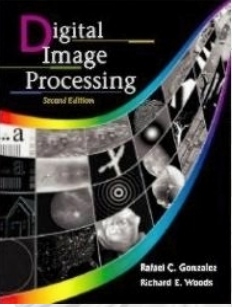# Chapter 9
Morphological Image Processing

- *Mathematic morphology*: a tool for extracting image components, such as boundaries, skeletons, and the convex hull.

- Morphological filtering

- Morphological Thinning

- Morphological Prunning

9.1 Introduction

- The language in mathematical morphology is *set theory*

- Sets in *mathematic morphology* represents objects in image.

- In binary images, the sets are members of the 2-D integer space Z^2, where each element of a set is a tuple (2-D vector) whose coordinates are the (*x, y*) coordinates of a black (or white) pixel in the image.

9.1 Preliminaries

- Let A be a set in Z^2, if $\boldsymbol{a}=(a_1, a_2)$ is an element of A then $\boldsymbol{a} \in A$.

- The set with no element is called the null or empty set and is denoted as $\varnothing$.

- If every element of a set A is also an element of B then A is a subset of B, denoted as $A \subseteq B$

- The *union*: $C = A \cup B$

- The *intersection*: $D = A \cap B$

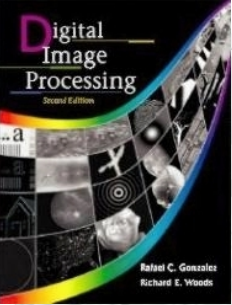- Two set are *mutually exclusive* or *disjoint* (they have no common element) then $A \cap B = \varnothing$

9.1 Preliminaries
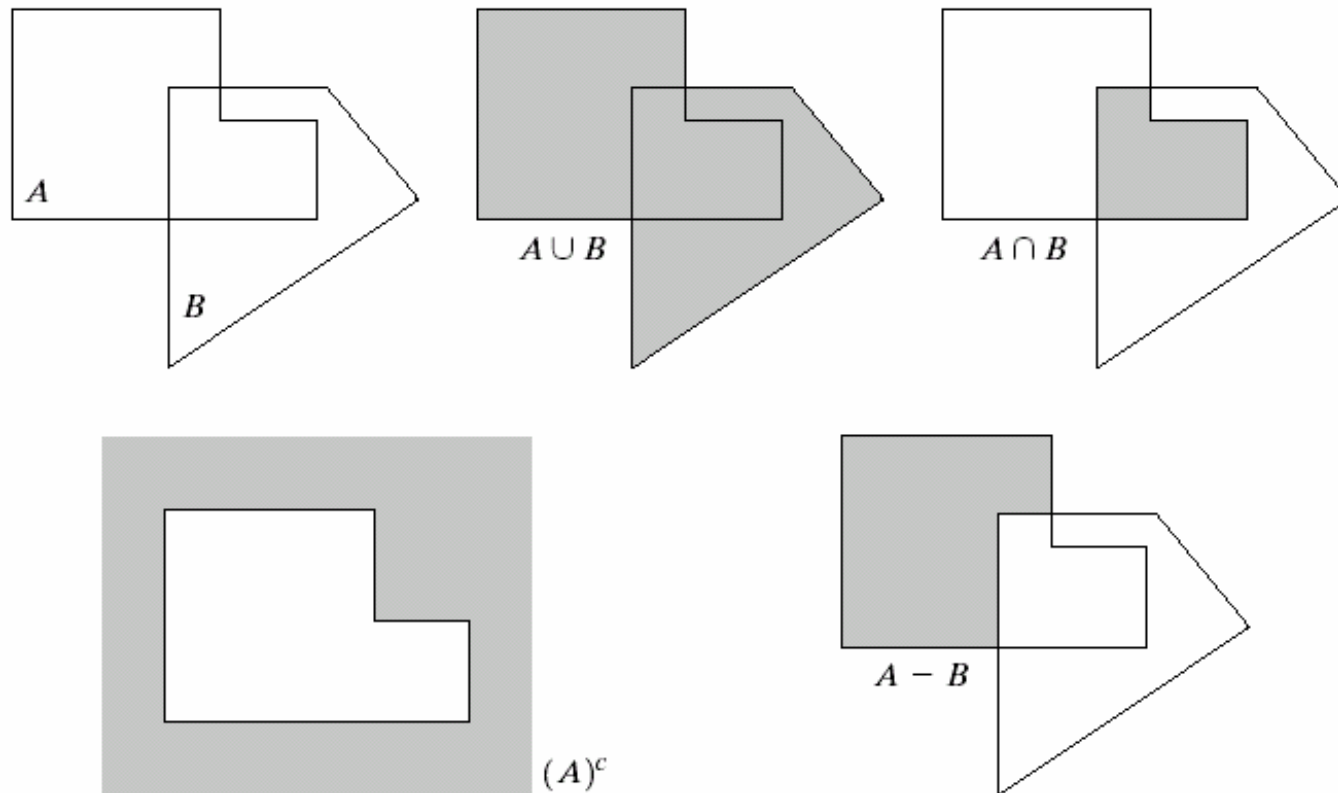
- The *complement* of a set A is the set of elements not contained in A as $A^c = \{w | w \notin A\}$

- The *difference* of two sets is the set of elements that belong to A but not to B, denoted as

$$A - B = \{w | w \in A, w \notin B\} = A \cap B^c$$

- The ***reflection*** of set A is $\hat{A} = \{w | w = -a, \text{ for } a \in A\}$

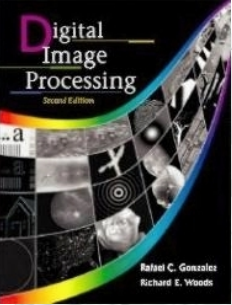- The ***translation*** of set A by a point $z = (z_1, z_2)$ as $(A)_z = \{c | c = a + z, \text{ for } a \in A\}$
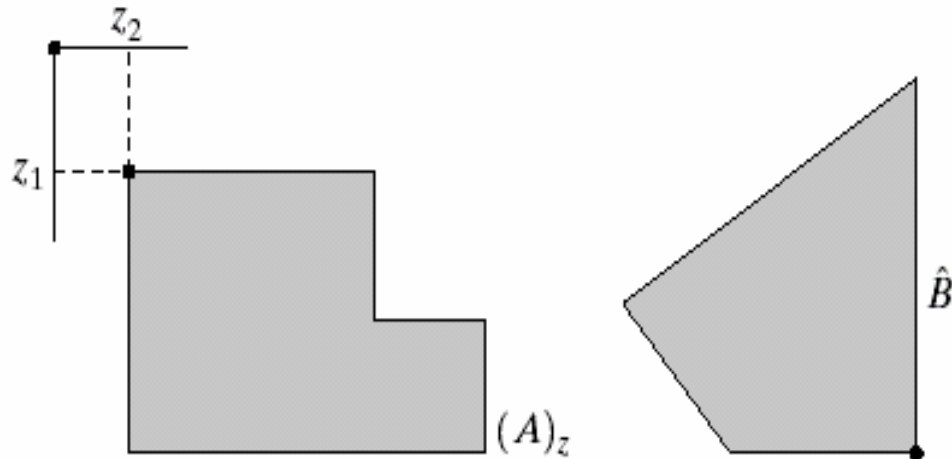
9.1 Preliminaries

a b c
d e

FIGURE 9.1
(a) Two sets A and B. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B.

9.1 Preliminaries

a b

FIGURE 9.2
(a) Translation of A by z.
(b) Reflection of B. The sets A and B are from Fig. 9.1.

9.1.2 Logic operation involving binary image

- Logic operation involving binary pixels and images.

- The principal logic operations are AND, OR, and NOT(complement).

- The *intersection* operation in set theory reduces to AND operation when the variables involved are binary.

9.1.2 Logic operation involving binary image

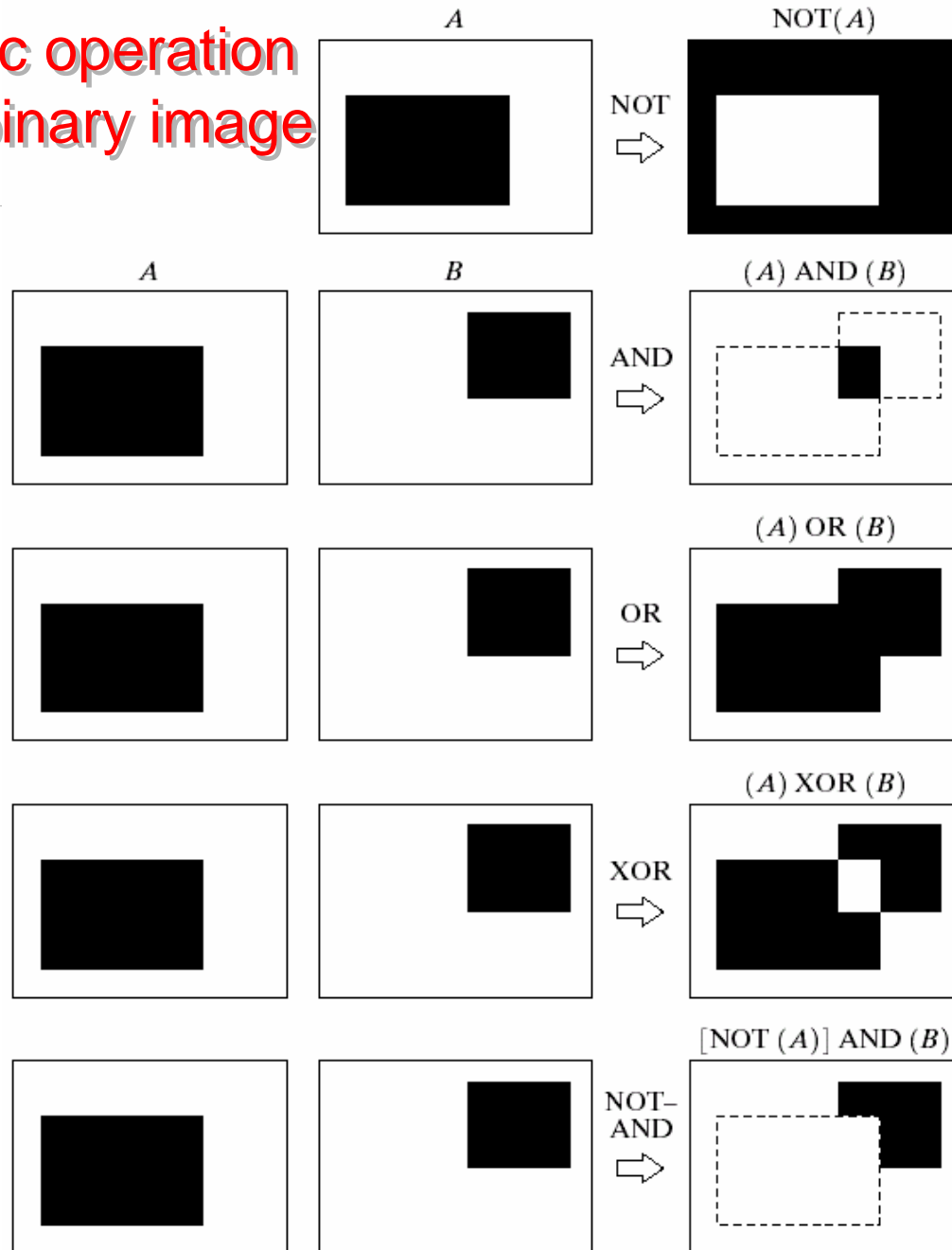**TABLE 9.1**
The three basic logical operations.

| p	q	p AND q (also $p \cdot q$)	p OR q (also $p + q$)	NOT (p) (also $\bar{p}$)
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

9.1.2 Logic operation involving binary image

FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.
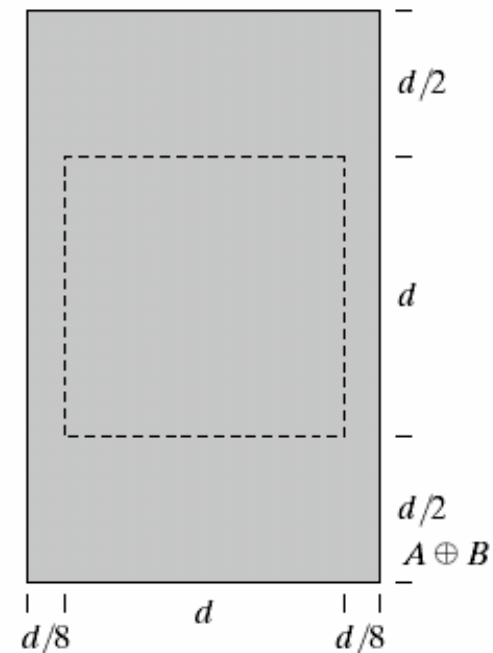
9.2 Dilation and Erosion

- The ***dilation*** and ***erosion*** are two fundamental operations in morphological processing

- The ***dilation*** of *A* by *B* is $A \oplus B = \{z | (\hat{B})_z \cap A \neq \varnothing\}$

- The set of all displacements z, such that $\hat{B}$ and *A* overlap by at least one element.

