Noniterative data inversion of phase retrieval by omega oscillating filtering for optical arbitrary waveform measurement

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We propose a noniterative data inversion process for the phase retrieval by omega oscillating filtering method that could measure both isolated attosecond pulses and periodic optical arbitrary waveform (OAW). The built-in phase modulation depth recovery not only prevents the need of independent calibration (a critical advantage in the extreme ultraviolet regime) but provides a self-consistency check for the data integrity. Our experiments successfully retrieved OAW with ~100% duty cycle in the near infrared regime.

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Optical arbitrary waveforms (OAW) [1] that can span the entire repetition period (100% duty cycle) have been applied to radiofrequency (RF) photonics [2] and generation of ultrahigh-rate pulse trains [3]. However, OAW cannot be fully characterized by conventional pulse measurement techniques that need to split the signal pulse into two isolated replicas [4–6]. Dual-quadrature spectral shearing interferometry [7], dual-quadrature spectral interferometry [8], and parallel optical homodyne detection followed by high-speed digitization [9] have been experimentally used in characterizing OAW in the near-infrared (NIR) regime. However, they are subject to lower sensitivity (due to the need of nonlinear optics) and requirement of well characterized reference pulse, respectively. These limitations would restrict their applications in measuring attosecond pulses in the extreme ultraviolet (EUV) regime. A novel OAW-compatible technique based on linear phase modulation was proposed and successfully measured NIR pulse train of 33% duty cycle [10,11]. The spectral phase is algebraically retrieved by using four power spectra taken under weak sinusoidal temporal phase modulation in phase increments of $\pi/2$. However, the phase modulation depth needs to be independently calibrated by CW interference or fitting the spectral shape of the RF-modulated CW light. These procedures need extra effort and are difficult to implement in the EUV regime. Phase retrieval by omega oscillation filtering (PROOF) [12] was recently developed and used in characterizing the record short (67 as) isolated EUV pulse [13]. In practice, PROOF is OAW compatible, free of nonlinear optics, and only needs a trivial reference (weak temporal phase modulation). In addition to these common merits enjoyed by the linear phase modulation method [10], PROOF is particularly attractive in measuring EUV waveforms, for it does not need to calibrate the phase modulation depth. However, the recursive and evolutionary phase-retrieval algorithms shown in [12] are subject to the π-phase ambiguity for adjacent frequency components (due to the inverse sine relation) and increased complexity, respectively. In this Letter, we propose a nonambiguous recursive data inversion process with built-in modulation depth recovery (or self-consistency check) for the PROOF method. Experiments were carried out for measuring OAW in the NIR regime, but the new approach is readily applicable to measurement of attosecond EUV pulses.

Assume the complex temporal and spectral envelopes of the unknown pulse as $a(t)$ and $A(\omega) \equiv F\{a(t)\} \equiv U(\omega)e^{j\psi(\omega)}$, where $U(\omega)$ (normalized to unit peak) can be obtained by an optical spectrum analyzer (OSA). A periodic temporal phase modulation $\phi(t) = \Phi\cos(\omega_{\mathrm{mod}}t)$ is applied to the variably delayed unknown pulse [Fig. 1(a)], where $\omega_{\mathrm{mod}}/(2\pi) = f_{\mathrm{mod}}$ has to match the repetition rate when measuring a periodic pulse train (discrete spectrum). The phase-modulated temporal envelope is approximated by

$$\begin{aligned} a'(t,\tau) &\equiv a(t-\tau)\times e^{j\phi(t)} \approx a(t-\tau)\times[1+j\phi(t)] \\ &= a(t-\tau)\times[1+j0.5\Phi(e^{j\omega_{\mathrm{mod}}t}+e^{-j\omega_{\mathrm{mod}}t})], \end{aligned}$$

