# CHAPTER 6

# *Data Encryption Standard (DES)*

(Solution to Practice Set)

## Review Questions

1. The block size in DES is 64 bits. The cipher key size is 56 bits. The round key size is 48 bits.
2. DES uses 16 rounds.
3. In the first approach, DES uses 16 mixers and 15 swappers in encryption or decryption algorithm; in the second (alternative approach), DES use 16 mixers and 16 swappers in encryption or decryption algorithm.
4. In DES, encryption or decryption uses $16 \times 2 + 2 = 34$ permutations, because each mixer uses two permutations and there are two permutations before and after the rounds. The round-key generator uses 17 permutation operations: one parity drop and 16 compression permutation operations for each round.
5. The total number of exclusive-or operations is $16 \times 2 = 32$, because each round uses two exclusive-or operations (one inside the function and one outside of the function).
6. The input to the function is a 32-bit word, but the round-key is a 48-bit word. The expansion permutation is needed to increase the number of bits in the input word to 48.
7. The cipher key that is used for DES include the parity bits. To remove the parity bits and create a 56-bit cipher key, a parity drop permutation is needed. Not only does the parity-drop permutation drop the parity bits, it also permutes the rest of the bits.
8. A weak key is the one that, after parity drop operation, consists either of all 0s, all 1s, or half 0s and half 1s. Each weak key is the inverse of itself: $E_k(E_k(P)) = P$. A semi-weak key creates only two different round keys and each of them is repeated eight times. The semi-weak round keys come in pairs, where a key in the pair is the inverse of the other key in the pair: $E_{k1}(E_{k2}(P)) = P$. A possible weak key is a key that creates only four distinct round keys; in other words, the sixteen round keys are divided into four groups and each group is made of four equal round keys.

**9.** Double DES uses two instances of DES ciphers for encryption and two instances of reverse ciphers for decryption. Each instance uses a different key, which means that the size of the key is 112 bits. However, double DES is vulnerable to meet-in-the-middle attack.

**10.** Triple DES uses three stages of DES for encryption and decryption. Two versions of triple DES are in use today: triple DES with two keys and triple DES with three keys. In triple DES with two keys, there are only two keys: $K_1$ and $K_2$. The first and the third stages use $K_1$; the second stage uses $K_2$. In triple DES with three keys, there are three keys: $K_1$, $K_2$, and $K_3$.

## Exercises

**11.**

**a.**

**Input: 1 1011 1** → **3, 11** → **Output: 03 (0011)**

**b.**

**Input: 0 0110 0** → **0, 6** → **Output: 09 (1001)**

**c.**

**Input: 0 0000 0** → **0, 0** → **Output: 04 (0100)**

**d.**

**Input: 1 1111 1** → **3, 15** → **Output: 09 (1001)**

**12.** The following table shows the output from all boxes. No pattern can be found:

| *Input* | *Box 1* | *Box 2* | *Box 3* | *Box 4* | *Box 5* | *Box 6* | *Box 7* | *Box 8* |
|---|---|---|---|---|---|---|---|---|
| **000000** | **1110** | **1111** | **1010** | **0111** | **0010** | **1100** | **0100** | **1101** |

**13.** The following table shows the output from all boxes. No pattern can be found:

| *Input* | *Box 1* | *Box 2* | *Box 3* | *Box 4* | *Box 5* | *Box 6* | *Box 7* | *Box 8* |
|---|---|---|---|---|---|---|---|---|
| **111111** | **1101** | **1001** | **1100** | **1110** | **0011** | **0011** | **1101** | **1011** |

**14.**

**a.** The following shows that 3 bits will be changed in the output.

| | | | | |
|---|---|---|---|---|
| **Input: 0 0000 0** | → | **00, 00** | → | **Output: 10 (1010)** |
| **Input: 0 0000 1** | → | **01, 00** | → | **Output: 13 (1101)** |

**b.** The following shows that 2 bits will be changed in the output.

| | | | | |
|---|---|---|---|---|
| **Input: 1 1111 1** | → | **03, 15** | → | **Output: 09 (1001)** |
| **Input: 1 1101 1** | → | **03, 14** | → | **Output: 14 (1100)** |

**15.**

**a.** The following shows that 2 bits will be changed in the output.

| | | | | |
|---|---|---|---|---|
| **Input: 0 0110 0** | → | **00, 06** | → | **Output: 03 (0011)** |
| **Input: 0 0000 0** | → | **00, 00** | → | **Output: 15 (1111)** |

**b.** The following shows that 2 bits will be changed in the output.

| | | | | |
|---|---|---|---|---|
| **Input: 1 1001 1** | → | **03, 09** | → | **Output: 06 (0110)** |
| **Input: 1 1111 1** | → | **03, 15** | → | **Output: 09 (1001)** |