- It can be rewritten as $A \oplus B = \{z | [(\hat{B})_z \cap A] \subseteq A\}$

- Set B is commonly referred to as the ***structuring element*** in dilation.

a b c
d   e
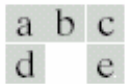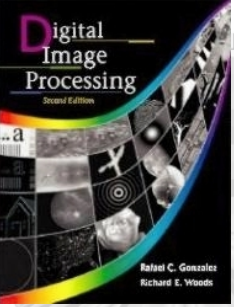
FIGURE 9.4
(a) Set A.
(b) Square
structuring
element (dot is
the center).
(c) Dilation of A
by B, shown
shaded.
(d) Elongated
structuring
element.
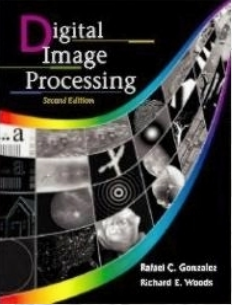(e) Dilation of A
using this
element.
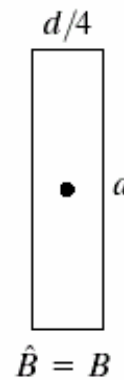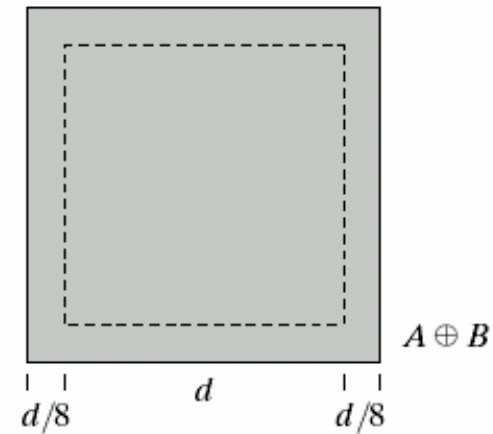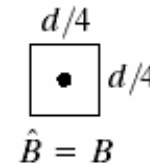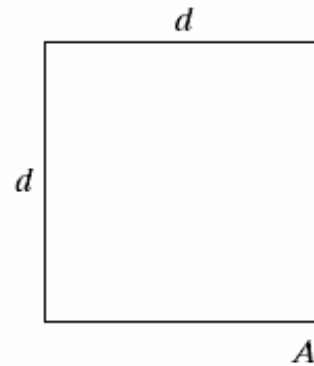
9.2 Dilation and Erosion

9.2 Dilation and Erosion

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

<pre>
a   c
   b
</pre>

FIGURE 9.5
(a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

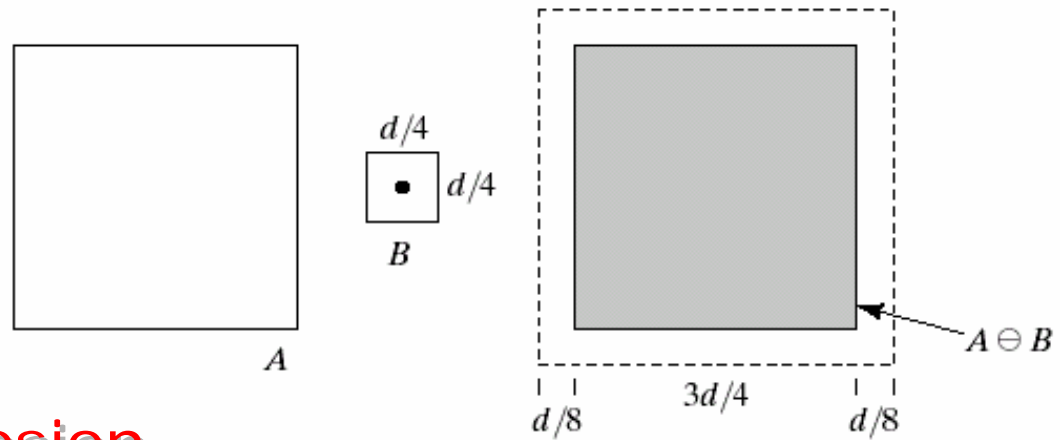| 0	1	0
1	1	1
0	1	0

9.2 Dilation and Erosion

- The ***erosion*** of A by B denoted as

$$A \ominus B = \{z | (B)_z \subseteq A\}$$

- The set of all points *z* such that B, translated by *z* in contained in A.

- ***Dilation*** and ***erosion*** are ***duals*** of each other

$$(A \ominus B)^c = A^c \oplus \hat{B}$$

- Starting with $(A \ominus B)^c = \{z | (B)_z \subseteq A\}^c$

- Then $(A \ominus B)^c = \{z | (B)_z \cap A^c = \varnothing\}^c = \{z | (B)_z \cap A^c \neq \varnothing\}$

- Therefore $(A \ominus B)^c = A^c \oplus \hat{B}$
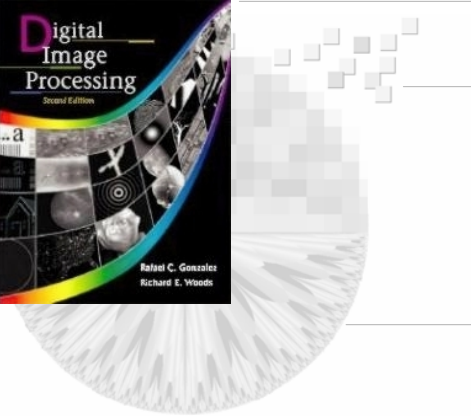
9.2 Dilation and Erosion
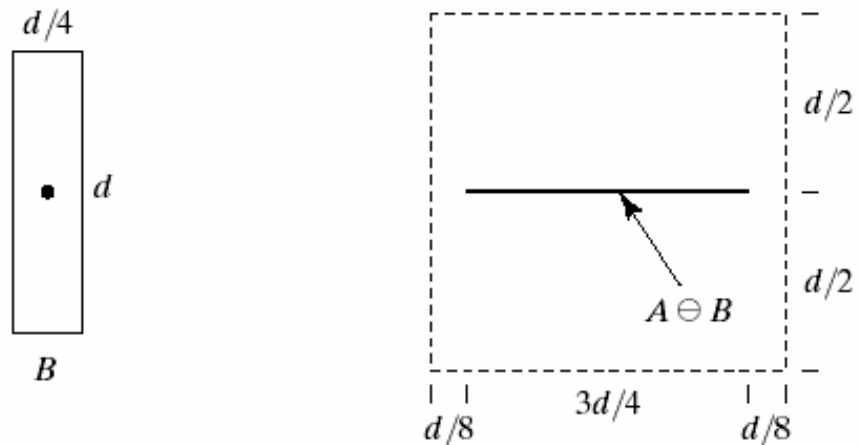
FIGURE 9.6 (a) Set A. (b) Square structuring element. (c) Erosion of A by B, shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.
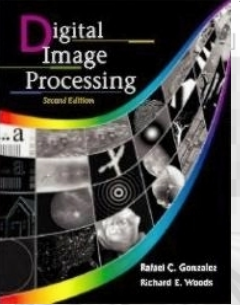
9.2 Dilation and Erosion

a b c

FIGURE 9.7 (a) Image of squares of size 1, 3, 5, 7, 9, and 15 pixels on the side. (b) Erosion of (a) with a square structuring element of 1's, 13 pixels on the side. (c) Dilation of (b) with the same structuring element.

9.3 Opening and Closing

- ***Opening*** smoothes the contour of an object, breaks narrow isthmuses and eliminates thin protrusion.

- Opening: A∘B=(A ⊖ B) ⊕ B

- Geometric interpretation for ***opening***: the boundary of A ∘ B is established by the point in B that reach the farthest into boundary of A as B is rolled around the inside of this boundary.

- Opening A by B is obtained by taking the union of all translates of B that fit into A.

$$A \circ B = \cup\{(B)_z | (B)_z \subseteq A\}$$

9.3 Opening and Closing

$$A \circ B = \cup\{(B)_z | (B)_z \subseteq A\}$$

Translates of B in A

B

a b c d
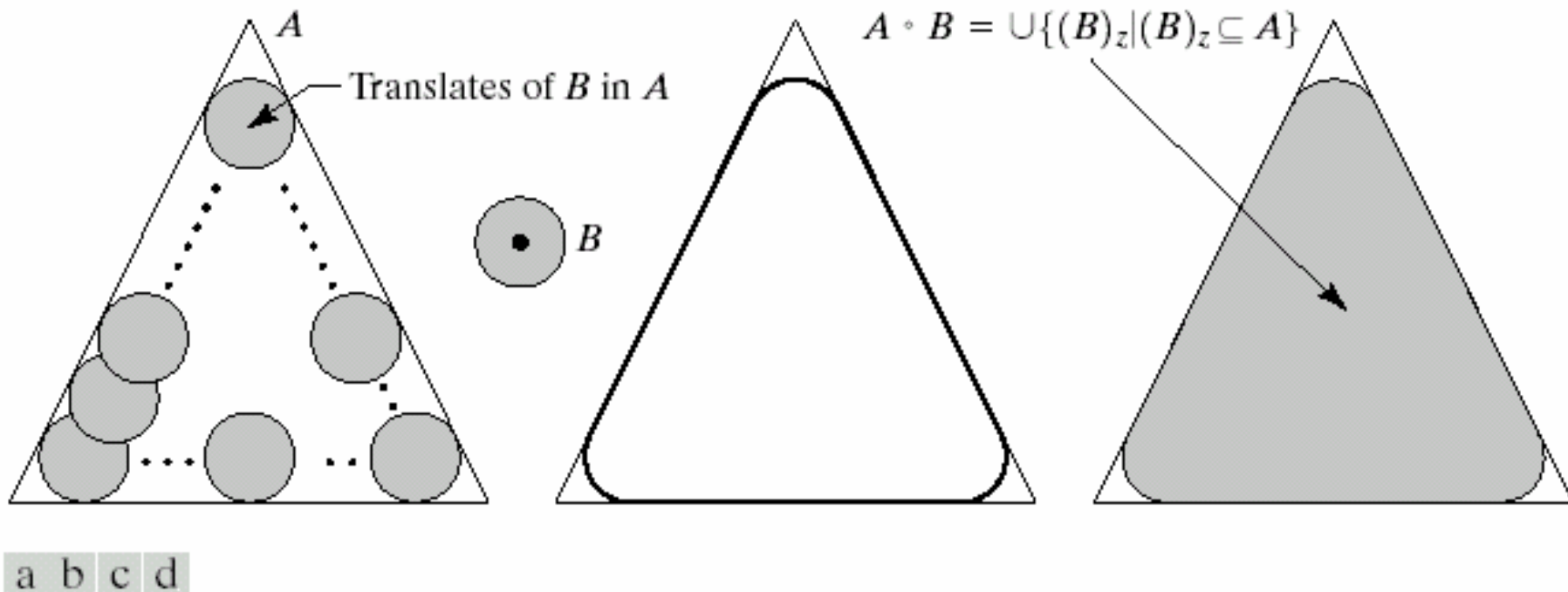
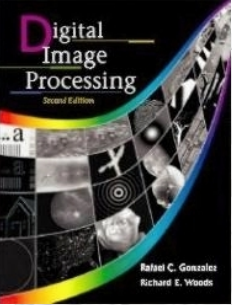**FIGURE 9.8** (a) Structuring element B "rolling" along the inner boundary of A (the dot indicates the origin of B). (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded).

9.3 Opening and Closing

- *Closing* also tends to smoothes contour, but it fuses narrow breaks and long thin gulfs, eliminate small holes, and fill gaps in the contour.

- Closing: $A \bullet B = (A \oplus B) \ominus B$

- Opening and closing are duals of each other.

- Geometrical interpretation of *closing*: a point w is an element of $A \bullet B$ if and only if $(B)_z \cap A \neq \varnothing$ for any translation of $(B)_z$ that contains w.

$$(A \bullet B)^c = A^c \circ \hat{B}$$

9.3 Opening and Closing
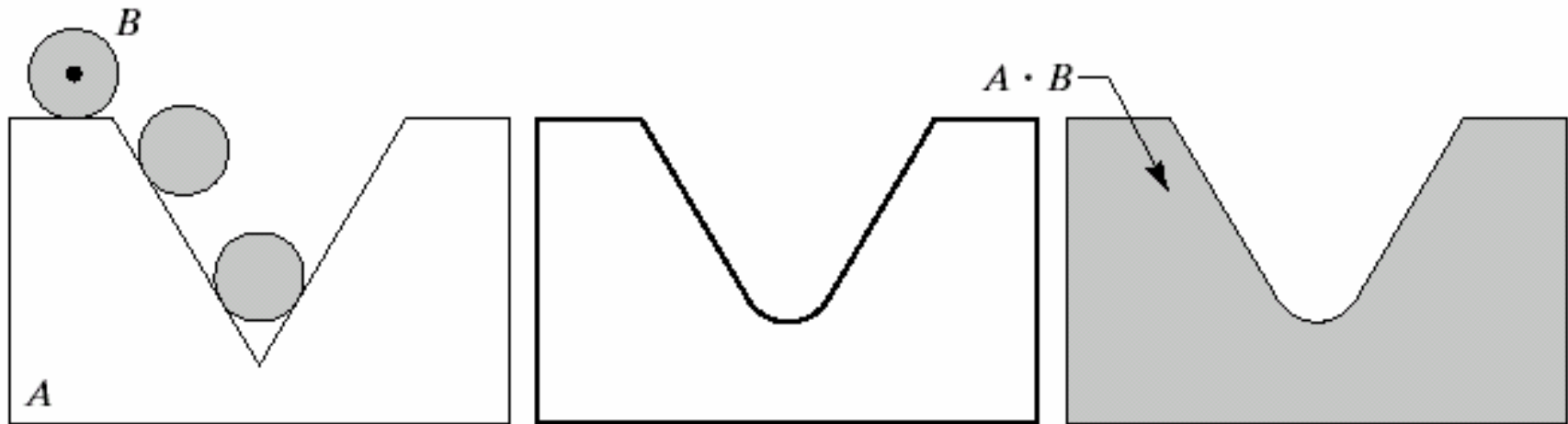
a b c

FIGURE 9.9 (a) Structuring element *B* "rolling" on the outer boundary of set *A*. (b) Heavy line is the outer boundary of the closing. (c) Complete closing (shaded).
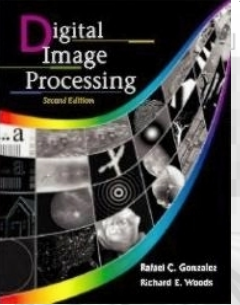
a
b c
d e
f g
h i

FIGURE 9.10
Morphological opening and closing. The structuring element is the small circle shown in various positions in (b). The dark dot is the center of the structuring element.