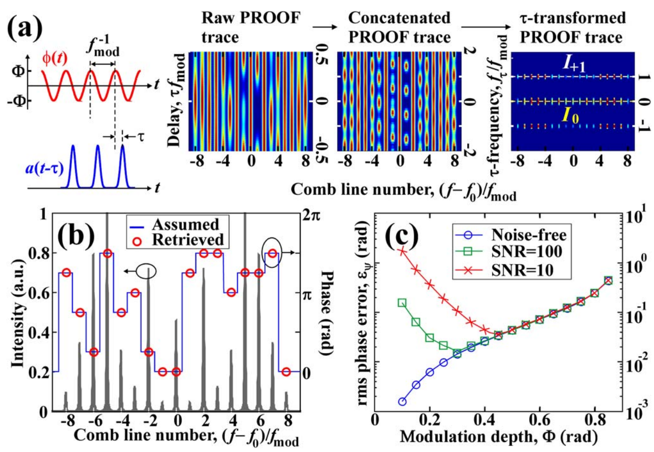

Fig. 1. (a) Schematic of noniterative PROOF method. (b) The power spectrum (shaded), the assumed (solid), and retrieved (open circles) spectral phases. (c) rms phase error versus phase modulation depth at SNR values of infinity (circles), 100 (squares), and 10 (crosses), respectively. All the figures are obtained by simulation with the frequency comb shown in (b) and $\Phi = 0.1$ rad.

if the modulation depth is weak ($\Phi \ll 1$). The PROOF trace $I(\omega, \tau)$ (normalized to unit peak) is formulated by $b \times I(\omega, \tau) \equiv |A'(\omega, \tau)|^2$, where $|A'(\omega, \tau)|^2 \equiv |F\{a'(t, \tau)\}|^2$. If a periodic OAW with ~100% duty cycle is measured (as in this work), the raw PROOF trace is concatenated along the τ axis to span multiple repetition periods to facilitate the subsequent demodulation process [Fig. 1(a)]. Fourier transform of $b \times I(\omega, \tau)$ with respect to τ gives three components centered at delay-angular frequencies $\omega_\tau (= 2\pi f_\tau)$ of 0, $\pm\omega_{\text{mod}}$, $\pm 2\omega_{\text{mod}}$ [Fig. 1(a)], where the DC component is

$$b \times I_0(\omega) = U^2(\omega) + 0.25\Phi[U^2(\omega - \omega_{\text{mod}}) + U^2(\omega + \omega_{\text{mod}})]. \tag{1}$$

As a result, the two unknown constants $\{\Phi, b\}$ can be solved by two algebraic equations arising from sampling Eq. (1) at two optical frequencies. The data integrity can be verified by comparing the solutions of $\{\Phi, b\}$ due to different sampling frequency pairs. The demodulated first-harmonic component (at $\omega_\tau = +\omega_{\text{mod}}$) of the scaled PROOF trace is

$$\begin{aligned} b \times I_{+1}(\omega) = & -j0.5\Phi \times U(\omega) \\ & \times \{U(\omega + \omega_{\text{mod}}) e^{j[\psi(\omega) - \psi(\omega + \omega_{\text{mod}})]} \\ & - U(\omega - \omega_{\text{mod}}) e^{j[\psi(\omega - \omega_{\text{mod}}) - \psi(\omega)]}\}. \end{aligned} \tag{2}$$

Assuming $\psi(0) = \psi(\omega_{\text{mod}}) = 0$ and substituting $\omega = 0$, ω_{mod} into Eq. (2), one can calculate two neighboring phases $\psi(-\omega_{\text{mod}})$ and $\psi(2\omega_{\text{mod}})$. The spectral phase function $\psi(\omega)$ (except for unimportant constant and linear components) can be reconstructed recursively.