**16.**

**a.** The following shows that two outputs are different.

| | | | | |
|---|---|---|---|---|
| **Input: 0 0110 0** | → | **00, 06** | → | **Output: 09 (1001)** |
| **Input: 1 1000 0** | → | **02, 08** | → | **Output: 15 (1111)** |

**b.** The following shows that two outputs are different.

| | | | | |
|---|---|---|---|---|
| **Input: 1 1001 1** | → | **03, 09** | → | **Output: 04 (0100)** |
| **Input: 0 0111 1** | → | **01, 07** | → | **Output: 03 (0011)** |

**17.** The following table shows 32 input pairs and 32 output pairs. The last column is the difference between the outputs.

| *Input pairs* | | *Output pairs* | | *d* |
|---|---|---|---|---|
| 000000 | 000001 | 0010 | 1110 | **1100** |
| 000010 | 000011 | 1100 | 1011 | **0111** |
| 000100 | 000101 | 0100 | 0010 | **0110** |
| 000110 | 000111 | 0001 | 1100 | **1111** |
| 001000 | 001001 | 0111 | 0100 | **0011** |
| 001010 | 001011 | 1010 | 0111 | **1001** |
| 001100 | 001101 | 1011 | 1101 | **0110** |
| 001110 | 001111 | 0110 | 0001 | **0111** |
| 010000 | 010001 | 1000 | 0101 | **1101** |
| 010010 | 010011 | 0101 | 0000 | **0101** |
| 010100 | 010101 | 0011 | 1111 | **1100** |
| 010110 | 010111 | 1111 | 1010 | **0101** |
| 011000 | 011001 | 1101 | 0011 | **1110** |
| 011010 | 011011 | 0000 | 1001 | **1001** |
| 011100 | 011101 | 1110 | 1000 | **0110** |
| 011110 | 011111 | 1001 | 0110 | **1111** |
| 100000 | 100001 | 0110 | 1011 | **1101** |
| 100010 | 100011 | 0010 | 1000 | **1010** |
| 100100 | 100101 | 0001 | 1100 | **1101** |
| 100110 | 100111 | 1011 | 0111 | **1100** |
| 101000 | 101001 | 1010 | 0001 | **1011** |
| 101010 | 101011 | 1101 | 1110 | **0011** |
| 101100 | 101101 | 0111 | 0010 | **0101** |
| 101110 | 101111 | 1000 | 1101 | **0101** |
| 110000 | 110001 | 1111 | 0110 | **1001** |
| 110010 | 110011 | 1001 | 1111 | **0110** |
| 110100 | 110101 | 1100 | 0000 | **1100** |
| 110110 | 110111 | 0101 | 1001 | **1100** |
| 111000 | 111001 | 0110 | 1010 | **1100** |
| 111010 | 111011 | 0011 | 0100 | **0111** |
| 111100 | 111101 | 0000 | 0101 | **0101** |
| 111110 | 111111 | 1110 | 0011 | **1101** |

If we sort the table on last column (output differences), we get the following: two (0011)'s, five (0101)'s, four (0110)'s, three (0111)'s, three (0111)'s, one (1010), one (1011), one (1100), five (1100)'s, four (1101)'s, one (1110), and two (1111)'s. **None of the group has a size larger than eight.**

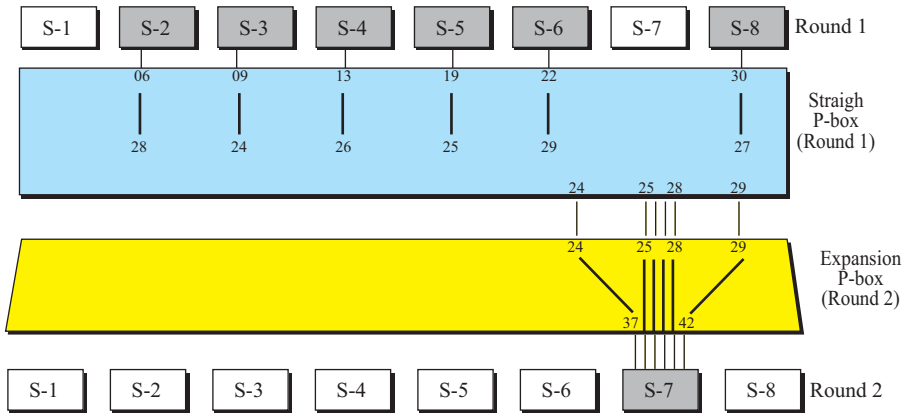
**18.** For S-Box 7, we set the first (leftmost) input bit to 0. We change the other five input bits. We then observe one of the output bits (we have chosen the third bit from the let) and count how many 0's and how may 1's we obtain for this bit. These two count must be close to each other. The following table shows inputs and outputs.