9.3 Opening and Closing

A

$A \ominus B$

$A \circ B = (A \ominus B) \oplus B$

$A \oplus B$

$A \cdot B = (A \oplus B) \ominus B$
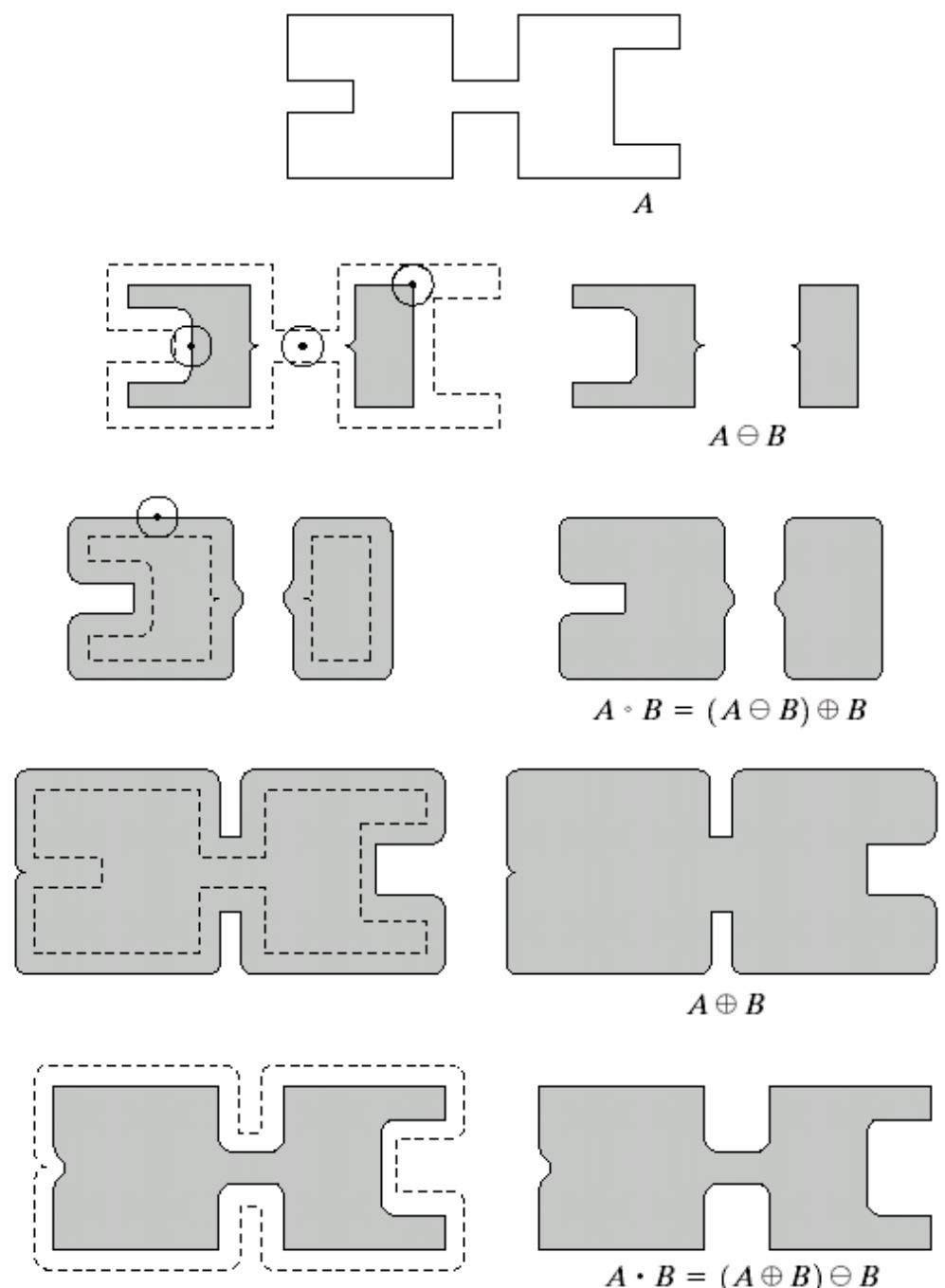
9.3 Opening and Closing

Properties of opening and closing

- A∘B is a subset of A

- If C is a subset of D, then C∘B is a subset of D∘B

- (A∘B)∘B= A∘B

- A is a subset of A•B

- If C is a subset of D, then C•B is a subset of D•B.

- (A•B)•B= A•B

$A \ominus B$

B

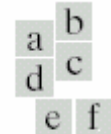| 1	1	1
1	1	1
1	1	1

a	b
d	c
e	f
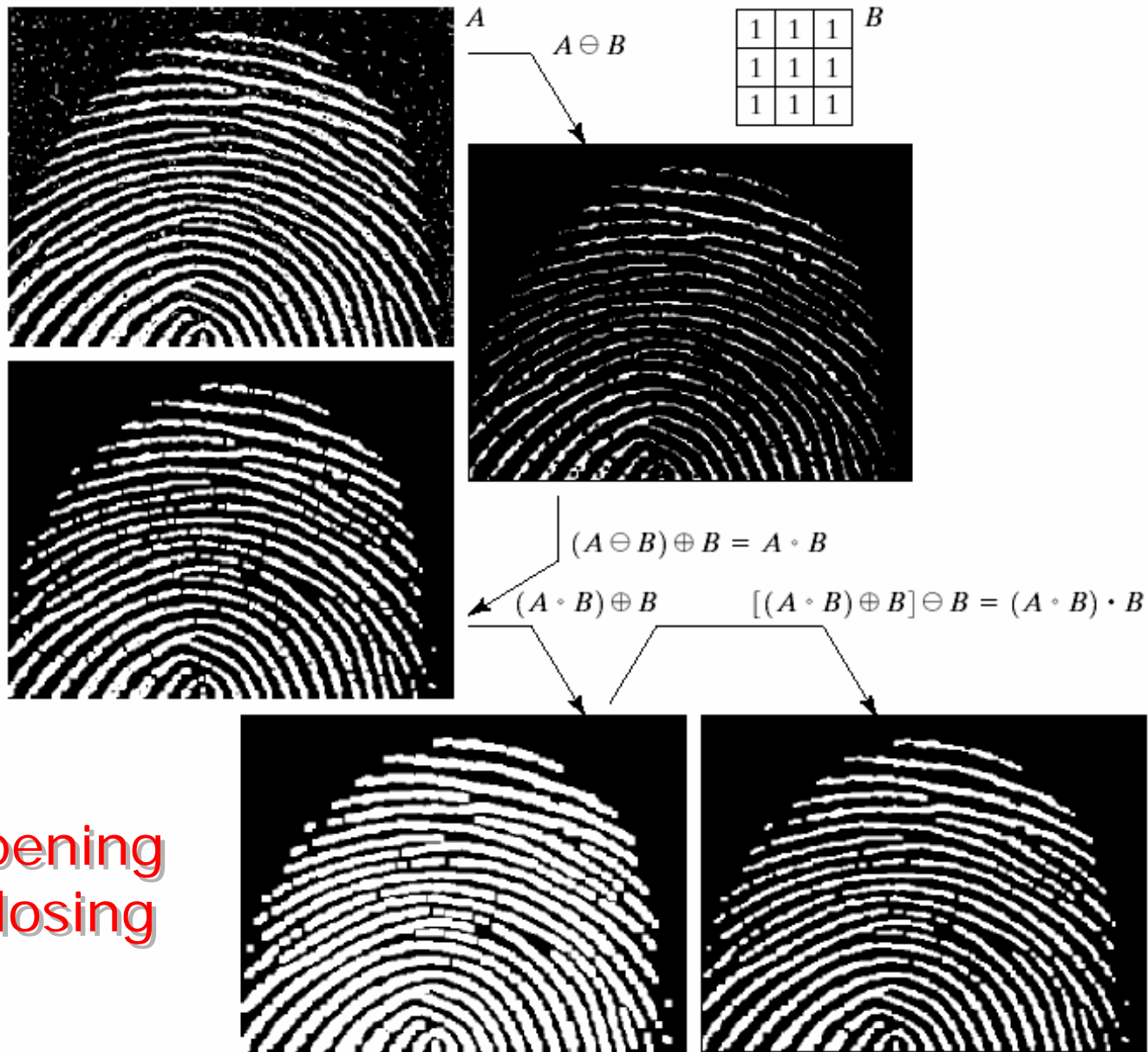
FIGURE 9.11
(a) Noisy image.
(c) Eroded image.
(d) Opening of A.
(d) Dilation of the opening.
(e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)

$(A \ominus B) \oplus B = A \circ B$

$(A \circ B) \oplus B$

$[(A \circ B) \oplus B] \ominus B = (A \circ B) \cdot B$

9.3 Opening and Closing

9.4 Hit or Miss Transformation
A tool for <u>shape detection</u>

- Goal: Find the ***location*** of the shape X in A=X$\cup$Y$\cup$Z
- Let X be enclosed by a small window W.
- W – X: the local background of X with respect to W
- AΘX may be viewed geometrically as the set of all locations of the origin X at which X found a ***match*** (or ***hit***) in A.
- Let B=(B_1, B_2), B_1=X, B_2=W – X
- The match of B in A is denoted as
  A$*$B= (AΘB_1)$\cap$(A$^c\Theta B_2$)=(AΘX)$\cap$[A$^c\Theta$(W – X)]
- By using the definition of set difference (*i.e.*,
- A – B=A$\cap$ B^c and the duality between the erosion and dilation, we have A$*$B= (AΘB_1) – (A$\oplus \hat{B}_2$)

9.4 Hit or Miss Transformation

a b
c d
e
f
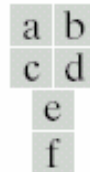
FIGURE 9.12
(a) Set A. (b) A window, W, and the local background of X with respect to W, $(W - X)$.
(c) Complement of A. (d) Erosion of A by X.
(e) Erosion of A^c by $(W - X)$.
(f) Intersection of (d) and (e), showing the location of the origin of X, as desired.

$A = X \cup Y \cup Z$

W

$(W - X)$

Origin

A^c

$(A \ominus X)$

$A^c \ominus (W - X)$

$A^c \ominus (W - X)$

$(A \ominus X) \cap (A^c \ominus [W - X])$
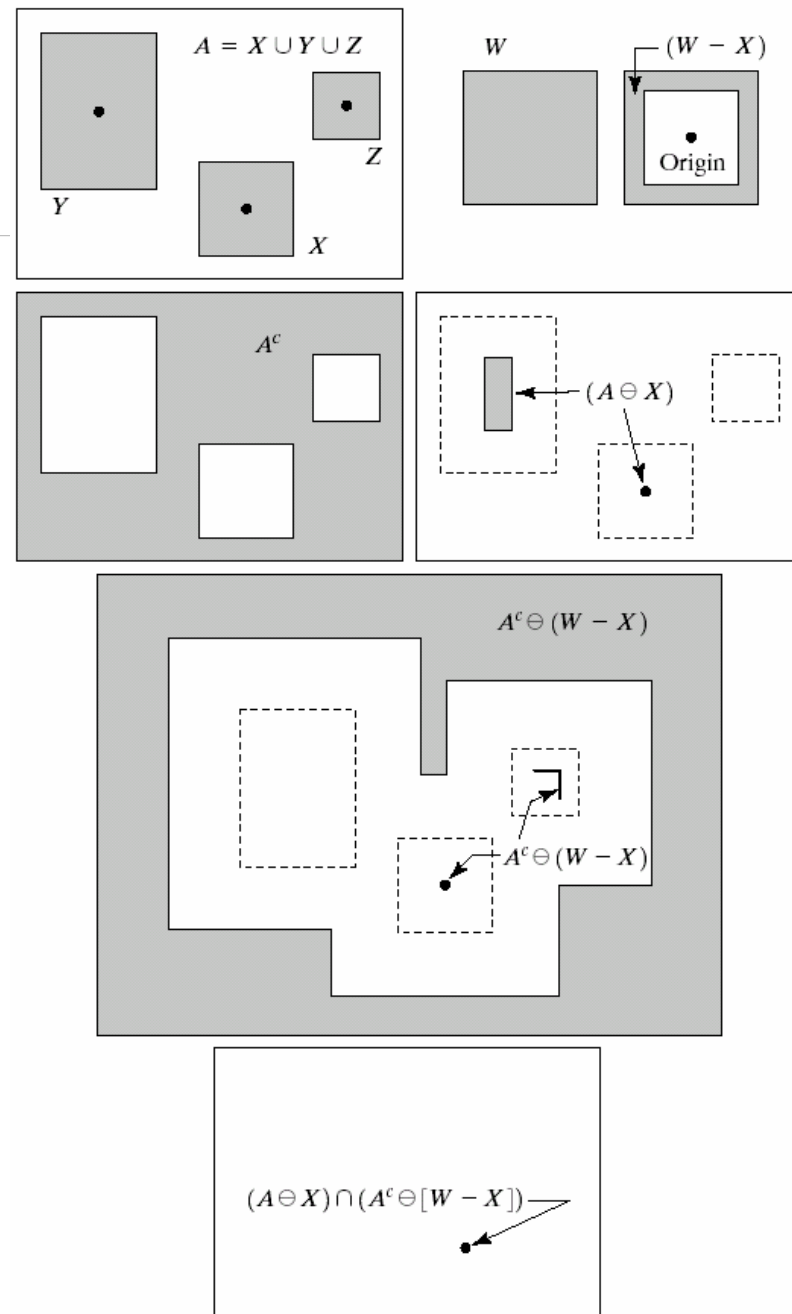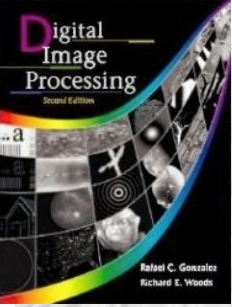
9.5 Basic Morphological Algorithm

- ***Operations***: extracting boundaries, connected components, the convex hull, and the skeleton of a region

- ***Examples***: ***Region filling, thinning, thickening, and pruning.***
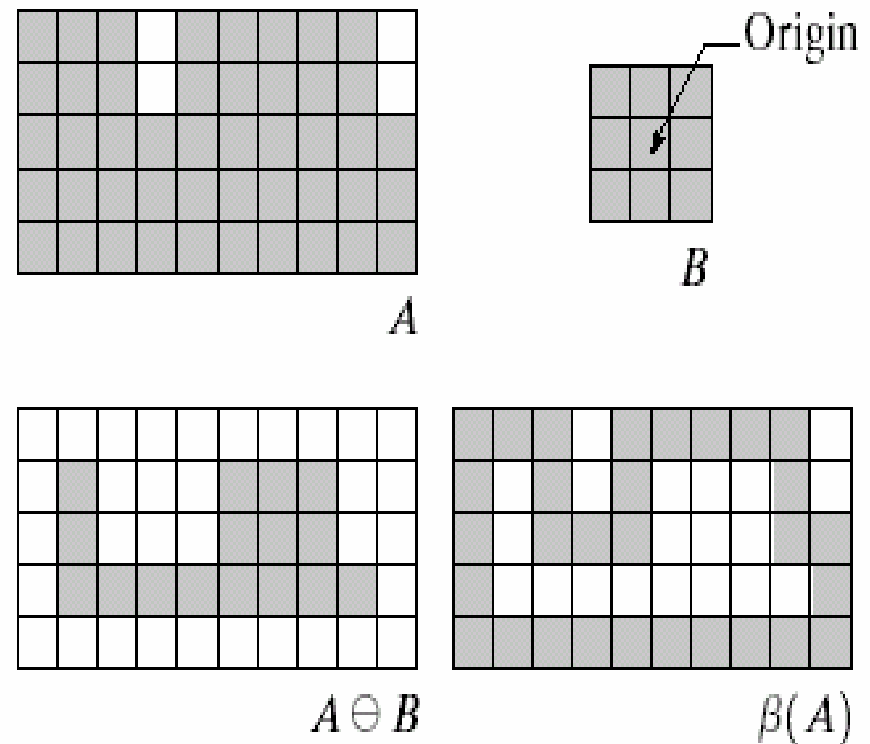
- ***Boundary extraction***: The boundary of set A: β(A) can be obtained by first eroding A by B and then performing the set difference between A and its erosion as

$$\beta(A) = A - (A \ominus B)$$

9.5 Basic Morphological Algorithm

a b
c d

FIGURE 9.13 (a) Set A. (b) Structuring element B. (c) A eroded by B. (d) Boundary, given by the set difference between A and its erosion.