A phase-modulated continuous-wave (PMCW) frequency comb with 20 GHz line spacing [3] was used in the simulation. Figure 1(a) shows the (noise-free) raw, concatenated, and transformed PROOF traces obtained by using $a'(t, \tau) = a(t - \tau) \times e^{j\phi(t)}$ and $\Phi = 0.1$ rad, where each τ- or ω_τ-dependent trace is individually normalized for the sake of visualization. The transformed trace exhibits three well-separated components around $\omega_\tau = 0$, $\pm\omega_{\text{mod}}$; even the waveform has ~100% duty cycle. The mean μ_Φ and standard deviation σ_Φ of the 16 solutions of Φ obtained by sampling Eq. (1) at 16 frequency pairs are 0.0997 rad and 5.77×10^{-4} rad, corresponding to a relative uncertainty ε_Φ ($\equiv \sigma_\Phi/\mu_\Phi$) of 0.58%. The accuracy of measurement is estimated by the intensity-weighted root mean square (rms) phase error ε_ψ [14]. As shown in Fig. 1(b), the difference between the assumed (solid) and retrieved (open circles) spectral phases under the conditions of noise-free and $\Phi = 0.1$ rad is negligible ($\varepsilon_\psi = 1.6 \times 10^{-3}$ rad). In the presence of noise, large phase error could arise if the modulation depth is too strong or too weak due to violation of the approximation [$e^{j\phi(t)} \approx 1 + j\phi(t)$] or low fringe visibility of the PROOF trace, respectively. Figure 1(c) illustrates the simulated phase error ε_ψ (5-time average) versus the modulation depth Φ at different signal-to-noise ratio (SNR) values (defined as the signal power to the noise power of the 17 comb lines) generated by the additive Poisson noise model [14]. Phase error as small as 0.036 rad can still be obtained at the minimum SNR (10) required by the iterative PROOF method [12]. A larger SNR permits a smaller ε_ψ-valley and a wider range of Φ within which the resulting error is less than a specified tolerance of ε_ψ.

Figure 2(a) shows our experimental setup. A 18 GHz sinusoidal signal from an RF function generator drives two phase modulators PM1, PM2. A CW laser (1 kHz linewidth, centered at $\lambda_0 = 1545$ nm, $f_0 = 194.2$ THz) is modulated by PM1, generating a PMCW frequency comb [shaded, Fig. 2(b)] with 11 spectral lines. The delay τ of the signal pulse train is scanned for one repetition period of 55.6 ps in increments of 1.39 ps by a line-by-line (LBL) pulse shaper [1–3]. The output pulse is modulated by PM2 coherently driven by the attenuated RF signal. The phase-modulated power spectra at different delays are recorded by an OSA to get the PROOF trace $I(\omega, \tau)$. Sampling the DC component of the PROOF trace at some frequency pair gives $\Phi = 0.27$ rad. The relative uncertainty ε_Φ of 10 retrieved modulation depths is 5.2%, proving the good integrity of the measured PROOF trace. The fluctuated spectral phase [circles, Fig. 2(b)] of the PMCW comb is measured by the noniterative PROOF, then compensated by the LBL pulse shaper. The experimentally measured intensity autocorrelation (IA) function of the phase-compensated pulse [solid, Fig. 2(c)] agrees well with that of the ideal transform-limited (TL) pulse obtained by simulation [dashed, Fig. 2(c)], confirming the high accuracy of the noniterative PROOF method in characterizing the ~100% duty cycle pulse [dotted, inset of Fig. 2(c)].

Two extra spectral phases $\psi_1(\omega)$, $\psi_2(\omega)$ were added to the compensated TL pulse train via the same LBL pulse shaper, and retrieved by the noniterative PROOF method. Figures 3(a) and 3(b) illustrate the experimental