| *Input* | *Output* | *Third bit* |
|---|---|---|
| **0** 0000 0 | 0 1 **0** 0 | 0 |
| **0** 0000 1 | 1 1 **0** 1 | 0 |
| **0** 0001 0 | 1 0 **1** 1 | 1 |
| **0** 0001 1 | 0 0 **0** 0 | 0 |
| **0** 0010 0 | 0 0 **1** 0 | 1 |
| **0** 0010 1 | 1 0 **1** 1 | 1 |
| **0** 0011 0 | 1 1 **1** 0 | 1 |
| **0** 0011 1 | 0 1 **1** 1 | 1 |
| **0** 0100 0 | 1 1 **1** 1 | 1 |
| **0** 0100 1 | 0 1 **0** 0 | 0 |
| **0** 0101 0 | 0 0 **0** 0 | 0 |
| **0** 0101 1 | 1 0 **0** 0 | 0 |
| **0** 0110 0 | 1 0 **0** 0 | 0 |
| **0** 0110 1 | 0 0 **0** 1 | 0 |
| **0** 0111 0 | 1 1 **0** 1 | 0 |
| **0** 0111 1 | 1 0 **1** 0 | 1 |
| **0** 1000 0 | 0 0 **1** 1 | 1 |
| **0** 1000 1 | 1 1 **1** 0 | 1 |
| **0** 1001 0 | 1 1 **0** 0 | 0 |
| **0** 1001 1 | 0 0 **1** 1 | 1 |
| **0** 1010 0 | 1 0 **0** 1 | 0 |
| **0** 1010 1 | 0 1 **0** 1 | 0 |
| **0** 1011 0 | 0 1 **1** 1 | 1 |
| **0** 1011 1 | 1 1 **0** 0 | 0 |
| **0** 1100 0 | 0 1 **0** 1 | 0 |
| **0** 1100 1 | 0 0 **1** 0 | 1 |
| **0** 1101 0 | 1 0 **1** 0 | 1 |
| **0** 1101 1 | 1 1 **1** 1 | 1 |
| **0** 1110 0 | 0 0 **0** 0 | 0 |
| **0** 1110 1 | 1 0 **0** 0 | 0 |
| **0** 1111 0 | 0 0 **0** 1 | 0 |
| **0** 1111 1 | 0 1 **1** 0 | 1 |

As the table shows we have **17 0's** and **15 1's**; close enough.

**19.** Figure S6.19 shows the situation. The inputs to S-box 7 in round 2 comes from six different S-boxes in round 1.

**Figure S6.19** *Solution to Exercise 19*




**a.** The six inputs to S-box 7 in round 2 come from six outputs (37, 38, 39, 40, 41, 42) of expansion permutation box.

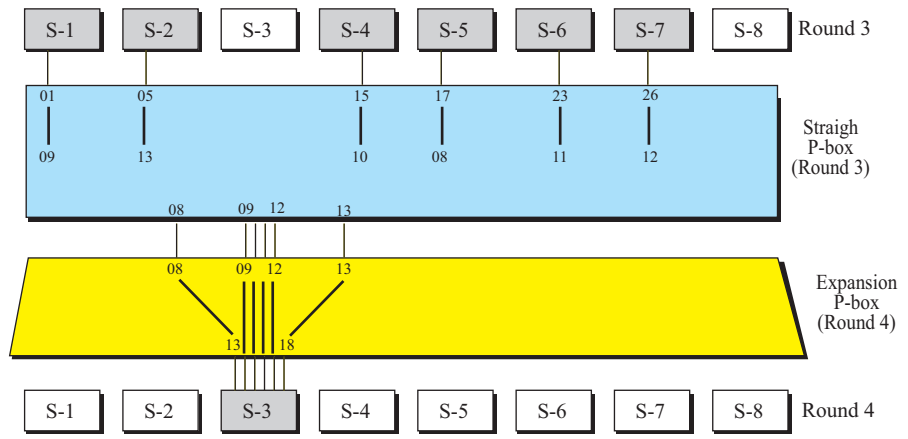
**b.** The above six outputs correspond to the six inputs (24, 25, 26, 27, 28, 29) in the expansion permutation box (See Table 6.2 in the textbook).

**c.** The above six inputs correspond to the six inputs (09, 19, 13, 30, 06, 22) inputs in the straight permutation box (See Table 6.11 in the textbook).

**d.** The above six inputs correspond to the outputs of six different S-Boxes (S-3, S-5, S-4, S-8, S-2, and S-6) in round 1.

**20.** Figure S6.20 shows the situation. The inputs to S-box 7 in round 2 comes from six S-boxes in round 1.

**Figure S6.20** *Solution to Exercise 20*



**a.** The six inputs to S-box 3 in round 4 come from six outputs (13, 14, 15, 16, 17, 18) of expansion permutation box in round 4.