A

Origin

B

$A \ominus B$

$\beta(A)$

9.5 Basic Morphological Algorithm

a b

FIGURE 9.14
(a) A simple
binary image, with
1's represented in
white. (b) Result
of using
Eq. (9.5-1) with
the structuring
element in
Fig. 9.13(b).

9.5 Basic Morphological Algorithm
Region Filling

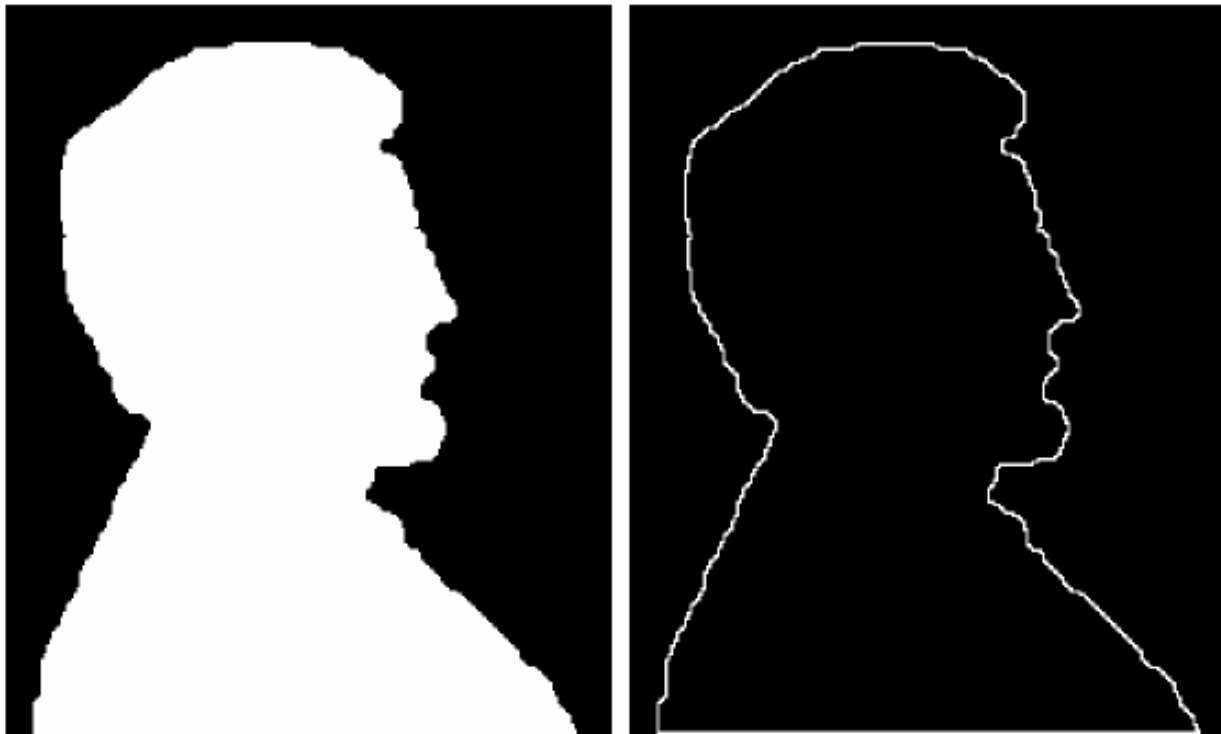- *A* denotes a set of containing a subset whose elements are 8-connected ***boundary points*** of a region.
- Beginning with a point *p* inside the boundary, the objective is to fill the entire region with 1's.
- Assume that all non-boundary points are labeled *0*.
- The filling iteration as

$$\mathbf{X}_k=(\mathbf{X}_{k-1}\oplus \boldsymbol{B})\cap \boldsymbol{A}^c \quad k=1,2,3,\ldots.$$

where $\mathbf{X}_0=p$ and $\boldsymbol{B}$ is the ***symmetric structure element***.

- The iteration stops at step *k* when $\mathbf{X}_k=\mathbf{X}_{k-1}$.
- The set union of $\mathbf{X}_k$ and *A* contains the filled set and it boundary.
- The dilation process $(\mathbf{X}_{k-1}\oplus B)$ is constrained by *A*, which limits the result to inside the region of interest.

9.5 Basic Morphological Algorithm

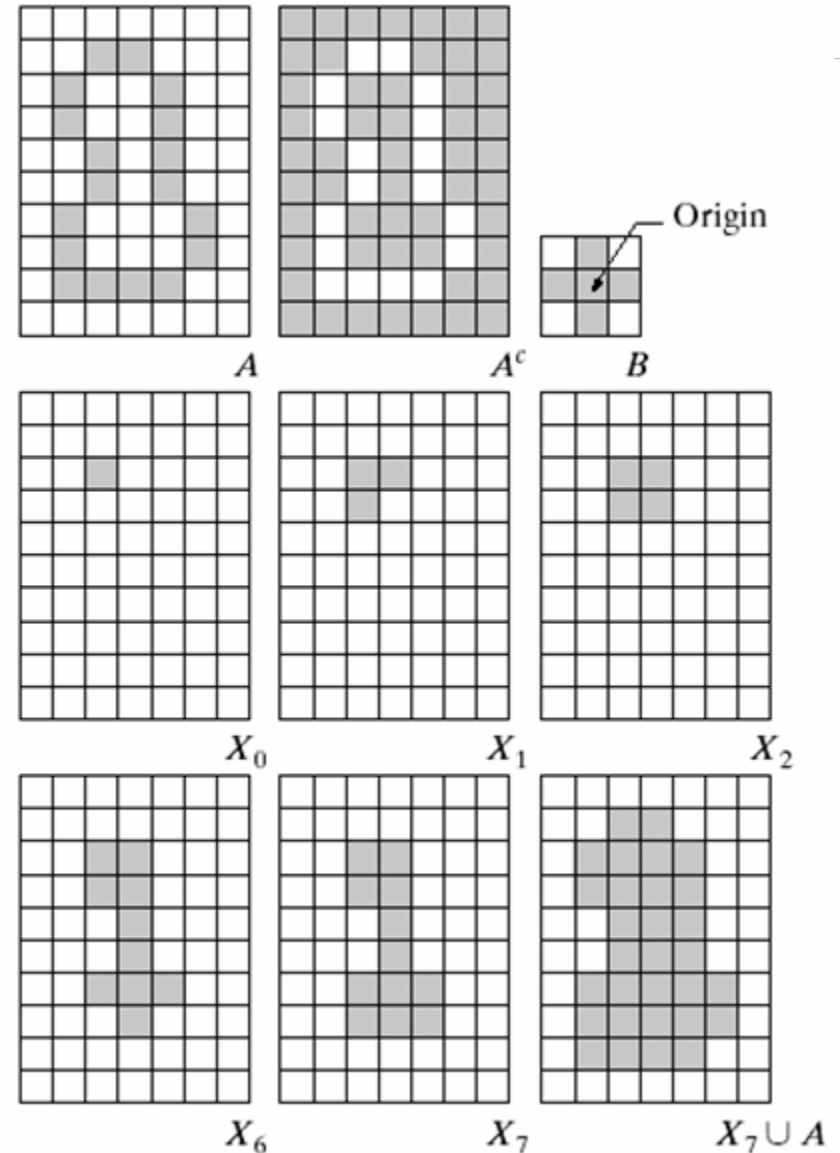| a	b	c
d	e	f
g	h	i

FIGURE 9.15
Region filling.
(a) Set A.
(b) Complement
of A.
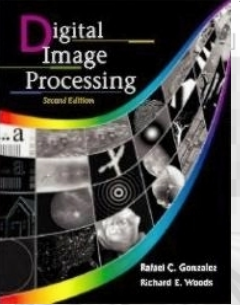(c) Structuring
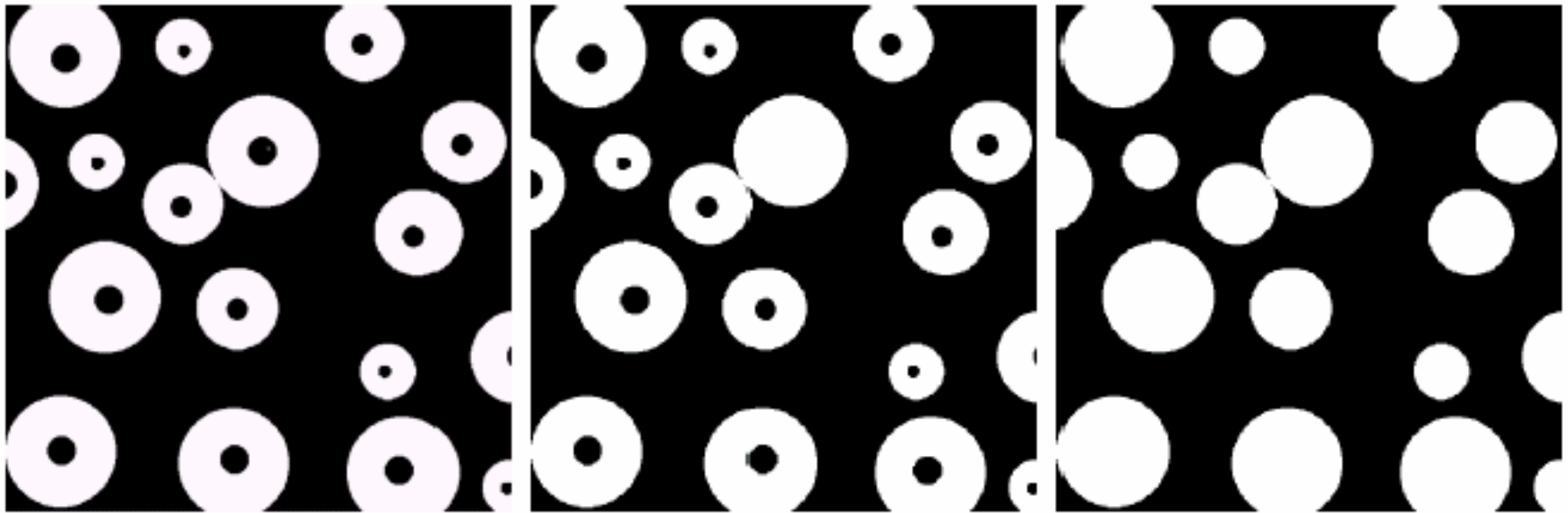element B.
(d) Initial point
inside the
boundary.
(e)–(h) Various
steps of
Eq. (9.5-2).
(i) Final result
[union of (a) and
(h)].

9.5 Basic Morphological Algorithm

a b c

FIGURE 9.16 (a) Binary image (the white dot inside one of the regions is the starting point for the region-filling algorithm). (b) Result of filling that region (c) Result of filling all regions.

9.5 Basic Morphological Algorithm

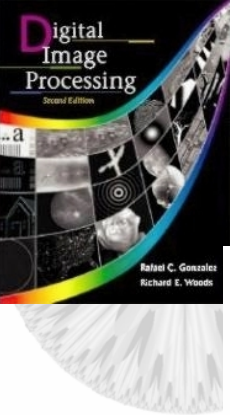***Extraction of connected components***:

- Let *Y* be a connected component contained in set *A* and point *p* of *Y* is known.

- The following iteration yields all the elements of Y:

$$X_k = (X_{k-1} \oplus B) \cap A \quad k=1,2,3,\ldots$$

where $X_0 = p$ and *B* is a suitable structuring element

- If $X_k = X_{k-1}$, then the iteration converges and we let $Y = X_k$.

