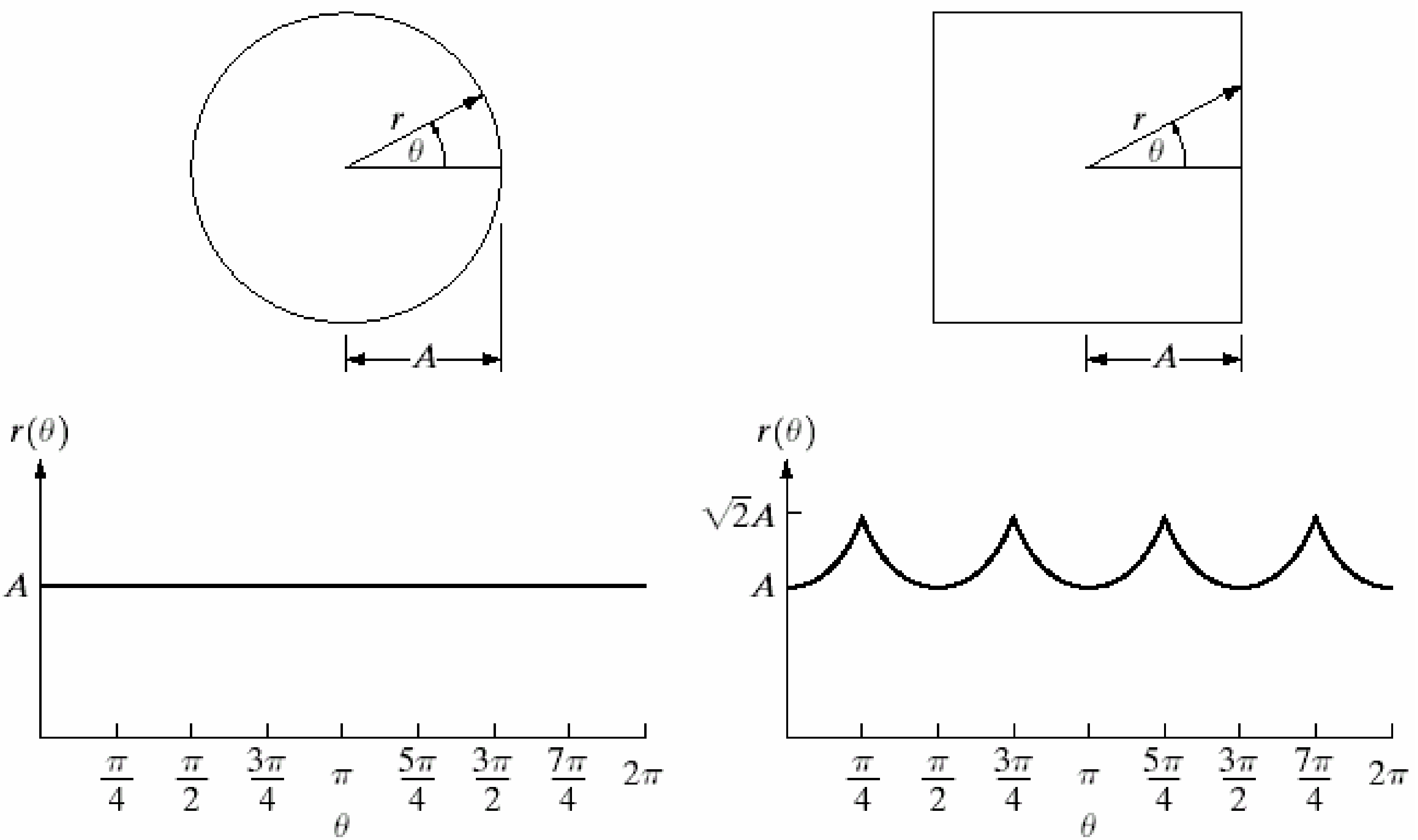
- **Signature**
- 1-D functional representation of a boundary.

  1) Plot the **distance** from the centroid to the boundary as a function of angles (Fig. 11.5), *i. e.*, r(θ).

  - Invariant to translation, but depend on the rotation and scaling.
  - Normalizing with respect to rotation.
  - Select the starting point as the point farthest to the centroid.

  2) Traverse the boundary and plot the **angle** between a line tangent to the boundary at that point and a reference line. Then use the **Slope density function**: (**histogram** of tangent-angle values) as signature.

11.1 Representation - Polygon approximation



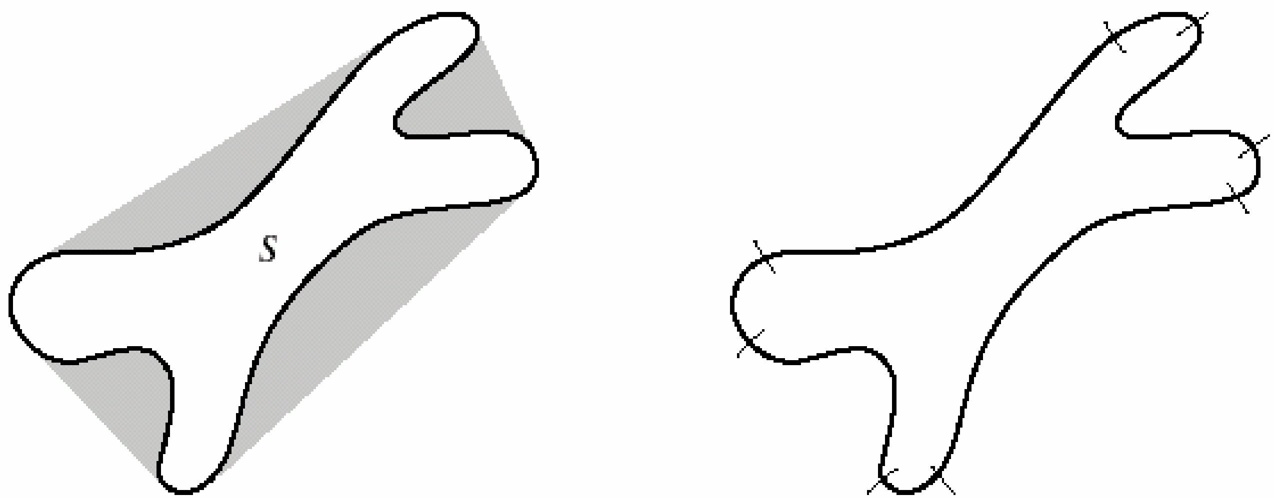
a b

FIGURE 11.5 Distance-versus-angle signatures. In (a) $r(\theta)$ is constant. In (b), the signature consists of the repetitions of the pattern $r(\theta) = A \sec\theta$ for $0 \leq \theta \leq \pi/4$ and $r(\theta) = A \csc\theta$ for $\pi/4 < \theta \leq \pi/2$.

11.1 Representation-boundary segment

- Convex hull ***H*** of an arbitrary set ***S*** is the smallest convex set containing S.
- The difference H – S is call ***convex deficiency D*** of the set S.
- The region boundary can be partitioned by following the contour of S and marking the points at which a transition is made into or out of a component of the ***convex deficiency***.
- The concept of ***convex hull*** and its ***deficiency*** are equally useful for describing an ***entire region***, as well as just its ***boundary***.

11.1 Representation-boundary segment



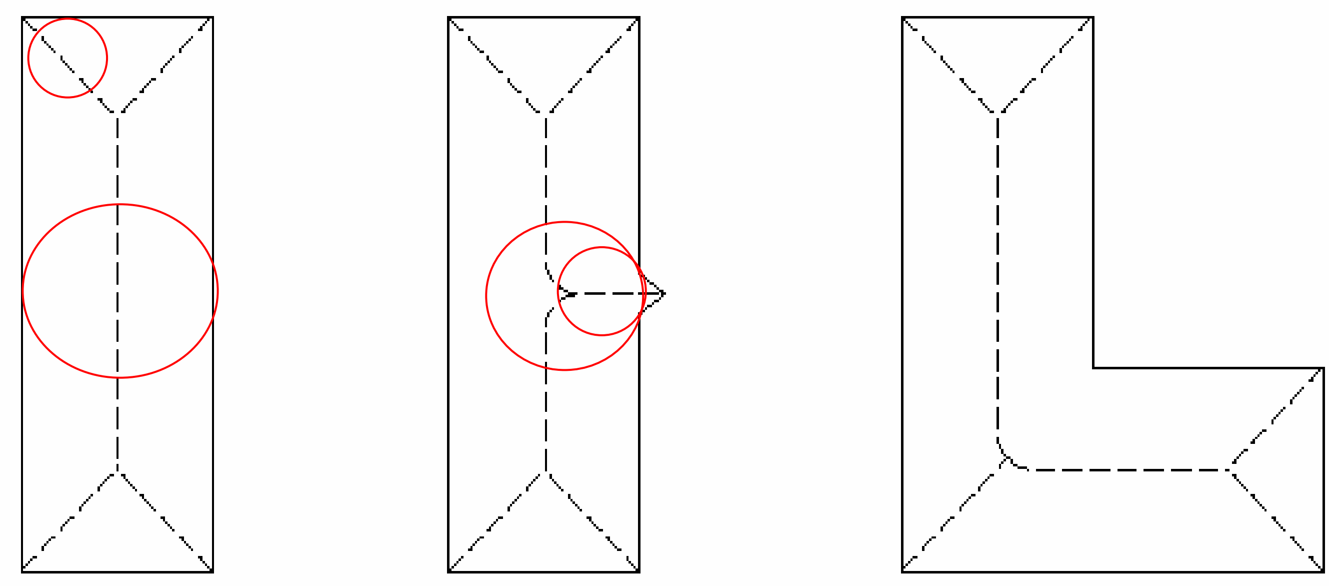
a b

FIGURE 11.6 (a) A region, *S*, and its convex deficiency (shaded). (b) Partitioned boundary.

11.1 Representation - Polygon approximation

- ***Skeleton*** of a region can be obtained by ***thinning*** algorithm
- ***Medial axis transformation*** (MAT):
  1) For each point in region ***R***, we find its closest neighbor in border ***B***.
  2) If ***p*** has more than one such neighbor, it is said to belong to the medial axis (skeleton) of ***R***.
- ***Thinning algorithm***: iteratively delete the edge points of a region subject to
  1) Does not remove the end points
  2) Does not break connectivity
  3) Does not cause excessive erosion of the region.

11.1 Representation - Polygon approximation



a b c

FIGURE 11.7 Medial axes (dashed) of three simple regions.

11.1 Representation - Polygon approximation

p_9	p_2	p_3
p_8	p_1	p_4
p_7	p_6	p_5

FIGURE 11.8 Neighborhood arrangement used by the thinning algorithm.

11.1 Representation - Polygon approximation

- **Thinning algorithm**

Step 1) flag a contour point p_1 for deletion if the following conditions are satisfied:

a) $2 \leq N(p_1) \leq 6$, where $N(p_1)$ is the number of neighbors of p_1.

b) $T(p_1)=1$, where $T(p_1)$ is number of 0-1 transitions in the ordered sequence $p_2, p_3, \ldots\ldots p_8, p_9, p_2$

c) $p_2 \bullet p_4 \bullet p_6 = 0$

d) $p_4 \bullet p_6 \bullet p_8 = 0$

If all conditions are satisfied, the point is flagged for deletion.

Step 2) Conditions (c) and (d) changed to

c') $p_2 \bullet p_4 \bullet p_8 = 0$

d') $p_2 \bullet p_6 \bullet p_8 = 0$

11.1 Representation - Polygon approximation

- **Thinning algorithm**
  1) Apply step 1 to flag border points for deletion
  2) Deleting the flagged point
  3) Apply step 2 to flag the remaining border points for deletion.
  4) Delete the flagged points

  The basic procedure is applied iteratively until no further points are deleted.
- Condition (a) is violated when p1 is the end point of a skeleton stroke.
- Condition (b) is violated when it is applied to points on stroke 1 pixel thick.

11.1 Representation - Polygon approximation

0	0	1
1	p_1	0
1	0	1

FIGURE 11.9 Illustration of conditions (a) and (b) in Eq. (11.1-1). In this case $N(p_1) = 4$ and $T(p_1) = 3$.

11.1 Representation - Polygon approximation

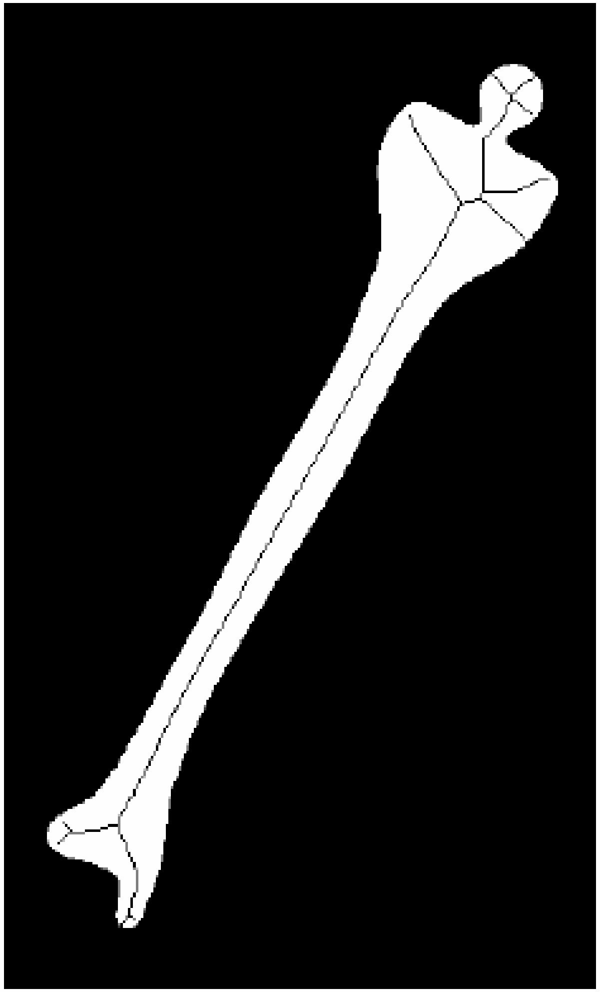


FIGURE 11.10 Human leg bone and skeleton of the region shown superimposed.

