# Self-referenced frequency comb measurement by using a polarization line-by-line pulse shaper

**Chi-Cheng Chen, Chen-Bin Huang, and Shang-Da Yang***

*Institute of Photonics Technologies, National Tsing Hua University, Hsinchu 30013, Taiwan*

**Corresponding author: shangda@ee.nthu.edu.tw*

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A polarization line-by-line pulse shaper is used for generation and noniterative spectral phase retrieval of optical arbitrary waveforms (OAWs) spanning over the entire repetition period. The method is completely reference-free, making it particularly attractive in measuring high repetition-rate OAW. © 2014 Optical Society of America

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The integration of line-by-line pulse shaping and optical frequency combs allows generation of optical arbitrary waveforms (OAWs) [1] with controllable ultrafast time-structure spanning the entire repetition period (100% duty cycle). OAWs have been applied to radio-frequency (RF) photonics [2], synthesis of sub-cycle optical fields [3], and generation and delivery of 496 GHz optical pulse trains over 25 km fiber links in the absence of dispersion compensation fiber through the temporal Talbot effect [4]. Furthermore, full vectorial OAW with time-varying state of polarization was recently demonstrated by using a polarization line-by-line pulse shaper [5].

However, OAW cannot be characterized by conventional pulse measurement techniques that need to split the signal pulse into two isolated replicas [6–9]. Dual-quadrature spectral interferometry [10] and parallel optical homodyne detection followed by high-speed digitization [11], have been used in OAW characterization, provided that a well characterized broadband optical reference pulse is available. The optical reference is reduced to a pair of coherent tones (probe) with the same mode spacing as the OAW, and specified relative phases $\Delta\phi$ in dual-quadrature spectral shearing interferometry (DQ-SSI) [12], where three-power spectra, due to sum-frequency generation (SFG) between the OAW signal and properly phased probe, are used in spectral phase recovery. In the previous implementation of DQ-SSI [12], the probe came from an optical delay line (to control $\Delta\phi$) and intensity modulation of a continuous-wave (CW) laser driven by the same RF source used in generating the OAW signal (to ensure the mutual coherence between the signal and the probe). Besides, the probe had to be frequency-detuned from the OAW signal such that the desired SFG spectrum could be spectrally separated from the two components due to individual frequency doubling in a chirped quasi-phase-matched waveguide. The spectral phases of OAW can also be noniteratively retrieved by applying RF sinusoidal temporal phase modulation to the OAW and acquiring the weakly modulated power spectra at discrete [13] or continuous [14] relative delays of within one repetition period. These linear techniques are highly sensitive, do not need any optical reference, and can be used in attosecond waveform characterizations in the extreme ultraviolet region [15]. Nevertheless, they, along with the previous implementation of DQ-SSI, become inadequate for OAW with large mode spacing (e.g., the 600 GHz Kerr comb [16]) due to the lack of a matching millimeter-wave source. Multiheterodyne techniques are linear, applicable to OAW with large mode spacing, and can work with reduced RF bandwidth. However, they need reference combs with different mode spacings and controllable carrier-envelope-offset frequencies [17].

In this Letter, we report on what is, to our best knowledge, the first self-referenced OAW measurement experiment using an orthogonally probed DQ-SSI system implemented by a polarization line-by-line pulse shaper. We know that a (scalar) pulse shaper has been applied to a variety of pulse measurement techniques [9,18–20] with the advantages of eliminating extra dispersion due to typical interferometer setup, simultaneous measurement and shaping, improved precision of system calibration, and ultrahigh sensitivity. Unfortunately, only the multiphoton intrapulse interference phase scan (MIIPS) [18] is theoretically OAW-compatible at the cost of a large amount of data (a series of 2D data sets) and requirement of iterative measurement/retrieval. In contrast, our method is OAW-compatible, applicable to frequency combs of large mode spacing, and inherits the above advantages of shaper-assisted measurement techniques.