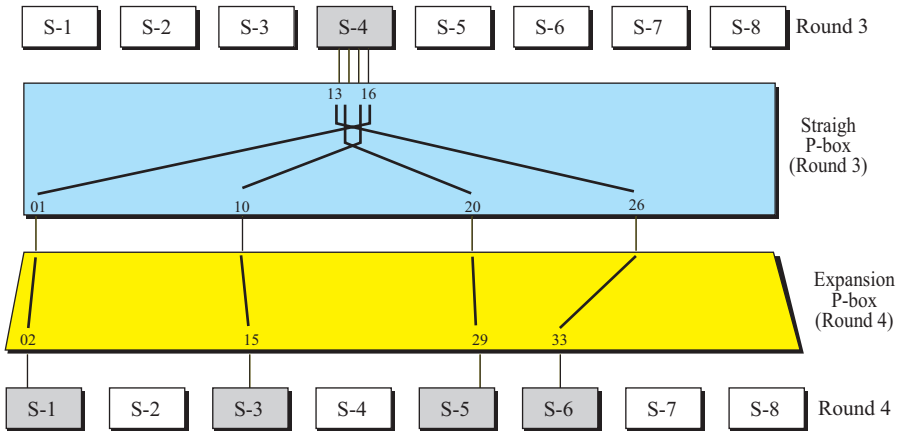
**b.** The above six outputs corresponds to the six inputs (08, 09, 10, 11, 12, 13) in the expansion permutation box in round 4 (See Table 6.2 in the textbook).

**c.** The above six inputs correspond to the six inputs (17, 01, 15, 23, 26, 05) inputs in the straight permutation box (See Table 6.11 in the textbook).

**d.** The above six inputs correspond to the outputs of six different S-Boxes (S-5, S-1, S-4, S-6, S-7, and S-2) in round 3. None of them come from S-3.

**21.** Figure S6.21 shows the situation. The outputs from S-box 4 in round 3 go to four S-boxes in round 4.

**Figure S6.21** *Solution to Exercise 21*



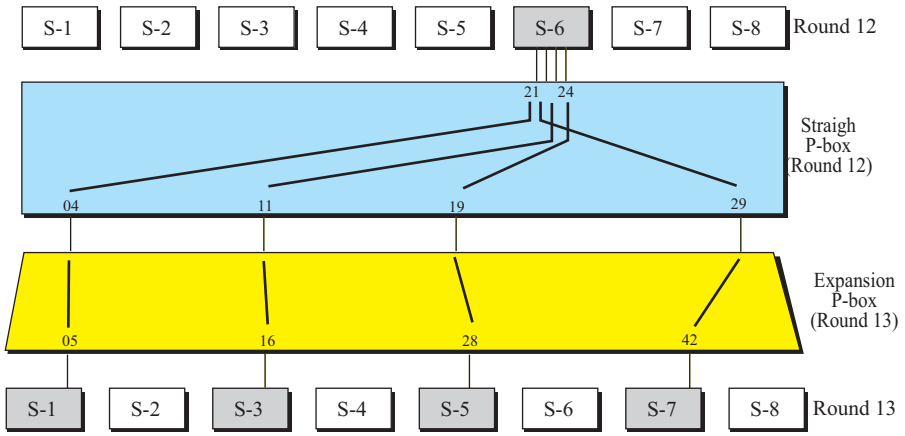

**a.** The four outputs from S-box 3 in round 4 go to six outputs (26, 20, 10, 01) of straight permutation box (See Table 6.1).

**b.** The above four outputs correspond to four outputs (33, 29, 15, 02) in the expansion permutation box (See Table 6.11).

**c.** The above four inputs corresponds to the inputs of four different S-Boxes (S-6, S-5, S-3, and S-1) in round 4.

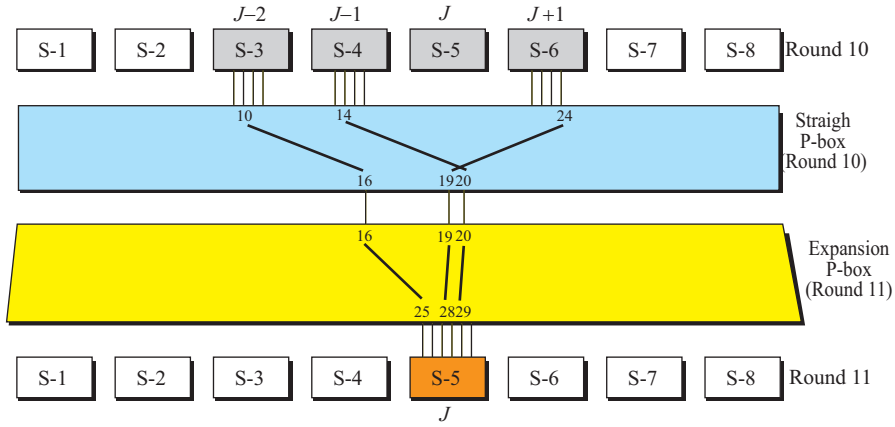
**22.** Figure S6.22 shows the situation. The outputs from S-box 6 in round 12 goes to four different S-boxes in round 13. None of them go to S-box 6. So the criterion cannot actually be tested by these exercise, but it does not violate the criterion either.