9.5 Basic Morphological ALgorithm
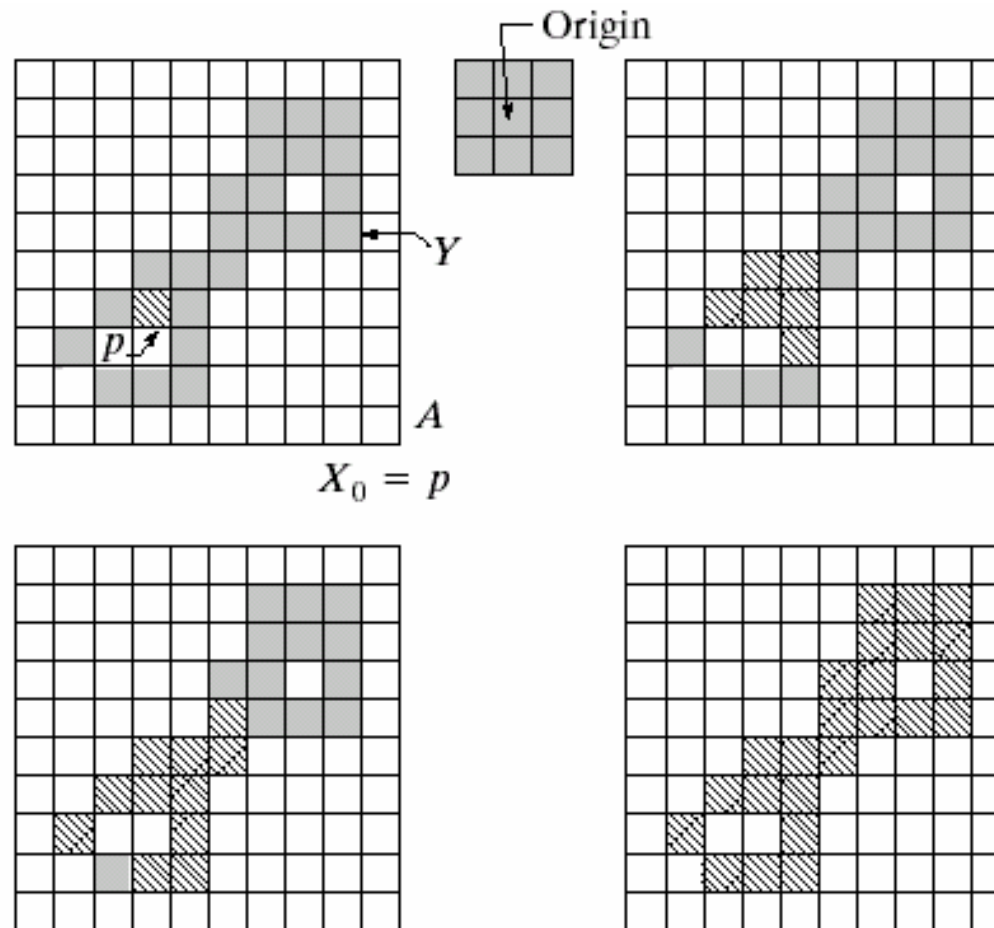
a b c
d   e

FIGURE 9.17 (a) Set A showing initial point p (all shaded points are valued 1, but are shown different from p to indicate that they have not yet been found by the algorithm). (b) Structuring element. (c) Result of first iterative step. (d) Result of second step. (e) Final result.

9.5 Basic Morphological Algorithm

a
b
c d

FIGURE 9.18
(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's.
(d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)
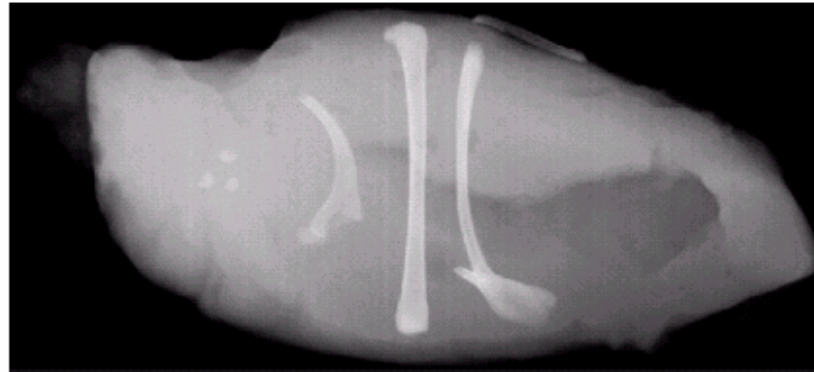
Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

9.5 Basic Morphological Algorithm-
Convex Hull

- A set *A* is said to be ***convex*** if the straight line segment joining any two points in *A* lines entirely within *A*.

- The *convex hull H* of an arbitrary set S is the smallest convex set containing S.

- The set difference *H-S* is called convex deficiency of S.

- Let B_i, *i=1~4* represent four structure elements, we have $X_k^i = (X_{k-1}^i * B^i) \cup A$ where *k=1, 2, 3,…* "*∗*" is the *hit-and-miss* operation, and $X_0^i = A$.

- Now let $D^i = X_{conv}^i$. Subscript "*conv*" indicates the convergence ($X_k^i = X_{k-1}^i$).

- The *convex hull* of *A* is $\quad C(A) = \bigcup_{i=1}^{4} D^i$

9.5 Basic Morphological ALgorithm
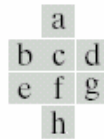
FIGURE 9.19
(a) Structuring elements. (b) Set A. (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.
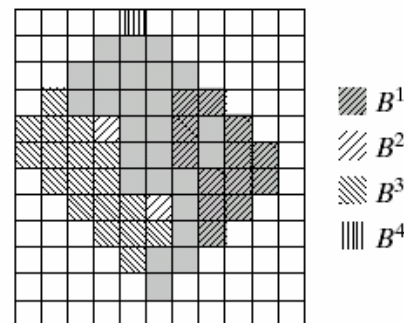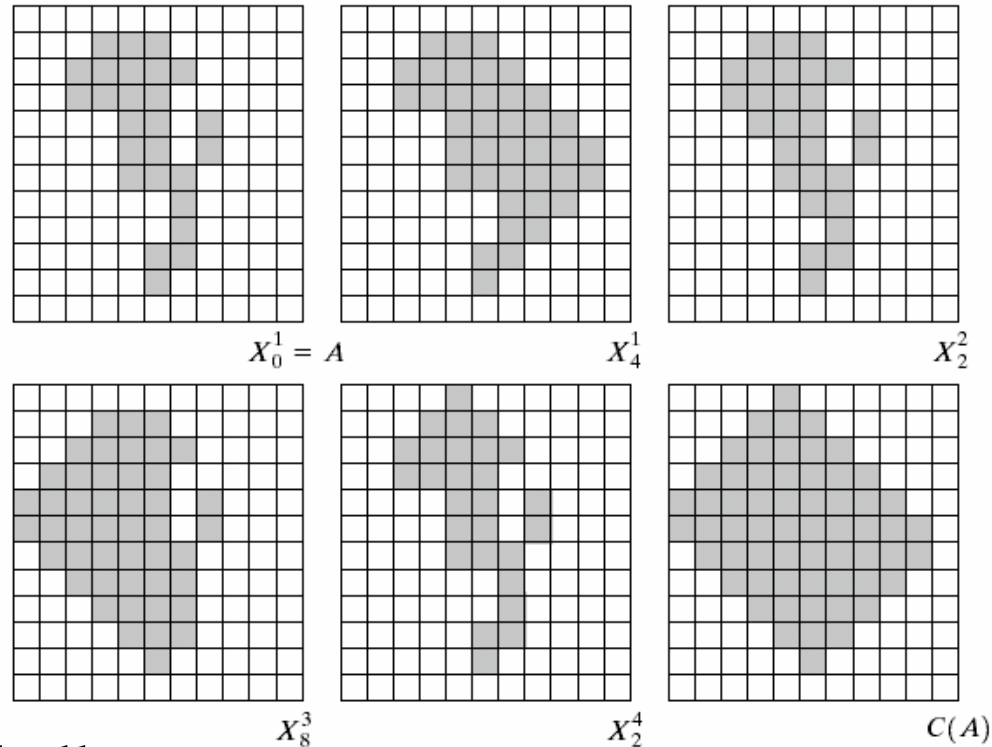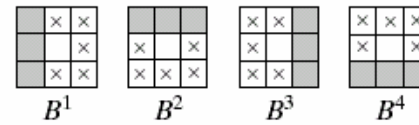
Shortcoming: the convex hull grows beyond the minimum dimensions required to guarantee convexity.

9.5 Basic Morphological Algorithm
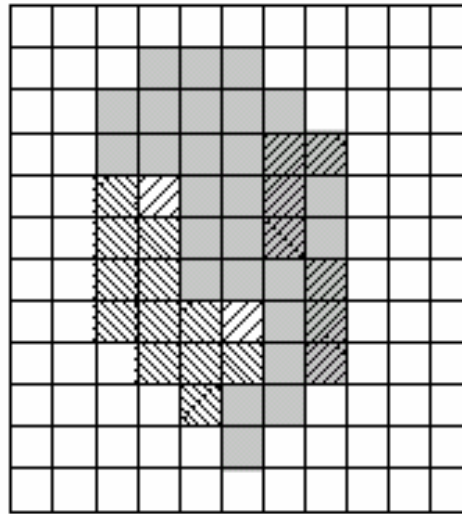
FIGURE 9.20 Result of limiting growth of convex hull algorithm to the maximum dimensions of the original set of points along the vertical and horizontal directions.

9.5 Basic Morphological Algorithm-
Thinning

- The thinning of a set A by a structuring element, denoted A⊗B, can be defined in terms of hit-or-miss operation:

$$A \otimes B = A - (A * B) = A \cap (A * B)^c$$

- Thinning A (***symmetrically***) is based on a sequence of structuring elements: $\{B\} = \{B^1, B^2, B^3, \dots B^n\}$ where B^i is a rotated version of B^{i-1}.

- ***Thinning*** by a sequence of structure elements as $A \otimes \{B\} = ((\dots.((A \otimes B^1) \otimes B^2)\dots.) \otimes B^n)$

9.5 Basic Morphological Algorithm
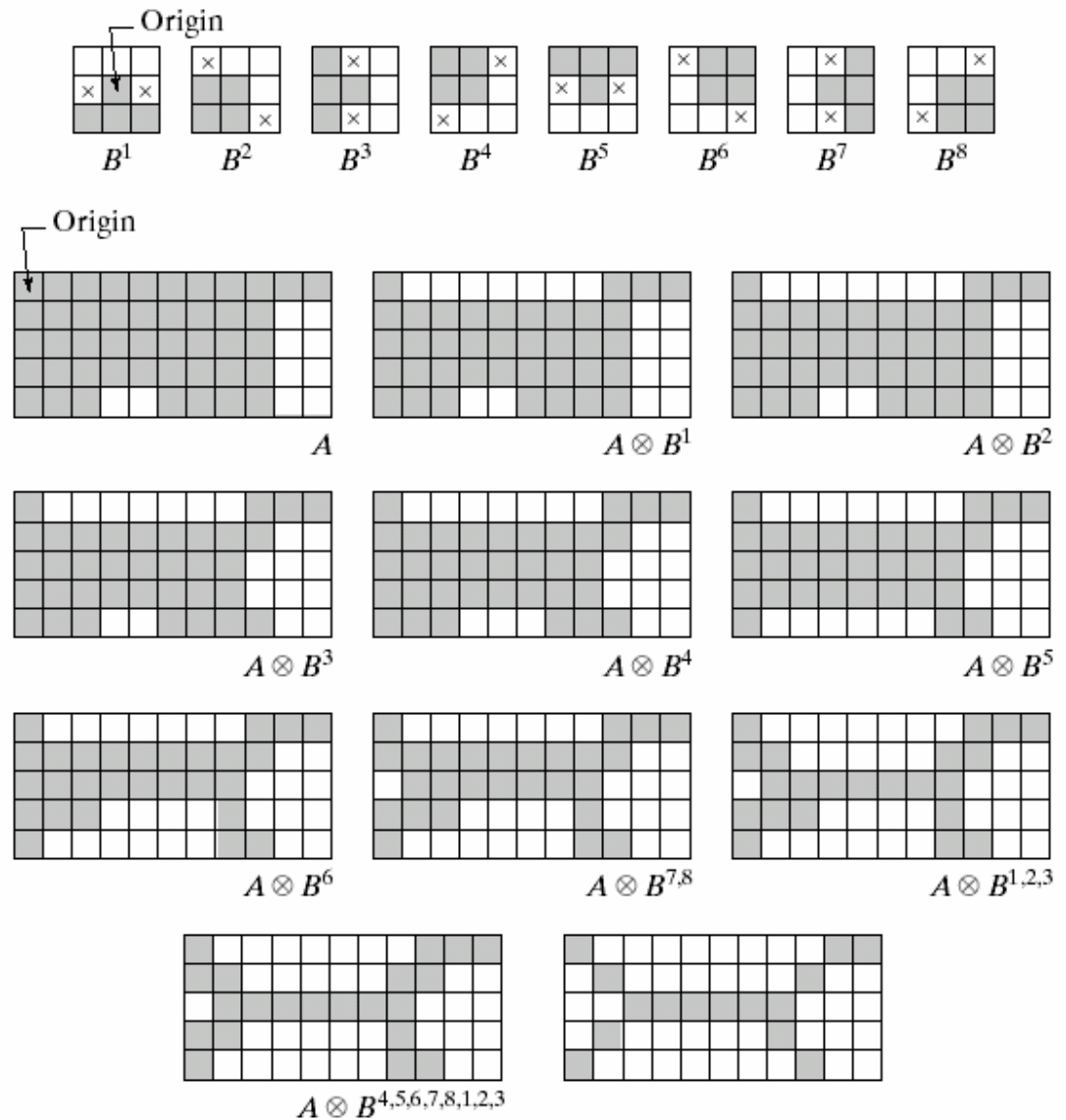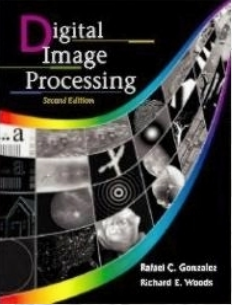
FIGURE 9.21 (a) Sequence of rotated structuring elements used for thinning. (b) Set *A*. (c) Result of thinning with the first element. (d)–(i) Results of thinning with the next seven elements (there was no change between the seventh and eighth elements). (j) Result of using the first element again (there were no changes for the next two elements). (k) Result after convergence. (l) Conversion to *m*-connectivity.

9.5 Basic Morphological Algorithm-Thickening

- Thickening is dual of thinning, and is defined as $A \odot B = A \cup (A*B)$ where B is a structure element suitable for thickening.

- The structure elements for thickening are the same as the ones for thinning with all *1's and 0's* interchanged.

- *Thickening* of A = *Thinning* the background of A and then *complement* the results (*see Fig. 9.22*)

9.5 Basic Morphological Algorithm
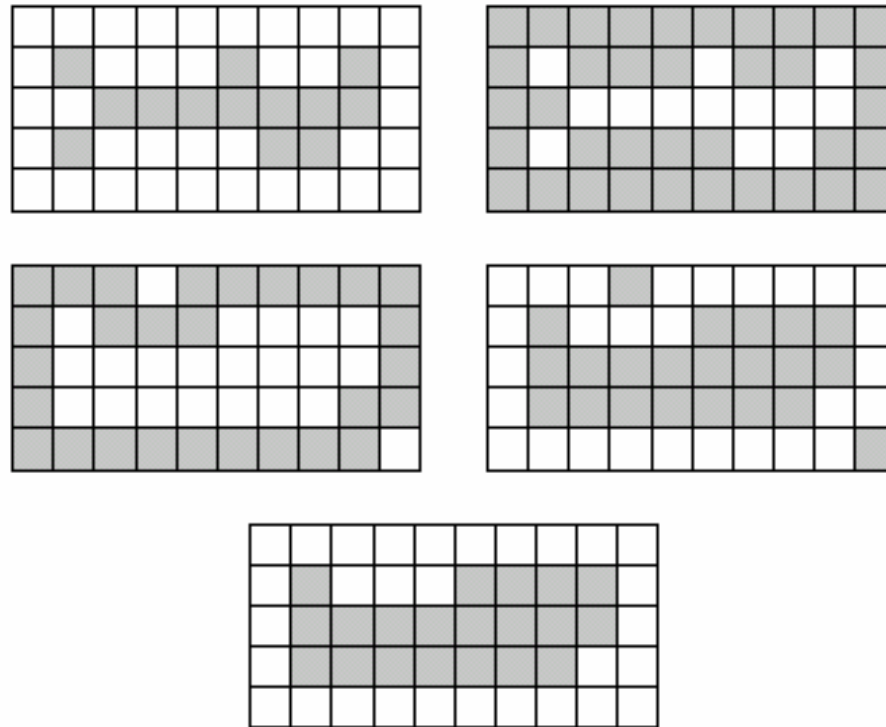
a b
c d
e

FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.