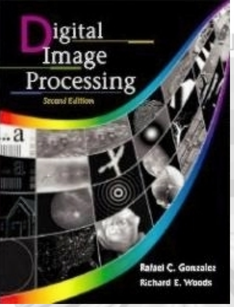
11.2 Boundary descriptor

- Simple descriptors

1) Length
2) Diameter: Diam(B)=*max*[D(p_i, p_j)] where p_i and p_j are points on the boundary.
3) Major axis and minor axis
4) Basic rectangle
5) Eccentricity = major axis/minor axis
6) Curvature: changes of slope.
7) Point *p* belongs to a segment which is convex if the change of slope at *p* is nonnegatoive and concave otherwise.
8) P is a *corner* depends on the curvature.

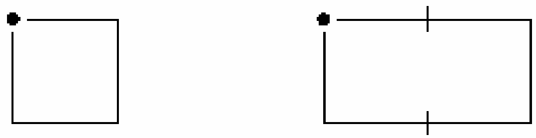
11.2 Boundary Description - shape number

- The ***first difference*** of a chain-coded boundary depends on the starting point.
- The ***shape number*** of a chain coded boundary is defined as the first difference of smallest magnitude.
- The ***difference*** of a chain code is independent of it rotation, it depend on the orientation of the grid.
- The order ***n*** of a shape number is defined as the number of digits in its representation.



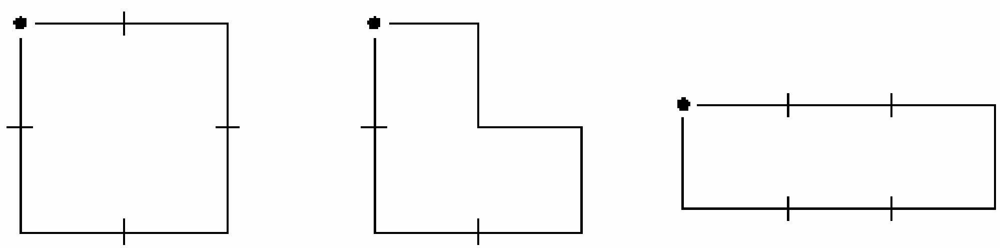
11.2 Boundary Description- shape number

Order 4 | Order 6



	Order 4	Order 6
Chain code:	0 3 2 1	0 0 3 2 2 1
Difference:	3 3 3 3	3 0 3 3 0 3
Shape no.:	3 3 3 3	0 3 3 0 3 3

Order 8



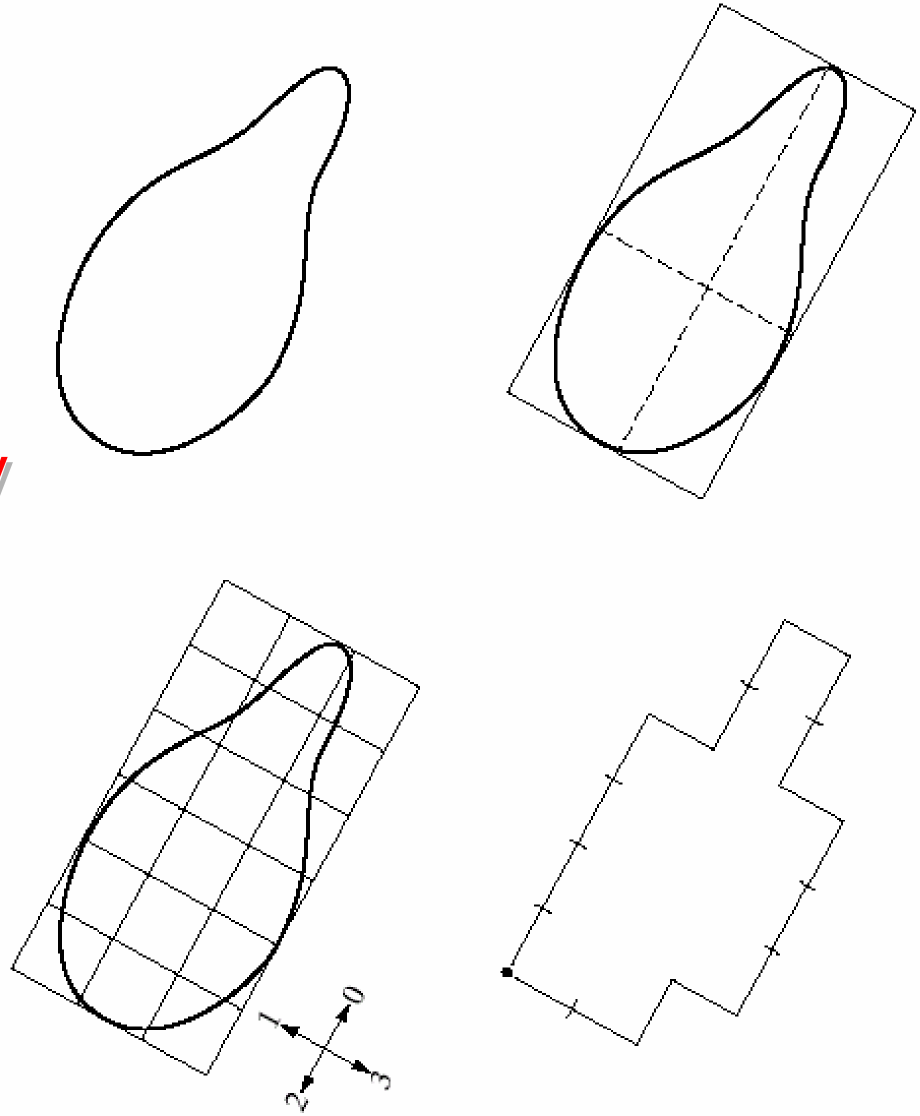
		Order 8	
Chain code:	0 0 3 3 2 2 1 1	0 3 0 3 2 2 1 1	0 0 0 3 2 2 2 1
Difference:	3 0 3 0 3 0 3 0	3 3 1 3 3 0 3 0	3 0 0 3 3 0 0 3
Shape no.:	0 3 0 3 0 3 0 3	0 3 0 3 3 1 3 3	0 0 3 3 0 0 3 3

FIGURE 11.11 All shapes of order 4, 6, and 8. The directions are from Fig. 11.1(a), and the dot indicates the starting point.

11.2 Boundary Description- shape number

- Example (Fig. 11.12)
- 1. Find the basic rectangle for n=18 (boundary)
- 2. Find the major and minor axis
- 3. Find the closest rectangle of order 18 is 3x6
- 4. obtain chain code
- 5. find the difference
- 6. find the shape no.

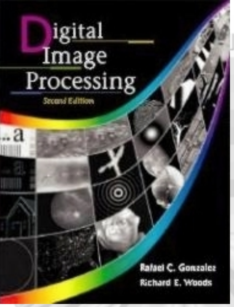
11.2 Boundary Description



a b
c d

FIGURE 11.12 Steps in the generation of a shape number.

Chain code:	0	0	0	0	3	0	0	3	2	2	3	2	2	2	1	2	1	1
Difference:	3	0	0	0	3	1	0	3	3	0	1	3	0	0	3	1	3	0
Shape no.:	0	0	0	3	1	0	3	3	0	1	3	0	0	3	1	3	0	3



11.2 Boundary Description–Fourier Descriptor

- For a K-point digital boundary, starting at an arbitrary point (x_0, y_0), K coordinate pairs (x_0, y_0), (x_{01}, y_{01}), ….,(x_{K-1}, y_{K-1}) are encountered in counterclockwise direction.
- Let $s(k)=[x(k), y(k)]$ for $k=0,1,\ldots K-1$,

  or $s(k)=x(k)+jy(k)$
- The *1-D DFT* of $s(k)$ is $a(u)=\frac{1}{K}\sum_{k=0}^{K-1} s(k)e^{-j2\pi uk/K}$
- The inverse *DFT* of $a(u)$ *is* $s(k)=\frac{1}{K}\sum_{k=0}^{K-1} a(u)e^{j2\pi uk/K}$

11.2 Boundary Description- Fourier Descriptor

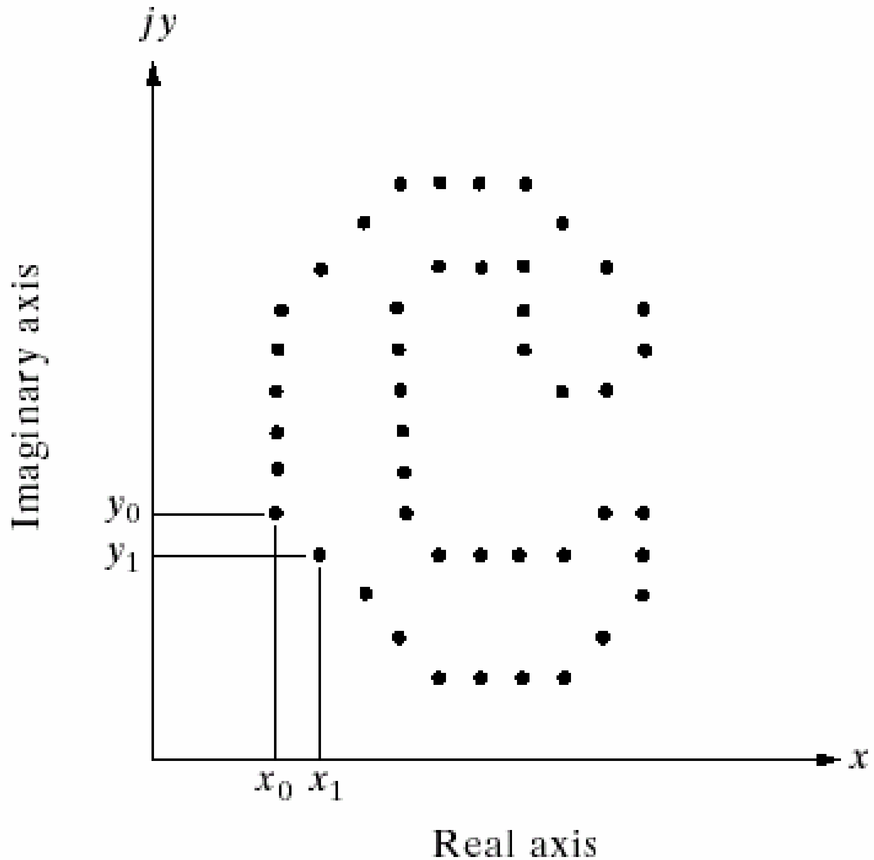


FIGURE 11.13 A digital boundary and its representation as a complex sequence. The points (x_0, y_0) and (x_1, y_1) shown are (arbitrarily) the first two points in the sequence.