In DQ-SSI, the signal pulse of complex spectral envelope $A(\omega) = |A(\omega)| \times \exp[j\psi(\omega)]$ interacts with two probe lines spectrally separated by $\Omega$ (spectral shear), producing an SFG spectrum:

$$S_i(\omega) = B(\omega) + M(\omega) \times \cos[\Delta\psi(\omega) + \Delta\phi_i], \qquad i = 1,2,3, \tag{1}$$

where $B(\omega) \equiv |A(\omega)|^2 + |A(\omega+\Omega)|^2$ and $M(\omega) \equiv 2|A(\omega) \times A(\omega+\Omega)|$ represent the background and modulation envelope spectra, and $\Delta\psi(\omega) \equiv \psi(\omega+\Omega) - \psi(\omega)$ and $\Delta\phi_i$ are the spectral phase difference function of the signal pulse and the relative phase between the two probe lines in the $i$th measurement, respectively. The spectral phase difference function $\Delta\psi(\omega)$ can be obtained by measuring three SFG spectra at three different probe phases $\Delta\phi_{1,2,3}$. For example, $S_{1,2,3}(\omega)$ become $B + M \times \cos(\Delta\psi)$, $B - M \times \sin(\Delta\psi)$, and $B - M \times \cos(\Delta\psi)$, if $\Delta\phi_{1,2,3}$ equals 0, $\pi/2$, $\pi$, respectively. The background

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spectrum $B(\omega)$ is calculated by $[S_1(\omega) + S_3(\omega)]/2$, from which one can, in turn, get $M \times \cos(\Delta\psi)$, $M \times \sin(\Delta\psi)$, $\tan(\Delta\psi)$, and $\Delta\psi(\omega)$. Finally, the spectral phase $\psi(\omega)$ can be unambiguously retrieved by concatenation with a resolution of $\Omega$. Instead of using an extra CW laser, RF function generator, intensity modulator, and optical delay line, as in [12], we propose to generate the precisely phased probe fields at an orthogonal polarization state (with respect to the signal) by a polarization line-by-line pulse shaper (at the cost of halved-available bandwidth). In this way, any OAW signal can be simultaneously measured and shaped without optical or RF reference, and thus, would be applicable to frequency combs of large mode spacing.

The experimental setup is shown in Fig. 1. A phase-modulated (PM) CW comb with 20 GHz mode spacing (50 ps repetition period) was generated by injecting a 1 kHz linewidth CW laser (NKT Adjustik), centered at 1545 nm, into a low-$V_\pi$ optical phase modulator. The signal pulse was polarization-controlled and sent into a polarization line-by-line pulse shaper consisting of a Wollaston prism, a two-lens telescope, a folded zero-dispersion compressor, and a liquid crystal spatial light modulator (SLM-640-D-NM, CRI). The polarization maintaining collimator and circulator suppressed the polarization cross-talk to improve the accuracy and repeatability of measurement. The y-polarization component was spectrally shaped to generate the two probe lines (with 35 dB sidemode suppression ratio) spaced by $\Omega = 2\pi \times 20$ GHz. The signal and the probe were mixed in a 2-mm-thick Type II BBO for SFG and measured by a home-made high-resolution (0.03 nm) spectrometer. In our experiments, the chosen power ratio between the signal and probe arms was 1.5–2 to ensure the highest signal-to-noise ratio and the best accuracy.

Figure 2 shows the measurement results of the residual spectral phase $\psi_{\text{res}}(\omega)$ of the raw PMCW comb. Figure 2(a) shows the SFG spectra measured at three probe phases $\Delta\phi$ of 0, $\pi/2$, and $\pi$, respectively. The experimentally measured $\psi_{\text{res}}(\omega)$ [Fig. 2(b), open circles] exhibits quasi-random feature, causing a temporal waveform filling the entire 50 ps repetition period [inset of Fig. 2(c), dashed]. By applying a spectral phase of $-\psi_{\text{res}}(\omega)$, we could compress the signal pulse close to its transform limit (TL) [inset of Fig. 2(c), solid]. The experimentally measured intensity autocorrelation (IA) function [Fig. 2(c), solid] is in good agreement with the simulated curve [Fig. 2(c), dashed] assuming a constant spectral phase, proving the integrity of our phase measurement.

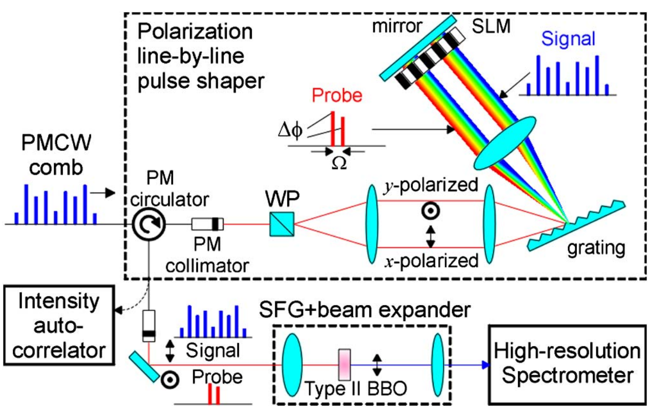


Fig. 1. Experimental setup. PC, polarization controller; PM, polarization maintaining; WP, Wollaston prism; SLM, spatial light modulator.