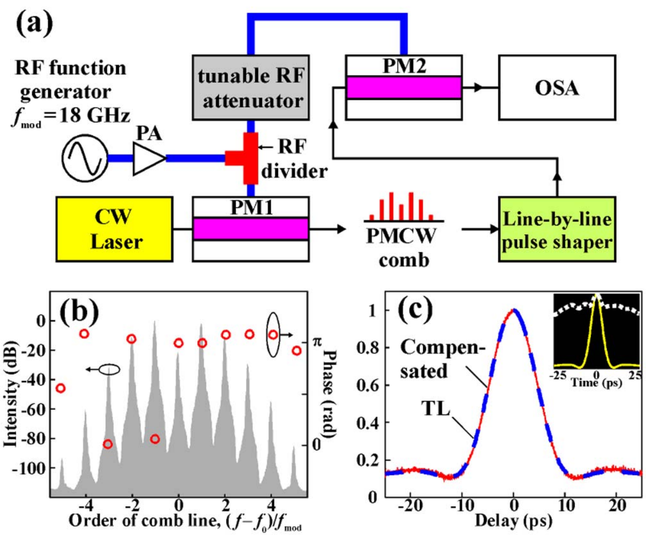

Fig. 2. (a) Experimental setup. PM1 and PM2, phase modulator; PA, power amplifier; OSA, optical spectrum analyzer. (b) Power spectrum (shaded) and the spectral phase retrieved by PROOF (open circles) of the PMCW comb. (c) The IA functions of the ideal transform-limited (TL) pulse (dashed) and the phase-compensated pulse (solid), respectively. The inset shows the temporal intensities of the TL (solid) and uncompensated (dotted) pulses, corresponding to duty cycles of 13% and ~100%, respectively.

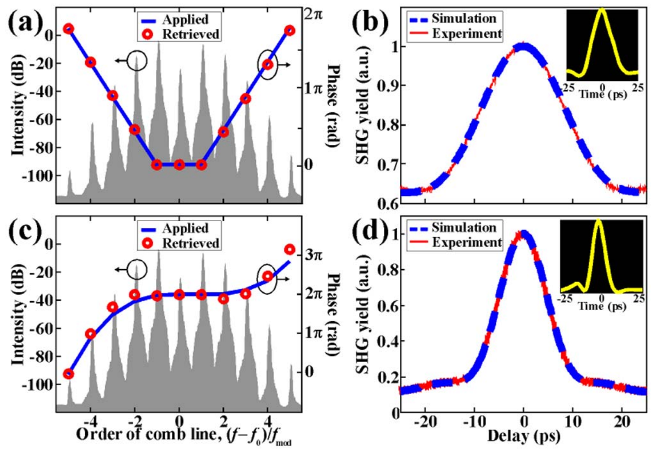

Fig. 3. (a) and (b) Measurement of $\psi_1(\omega)$. (a) Power spectrum (shaded). The applied (solid) and retrieved (circles) spectral phases. (b) Simulated (dashed) and experimentally measured (solid) IA functions. The inset shows the temporal intensity. (c), (d) Counterparts of (a), (b) for the measurement of $\psi_2(\omega)$.

results of a funnel-shaped phase $\psi_1(\omega)$ [solid, Fig. 3(a)], corresponding to a pulse train of 32% duty cycle [inset of Fig. 3(b)]. The retrieved phase modulation depth and its relative uncertainty are $\Phi = 0.29$ rad and $\varepsilon_\Phi = 3.0\%$, respectively. The reconstructed spectral phase [open circles, Fig. 3(a)] agrees well with the applied one (solid), corresponding to an rms phase error of $\varepsilon_\psi = 0.061$ rad (0.046 rad estimated by simulation under $\Phi = 0.29$ rad). The reliability of phase retrieval is independently confirmed by the good agreement between the simulated and experimentally measured IA functions [Fig. 3(b)]. Figures 3(c) and 3(d) show the measurement results of a third-order polynomial phase $\psi_2(\omega) = -(6\ \mathrm{ps}^2)\omega^2 + (25\ \mathrm{ps}^3)\omega^3$ [solid, Fig. 3(c)], corresponding to a pulse train of 16% duty cycle [inset of Fig. 3(d)]. Noniterative PROOF retrieval gives $\Phi = 0.48$ rad, $\varepsilon_\Phi = 3.4\%$, and a spectral phase curve [open circles, Fig. 3(c)] with $\varepsilon_\psi = 0.17$ rad (0.11 rad estimated by simulation under $\Phi = 0.48$ rad). The good agreement between the simulated and experimentally measured IA functions [Fig. 3(d)] further confirms the accuracy of phase retrieval.

In summary, we proposed a noniterative data inversion process for the PROOF method, which can be used in measuring both isolated pulses and periodic OAW in the NIR and EUV regimes. The built-in phase modulation depth recovery is not only eliminates the need of independent calibration (a critical advantage in the EUV regime) but provides a self-consistency check for the data integrity. The method was experimentally verified by measuring three NIR periodic pulse trains with duty cycles up to ~100%.

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