11.2 Boundary Description - Fourier Descriptor

- If only the first P coefficients (P<K) are used then

$$\hat{s}(k) = \sum_{u=0}^{P-1} a(u) e^{j2\pi uk/K}$$

- The coefficients $\{a(u)\}$ carry shape information which are insensitive to translation, rotation, and scale change of the shape.
- The descriptors are insensitive to the change of starting point.
- Rotation of a point by an angle θ about the origin of the complex plane is accomplish by multiplying the point by $e^{j\theta}$.
- The rotated sequence $s(k)e^{j\theta}$ whose Fourier descriptors are

$$a_r(u) = \frac{1}{K} \sum_{k=0}^{K-1} s(k) e^{j\theta} e^{-j2\pi uk/K} = a(u) e^{j\theta}$$

11.2 Boundary Description - Fourier-Descriptor

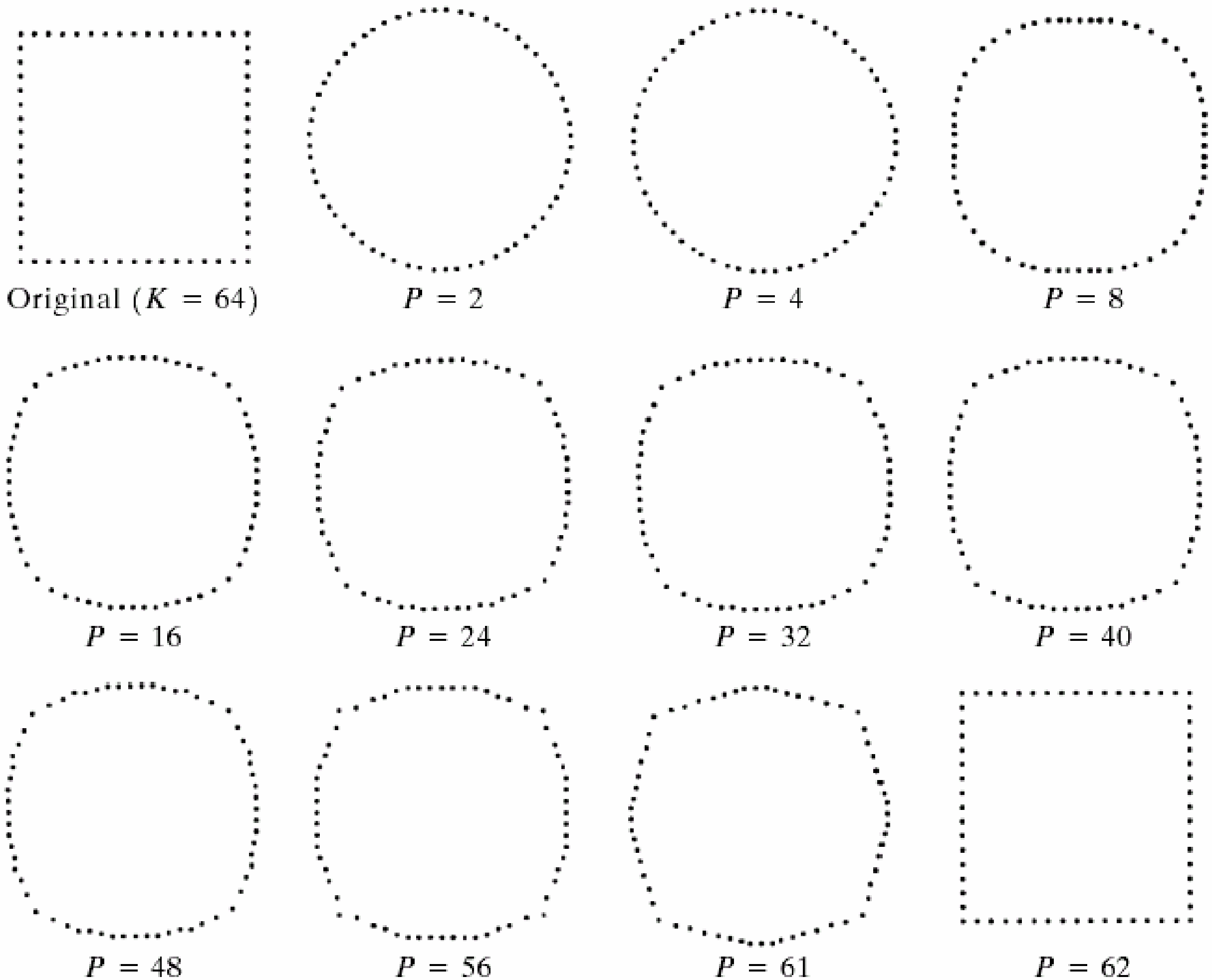
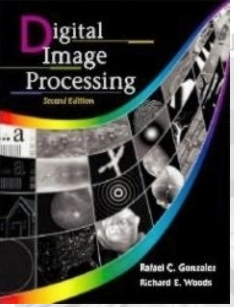


FIGURE 11.14 Examples of reconstruction from Fourier descriptors. P is the number of Fourier coefficients used in the reconstruction of the boundary.



11.2 Boundary Description -Fourier-Descriptor

Transformation	Boundary	Fourier Descriptor
Identity	$s(k)$	$a(u)$
Rotation	$s_r(k) = s(k)e^{j\theta}$	$a_r(u) = a(u)e^{j\theta}$
Translation	$s_t(k) = s(k) + \Delta_{xy}$	$a_t(u) = a(u) + \Delta_{xy}\delta(u)$
Scaling	$s_s(k) = \alpha s(k)$	$a_s(u) = \alpha a(u)$
Starting point	$s_p(k) = s(k - k_0)$	$a_p(u) = a(u)e^{-j2\pi k_0 u/K}$

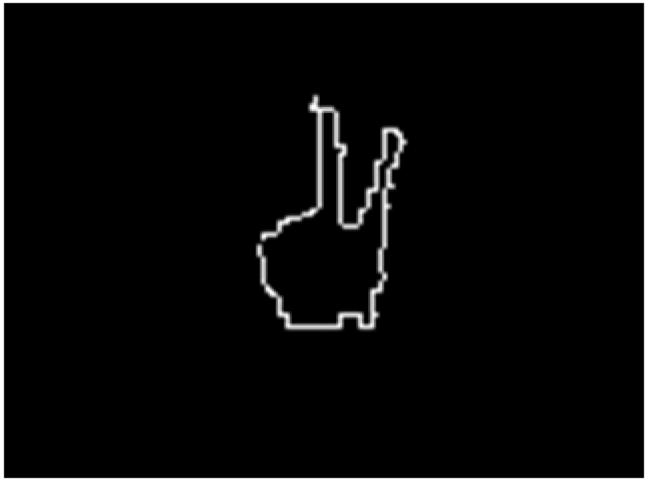
TABLE 11.1 Some basic properties of Fourier descriptors.

1) Translation: $s_t(k)=s(k)+\Delta_{xy}=[x(k)+\Delta x]+j[y(k)+\Delta y]$

2) Change the starting point of the sequence to $k=k_0$ *from* $k=0$ *as*

$s_p(k)=s(k-k_0)=x(k-k_0)+j\,y(k-k_0)$

11.2 Boundary Description – Fourier Descriptor



The contour of hand silhouette.

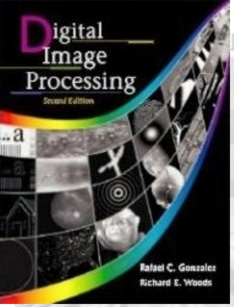
11.2 Fourier Descriptor

- Fourier series of a sequence of points $\{x(m), y(m)\}$ can be defined as

$$x(m) = \sum a(n) e^{j2\pi nm/N} \quad y(m) = \sum b(n) e^{j2\pi nm/N}$$

where a(n) and b(n) are the Fourier coefficient

$$a(n) = \sum_{m=1}^{N} x(m) e^{-j2\pi nm/N} \quad b(n) = \sum_{m=1}^{N} y(m) e^{-j2\pi nm/N}$$



11.2 Fourier Descriptor

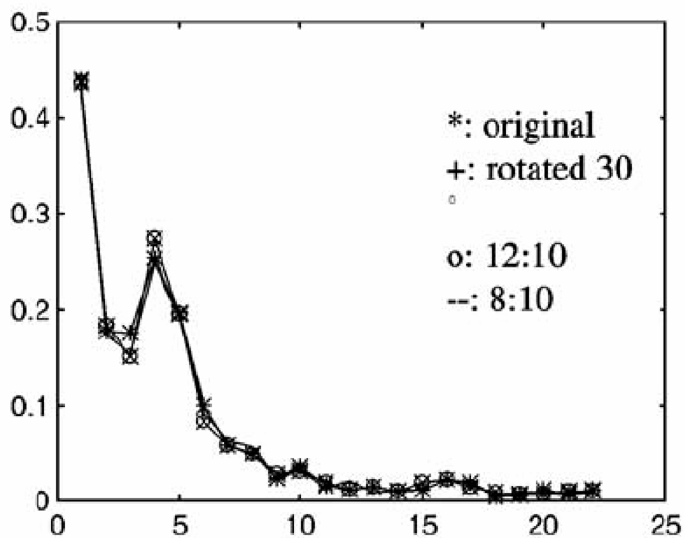
- Assuming local variation of hand shape is smooth so that the higher order terms of the Fourier descriptor are not necessary.
- To normalize the size of hand gesture we let S(n)=r(n)/r(1) (normalization), and we have

$$r(n) = \sqrt{|a(n)|^2 + |b(n)|^2} \qquad n=1,2,\ldots,22$$

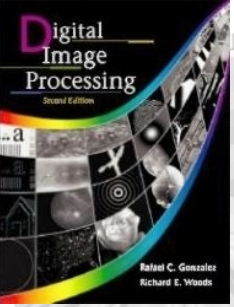
- Using 22 harmonics of the FD's coefficient, $S(n)$, is enough to describe the macroscopic information of the hand shape.
- FD is translation, rotation, and scaling invariance.

11.2 Fourier Descriptor

(a) Original image (b) Rotation 90° (c) Zoom in 12:10 (d) Zoom out 8:10



(e) Fourier descriptor vectors of the four different shapes without the first term.



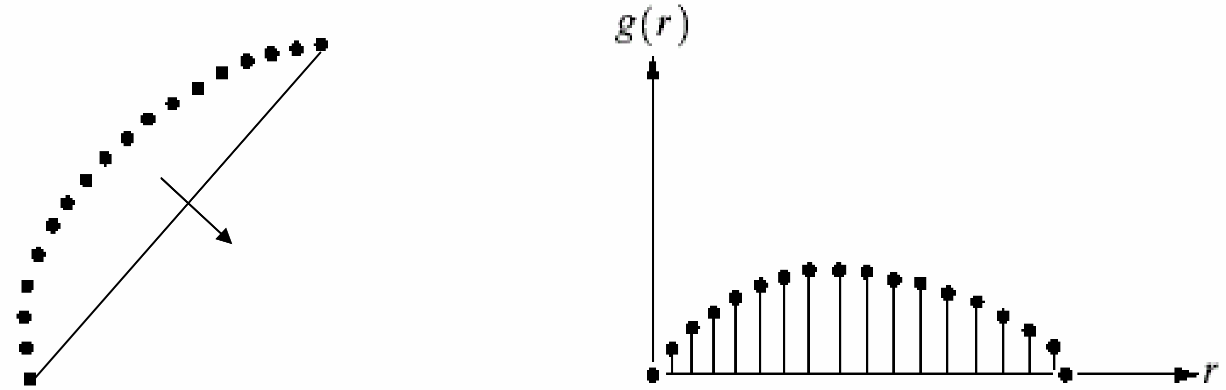
11.2 Boundary Description -Statistical moment

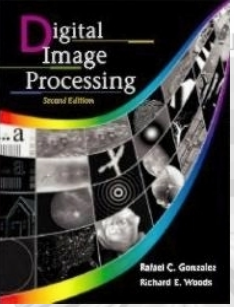
- The shape of boundary segments can be described quantitatively by using simple statistical moments such as *mean*, *variance*, and *higher-order moments*.
- Figure 11.5 represented as 1-D function $g(r)$.
- Treat the amplitude of g as a discrete random variable v and form an amplitude histogram $p(v_i)$, $i=0,1,\ldots A-1$, where A is the number of discrete amplitude increments in which we divide the amplitude scale.

11.2 Boundary Description -Statistical moment

a b

FIGURE 11.15
(a) Boundary segment.
(b) Representation as a 1-D function.





11.2 Boundary Description -Statistical moment

- The *nth* moment of *v* about its mean *m* is

$$\mu_n(v) = \sum_{i=0}^{A-1} (v_i - m)^n p(v_i)$$

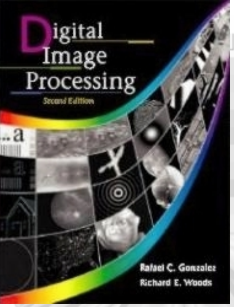
where the mean is $$m = \sum_{i=0}^{A-1} v_i p(v_i)$$

- The m is the mean and μ_2 is the variance.

11.2 Boundary Description -Statistical moment

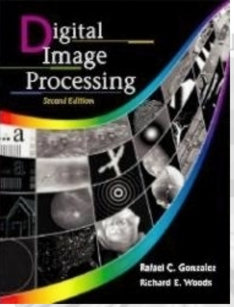
- An alternative approach is normalize $g(r)$ to unit area and treat it as histogram.
- $g(r_i)$ is treated as the probability of value r_i occuring.
- The moments are $\mu_n(r) = \sum_{i=0}^{K-1} (r_i - m)^n g(r_i)$

where $m = \sum_{i=0}^{K-1} r_i g(r_i)$

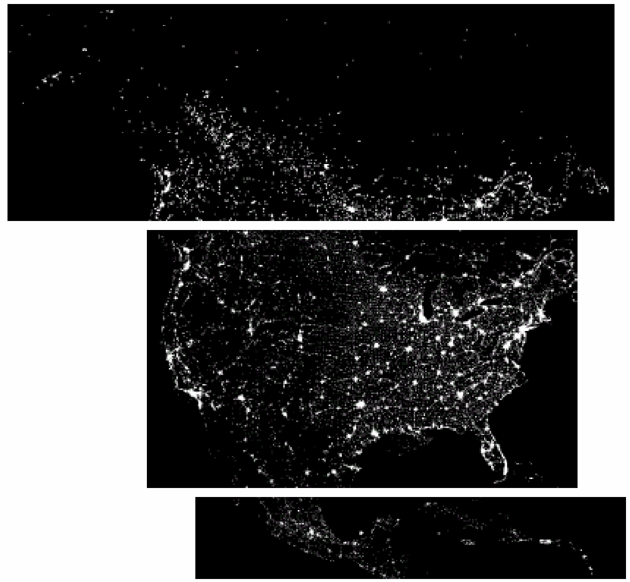


11.3 Regional Descriptors -Simple Descriptor

- Area is the number of pixels in the regions
- Perimeter is the length of the boundary.
- Compactness=(perimeter)^2/area.