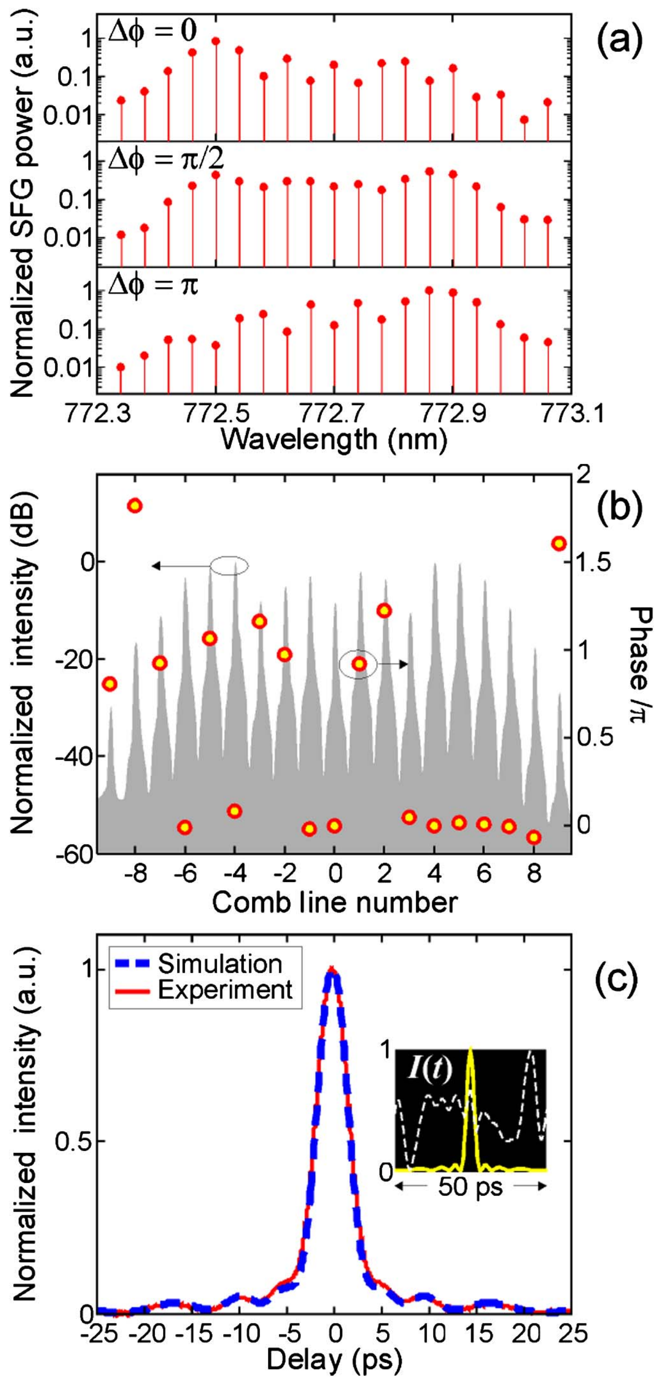


Fig. 2. Measurement of the raw PMCW comb. (a) The SFG spectra measured at three different probe phases. (b) Power spectrum (shaded) measured by an OSA and the spectral phases retrieved by DQ-SSI (open circles). (c) The IA functions of an ideal transform-limited (TL) pulse (dashed) and the experimentally phase-compensated pulse (solid), respectively. The inset shows the temporal intensities of the uncompensated (dashed) and compensated (solid) pulses, where the former exhibits ~100% duty cycle.