**Figure S6.22** *Solution to Exercise 22*




**23.** Figure S6.23 shows the situation. We assume $j = 5$.

**Figure S6.23** *Solution to Exercise 23*



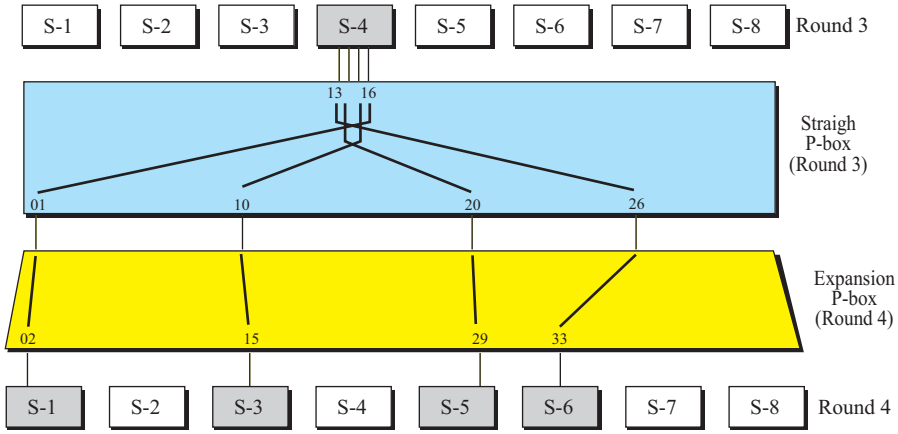

**a.** One output from S-box 3 (output 10) goes to the first input of S-box 5 (input 25).

**b.** An output from S-box 4 (output 14) goes one the last two inputs of S-box 6 (input 29).

**c.** An output from S-box 6 (output 24) goes to one of the middle input of S-box 5 (input 19).

**24.** The solution can be found in Figure S6.24. The outputs of S-4 is distributed between S-1, S-3, S-5, and S-7 in the next round.

**Figure S6.24** *Solution to Exercise 24*




**a.** The output 16 goes to bit 2 (one of the first two bits of S-box 1).

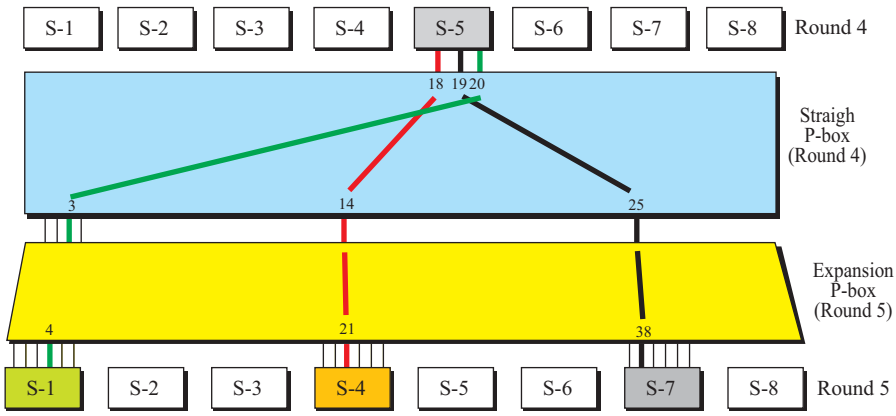
**b.** The output 14 goes to bit 29 (the last bit of S-box 5).