11.3 Regional Descriptors

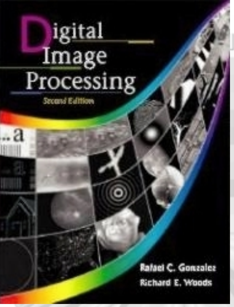


Region no, (from top)	Ratio of lights per region to total lights
1	0.204
2	0.640
3	0.049
4	0.107

FIGURE 11.16 Infrared images of the Americas at night. (Courtesy of NOAA.)

11.3 Regional Descriptors -Topological Descriptor

- Topology is the study of properties of a figure that are unaffected by any deformation (rubber-sheet distortion).
- The number of ***holes***: H
- The number of ***connected components***: C
- ***Euler number*** E: $E=C-H$.



11.3 Regional Descriptors -Topological Descriptor

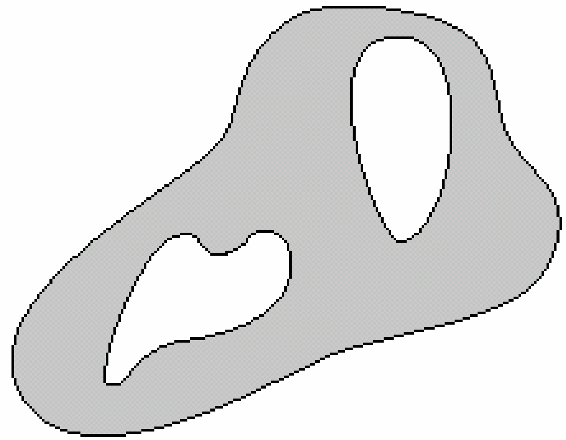


FIGURE 11.17 A region with two holes.

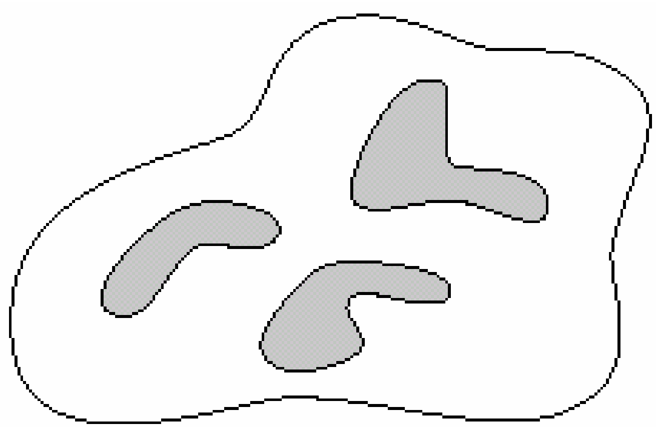


FIGURE 11.18 A region with three connected components.

11.3 Regional Descriptors -Topological Descriptor

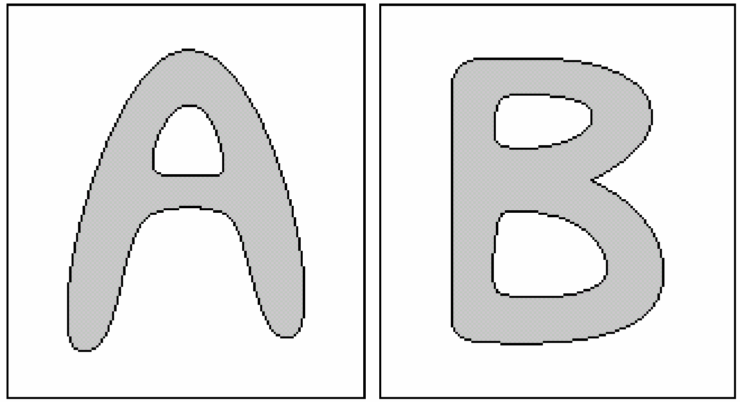
- Regions represented by straight-line segments (polygonal networks), such as Fig. 11.20, has the following relationship in topology as

$$E=V-Q+F=C-H$$

where V is the number of vertices and Q is the number of edges.

- Segmentation is based on the thresholding.
- How the connected components can be used to "finish" the segmentation.
- Figure 11.21(b) has 1591 connected components, C=1591, and its Euler number E=1552, and H=39. Figure 11.21(c) shows the connected component with 8479 elements

11.3 Regional Descriptors -Topological Descriptor



a b

FIGURE 11.19 Regions with Euler number equal to 0 and −1, respectively.

$$V-Q+F = C-H = E$$

$$7-11+2 = 1-3 = -2$$

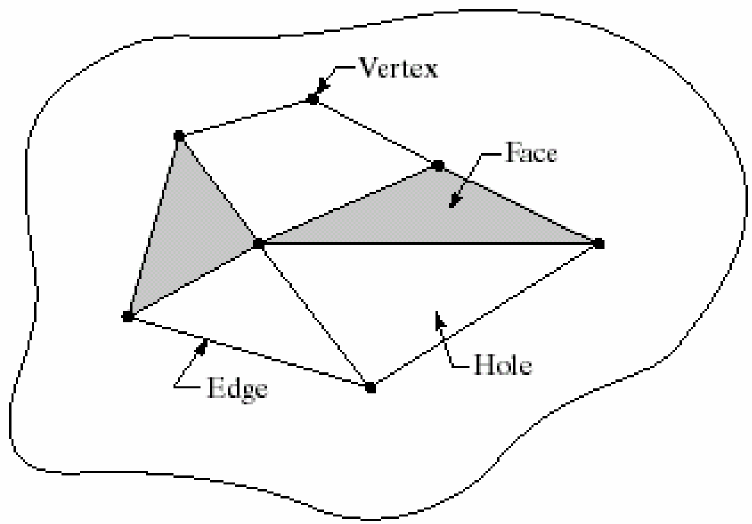
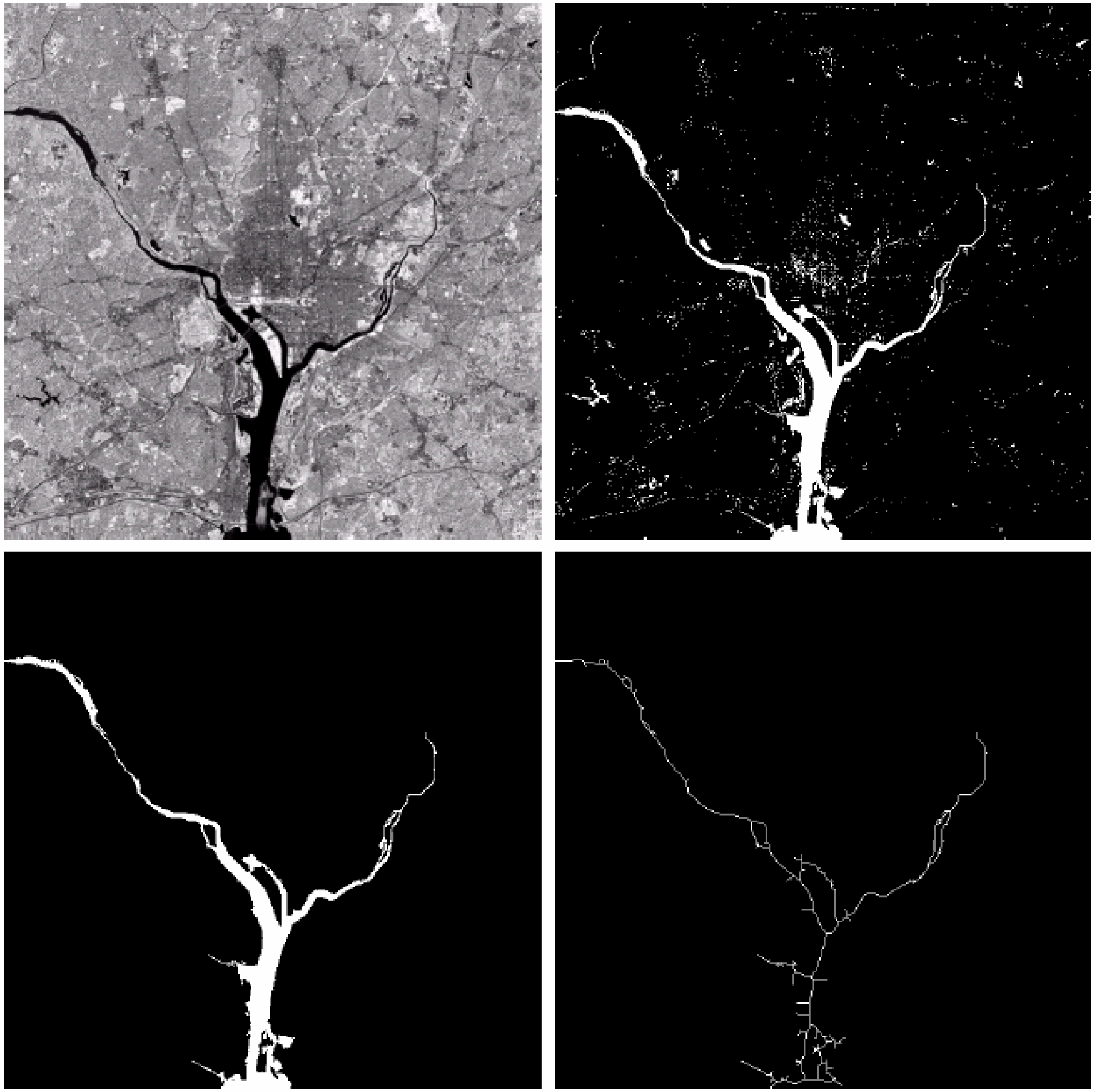


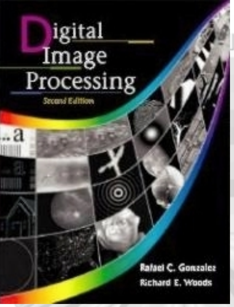
FIGURE 11.20 A region containing a polygonal network.

11.3 Regional Descriptors -Topological Descriptor



a b
c d

FIGURE 11.21 (a) Infrared image of the Washington, D.C. area. (b) Thresholded image. (c) The largest connected component of (b). Skeleton of (c).



11.3 Regional Descriptors -Texture

- The texture measurement provides the properties such as ***smoothness, coarseness, and regularity***.
- Three principal approaches: *statistical, structure, and spectral*.
- ***Statistical approaches:***

Let z be a random variable and $p(z_i)$, $i=0,1,\ldots L-1$ is the corresponding histogram, L is the number of gray-levels.

The nth moment of z about the mean (m) is

$$\mu_n(z) = \sum_{i=0}^{L-1} (z_i - m)^n g(z_i)$$

The second moment μ_2 (=variance σ^2) can be used to define the measure R as

$$R = 1 - \frac{1}{1+\sigma^2(z)}$$

R=0 (for constant density, σ =0), R→1 (for large σ)

11.3 Regional Descriptors -Texture

- ***Statistical approach***

The 2nd moment μ_2 (=variance σ^2) is used to measure the contrast.

The 3rd moment μ_3 is used to measure the skewness of the histogram.

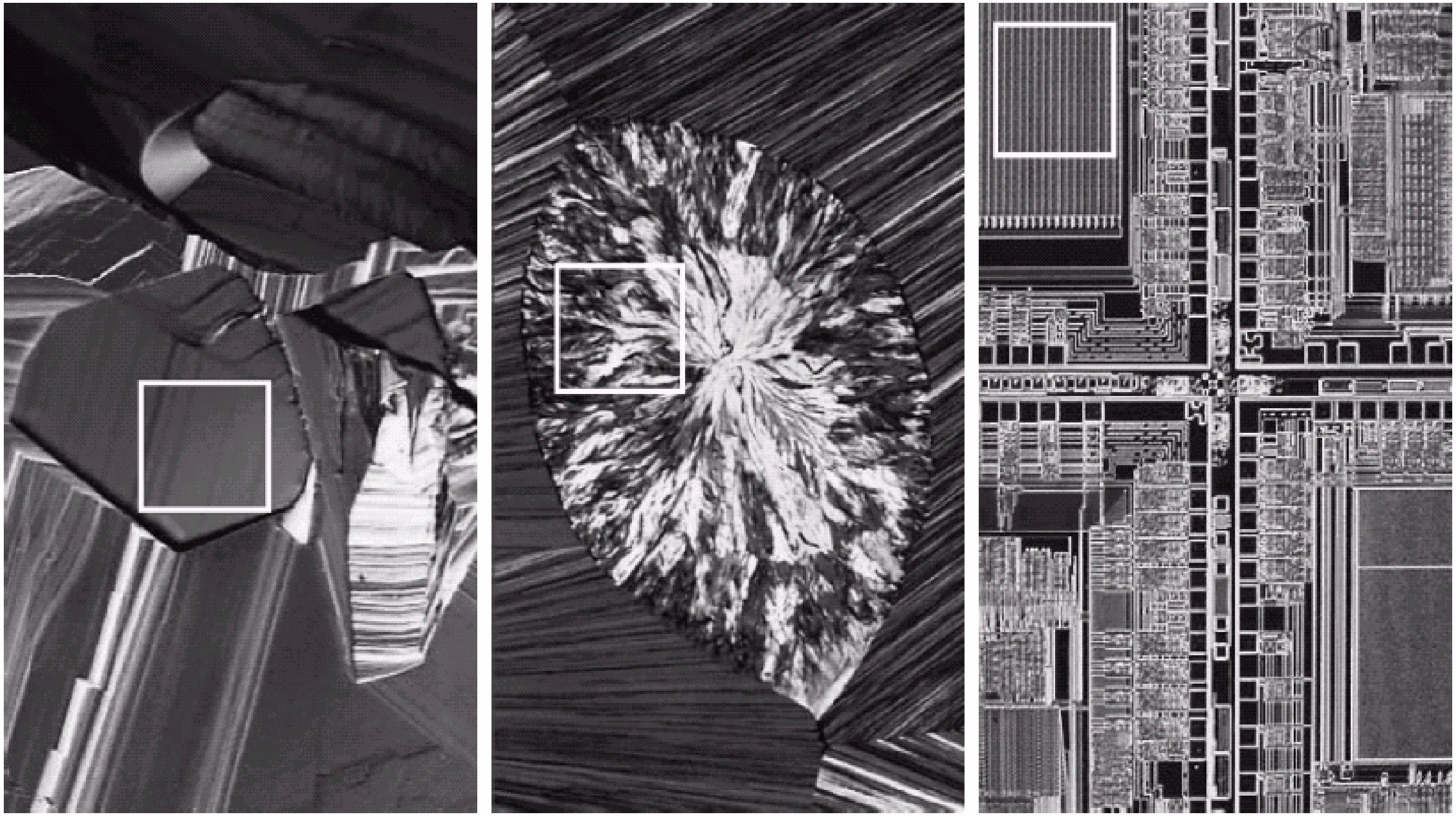
The 4th moment μ_4 is used to measure the relative flatness of the histogram.

