# Arbitrary complex phase-matching spectral grid designed by iterative domino method

Jui-Yu Lai,<sup>1,\*</sup> Cheng-Wei Hsu,<sup>1</sup> Ning Hsu,<sup>2</sup> Yen Hung Chen,<sup>2</sup> and Shang-Da Yang<sup>1</sup>

<sup>1</sup>Institute of Photonics Technologies, National Tsing Hua University, Hsinchu 30013, Taiwan <sup>2</sup>Department of Optics and Photonics, National Central University, Jhongli 320, Taiwan

**Abstract:** We designed complex phase-matching spectral grid by iterative domino algorithm, and analytically proved that it can achieve the maximum conversion efficiency. Experimentally measured conversion efficiencies agreed well with the numerical results. © 2011 Optical Society of America

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## **1. Introduction**

Quasi-phase matching (QPM) is a key technique in wavelength conversion processes, which allows for the use of the highest nonlinear susceptibility component and engineering of phase matching (PM) spectrum. Some applications, such as gas sensing using several absorption lines [1] and generation of multi-octave-spanning laser harmonics [2] by monolithic QPM gratings, have been demonstrated. There exist some methods, such as aperiodic optical superlattice (AOS) optimized by simulated annealing (SA) [3], and nonperiodic optical superlattice (NOS) optimized by genetic algorithm (GA) [4], capable of designing arbitrary efficiency distribution for PM spectral grid. Previously we numerically demonstrated a new scheme, i.e. hyperfine aperiodic optical superlattice (HAOS) by iterative domino (ID) algorithm, achieving unprecedented overall conversion efficiency and spectral fidelity [5]. In this manuscript, we report on advances of HAOS+ID in three aspects. (1) Analytically proving that the ID algorithm can approach the limit of overall conversion efficiency. (2) Experimentally measuring the conversion efficiency spectra of QPM gratings designed by HAOS+ID, NOS+GA, AOS+SA, respectively. The overall conversion efficiency of HAOS is 17% and 34% higher than those of NOS and AOS. (3) Extending the design to arbitrary efficiency and phase distribution, which will enable temporal shaping of nonlinear signal waveforms.

# 2. Theory and experiment

An HAOS consists of N unit blocks of the same length dx (crystal length  $L=N\times dx$ ), where the block orientations are determined by the ID algorithm to approach a target PM spectral grid. The second-harmonic efficiency at fundamental wavelength  $\lambda$  is:

$$\eta(\lambda) \propto |G(\lambda)|^2, \ G(\Delta k) = \sum_{n=1}^N z_n(\Delta k), \ z_n(\Delta k) = \delta_n \times \left(e^{i\Delta k \cdot x_n} - e^{i\Delta k \cdot x_{n-1}}\right) / (\Delta k \times L), \tag{1}$$

where  $\Delta k$  is the corresponding wavevector mismatch,  $\delta_n = \pm 1$  is the orientation of the *n*th unit block ( $x \in [x_{n-1}, x_n = n \times dx]$ ). The ID algorithm can calculate the fitness values due to  $\delta_n = 1$  and  $\delta_n = -1$  by one subtraction (without the lengthy summation of *N* numbers), greatly enhancing the computation efficiency (100s times faster than GA, SA in accomplishing the same performance).



Fig. 1. (a) The 2-peak target. (b) The arc vectors due to the nth unit block. (c) The mismatch vectors G(\Delta k\_i) due to the first 6 unit blocks oriented as indicated.

A key question for all QPM design methods is whether the maximum conversion efficiency can be achieved. Here we use a target PM spectrum with two equally high peaks at wavelengths  $\lambda_i$  (corresponding to  $\Delta k_i$ , i=1, 2) as an example (Fig. 1a). The *n*th unit block contributes to  $G(\Delta k)$  of the *i*th peak by a complex number  $z_n(\Delta k_i)$ , which can be represented by an arc vector of a circle  $C_i$  of radius  $(\Delta k_i \times L)^{-1}$  on the complex plane (Fig. 1b). All the arc vectors have the same length dx/L (for  $x_n \cdot x_{n-1} = dx$ ), while the phase (angle) depends on the unit block position  $x_n$  and its orientation  $\delta_n$ . The achieved efficiency of the 1st (2nd) peak depends on the summation of N vectors with angles  $\theta_n$ . ( $\phi_n$ ), while the ID algorithm intends to maximize the vector lengths  $|G(\Delta k_i)|$  for  $\eta \propto |G(\Delta k_1)|^2 + |G(\Delta k_2)|^2$  (Fig. 1c). If the crystal length L is large, a double integral of continuous variables  $\theta$  and  $\phi$  can replace the discrete vector summation to calculate the maximum (normalized) overall conversion efficiency:

