Even-order Spectral Phase Retrieval by Modified Interferometric Field Autocorrelation Trace

Shang-Da Yang, Yong-Yuan Huang
Institute of Photonics Technologies, National Tsing Hua University, Hsinchu 30013, Taiwan
shangda@ee.nthu.edu.tw, Phone: (886)-3-5742583, Fax: (886)-3-5751113

Abstract: We proposed analytic retrieval of all even-order spectral phase components by measuring modified interferometric field autocorrelation trace using thick nonlinear crystals. This new scheme is attractive in measuring ultraweak ultrashort optical pulses.

1. Introduction:

Determination of spectral phase of ultrashort optical pulses is essential in a variety of applications, such as coherent control, nonlinear spectroscopy [1], and adaptive pulse compression [2]. For lack of electronic response in the femtosecond time scale, Michelson interferometer (MI) in collaboration with nonlinear optical effects are usually employed to achieve ultrafast signal gating and pulse characterizations. For example, intensity autocorrelation [3], modified spectrum autointerferometric correlation (MOSAIC) [4], and frequency-resolved optical gating (FROG) [5] have been used to measure pulse width, temporal phase variation, and complete intensity and phase profiles, respectively. All these existing techniques, however, require a nonlinear phase-matching (PM) spectrum that is broader than the input pulse bandwidth. As a result, thin nonlinear crystals are routinely used in measuring extremely short pulses, which seriously restricts the measurement sensitivity. In this work, we report analytic determination of all even-order spectral phase components by measuring modified interferometric field autocorrelation (MIFA) trace using a δ-like PM spectrum. This new scheme uncovers that spectral phase retrieval is possible by pushing the nonlinear PM bandwidth to both extremes. It could largely improve the measurement sensitivity and eliminate the complexity of engineering broad PM response [3] by simply using thick nonlinear crystals.

2. Theory:

Fig. 1 illustrates the schematic diagram of MIFA trace and even-order spectral phase measurements. A pulse of complex envelope $a(t)$ and carrier angular frequency ω_0 ($=2\pi f_0$) is sent into a collinear MI to produce a pulse pair with time delay τ: $a_\omega(t,\tau)=a(t)+a(t-\tau)\,e^{-j\omega_0\tau}$. The output second-harmonic (SH) spectral envelope $A_{2\omega}(\omega,\tau)$ due to a thick nonlinear crystal with sufficiently narrow PM power spectrum is proportional to $F\{a_\omega^2(t,\tau)\}\big|_{\omega=0}=\int_{-\infty}^{\infty}a_\omega^2(t,\tau)dt$, where $F\{\}$ represents Fourier transform. By measuring the average SH power as a function of delay τ: $\int_{-\infty}^{\infty}|A_{2\omega}(\omega,\tau)|^2 d\omega$, we can obtain the MIFA trace:

$$S_{MIFA}(\tau)\propto\ 1+2|G_1'(\tau)|^2+4\,\mathrm{Re}\{G_1'(\tau)\}\cos(\omega_0\tau)+\cos(2\omega_0\tau) \tag{1}$$

where the modified field autocorrelation function is defined as [6]:

$$G_1'(\tau)\equiv\langle a(t)a(t-\tau)\rangle\big/\langle a^2(t)\rangle=|G_1'(\tau)|\,e^{j\phi_G(\tau)} \tag{2}$$

which is different from the standard field autocorrelation function: $\langle a(t)a^*(t-\tau)\rangle\big/\langle |a(t)|^2\rangle$. Fourier transform of eq. (1) leads to five spectral components centered at delay angular frequencies of $\Omega=0$, $\pm\omega_0$, and $\pm 2\omega_0$, respectively. By extracting the components centered at $\Omega=0$ and $\Omega=\omega_0$, we can get $|G_1'(\tau)|$ and $\mathrm{Re}\{G_1'(\tau)\}$, from which the phase of $G_1'(\tau)$ can be evaluated by: $\phi_G(\tau)=\cos^{-1}[\mathrm{Re}\{G_1'(\tau)\}/|G_1'(\tau)|]$. According to eq. (2), the Fourier transform of complex function $G_1'(\tau)$ becomes:

$$\tilde{G}_1'(\omega)\propto A(\omega)\cdot A(-\omega)=|A(\omega)\cdot A(-\omega)|\cdot\exp[j(\psi(\omega)+\psi(-\omega))] \tag{3}$$

where $A(\omega)\equiv F\{a(t)\}=|A(\omega)|\cdot\exp[j\psi(\omega)]$ represents the complex spectral envelope, and the angle of $\tilde{G}_1'(\omega)$ is twice the even-order spectral phase: $\psi_e(\omega)\equiv[\psi(\omega)+\psi(-\omega)]/2$. The $\cos^{-1}(\,)$ function used to determine $\phi_G(\tau)$ results in an ambiguity of sign, which subsequently causes time-reversal ambiguity as in second-harmonic generation FROG [5]. Since $A_{2\omega}(\omega=0,\tau=0)=A(\omega)\otimes A(\omega)\big|_{\omega=0}$, all spectral components of $A(\omega)$ contribute to the SH spectral center, which enables our scheme to retrieve even-order spectral phase.