The measure of "uniformity" of the histogram as $U = \sum_{i=0}^{L-1} p^2(z_i)$

The average entropy measure as $e = -\sum_{i=0}^{L-1} p(z_i) \log_2 p(z_i)$

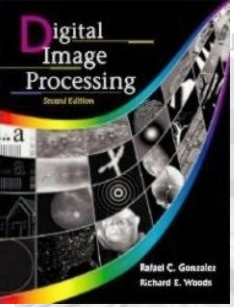
This approach measure no information regarding to the relative position of pixels with respect to each other.

11.3 Regional Descriptors -Texture



a b c

FIGURE 11.22 The white squares mark, from left to right, smooth, coarse, and regular textures. These are optical microscope images of a superconductor, human cholesterol, and a microprocessor. (Courtesy of Dr. Michael W. Davidson, Florida State University.)



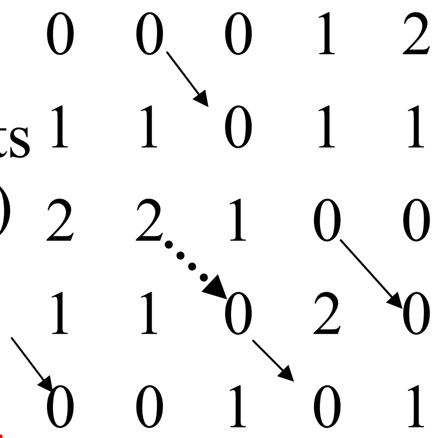
11.3 Regional Descriptors -Texture

TABLE 11.2
Texture measures for the subimages shown in Fig. 11.22.

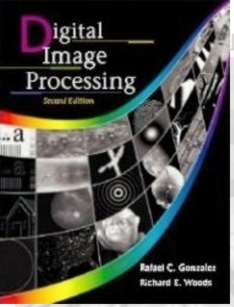
Texture	Mean	Standard deviation	*R* (normalized)	Third moment	Uniformity	Entropy
Smooth	82.64	11.79	0.002	−0.105	0.026	5.434
Coarse	143.56	74.63	0.079	−0.151	0.005	7.783
Regular	99.72	33.73	0.017	0.750	0.013	6.674

11.3 Regional Descriptors -Texture

- Let ***P*** be a position operator, ***A*** be a k×k matrix whose element a_{ij} is the number of times that points with gray level z_i,occur (in position specified by ***P***) relative to points with gray level z_j, with $1 \leq i, j \leq k$.
- For example, an image with $z_1=0$, $z_2=1$, $z_3=2$ as
- Define the position operator ***P*** as "***one pixel below and one-pixel to the right*** yields a 3×3 matrix **A** as
- a_{11} is the number of times that a point with level z_1=0 appears related with another point ***of the same level***
- a_{13} is the number of times that a point with level z_1=0 appears related with another point with gray-level z_3=2



$$A = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 0 & 2 & 0 \end{bmatrix}$$



11.3 Regional Descriptors -Texture

- Let *n* be the number of point pairs in the image that satisfy P (*n=16*).
- If ***C***=***A***/n then c_{ij} is the estimate of the joint probability that a pair of points satisfying ***P*** will have values (z_i, z_j)
- The matrix **C** is called ***gray-level co-occurence matrix***.
- ***C*** depends on ***P***.
- To analyze a given ***C*** to categorize the texture of region over which ***C*** was computed.

11.3 Regional Descriptors -Texture

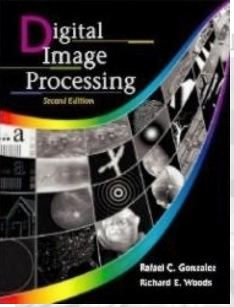
- A set of descriptors based on **C** are

1) Maximum probability
2) Element difference moment of order k
3) Inverse element difference moment of order k
4) Uniformity
5) Entropy

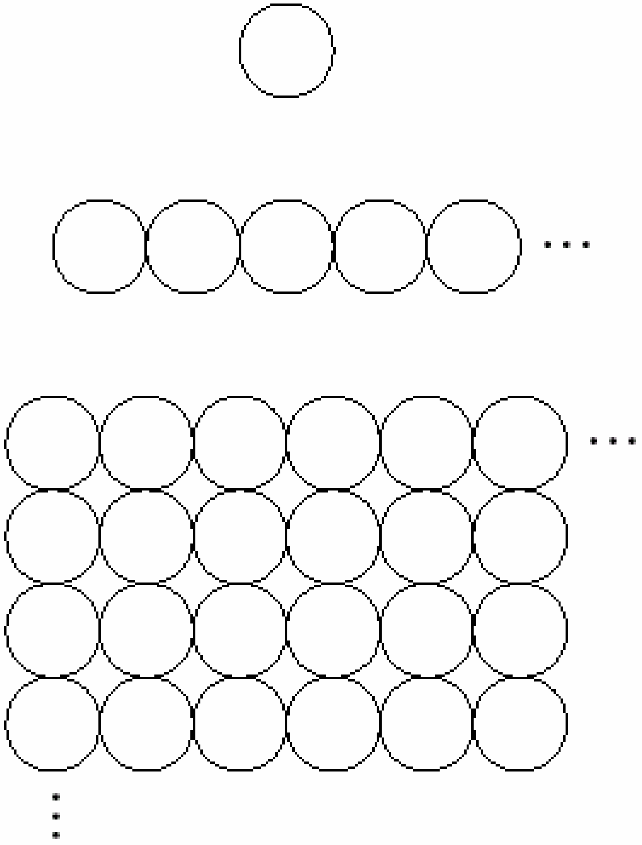
11.3 Regional Descriptors -Texture

- ***Structural approach***: a simple "texture" primitive can be used to form more complex texture pattern.

1) Define a rule of the form : $S \rightarrow aS$, which indicates that the symbol S may be written as aS.
2) Let a represents a circle, and the meaning of "***circles to the right***" is assign a string of the form *aaa*..., and the rule $S \rightarrow aS$ generates *Fig11.23(b)*.
3) Define new rules: $S \rightarrow bA$, $A \rightarrow cA$, $A \rightarrow c$, $A \rightarrow bS$, $S \rightarrow a$, where b represents "***circle down***" and c means "***circle to the left***"
4) Generate a string of the form *aabccbaa* that corresponding to a 3×3 matrix of circles.

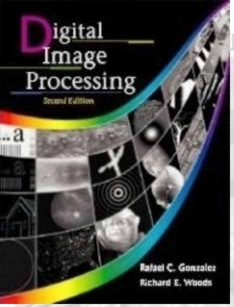


11.3 Regional Descriptors -Texture



a
b
c

FIGURE 11.23
(a) Texture primitive.
(b) Pattern generated by the rule $S \rightarrow aS$.
(c) 2-D texture pattern generated by this and other rules.



11.3 Regional Descriptors -Texture

- ***Spectral approach***
- Fourier spectrum is suitable for describing the directionality of periodic in 2-D image.
- Three features in Fourier spectrum:

1) ***Prominent peaks*** give the principal direction of the texture patterns.
2) The ***location of the peaks*** give the fundamental spatial period of the patterns.
3) By filtering the periodic component, the other non-periodic pattern can be described by statistical technique.

11.3 Regional Descriptors -Texture

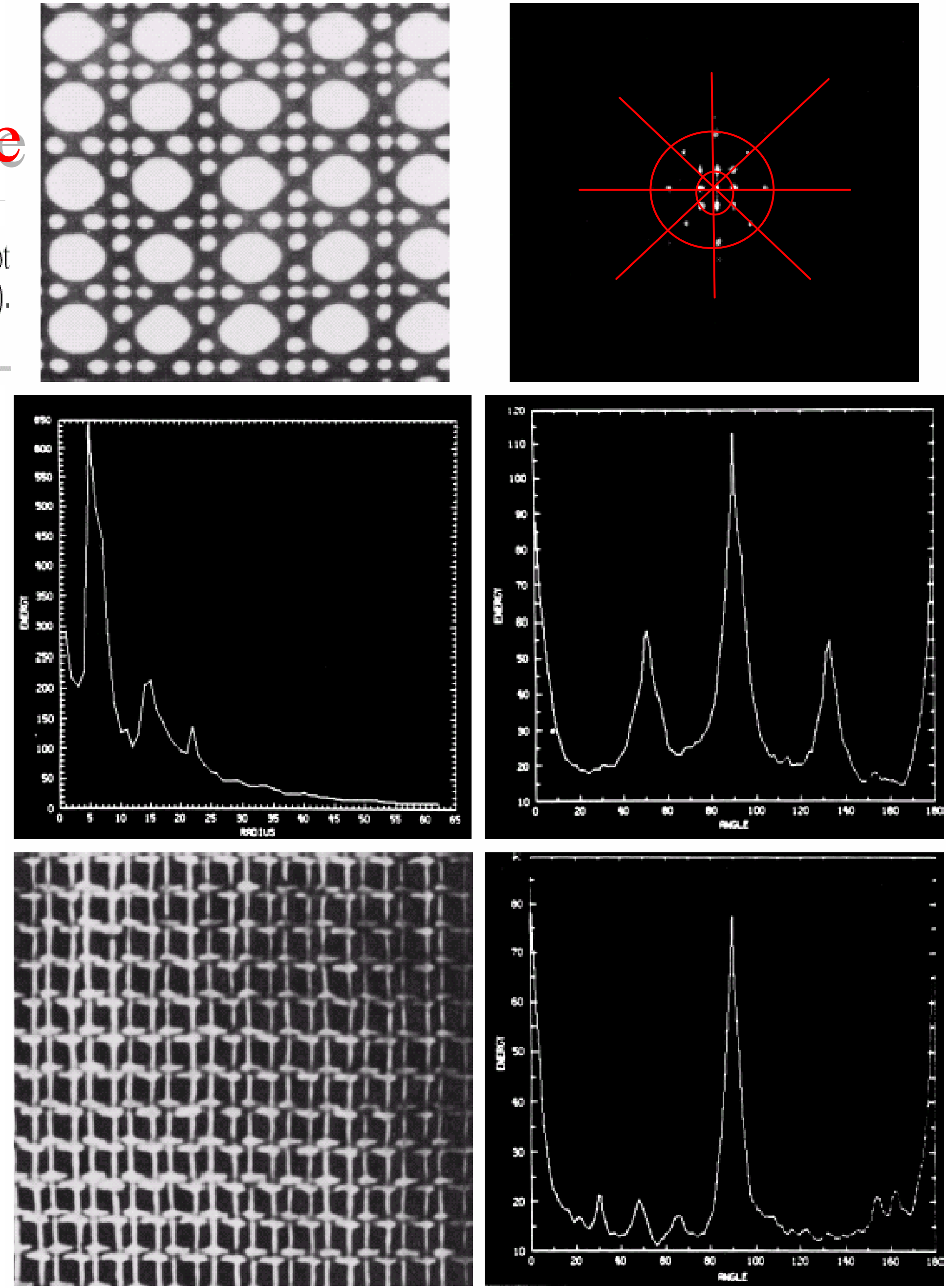
- ***Spectral approach***
- Express the spectral in polar coordinates as $S(r, \theta)$.
- For each direction θ, we have a 1-D expression of the spectrum as $S_\theta(r)$.
- Global description as $S(r)=\Sigma_\theta S_\theta(r)$.
- For each frequency r, we have a 1-D expression of the spectrum as $S_r(\theta)$.
- Global description as $S(\theta)=\Sigma_r S_r(\theta)$.
- Constitute $[S(r),S(\theta)]$ for each pair of (r, θ)

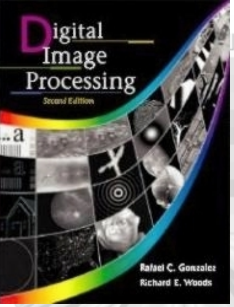
11.3 Regional Descriptors -Texture

a b
c d
e f

FIGURE 11.24 (a) Image showing periodic texture. (b) Spectrum. (c) Plot of $S(r)$. (d) Plot of $S(\theta)$. (e) Another image with a different type of periodic texture. (f) Plot of $S(\theta)$. (Courtesy of Dr. Dragana Brzakovic, University of Tennessee.)