$$\eta_{\max} = \left(16\int_0^{\pi/2}\int_{-\phi}^{\pi/2}\sin\phi d\theta d\phi - 16\int_0^{\pi/2}\int_{-\pi/2}^{-\phi}\sin\phi d\theta d\phi\right) / \left(\int_0^{\pi}\int_{-\pi}^{\pi}\sin\phi d\theta d\phi - \int_{-\pi}^0\int_{-\pi}^{\pi}\sin\phi d\theta d\phi\right) = 8/\pi^2,$$

where the denominator means the maximum efficiency of single-peak target spectrum. As a proof of this result, we designed an HAOS consisting of 218,000 unit blocks (dx=0.1 µm, L=21.8 mm) to achieve two PM peaks at  $\lambda_1$ =1547 nm and  $\lambda_2$ =1553 nm. The achieved overall conversion efficiency is  $\eta_{tor}$ =0.8105, very close to the theoretical limit of 8/ $\pi^2$ =0.8106.

We used photolithographic patterning and electric-field poling techniques to fabricate a 18.9-mm-long congruent lithium niobate QPM sample based on our simulation results of HAOS+ID, NOS+GA, and AOS+SA [5]. A reference QPM grating of constant period  $\Lambda$ =18.8 µm was fabricated on each sample such that the normalized conversion efficiency can be experimentally measured. Fig. 2 shows the target (open circles) and experimentally measured PM tuning curves of HAOS (solid), NOS (dashed-dotted), and AOS (dashed), respectively. The overall conversion efficiency and average shape error of HAOS are  $\eta_{tot}$ =0.94,  $\Delta\eta$ =2.5%, well consistent with the simulation results ( $\eta_{tot}$ =0.94,  $\Delta\eta$ =4×10<sup>-5</sup>) and significantly outperform NOS ( $\eta_{tot}$ =0.80,  $\Delta\eta$ =5.5%) and AOS ( $\eta_{tot}$ =0.70,  $\Delta\eta$ =6.2%). The discrepancy of average shaper error between simulation and experiment is primarily due to the random duty cycle error during the fabrication process.





Fig. 2. Experimentally measured PM tuning curves designed for 5-peak target spectrum (open circles) by HAOS+ID (solid), NOS+GA (dashed-dotted), and AOS+SA (dashed), respectively.

Fig. 3. Target (open circles) and simulation result (arrows) of five complex PM peaks with V-shaped relative efficiencies and  $\pi/4$  phase shifts represented by the radius and angle of the polar coordinates.

The HAOS+ID method can be generalized to the optimization of arbitrary efficiency and phase distribution by using complex target spectrum and fitness function. As a proof of concept, we designed an HAOS consisting of 94,500 unit blocks (dx=0.2µm, L=18.9 mm) to achieve 5 PM peaks with V-shaped efficiency distribution (same as open circles in Fig. 2) and constant  $\pi/4$  relative phase shift between neighboring peaks. As shown in Fig. 3, it achieved nearly perfect phase shifts with errors in the order of 10<sup>-4</sup> rad at the cost of degraded overall conversion efficiency and average shape error ( $\eta_{tor}$ =0.74,  $\Delta\eta$ =2.61%, vs.  $\eta_{tor}$ =0.94,  $\Delta\eta$ ~10<sup>-5</sup> in the efficiency-only design). The degradation is a natural result of increased complexity of the target PM spectrum. There will be no room for QPM design if a continuous complex target PM spectrum is specified.

### **3.** Conclusions

We designed a QPM sample to achieve 5 PM peaks with V-shaped relative efficiencies by HAOS+ID method, and experimentally confirmed its validity. The achieved overall conversion efficiency and spectral fidelity are better than those of NOS+GA and AOS+SA in theory and experiment. We also analytically proved (by the model of 2-peak target spectrum) that the ID algorithm can achieve the limit of overall conversion efficiency. This work was supported by the National Science Council of Taiwan under grants NSC 100-2221-E-007-093-MY3, 99-2120-M-007-010, and by the National Tsing Hua University under grant 100N2081E1.

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