9.5 Basic Morphological Algorithm-
Skeletons

- The ***skeleton*** of set A is denoted as $S(A)$ as shown in fig. 9.23. It has the properties as

(a) If z is a point of $S(A)$, and $(D)_z$ is the largest disk centered at z and contained in A. If one can not find a larger disk, then $(D)_z$ is called a ***maximum disk***.

(b) The $(D)_z$ touches the boundary of A at two or more different places.

- The skeleton of A can be expressed in terms of ***erosions*** and ***openings***, as

$$S(A) = \bigcup_{k=0}^{K} S_k(A) \text{ with } S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

where B is a structure element, and $(A \ominus kB)$ indicated k successive erosions of A as

$$(A \ominus kB) = (\ldots.((A \ominus B) \ominus B) \ominus \ldots)\ominus B$$

9.5 Basic Morphological Algorithm-Skeletons

- *K* is the last iteration step before *A* erodes to an empty set.

- In other words, $K=max\{k\ /(A\Theta kB)\neq\varnothing\}$.

- S(*A*) is a union of the skeleton subset $S_k(A)$

- *A* can be **reconstructed** from these subsets as

$$A=\bigcup_{k=0}^{K}(\,S_k(\,A\,)\oplus kB\,)$$

- where $S_k(A)\oplus kB$ denote the *k* successive dilations of $S_k(A)$ as

$$S_k(A)\oplus kB=((.....(S_k(A)\oplus B)\oplus B)\oplus ....)\oplus B$$

9.5 Basic Morphological Algorithm

a b
c d

FIGURE 9.23
(a) Set A.
(b) Various positions of maximum disks with centers on the skeleton of A.
(c) Another maximum disk on a different segment of the skeleton of A.
(d) Complete skeleton.

9.5 Basic Morphological Algorithm- skeleton

FIGURE 9.24 Implementation of Eqs. (9.5-11) through (9.5-15). The original set is at the top left, and its morphological skeleton is at the bottom of the fourth column. The reconstructed set is at the bottom of the sixth column.

9.5 Basic Morphological Algorithm-
Pruning

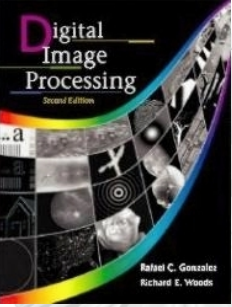- Pruning method is essential process to "***clean-up***" the *parasitic components* after thinning and skeletonizing algorithms.

- Use thinning to detect the end point

$$X_1 = A \otimes \{B\}$$

where $\{B\}$ is a set of ***structure elements***

- *Restore* the character to its original form

    $\rightarrow$ it requires forming a set X_2 containing end points in X_1 (*Fig. 9.25(e)*)
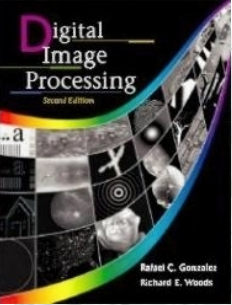
$$X_2 = \bigcup_{i=1}^{8} (X_1 * B^k)$$

9.5 Basic Morphological Algorithm-
Pruning

- Apply dilation of the end point three times and use *A* as a delimeter as (fig. 9.25(f))

$$X_3 = (X_2 \oplus H) \cap A$$

- Where *H* is a *3x3* structure element of *1's*

- Finally *the union of* X_3 and X_1 → yields the desire result.
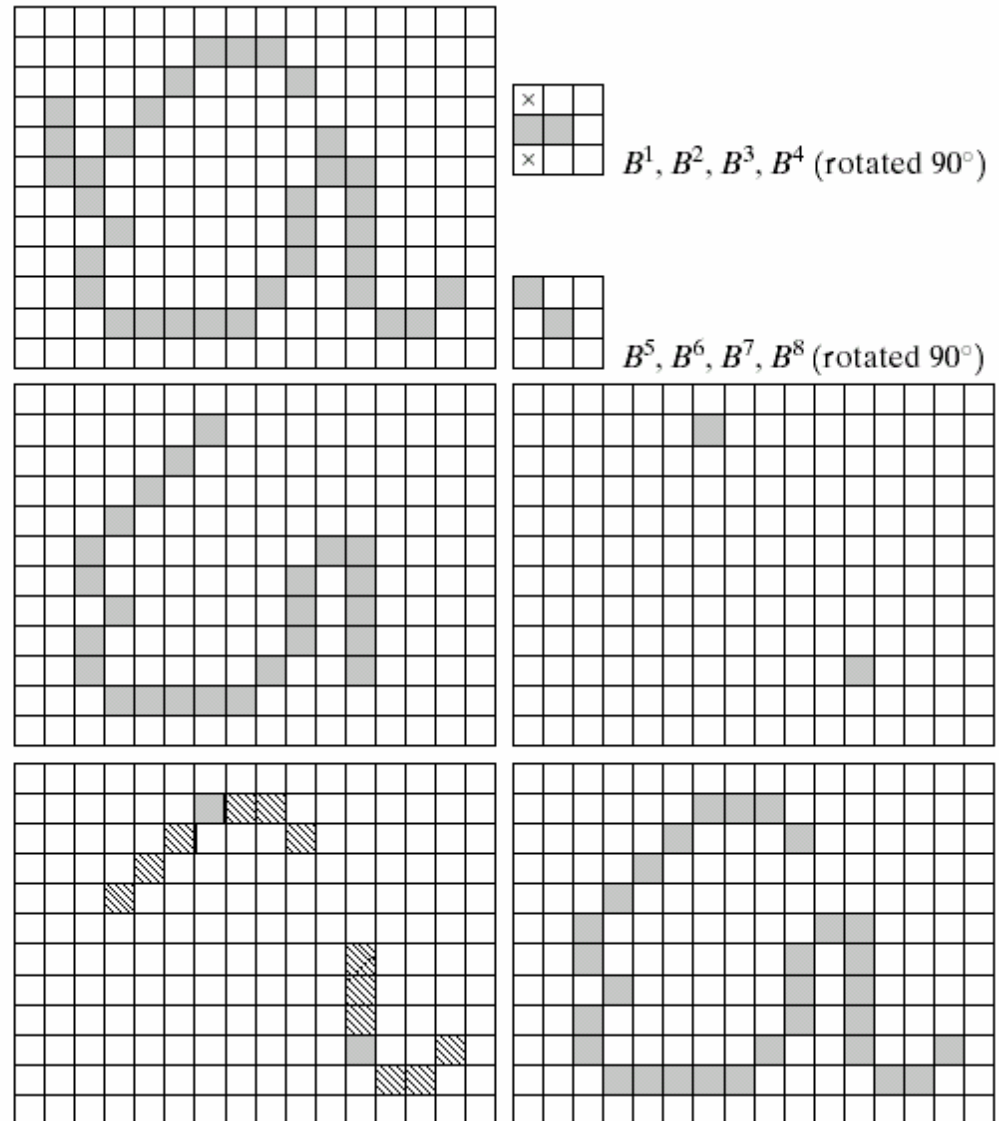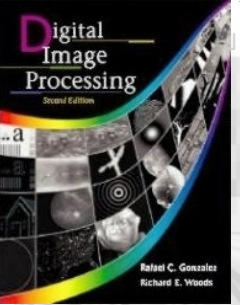
- $X_4 = X_3 \cup X_1$

9.5 Basic Morphological Algorithm-Pruning

a b
c
d e
f g

FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.
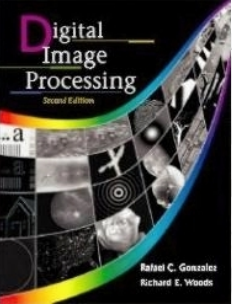

B^1, B^2, B^3, B^4 (rotated 90°)

B^5, B^6, B^7, B^8 (rotated 90°)

9.5 Basic Morphological Algorithm

TABLE 9.2

Summary of morphological operations and their properties.

Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Translation	$(A)_z = \{w \mid w = a + z, \text{ for } a \in A\}$	Translates the origin of A to point z.
Reflection	$\hat{B} = \{w \mid w = -b, \text{ for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w \mid w \notin A\}$	Set of points not in A.
Difference	$A - B = \{w \mid w \in A, w \notin B\}$ $= A \cap B^c$	Set of points that belong to A but not to B.
Dilation	$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \varnothing\}$	"Expands" the boundary of A. (I)
Erosion	$A \ominus B = \{z \mid (B)_z \subseteq A\}$	"Contracts" the boundary of A. (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smoothes contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smoothes contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)

9.5 Basic Morphological Algorithm

TABLE 9.2
Summary of
morphological
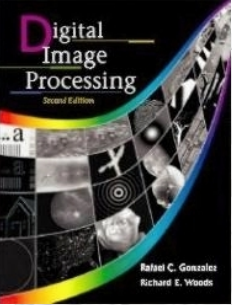results and their
properties.
(continued)

Hit-or-miss transform	$A \circledast B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match ("hit") in A and B_2 found a match in A^c.
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A. (I)
Region filling	$X_k = (X_{k-1} \oplus B) \cap A^c; X_0 = p$ and $k = 1, 2, 3, \ldots$	Fills a region in A, given a point p in the region. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A; X_0 = p$ and $k = 1, 2, 3, \ldots$	Finds a connected component Y in A, given a point p in Y. (I)
Convex hull	$X_k^i = (X_{k-1}^i \circledast B^i) \cup A; i = 1, 2, 3, 4;$ $k = 1, 2, 3, \ldots; X_0^i = A;$ and $D^i = X_{conv}^i.$	Finds the convex hull $C(A)$ of set A, where "conv" indicates convergence in the sense that $X_k^i = X_{k-1}^i.$ (III)

9.5 Basic Morphological Algorithm

| Operation	Equation	Comments (The Roman numerals refer to the structuring elements shown in Fig. 9.26).
Thinning	$A \otimes B = A - (A \circledast B)$ $\quad = A \cap (A \circledast B)^c$ $A \otimes \{B\} =$ $((\dots((A \otimes B^1) \otimes B^2)\dots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$	Thins set A. The first two equations give the basic definition of thinning. The last two equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \circledast B)$ $A \odot \{B\} =$ $((\dots(A \odot B^1) \odot B^2 \dots) \odot B^n)$	Thickens set A. (See preceding comments on sequences of structuring elements.) Uses IV with 0's and 1's reversed.

9.5 Basic Morphological Algorithm

TABLE 9.2

Summary of morphological results and their properties. *(continued)*

|		
Skeletons	$S(A) = \bigcup_{k=0} S_k(A)$ $S_k(A) = \bigcup_{k=0}^{K} \{(A \ominus kB) - [(A \ominus kB) \circ B]\}$ Reconstruction of A: $A = \bigcup_{k=0}^{K} (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A. The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the kth iteration of successive erosion of A by B. (I)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^{8} (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.

9.5 Basic Morphological Algorithm
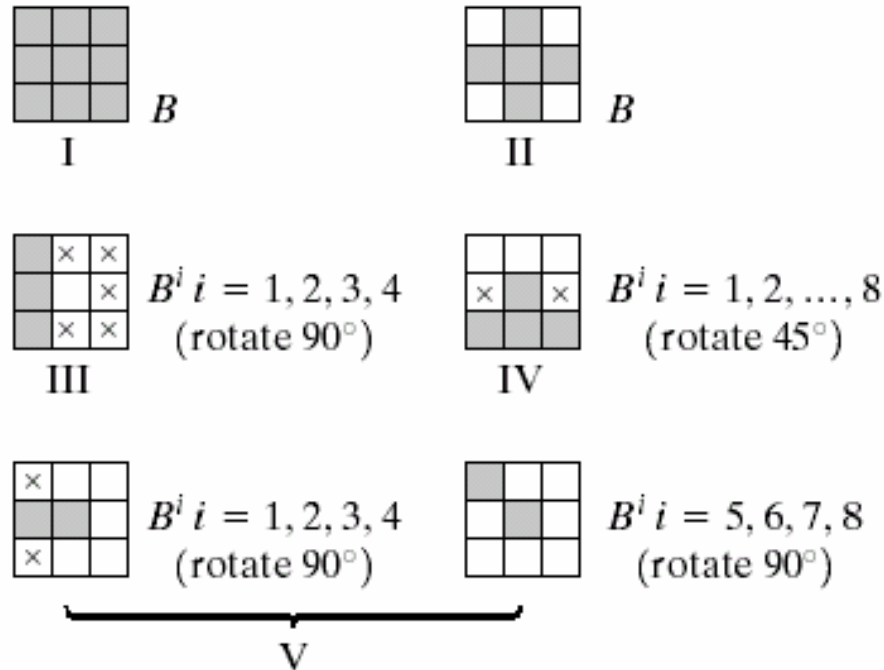
FIGURE 9.26 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the ×'s indicate "don't care" values.