S(r)





11.3 Regional Descriptors -Texture

- ***Moment of two dimensional functions***
- For 2-D continuous function $f(x, y)$, the moment of order $(p+q)$ is defined as

$$m_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^p y^q f(x,y)dxdy = \sum_x \sum_y x^p y^q f(x,y)$$

- The central moments are

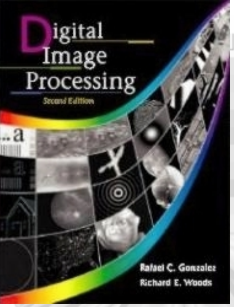
$$\mu_{pq} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\bar{x})^p (y-\bar{y})^q f(x,y)dxdy$$

. or

$$\mu_{pq} = \sum_x \sum_y (x-\bar{x})^p (y-\bar{y})^q f(x,y)$$

where $\bar{x} = m_{10}/m_{00}$ and $\bar{y} = m_{01}/m_{00}$

The central moments are μ_{00},($=m_{00}$), μ_{10} (=0), μ_{01}(=0), μ_{11}, μ_{20}, μ_{02}, μ_{21}, μ_{12},….



11.3 Regional Descriptors -Texture

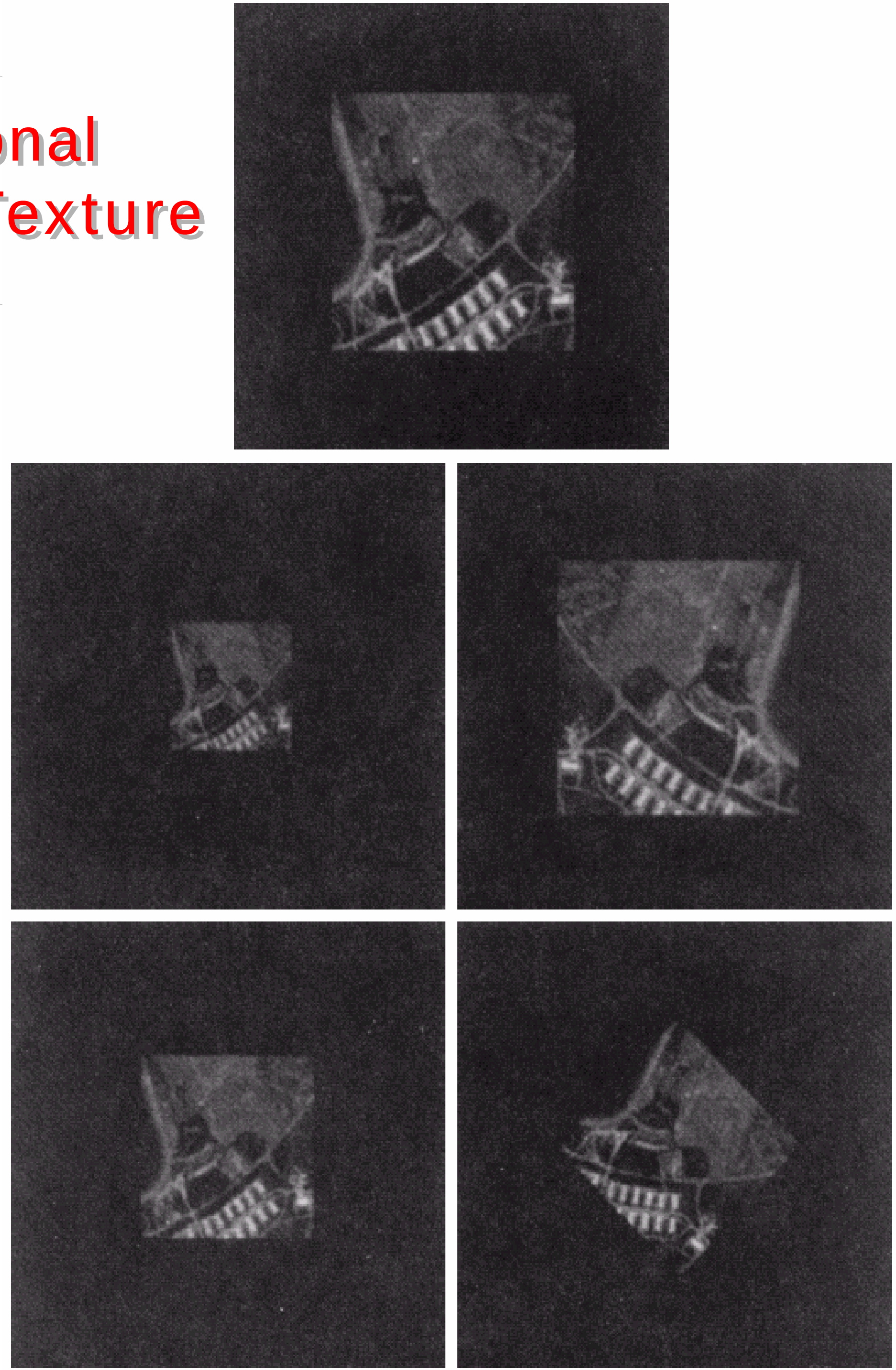
- The ***normalized central moment*** is defined as

$$\eta_{pq} = \mu_{pq} / \mu_{00}^{\gamma}$$

where $\gamma = (p+q)/2+1$, for $p+q=2, 3, \ldots$

- Seven invariant moments $\Phi_1, \ldots.\ \Phi_7$ are shown in textbook
- Examples of the invariant moments are shown in Figure 11.25.

11.3 Regional Descriptors -Texture



a
b c
d e

FIGURE 11.25 Images used to demonstrate properties of moment invariants (see Table 11.3).

11.3 Regional Descriptors -Texture

Invariant (Log)	Original	Half Size	Mirrored	Rotated 2°	Rotated 45°
ϕ_1	6.249	6.226	6.919	6.253	6.318
ϕ_2	17.180	16.954	19.955	17.270	16.803
ϕ_3	22.655	23.531	26.689	22.836	19.724
ϕ_4	22.919	24.236	26.901	23.130	20.437
ϕ_5	45.749	48.349	53.724	46.136	40.525
ϕ_6	31.830	32.916	37.134	32.068	29.315
ϕ_7	45.589	48.343	53.590	46.017	40.470

TABLE 11.3
Moment invariants for the images in Figs. 11.25(a)-(e).

11.4 Use of Principal Component Description

- Treat the vectors **x** as a random quantity.
- The mean vector is $\mathbf{m}_x = E\{\mathbf{x}\}$
- The covariance matrix: $\mathbf{C}_\mathbf{x} = E\{(\mathbf{x} - \mathbf{m}_x)(\mathbf{x} - \mathbf{m}_x)^T\}$ which is real and symmetric.
- c_{ii} is variance of x_i, and c_{ij} is the covariance between elements x_i and x_j .
- If element x_i and x_j are uncorrelated then $c_{ij}=c_{ji}=0$.

11.4 Use of Principal Component Description

- For K vector samples from random population, the mean vector is

$$\mathbf{m}_x = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k$$

- By expanding the product $(\mathbf{x} - \mathbf{m}_x)(\mathbf{x} - \mathbf{m}_x)^T$, the covariance matrix can be approximated as

$$\mathbf{C}_x = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^T - \mathbf{m}_k\mathbf{m}_k^T$$

11.4 Use of Principal Component Description

- Example 11.9. $\mathbf{x}_1=[0, 0, 0]^T$, $\mathbf{x}_2=[1, 0, 0]^T$ $\mathbf{x}_3=[1, 1, 0]^T$ $\mathbf{x}_4=[1, 0, 1]^T$.
- We may compute $\mathbf{m}_x$ and $\mathbf{C}_x$ as

$\mathbf{m}_x=1/4[3, 1, 1]^T$

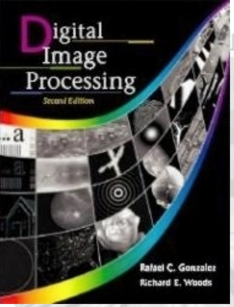
$$\mathbf{C}_x = \frac{1}{16}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

- The diagonal terms indicate that the three components of the vectors have the same variance.
- x_1 and x_2, x_1 and x_3 are positive related.
- x_2 and x_3 are negative related.

11.4 Use of Principal Component Description

- Because $\boldsymbol{C}_x$ is real and symmetric, we may find a set of n orthonormal eigenvectors.
- Let $\boldsymbol{e}_i$ and λ_i, $i=1, 2,\ldots n$ be the eigenvectors and eigenvalues of $\boldsymbol{C}_x$, with $\lambda_i \geq \lambda_{i+1}$.
- Let **A** be the matrix whose rows are formed from the eigenvectors of $\mathbf{C}_x$ ordered so that the first row of **A** is eigenvector corresponding to the largest eigenvalue, and the last row is the eigenvector corresponding to the smallest eigenvalue.
- Suppose **A** is used as a transformation matrix to map the **x**'s into vector denoted by **y**'s as follows:

$$\mathbf{y}=\mathbf{A}(\mathbf{x}-\mathbf{m}_x)$$



11.4 Use of Principal Component Description

- The above expression is called ***Hotelling transform*** or ***Principal component transform.***
- $\mathbf{m}_y = E\{\mathbf{y}\} = 0$
- $\mathbf{C}_y$ is $= \mathbf{A}\mathbf{C}_x\mathbf{A}^T$.
- $\mathbf{C}_y$ is a diagonal matrix.

$$\mathbf{C}_y = \begin{bmatrix} \lambda_1 & & & & 0 \\ & \lambda_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ 0 & & & & \lambda_n \end{bmatrix}$$

- The reconstruction of $\mathbf{x}$ is $\mathbf{x} = \mathbf{A}^T\mathbf{y} + \mathbf{m}_x$

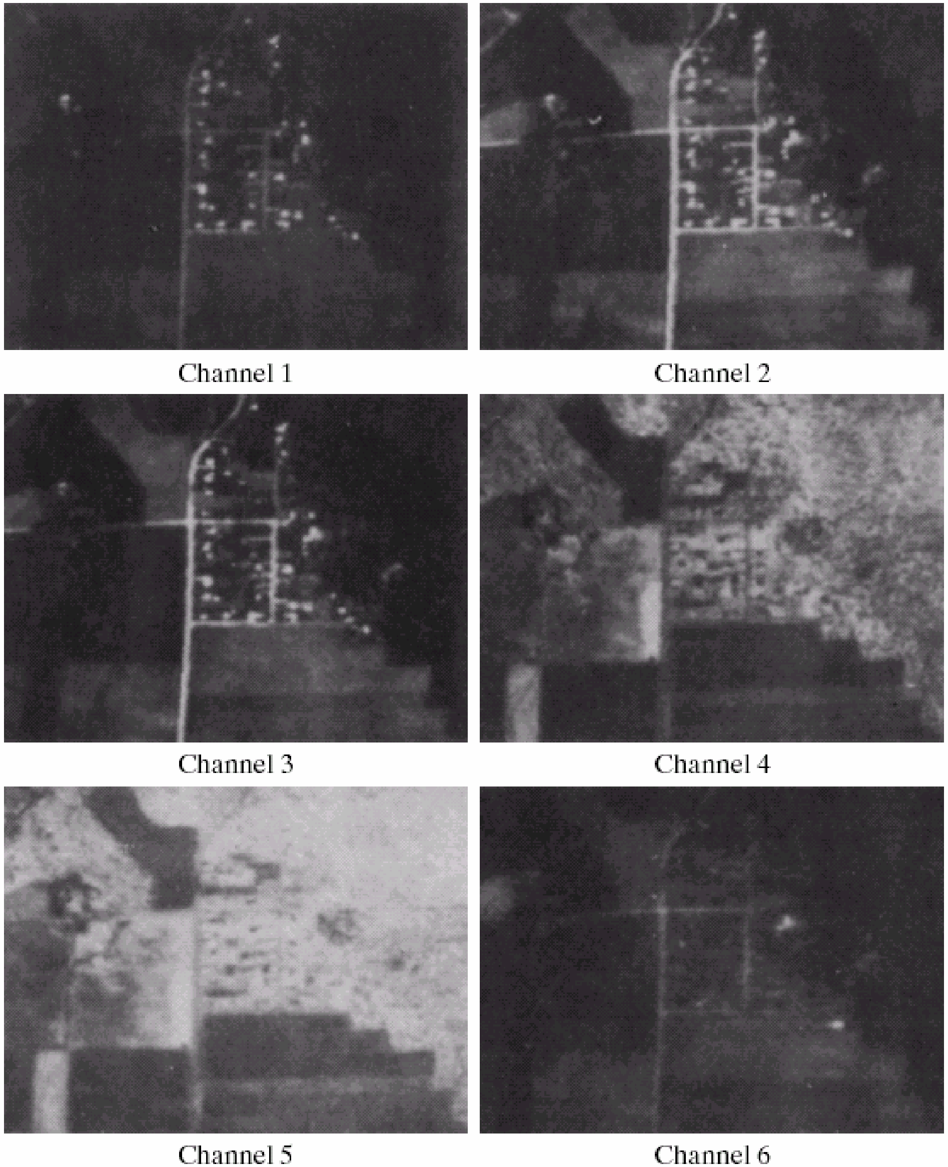
11.4 Use of Principal Component Description

- Instead of using all eigenvectors of $\mathbf{C}_x$, we form matrix $\mathbf{A}_k$ from *k* eigenvector corresponding to *k* largest eigenvalues.
- $\mathbf{A}_k$ is a transformation matrix of order $k \times n$.
- The $\mathbf{y}$ vector would be *k* dimension.
- The reconstructed vector is no longer exact as

$$\hat{\mathbf{x}} = \mathbf{A}_k^T \mathbf{y} + \mathbf{m}_\mathbf{x}$$

11.4 Use of Principal Component Description

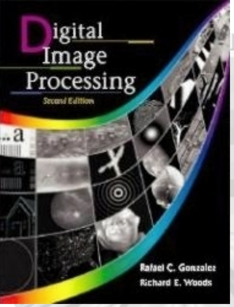
FIGURE 11.26 Six spectral images from an airborne scanner. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)



11.4 Use of Principal Component Description

TABLE 11.4
Channel numbers and wavelengths.