**c.** The output 15 goes to bit 15 (one of middle bit of S-box 3).

**d.** The output 13 goes to bit 33 (one of the middle bit of S-6).

**25.** Figure S6.25 will help us in this problem.

**Figure S6.25** *Solution to Exercise 25*



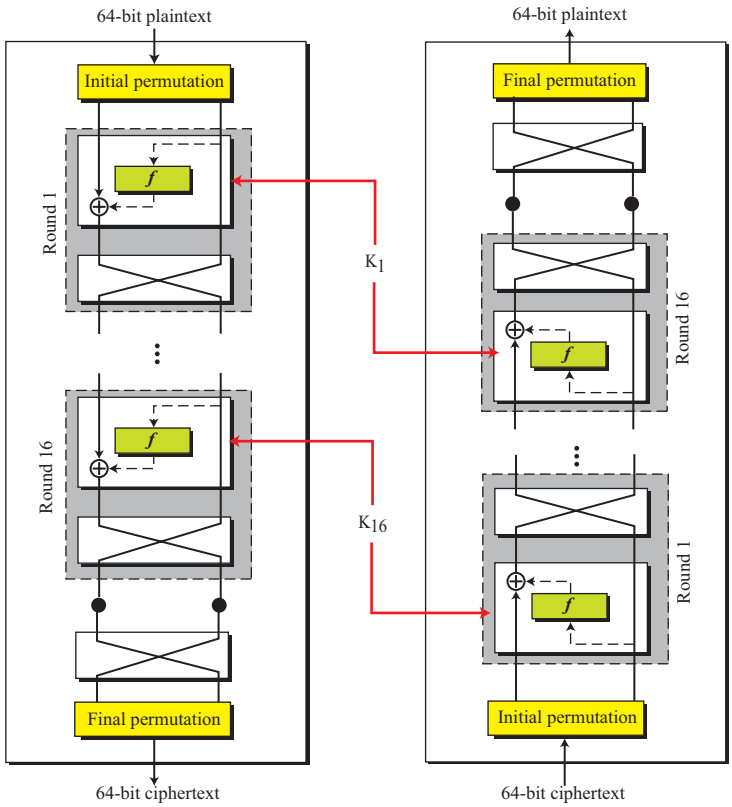
**a.** Only output 19 form S-box 5 (in round 4) goes to S-box 7 (in round 5). However, the input 38 is not a middle input, so the criterion does not apply. This answers the question about this exercise, but we do some more investigations.

**b.** Output 18 from S-box 5 goes a middle input (21) in S-box 4 in the next round. To check the criteria, we need to see if any input from S-box 4 goes to a middle input in next round. Looking at Figure S6.24, we can see that this is not the case. The criterion applies here.

**c.** We also observe that output 19 from S-box 5 goes a middle input in S-box 1 (input 4). However, none of the inputs from S-box 1 in round 4 goes to a middle input of S-box 5 in the next round. Only one output from S-box 1 (output 2 goes to S-box 5, input 26, but it is not a middle input). The criterion applies here.

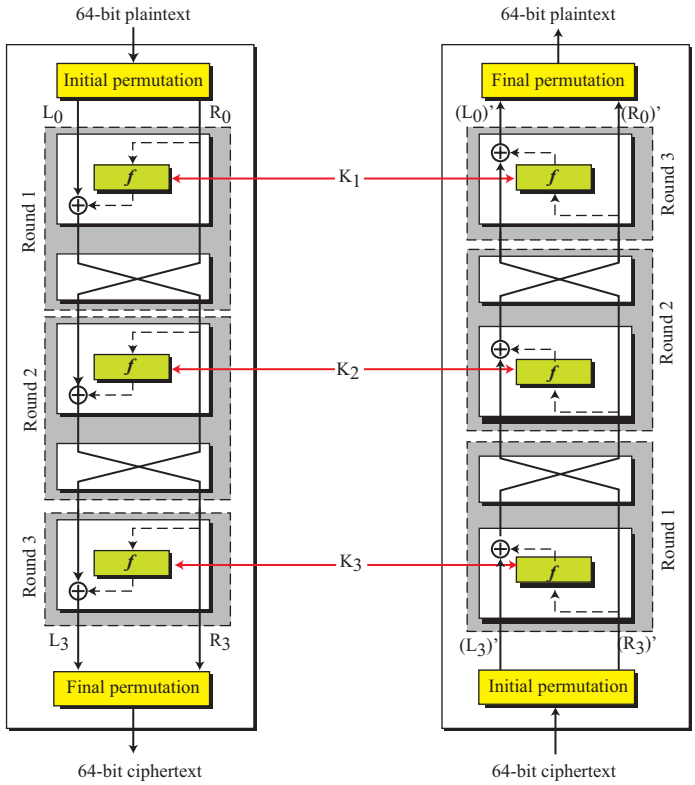
**26.** Figure S6.26 shows the alternative approach.

**Figure S6.26** *Solution to Exercise 26*



**27.** Figure S6.27 shows a three-round cipher. We prove the equalities between the L's and R's from bottom to top. We have labeled each left section in the encryption $L_i$ and in the decryption $(L_i)'$; we have labeled each right sections $R_i$ in the encrypting and $(R_i)'$ in decryption.

**Figure S6.27** *Solution to Exercise 27*




**a.** Since final permutation and initial permutations are inverse of each other (if there is no corruption during transmission), we have

$$\mathbf{(L_3)' = (L_3)} \qquad \mathbf{(R_3)' = (R_3)}$$

**b.** Using the previous equalities and the relations in the mixers, we have

$$(L_2)' = (R_3)' = R_3 = R_2$$

$$\mathbf{(L_2)' = R_2}$$

$$(R_2)' = (L_3)' \oplus \boldsymbol{f}[(R_3)', K_3]$$

$$(R_2)' = L_3 \oplus \boldsymbol{f}[R_3, K_3]$$

$$(R_2)' = L_2 \oplus \boldsymbol{f}[R_2, K_3] \oplus \boldsymbol{f}[R_2, K_3]$$

$$\mathbf{(R_2)' = L_2}$$

**c.** Using the previous equalities and the relations in the mixers, we have

$$(L_1)' = (R_2)' = L_2 = R_1$$

$$\mathbf{(L_1)' = R_1}$$

$$(R_1)' = R_2 \oplus \boldsymbol{f}[L_2, K_2]$$
$$(R_1)' = R_2 \oplus \boldsymbol{f}[R_1, K_2]$$
$$(R_1)' = L_1 \oplus \boldsymbol{f}[R_1, K_1] \oplus \boldsymbol{f}[R_1, K_1]$$
$$\mathbf{(R_1)' = L_1}$$