9.6 Extension to Gray-Level Image -Dilation

- ***Gray-scale dilation*** of *f* by *b* is defined as

$(f \oplus b)(s, t) = \max\{f(s-x, t-y) + b(x, y)/(s-x), (t-y) \in D_f; (x, y) \in D_b\}$

where D_f and D_b are the domain of *f* and *b*

- Simplified 1-D function as

$(f \oplus b)(s) = \max\{f(s-x) + b(x)/(s-x) \in D_f; x \in D_b\}$

9.6 Extension to Gray-Level Image

a b
c d
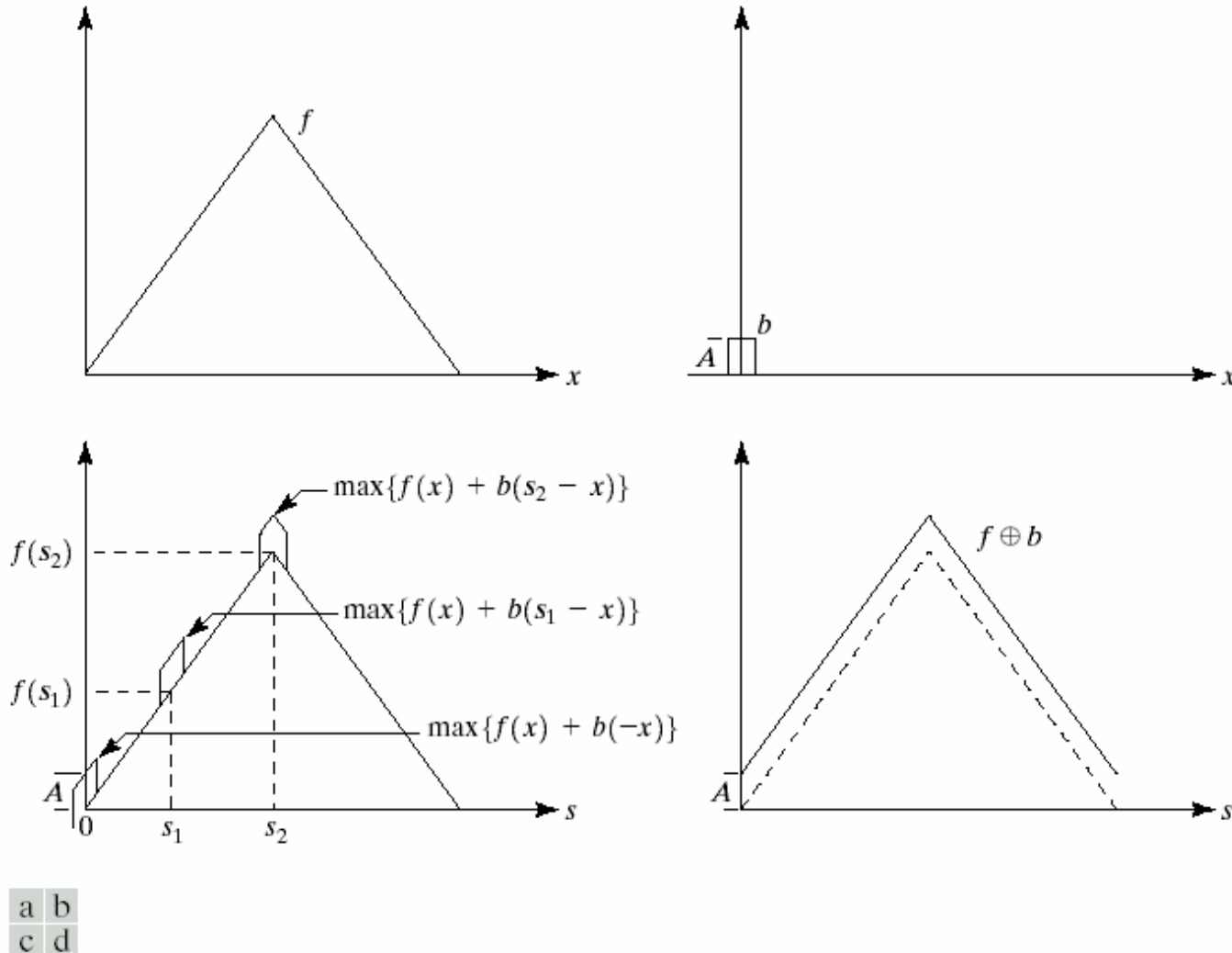
FIGURE 9.27 (a) A simple function. (b) Structuring element of height A. (c) Result of dilation for various positions of sliding b past f. (d) Complete result of dilation (shown solid).

9.6 Extension to Gray-Level Image -Erosion

- ***Gray-scale erosion*** of f by b is defined as

$(f\,\Theta b)(s,\,t)=\min\{f(s+x,\,t+y)-b(x,\,y)/(s+x),\,(t+y)\in D_f;$
$(x,\,y)\in D_b\}$

where D_f and D_b are the domain of f and b

- Simplified 1-D function as

$(f\,\Theta\,b)(s)=\min\{f(s+x)-b(x)/(s+x)\in D_f;\,x\in D_b\}$
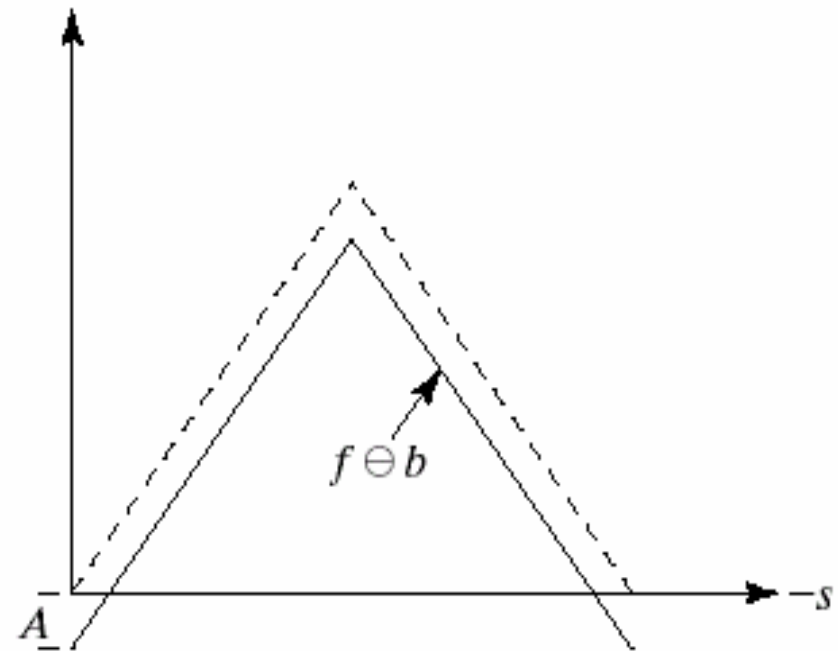
$(f\,\Theta b)^c(s,\,t)=(f^c\oplus\hat{b})(s,\,t)$
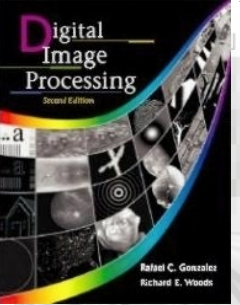
where $f^c=\,-f(x,y)$ and $\hat{b}=b(-x,\,-y)$

9.6 Extension to Gray-Level Image
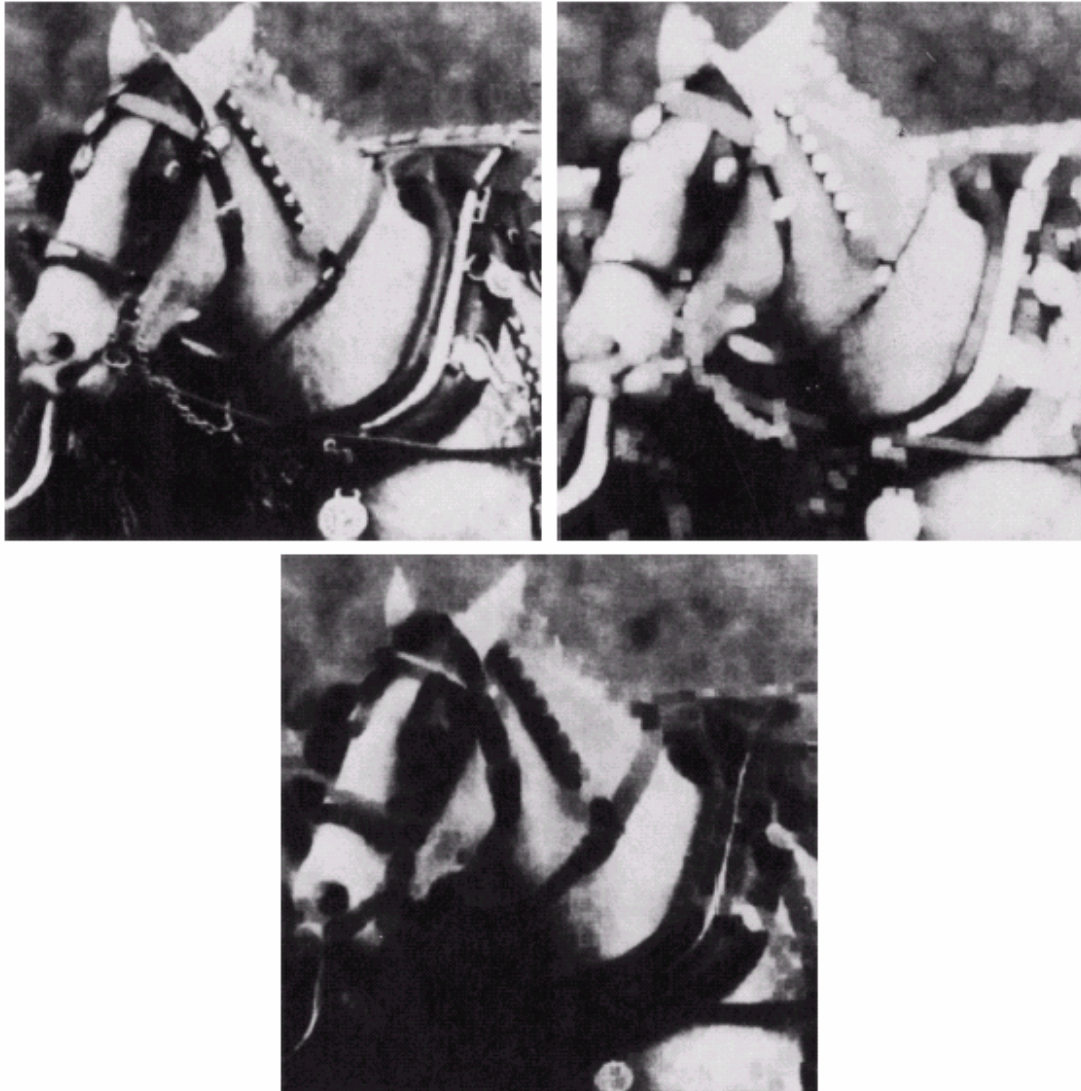
FIGURE 9.28
Erosion of the
function shown in
Fig. 9.27(a) by the
structuring
element shown in
Fig. 9.27(b).

$f \ominus b$

9.6 Extension to Gray-Level Image

a b
c

FIGURE 9.29
(a) Original
image. (b) Result
of dilation.
(c) Result of
erosion.
(Courtesy of
Mr. A. Morris,
Leica Cambridge,
Ltd.)

9.6 Extension to Gray-Level Image- opening and closing

- Opening: $f \circ b = (f \Theta b) \oplus b$
- Closing: $f \bullet b = (f \oplus b) \Theta b$
- $(f \bullet b)^c = f^c \circ \hat{b}$
- $f^c = -f(x, y)$ and $\hat{b} = b(-x, -y)$
- $-(f \bullet b) = -f \circ \hat{b}$
- Viewing $f(x, y)$ in 3-D perspective as a 2-D surface.
- Opening f by a spherical structure element, b, may be interpreted geometrically as the process of pushing the ball against the underside of the surface

9.6 Extension to Gray-Level Image-opening and closing

- The opening of *f* by *b* is the surface of the highest points reached by any part of the sphere as it slides over the entire *under-surface* of *f*.

- Opening is to remove the *light details* of the image.

- The closing operation can be viewed as slide the ball on the top of the surface.

- Closing is to remove the *dark details* of the image.

9.6 Extension to Gray-Level Image-opening and closing
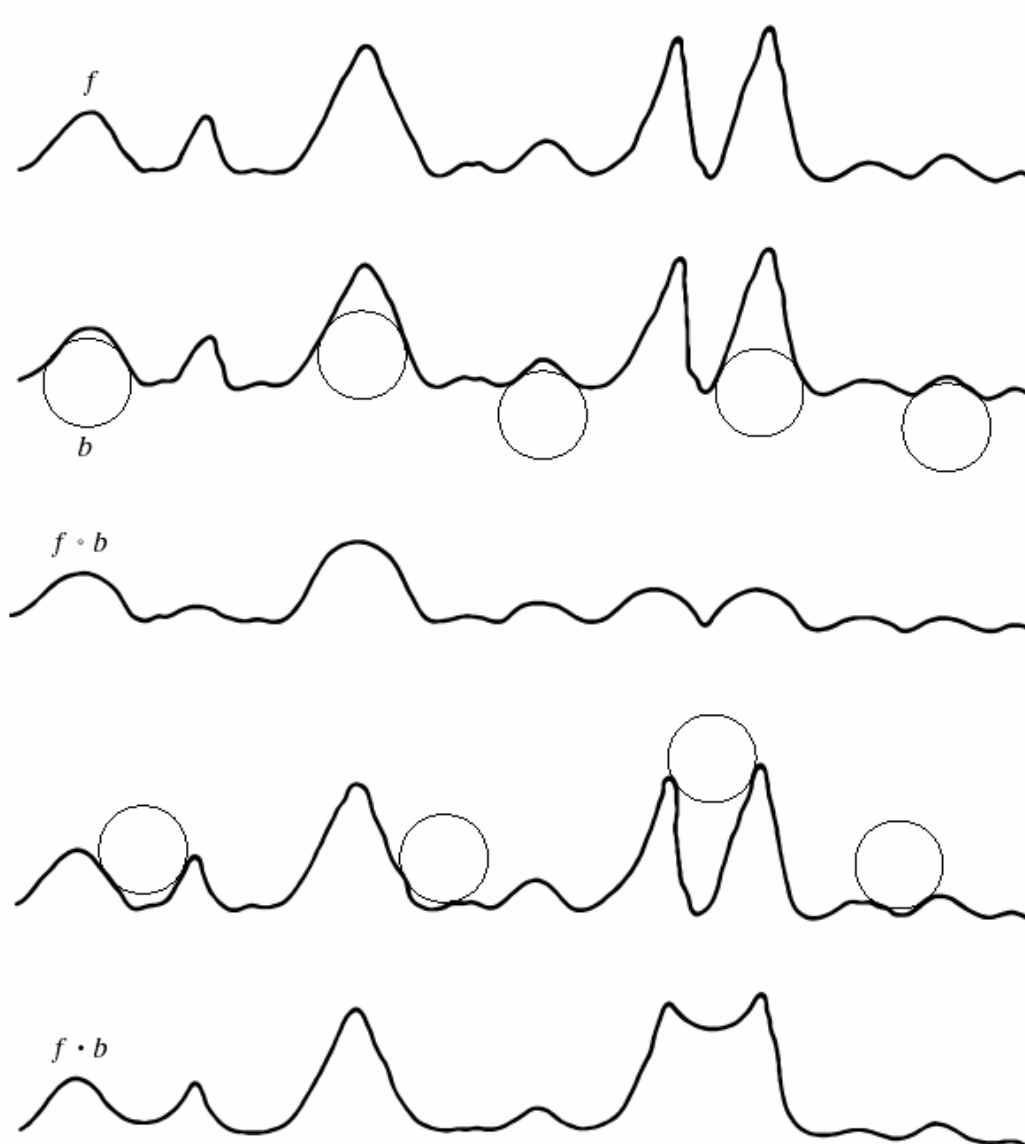
- *The opening operation properties*