Channel	Wavelength band (microns)
1	0.40–0.44
2	0.62–0.66
3	0.66–0.72
4	0.80–1.00
5	1.00–1.40
6	2.00–2.60



11.4 Use of Principal Component Description

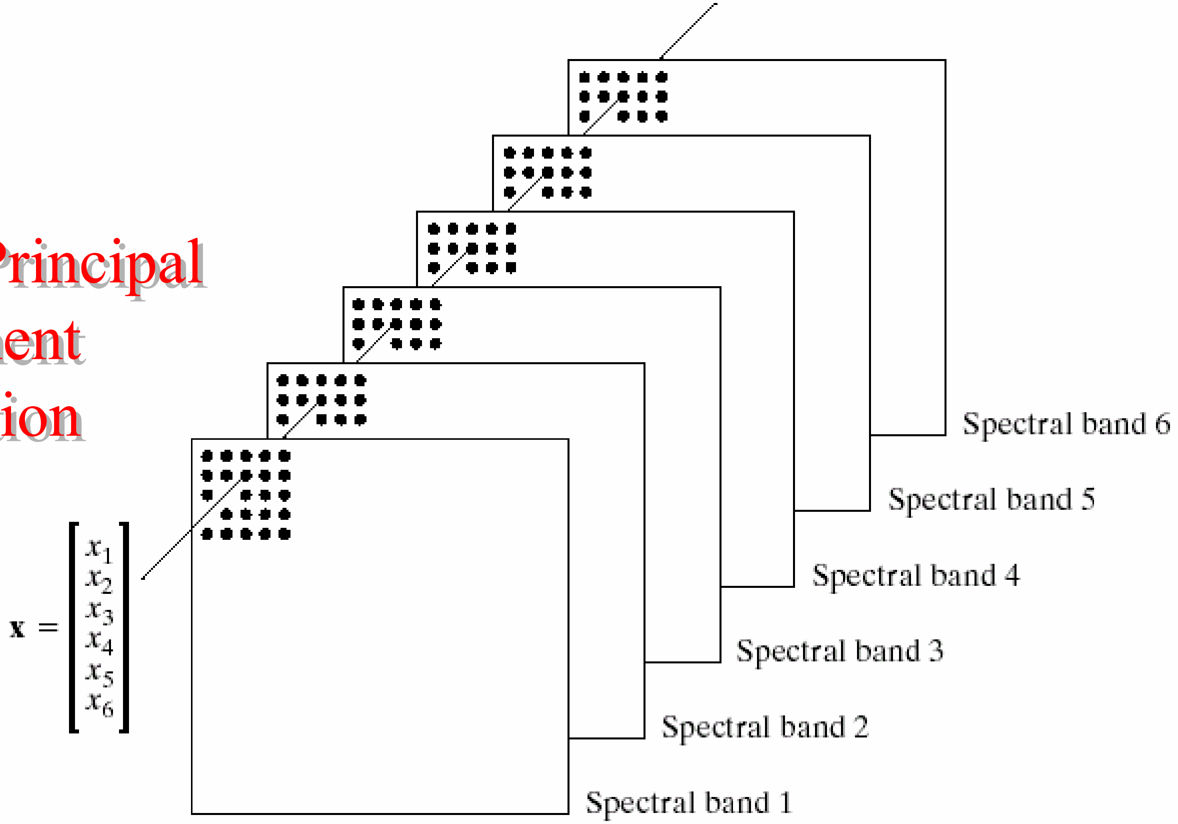


FIGURE 11.27 Formation of a vector from corresponding pixels in six images.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
3210	931.4	118.5	83.88	64.00	13.40

TABLE 11.5 Eigenvalues of the covariance matrix obtained from the images in Fig. 11.26.

11.4 Use of Principal Component Description

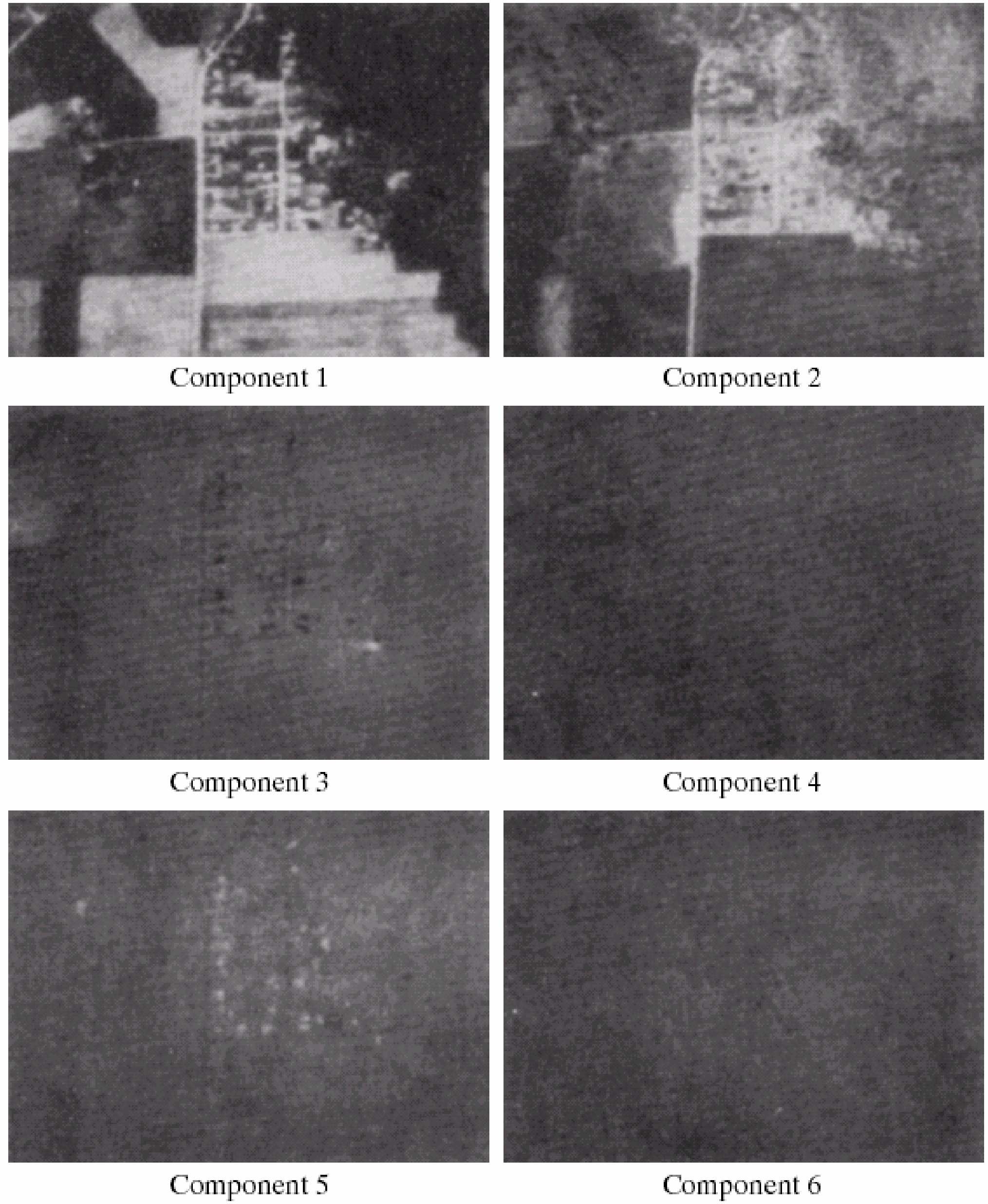
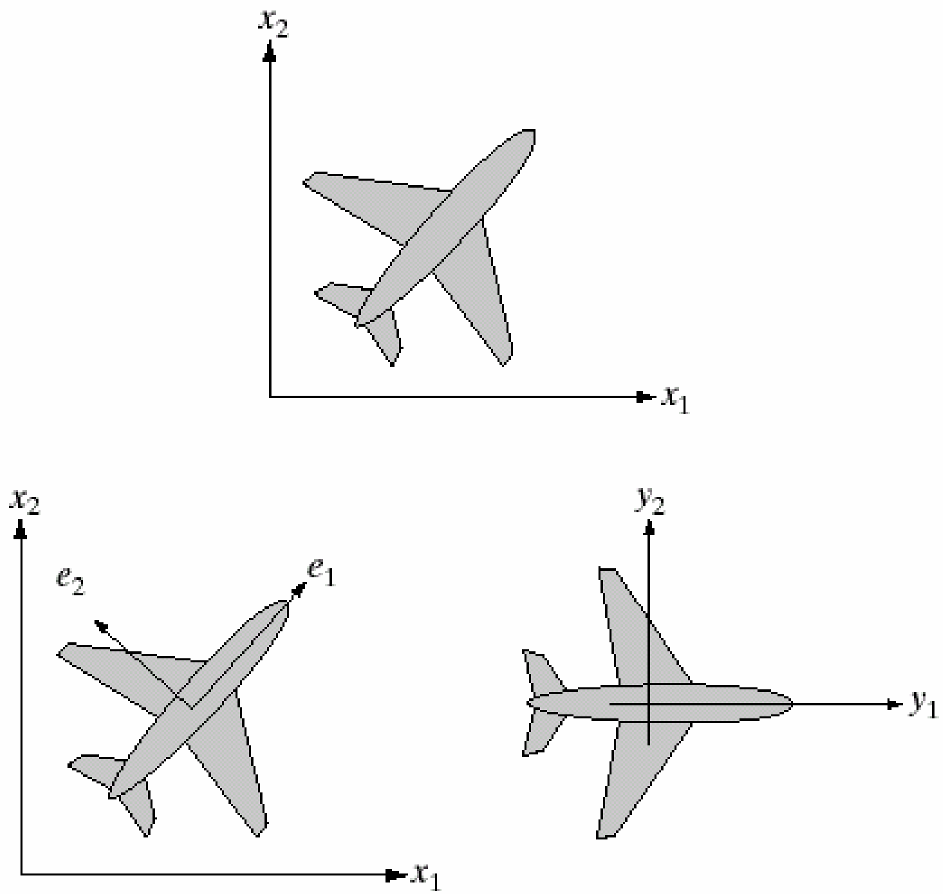


FIGURE 11.28 Six principal-component images computed from the data in Fig. 11.26. (Courtesy of the Laboratory for Applications of Remote Sensing, Purdue University.)

11.4 Use of Principal Component Description



a
b c

FIGURE 11.29 (a) An object. (b) Eigenvectors. (c) Object rotated by using Eq. (11.4-6). The net effect is to align the object along its eigen axes.