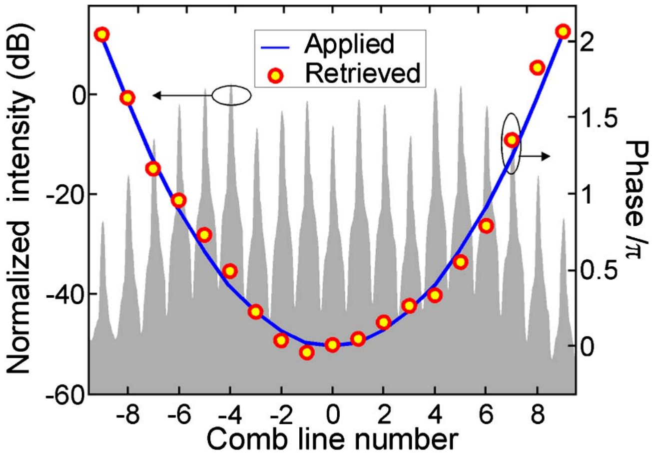


Fig. 3. Power spectrum (shaded), the target (solid) and the retrieved (circles) quadratic spectral phases.

The accuracy of measurement can be estimated by the weighted root-mean-square (rms) phase error [21]:

$$\varepsilon_{\mathrm{rms}} = \sqrt{\sum_i [\psi_i - \psi_i']^2 \times I_i^2 / \sum_i I_i^2}, \tag{2}$$

where $I_i, \psi_i$, and $\psi_i'$ represent the spectral intensity, the correct and retrieved spectral phases of the $i$th comb line, respectively. Three nontrivial spectral phase functions were employed to investigate the accuracy of polarization shaper-assisted DQ-SSI. In the first case, we applied a quadratic spectral phase $\psi(\omega) = (-D_g/2)\omega^2$, corresponding to a group delay dispersion (GDD) of $D_g = -10\ \mathrm{ps}^2$ (Fig. 3, solid), to the compensated (nearly TL) pulse train via the same polarization line-by-line pulse shaper for DQ-SSI measurement. The retrieved spectral phases had a rms error $\varepsilon_{\mathrm{rms}}$ of 0.24 rad (Fig. 3, open circles), mainly attributed to the finite spectral resolution of the homemade spectrometer.

The second case was a sinusoidal phase of the form (1.5 rad) × sin($\tau\omega$) with $\tau$ equal to 6 ps [Fig. 4(a), solid], corresponding to a series of peaks equally spaced by 6 ps in the time domain [inset of Fig. 4(b)]. This type of periodic waveforms is useful in background-free, high-resolution, single-pulse coherent anti-Stokes–Raman spectroscopy [22]. The experimentally reconstructed spectral phases [Fig. 4(a), circles] agree well with the applied counterparts ($\varepsilon_{\mathrm{rms}} = 0.26$ rad). The experimentally measured IA function [Fig. 4(b), solid]

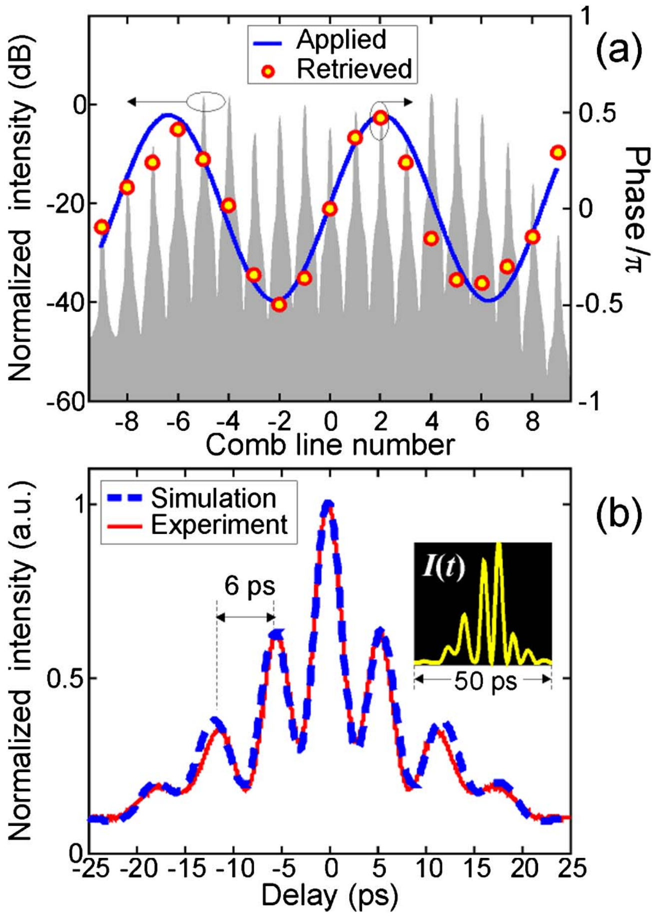


Fig. 4. Measurement of a sinusoidal spectral phase. (a) Power spectrum (shaded), the target (solid) and the retrieved (circles) spectral phases. (b) The IA functions obtained by simulation (dashed) and experiment (solid). The inset shows the corresponding temporal intensity.