  1) $(f \circ b) \sqsubseteq f$

  2) If $f_1 \sqsubseteq f_2$ then $(f_1 \circ b) \sqsubseteq (f_2 \circ b)$

  3) $(f \circ b) \circ b = f \circ b$

  where $e \sqsubseteq r$ indicates that the domain of e is a subset of the domain of r, and also that $e(x,y) \leq r(x,y)$ in the domain of e
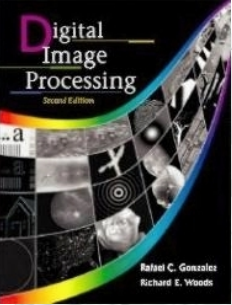
9.6 Extension to Gray-Level Image

a
b
c
d
e

FIGURE 9.30
(a) A gray-scale scan line.
(b) Positions of rolling ball for opening.
(c) Result of opening.
(d) Positions of rolling ball for closing. (e) Result of closing.

9.6 Extension to Gray-Level Image

a b

FIGURE 9.31 (a) Opening and (b) closing of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)
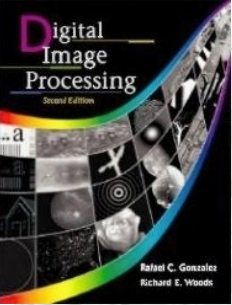
9.6 Extension to Gray-Level Image

- Morphological smoothing: apply opening and then closing to remove the bright and dark noise.

- Morphological Gradient

$$g = (f \oplus b) - (f \ominus b)$$
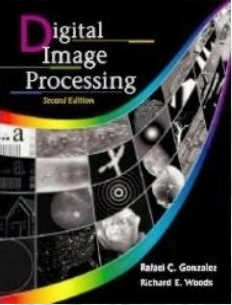
- Top-hat transformation

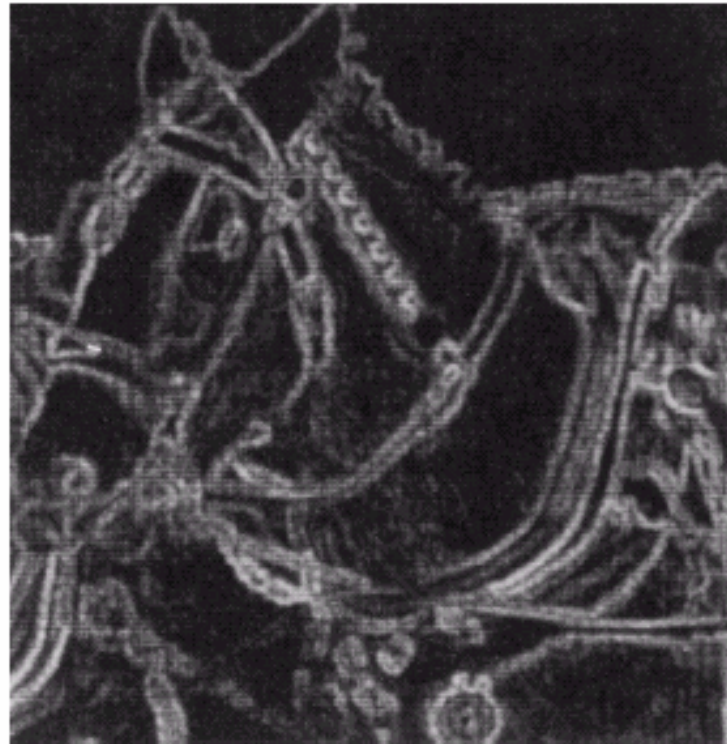$$h = f - (f \circ b)$$

9.6 Extension to Gray-Level Image

FIGURE 9.32 Morphological smoothing of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)
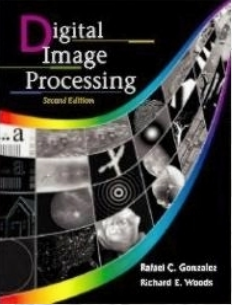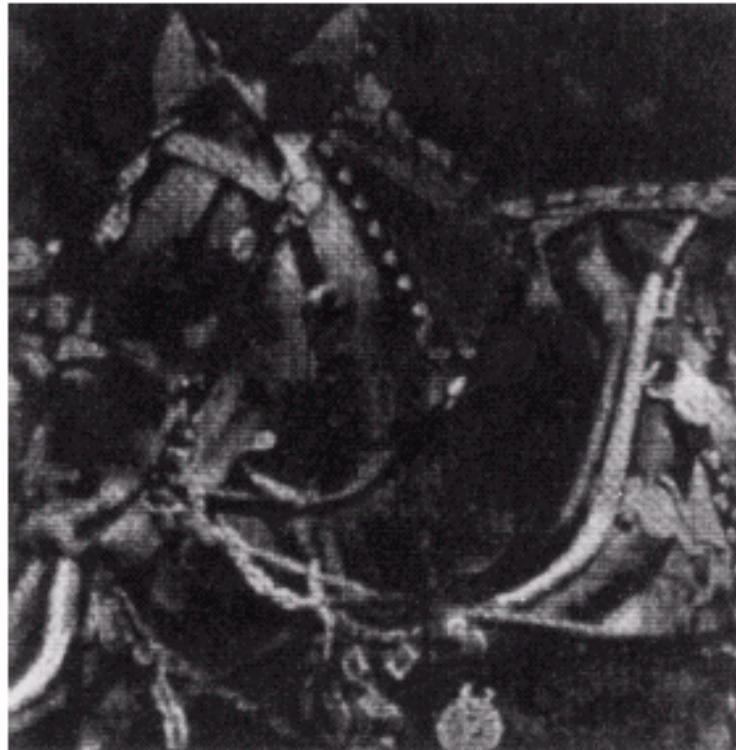
9.6 Extension to Gray-Level Image

FIGURE 9.33 Morphological gradient of the image in Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

9.6 Extension to Gray-Level Image

FIGURE 9.34 Result of performing a top-hat transformation on the image of Fig. 9.29(a). (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

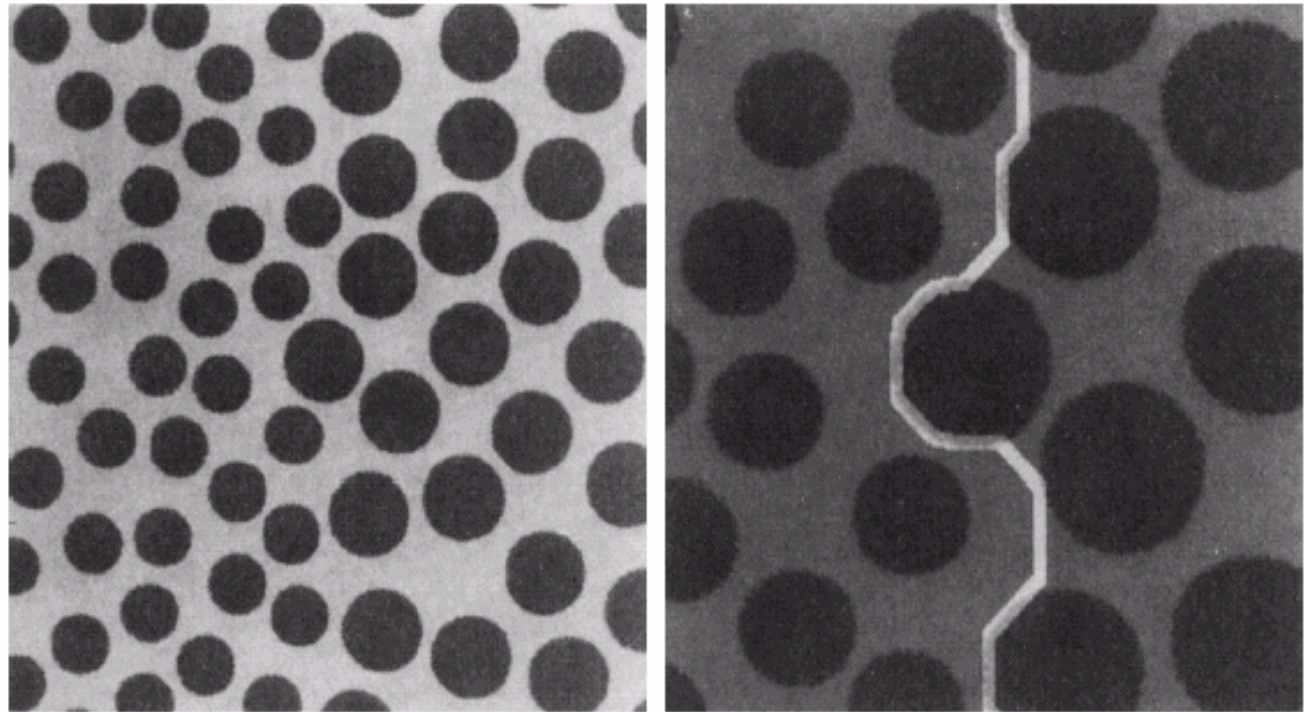# 9.6 Extension to Gray-Level Image

- Textural Segmentation:

1) *Close* the input image by using successively larger structure elements.

2) When the structure element=small blobs, they are removed and leaving only light background.

3) A single *opening* is performed with a structure element that is large in relation to the separation between large blobs

4) Remove light patches between the blobs, and leave a dark region on the right.

5) A light region on the left and dark region on the right.

6) A simple threshold then yields the boundary between two texture regions.

9.6 Extension to Gray-Level Image

a b

FIGURE 9.35
(a) Original image. (b) Image showing boundary between regions of different texture. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)

9.6 Extension to Gray-Level Image

- Granulometry: determining the size distribution of particles in an images:

  1) Opening with increasing size structure elements.

  2) Each difference between the original and the opened image is computed after each pass.

  3) These differences are normalized and used to construct a histogram of particle-size distribution.

9.6 Extension to Gray-Level Image

Three predominant particle sizes

difference
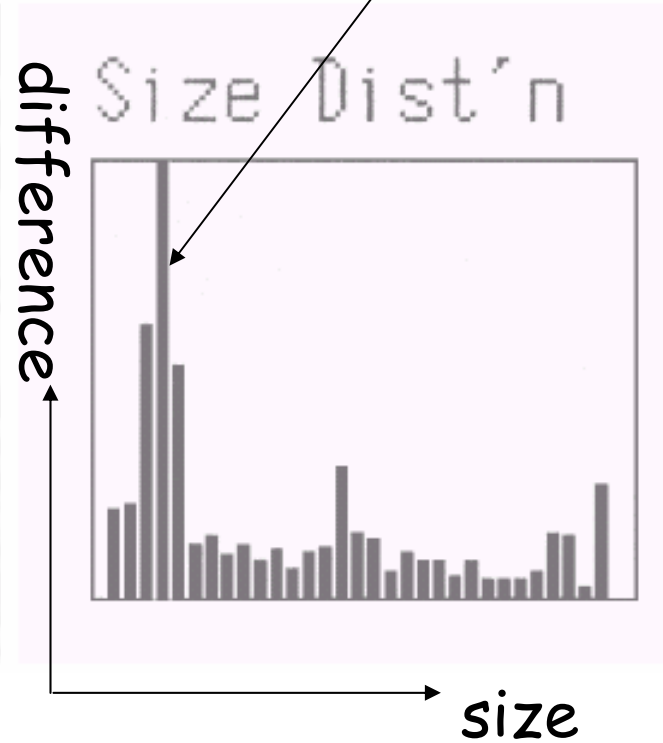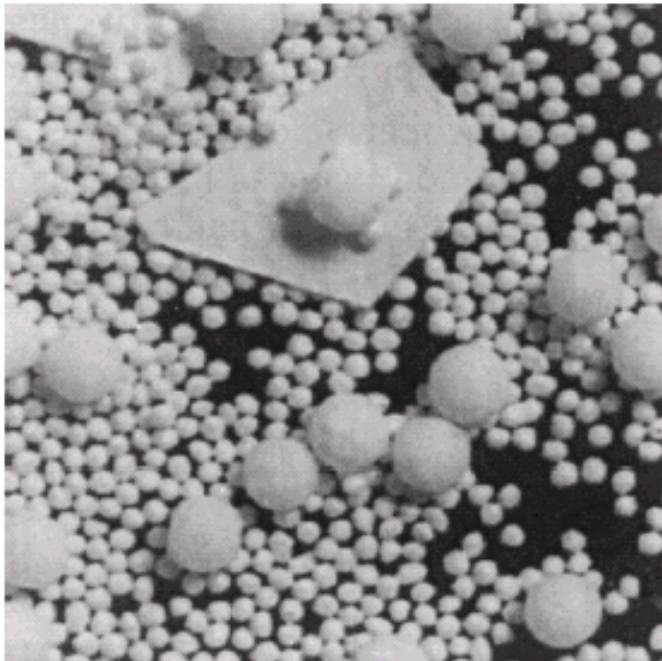
Size Dist'n

size

a b

FIGURE 9.36
(a) Original image consisting of overlapping particles; (b) size distribution. (Courtesy of Mr. A. Morris, Leica Cambridge, Ltd.)