Robustness Enhancement of Iteration-free Spectral Phase Retrieval by Interferometric Second-harmonic Trace

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Abstract: We theoretically demonstrated a new multi-slice scheme that could suppress the noise-induced spectral phase error in measurement of electric field by interferometric spectral trace observation (MEFISTO) by eightfold or better without measuring additional data.

OCIS codes: (320.7100) Ultrafast measurements, (190.7110) Ultrafast nonlinear optics, (120.3180) Interferometry

1. Introduction:

Measuring the complex field of femtosecond pulses is essential in ultrafast signal generation and processing, especially when nearly transform-limited or precisely shaped pulses are involved [1-2]. Among the existing techniques that can deliver intensity and phase of signal pulses, frequency-resolved optical gating (FROG) [3] is especially popular because of its robustness against system error and measurement noise. An interferometric extension of FROG, namely measurement of electric field by interferometric spectral trace observation (MEFISTO), was recently proposed for spectral phase retrieval [4-5]. The primary advantages of this new scheme over conventional FROG are twofold: (a) the data acquisition uses a collinear configuration, which permits the employment of straight waveguide as highly efficient second-harmonic (SH) converter [6]; (b) the phase retrieval requires no iteration and can be updated much faster. Although MEFISTO trace still provides the self-consistency checks to correct system error [4], its noise response remains unclear. In this work, we numerically analyzed the robustness of MEFISTO against measurement noise. By introducing multi-slice procedure in phase recovery, we greatly suppressed the noise-induced error in MEFISTO, making it practically useful in real applications.

2. Theory:

Fig.1 illustrates the schematic of FROG and MEFISTO measurements. A pulse of complex envelope $a(t)$ and carrier frequency f_0 is sent into a collinear Michelson interferometer (MI) to produce a pulse pair with variable delay t. By passing the pulse pair through a nonlinear crystal and measuring the output SH power spectrum for each delay t, we can obtain an interferometric trace $I^{SHG}(f,t)$, where f represents frequency. The Fourier transform of the trace with respect to t, $Y^{SHG}(f,k) \equiv F_t\{I^{SHG}(f,t)\}$, consists of five spectral components centered at delay-frequencies of k=0, $\pm f_0$, and $\pm 2f_0$, respectively. FROG measurement uses the entire band around k=0 to retrieve the complex field [3]. In contrast, MEFISTO only takes two neighboring slices from $Y^{SHG}(f,k)$ near $k=f_0$ (typically at $k=f_0$, and $f_0-\Delta f$, where $\Delta f = T^{-1}$, T is the size of t-window) to evaluate the differential spectral phase $\Delta f(f) \equiv f(f+\Delta f) - f(f)$ [4]:

$$\Delta f(f) = \pm\cos^{-1}[\Omega(f,k=f_0)] \mp \cos^{-1}[\Omega(f,k=f_0-\Delta f)] + f(0) - f(-\Delta f) \qquad (1)$$

where $\Omega(f,k) \equiv Y^{SHG}(f,k)/[4U_{SHG}(f)U(f+f_0-k)U(k-f_0)]$, $A(f) \equiv F_t\{a(t)\} = U(f)\exp[jf(f)]$, and $U_{SHG}(f) \equiv |F_t\{a^2(t)\}|$. Spectral phase profile $f(f)$ can be uniquely determined by eq. (1) except for a linear term and ambiguity of sign [4].

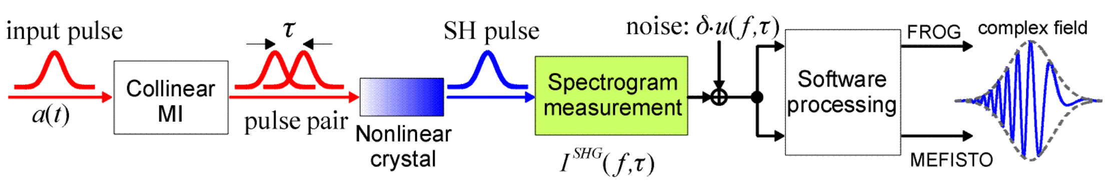

Fig. 1. Schematic of FROG and MEFISTO measurements. MI: Michelson interferometer.

3. Simulation and Discussion:

Our simulation assumed a linearly chirped Gaussian pulse centered at 1.56-mm wavelength (f_0=192 THz): $A(f)$= $\exp[-(1+j\cdot 1.12)(f/W)^2]$, where W=15.9 THz. The noise response was analyzed by adding some random noise to the interferometric trace: $\tilde{I}^{SHG}(f,t) = I^{SHG}(f,t) + d \cdot u(f,t)$, where d denotes the noise amplitude relative to the peak of