**d.** Using the previous equalities and the relations in the mixers, we have

$$(L_0)' = (L_1)' \oplus \boldsymbol{f}[(R_1)', K_1]$$
$$(L_0)' = R_1 \oplus \boldsymbol{f}[L_1, K_1]$$
$$(L_0)' = L_0 \oplus \boldsymbol{f}[L_0, K_1] \oplus \boldsymbol{f}[L_0, K_1]$$
$$\mathbf{(L_1)' = L_0}$$

$$(R_0)' = (R_1)' = L_1 = R_0$$

$$\mathbf{(R_0)' = R_0}$$

We have proved that $\mathbf{(L_1)' = L_0}$ and $\mathbf{(R_0)' = R_0}$. Since the final permutation and initial permutation are inverse of each other, the plaintext created at the destination is the same as the plaintext started at the source.

**28.** If we create a table of input and output, we can answers the three questions.

| In | Out | In | Out |
|---|---|---|---|
| 14 | 01 | 23 | 13 |
| 17 | 02 | 19 | 14 |
| 11 | 03 | 12 | 15 |
| 24 | 04 | 04 | 16 |
| 01 | 05 | 26 | 17 |
| 05 | 06 | 08 | 18 |
| 03 | 07 | 16 | 19 |
| 28 | 08 | 07 | 20 |
| 15 | 09 | 27 | 21 |
| 06 | 10 | 20 | 22 |
| 21 | 11 | 13 | 23 |
| 10 | 12 | 02 | 24 |

| In | Out | In | Out |
|---|---|---|---|
| 41 | 25 | 44 | 37 |
| 52 | 26 | 49 | 38 |
| 31 | 27 | 39 | 39 |
| 37 | 28 | 56 | 40 |
| 47 | 29 | 34 | 41 |
| 55 | 30 | 53 | 42 |
| 30 | 31 | 46 | 43 |
| 40 | 32 | 42 | 44 |
| 51 | 33 | 50 | 45 |
| 45 | 34 | 36 | 46 |
| 33 | 35 | 29 | 47 |
| 48 | 36 | 32 | 48 |

**a.** The missing inputs are 09, 18, 22, 25, 35, 38, 43 and 54.

**b.** From above table, we can see that the left 24 bits come from the left 28 bits (except bits 09, 18, 22, and 25, which are blocked).

**c.** From the above table, we can see the right 24 bits come from the right 28 bits (except bits 35, 38, 43, and 54, which are blocked).

**29.** The following shows the result:

$$(1066\ 0099\ 0088\ 0088)_{16}$$

**30.** The following shows the result:

$$(0F55\ AAFF\ 0F55\ AAFF)_{16}$$

**31.** The first round key is

$$(1437\ 4013\ 3784)_{16}$$

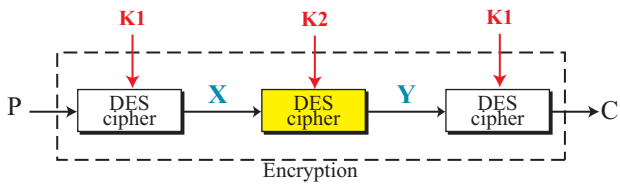
**32.** The following shows the effect:

| | Originals | | | | Complements | | | |
|---|---|---|---|---|---|---|---|---|
| Plaintext: | 0000 | 0000 | 0000 | 0000 | **FFFF** | **FFFF** | **FFFF** | **FFFF** |
| Key: | 0000 | 0000 | 0000 | 0000 | **FFFF** | **FFFF** | **FFFF** | **FFFF** |
| Ciphertext: | 0808 | 02AA | AA02 | A8AA | **F7F7** | **FD55** | **55FD** | **5755** |

When we complement the plaintext and the key, the ciphertext is complemented.