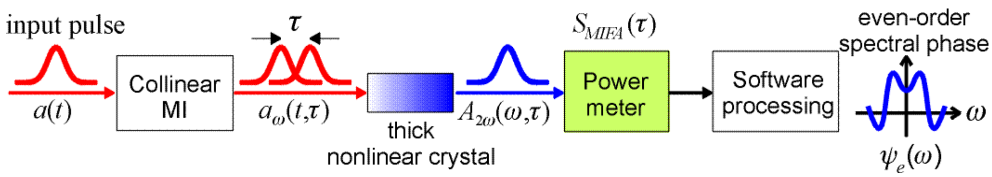

Fig. 1. Schematic diagram of MIFA trace and even spectral phase measurements. MI: Michelson interferometer.

3. Simulation and Discussion:

We assumed a Gaussian spectral amplitude $|A(f)|=\exp[-2\ln 2(f/\Delta)^2]$ and a trapezoidal spectral phase (containing infinitely many even-order components) to verify the feasibility of MIFA scheme, where Δ denotes the full-width at half maximum (FWHM) of $|A(f)|^2$. The performance of spectral phase retrieval is quantitatively measured by the root-mean-square (rms) error ε defined in [7]. As shown in Fig. 2, the retrieved phase (circles) is agrees well with the actual one (solid), corresponding to a small error of ε=4.7×10^{-3} rad. Fig. 3 demonstrates the weak dependence of rms error on nonzero PM bandwidth Δ_{PM} (defined as the FWHM of a sinc^2 PM curve arising from uniform nonlinear crystal), where a linearly chirped Gaussian pulse $A(f)=\exp[-2\ln 2(1+j\alpha)(f/\Delta)^2]$ is used. For example, the phase error is only ~10^{-2} rad at α=3 (diamonds) even the PM bandwidth Δ_{PM} is as broad as one-fifth of the power spectral FWHM Δ. For a pulse with stronger chirp rate α, both the rms error ε and the phase variation δ within a spectral range of Δ (in this model, δ~0.35α radian) will be greater. We found that a PM bandwidth satisfying $\Delta_{PM}\leq 0.1\Delta$ is sufficient to perform accurate measurement with $\varepsilon\leq 0.01\delta$ for α=0.5~8. We also examined the influences of fringe jitter and unbalanced power splitting ratio arising from an imperfect MI by using the chirped Gaussian pulse with α=2 (δ=0.7 rad). Our analysis (not shown here) concluded that the MIFA scheme is robust against fringe jitter (ε~10^{-3} rad when the standard deviation of the normally distributed random delay is 20% of the Nyquist delay step 0.25/f_0), but requires good calibration to balance the powers of the two MI arms (ε~10^{-2} rad if the powers differ by 5%).

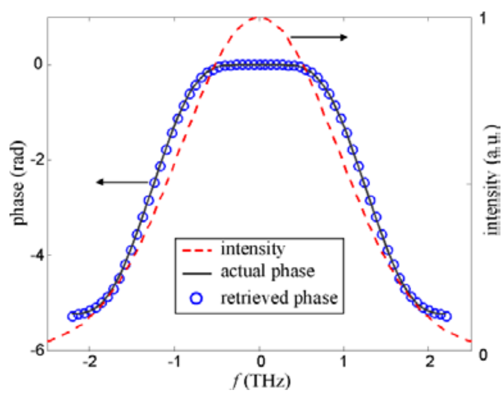

Fig. 2. Actual (solid) and retrieved (circles) spectral phase profiles assuming Gaussian power spectrum (dashed). The corresponding error is 4.7×10^{-3} rad.

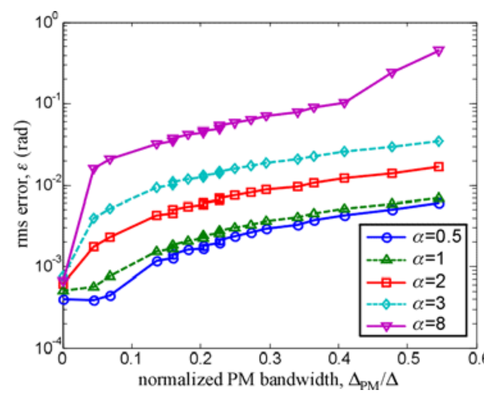

Fig. 3. rms error versus normalized PM bandwidth by assuming linearly chirped Gaussian pulses at different chirp rates.

4. Conclusion:

We have numerically demonstrated that correlation measurement by using thick nonlinear crystals enables direct retrieval of even-order spectral phase profile, which is attractive in measuring ultraweak ultrashort optical pulses. Combining with other constraints, such as intensity autocorrelation or SH power spectrum, odd-order phase components can also be determined by optimization techniques.

References:

[1]. N. Dudovich, D. Oron, Y. Silberberg, Nature **418**, 512-514 (2002).
[2]. M. Yamashita, K. Yamane, R. Morita, IEEE J. Sel. Topics Quantum Electron., **12**, 213-222 (2006).
[3]. S. -D. Yang, A. M. Weiner, K. R. Parameswaran, M. M. Fejer, Opt. Lett. **29**, 2070-1072 (2004).
[4]. D. A. Bender, M. P. Hasselbeck, M. Sheik-Bahae, Opt. Lett., **31**, 122-124 (2006).
[5]. R. Trebino, Kluwer Academic Publishers, Boston, MA, 2000.
[6]. S. -D. Yang, H. Miao, Z. Jiang, A. M. Weiner, K. R. Parameswaran, M. M. Fejer, submitted to Appl. Optics.
[7]. D.N. Fittinghoff, K.W. DeLong, R. Trebino, C.L. Ladera, J.Opt.Soc.Am.B, **12**, 1955-1967 (1995).