11.5 Relational Description

- Rules for describing the context of relation.
- Apply equally to boundaries and regions.
- Define two primitives *a* and *b* as shown in Fig. 11.30.
- We define rewriting rules as

  (a) $S \rightarrow aA$

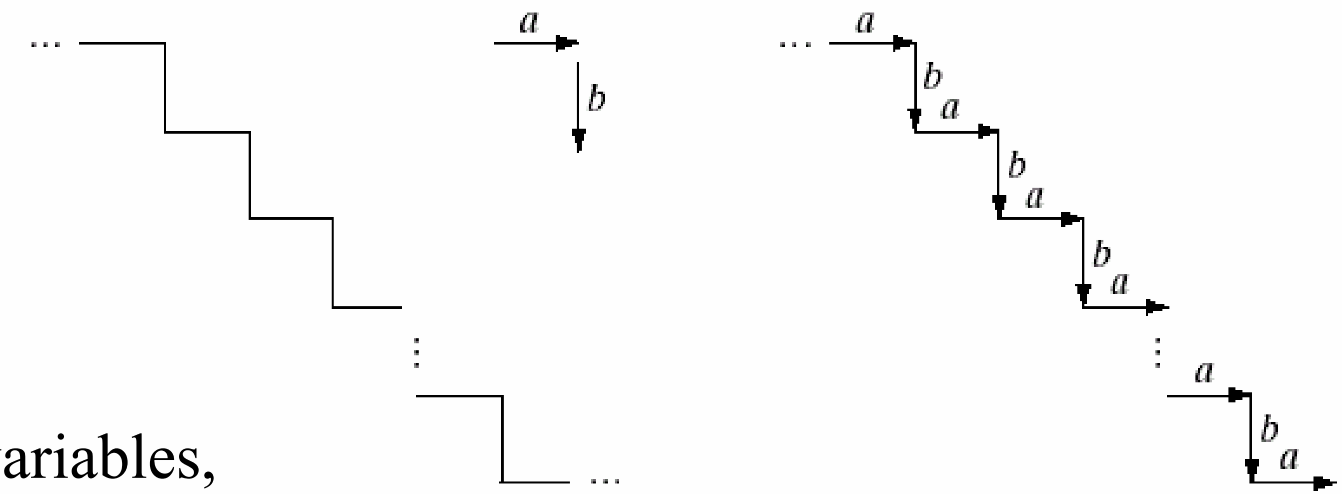
  (b) $A \rightarrow bS$

  (c) $A \rightarrow b$.

  where *A* and *S* are variables, and the elements *a* and b are constant corresponding o the primitives.

Rule 1 indicates the staring symbols *S* can be replaced by *a*A.

11.5 Relational Description



a b

FIGURE 11.30 (a) A simple staircase structure. (b) Coded structure.

Let A and S are variables, define rewriting rules as

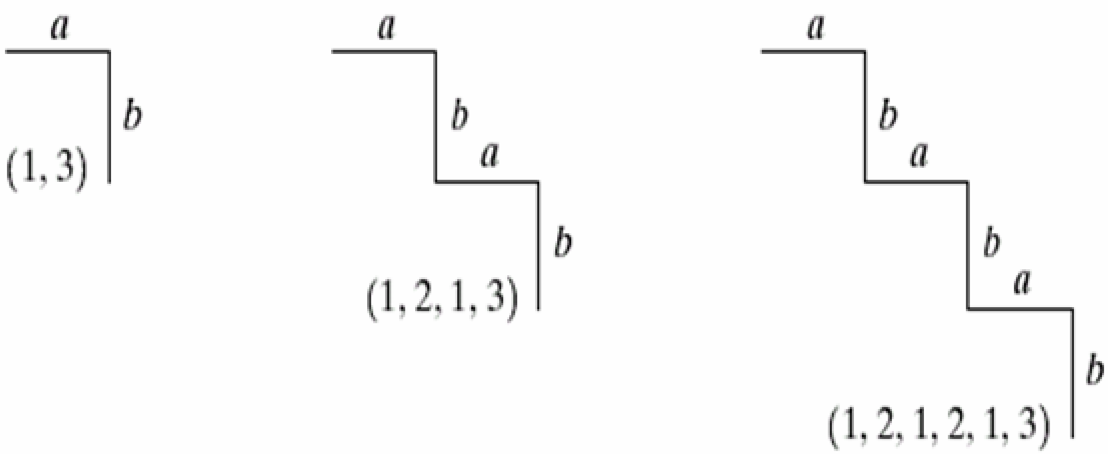
(a) $S \rightarrow aA$

(b) $A \rightarrow bS$

(c) $A \rightarrow b.$

11.5 Relational Description

FIGURE 11.31 Sample derivations for the rules $S \rightarrow aA$, $A \rightarrow bS$, and $A \rightarrow b$.



11.5 Relational Description

- For 2-D object description, we follow the contour of an object and code the result with segments of specific direction and/or length as shown in Figure 11.32.

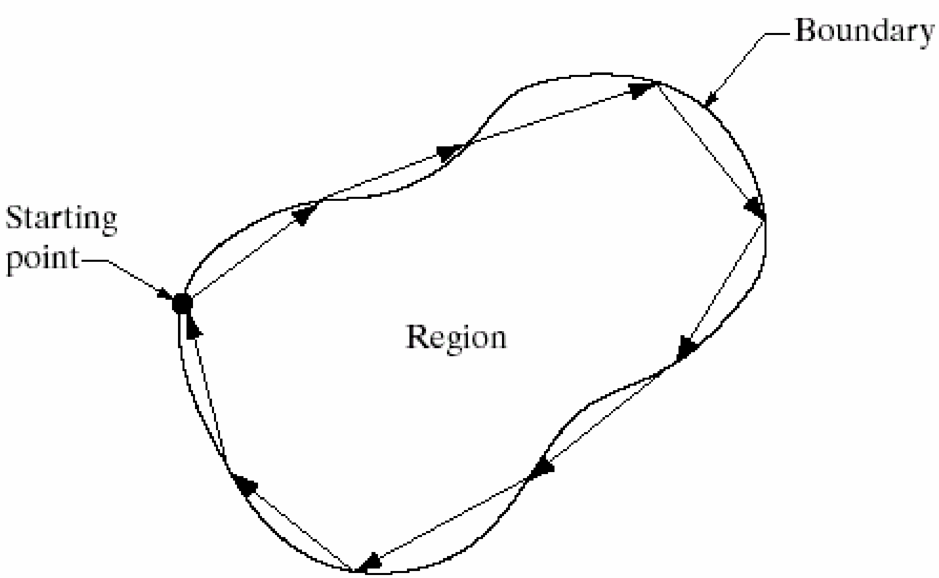
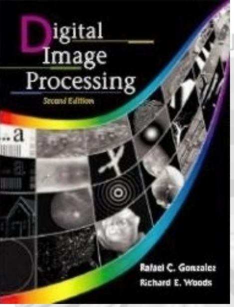
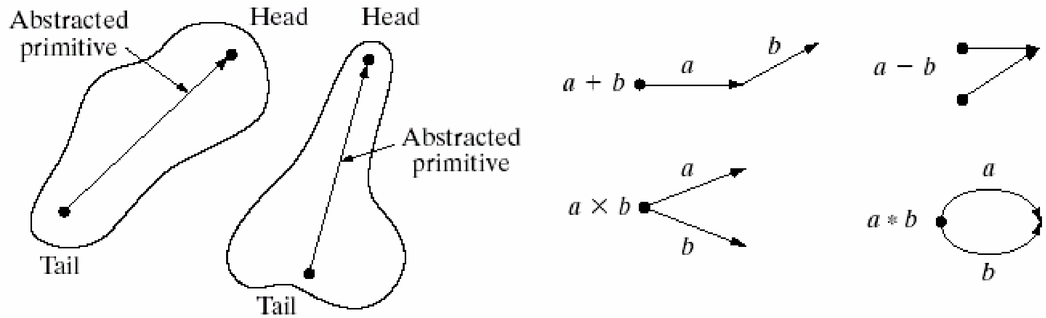


FIGURE 11.32 Coding a region boundary with directed line segments.

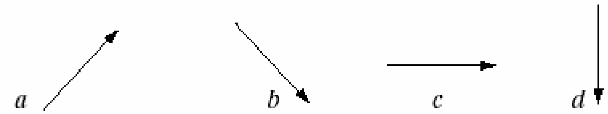
11.5 Relational Description

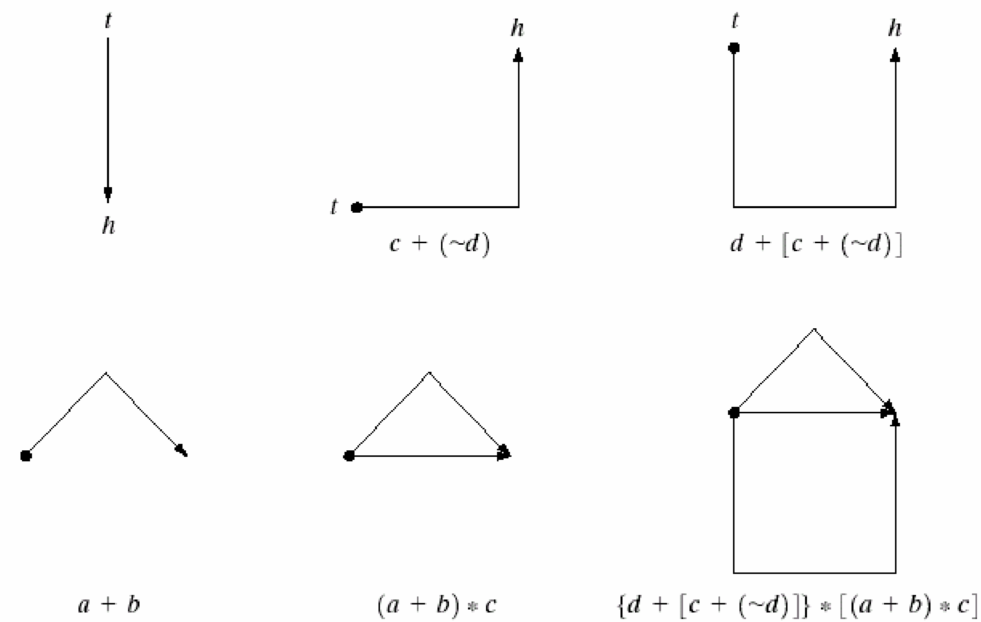
- Another description is to describe the sections of an image (small homogeneous region) by direct line segments, which can be joined in other ways besides head-to-tail connections as shown in Figure 11.33.
- Sting descriptions are best suited for applications in which connectivity of primitives can be expressed in a head-to-tail or other connected manner.





11.5 Relational Description





a	b
	c
	d

FIGURE 11.33 (a) Abstracted primitives. (b) Operations among primitives. (c) A set of specific primitives. (d) Steps in building a structure.

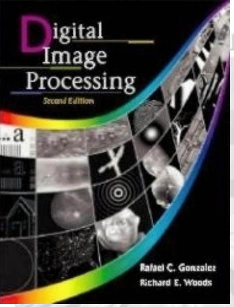
11.5 Relational Description

- Sometimes regions may not be contiguous, and we use Tree to describe such regions.
- A tree T is a finite set of one or more nodes for which

  a) there is a unique node $ designated the root

  b) the remaining nodes are partitioned into *m* disjoint sets T_1, ….T_m, each of which in turn is a tree called a subtree of T.

The tree frontier is a set of nodes at the bottom of the tree (the leaves), taken in order from left to right, (see Figure 11.34).



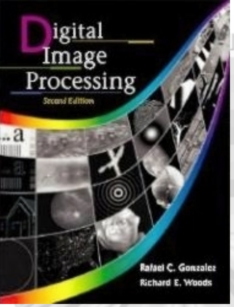
11.5 Relational Description

Two types of information in a tree

a) information about a node

b) information relating a node to its neighbors

For image description, the 1st type of information identifies an image structure, whereas the 2nd type of information defines the physical relationship of that substructure to other substructure.



11.5 Relational Description

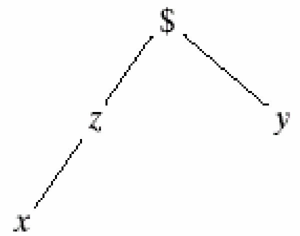
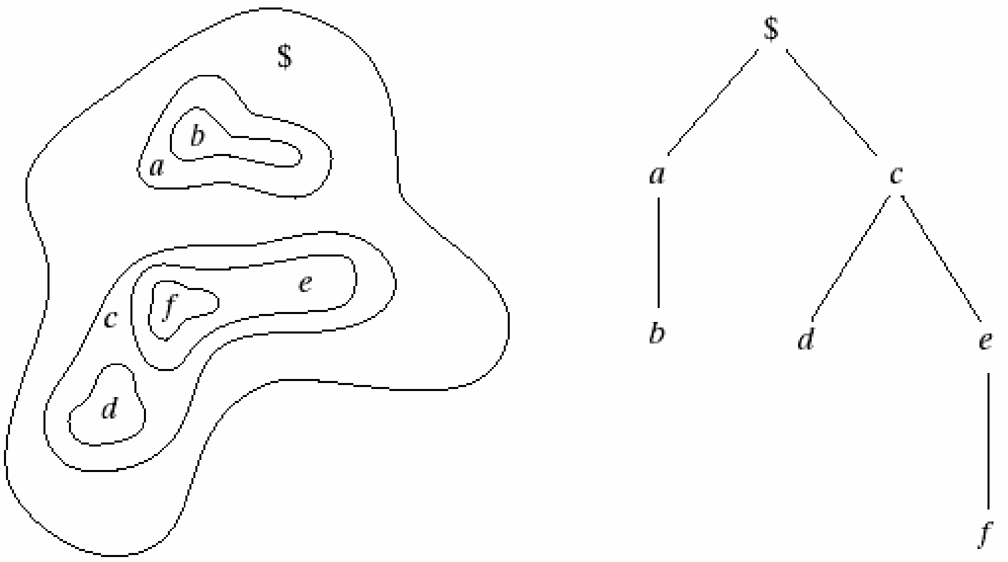


FIGURE 11.34 A simple tree with root $ and frontier *xy*.



a b

FIGURE 11.35 (a) A simple composite region. (b) Tree representation obtained by using the relationship "inside of."