**33.** Figure S6.33 shows the encryption using 3DES with two keys, in which X or Y are the intermediate texts.

**Figure S6.33** *Solution to Exercise 33*




Van Oorschot and Wiener have devised a meet-in-the-middle attack on the above configuration. The attack is basically a known-plaintext attack. It follows the steps shown below:

**a.** Eve intercepts *n* plaintext/ciphertext pairs and stores them in a table which is sorted on values of P as shown below:

| *Plaintext* | *Ciphertext* |
|---|---|
| $P_1$ | $C_1$ |
| $P_2$ | $C_2$ |
| … | … |
| $P_n$ | $C_n$ |

Table 1: *n* P/C pairs

**b.** Eve now chooses a value for X (see Figure S6.33) and uses the decryption algorithm, and all $2^{56}$ possible K1's values to create $2^{56}$ different P values as shown below:

$$P_1 = D(K1_1, X) \quad P_2 = D(K1_2, X) \quad \dots \quad P_m = D(K1_m, X) \qquad \text{where } m = 2^{56}$$

**c.** If a value of P created in step *b* matches one of the value of P in Table 1, Eve uses the corresponding value for K1 (from the list in step b) and the value of C from Table 1 and calculates a value for second intermediate text Y = D (K1, C). Now Eve creates a second table, Table 2, which is sorted on the value of Y. Eve has now *r* possible candidate for K1 keys.

| Y | K1 |
|---|---|
| $Y_1$ | $K1_1$ |
| $Y_2$ | $K1_2$ |
| … | … |
| $Y_r$ | $K1_r$ |

Table 2: *r* Y/K1 pairs

**d.** Eve now searches for K2. For each $2^{56}$ possible values of K2, Eve uses the decryption algorithm and the value of X chosen in step *b* to create $2^{56}$ different values for Y's.

$$Y_1 = D(K2_1, X) \quad Y_2 = D(K2_2, X) \quad \dots \quad Y_m = D(K2_m, X) \qquad \text{where } m = 2^{56}$$

If a value of $Y_i$ created in this step matches one of the value in Table 2, Eve have found a pair of keys: K1 is extracted from Table 2 and K2 is extracted from the decryption algorithm that matches the value of Y in Table 2.

**e.** Now Eve tests pairs of K1/K2 values on more intercepted plaintext/ciphertext. If there is matching, Eve has found the keys; if there is no match, Eve needs to repeat step *b* to *e* using a different value of X.

**34.**

```
permute (n, m, inBlock[1 … n], outBlock[1 … m], permutationTable[1 … m])
{
        i ← 1
        while (i ≤ m)
        {
                outBlock[i] ← inBlock[permutationTable[i]]
                i ← i + 1
        }
        return
}
```

**35.**

```
split (n, m, inBlock[1 … n], leftBlock[1 … m], rightBlock[1 … m])
{
        i ← 1
        while (i ≤ m)
        {
                leftBlock[i] ← inBlock[i]
                rightBlock[i] ← inBlock[i + m]
                i ← i + 1
        }
        return
}
```

**36.**

```
combine (n, m, leftBlock[1 … n], rightBlock[1 … n], outBlock[1 … m])
{
        i ← 1
        while (i ≤ m)
        {
                outBlock[i] ← leftBlock[i]
                outBlock[i + m] ← rightBlock[i]
                i ← i + 1
        }
        return
}
```

**37.**

```
exclusiveOr (n, firstBlock[1 … n], secondBlock[1 … n], outBlock[1 … n])
{
        i ← 1
        while (i ≤ n)
        {
                outBlock[i] ← firstBlock [i] ⊕ secondBlock[i]
                i ← i + 1
        }
        return
}
```

**38.**

```
cipher (plainBlock[1 … 64], RoundKeys[1 … 16][1 … 48], cipherBlock[1 … m64])
{
        permute (64, 64, plainBlock, inBlock, InitialPermutationTable)
        split (64, 32, inBlock, rightBlock, leftBlock)
        for (round = 1 to 16)
        {
                mixer (leftBlock, rightBlock, RoundKey[round])
                swapper (leftBlock, rightBlock)
        }
        swapper (leftBlock, rightBlock)
        combine (32, 64, leftBlock, rightBlock, outBlock)
        permute (64, 64, outBlock, cipherBlock, FinalPermutationTable)
}
```

**39.** We have added one extra parameter ED (encrypt/decrypt). If ED = E, we do encryption; if ED = D, we do decryption.

```
cipher (ED, plainBlock[1…64], RoundKeys[1…16][1…48], cipherBlock[1 … m64])
{
        permute (64, 64, plainBlock, inBlock, InitialPermutationTable)
        split (64, 32, inBlock, rightBlock, leftBlock)
        for (round = 1 to 16)
        {
                if (ED  = E)
                        mixer (leftBlock, rightBlock, RoundKey[round]
                if (ED  = D)
                        mixer (leftBlock, rightBlock, RoundKey[16 – round]
                if (round!= 16)
                        swapper (leftBlock, rightBlock)
        }
        combine (32, 64, leftBlock, rightBlock, outBlock)
        permute (64, 64, outBlock, cipherBlock, FinalPermutationTable)
}
```