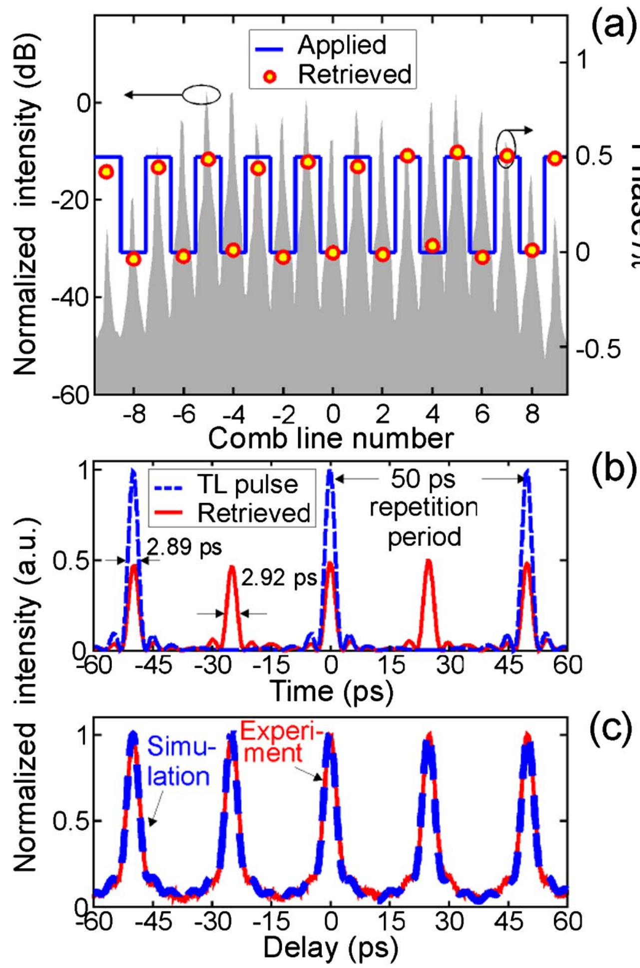


Fig. 5. Measurement of the repetition-rate-doubled pulses. (a) Power spectrum (shaded), the target (solid) and the retrieved (circles) spectral phases. (b) Temporal intensities of the retrieved repetition-rate-doubled (solid) and simulated TL (dashed) pulse trains. (c) The IA functions of the simulated (dashed) and experimental repetition-rate-doubled pulses (solid), respectively.

clearly resolves the 6 ps peak spacing and matches well the simulated curve (dashed).

One of the unique features of OAW is that the intensity repetition-rate can be multiplied via the temporal Talbot effect [4,5]. In the third case, the intensity repetition-rate was doubled [Fig. 5(b)] by introducing alternating $\{0, \pi/2\}$ phases to the compensated PMCW comb [Fig. 5(a), solid]. The strong phase jumps can be accurately retrieved by our self-referenced DQ-SSI scheme [Fig. 5(a), circles] with a small rms phase error ($\varepsilon_{\mathrm{rms}} = 0.08$ rad). Figure 5(b) illustrates the temporal intensities of the repetition-rate doubled (solid) and the TL (dashed) pulse trains obtained by DQ-SSI and simulation, respectively. The corresponding pulse widths are 2.92 and 2.89 ps, confirming the temporal self-image relation. The experimentally measured IA function [Fig. 5(c), solid] exhibits the halved-repetition period, in good agreement with the simulated one (dashed).

In summary, we experimentally demonstrated a self-referenced DQ-SSI scheme that can, for the first time to our best knowledge, algebraically measure ultrahigh repetition-rate OAWs. The apparatus can be precisely calibrated to accurately recover complicated spectral phases. Compared with the previous DQ-SSI implementation, our scheme does not need an extra CW laser, RF function generator, intensity modulator, nor an optical delay line, and is applicable to high repetition-rate frequency combs.

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