$I^{SHG}(f,t)$, and $u(f,t)$ was implemented by a random matrix uniformly distributed between 0 and 1. The contaminated trace $\tilde{I}^{SHG}(f,t)$ was transformed into $\tilde{Y}^{SHG}(f,k)$, then properly sampled (low-pass filtered) to feed the MEFISTO (FROG) program. The performance of spectral phase retrieval is quantitatively measured by the normalized root-mean-square (RMS) error $e \equiv \left[\sum_i (\tilde{f}_i - f_i)^2 \cdot U_i^2 \Big/ \sum_i f_i^2 \cdot U_i^2 \right]^{1/2}$, where subscript i denotes function value at the i-th sampling frequency, $\tilde{f}$ is the retrieved spectral phase, and $U^2(f)$ is the fundamental spectral intensity used as weighting function. The reliability of MEFISTO program was confirmed by the negligible error ($e = 7\times10^{-5}$) derived in the absence of noise. Fig.2 illustrates the RMS error e versus noise amplitude d for FROG and MEFISTO. FROG (dotted) shows excellent noise resistance ($e < 0.2$ for $d < 9\%$), because the built-in redundancy and iteration can exclude unrealistic solution corresponding to the noise-contaminated trace [3]. In contrast, error of standard MEFISTO (dash-dot) grows rapidly with the increase of noise ($e \approx 1$ when $d \approx 4\%$), for eq. (1) uses very limited spectrogram data and is inherently vulnerable to the measurement noise. By sampling multiple slices from $\tilde{Y}^{SHG}(f,k)$ at $k = f_0 \pm n\Delta f$ (n=0, 1, 2,..), and averaging all the retrieved spectral phase profiles (weighted by the area of $\tilde{Y}^{SHG}$), we could suppress the error without measuring additional data. For example, using 8 slices (solid) could reduce e by eightfold when d=9%. However, the error suppression will saturate when the sampled slices cover a delay-frequency range Δk much wider than the input spectrum. Fig. 3 shows that using more than 12 slices ($\Delta k \approx 36$ THz, about twice the FWHM of input power spectrum) could start to degrade the error. Better suppression of error can be achieved by decreasing Δf (step size of k), such that more slices are applicable within the same bandwidth. This will require a broader t-window, but we can employ the down-sampling technique [7] to reduce the number of samplings along the t-axis. We also examined the performance of multi-slice MEFISTO against multiplicative noise: $\tilde{I}^{SHG}(f,t) = I^{SHG}(f,t)\times[1 + d \cdot u(f,t)]$. The result (not shown here) shows that multi-slice procedure performs well in the presence of strong multiplicative noise (e.g. using 8 slices could reduce e by threefold when d=9%), but is less effective when noise is weak.

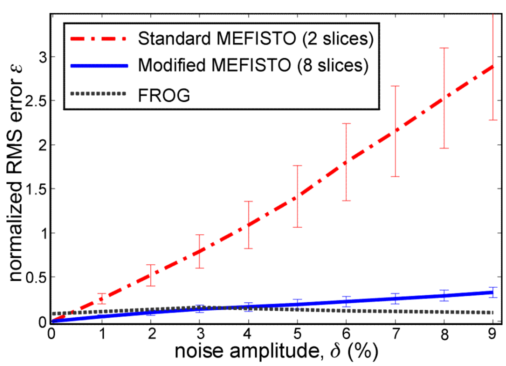

Fig. 2. Noise response of FROG and MEFISTO. The length of error bar represents the standard deviation of five data points.

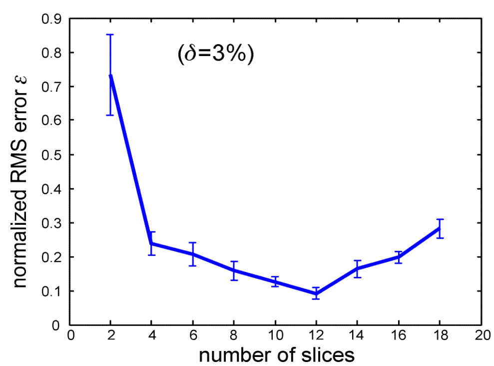

Fig. 3. RMS error versus the number of slices used to retrieve spectral phase at fixed noise amplitude d=3%. The spacing between adjacent slices is Δf=3.3 THz, and input spectral FWHM is 18.7 THz.

4. Conclusion:

We have shown that standard MEFISTO is subject to measurement noise for lack of built-in redundancy and iteration in the phase retrieval process. By making use of the data redundancy in MEFISTO traces, our multi-slice scheme can substantially suppress the noise-induced error by eightfold or better (if broader t-window is used). For typical measurement systems with moderate noise amplitude (say <10% of the spectrogram peak), the iteration-free multi-slice MEFISTO scheme can be as robust as FROG.

References:

[1]. B. C. Thomsen, D. A. Reid, R. T. Watts, L. P. Barry, J. D. Harvey, IEEE Trans. Instrum. Meas. **53**, 186 (2004).

[2]. Z. Jiang, D. S. Seo, D. E. Leaird, R. V. Roussev, C. Langrock, M. M. Fejer, A. M. Weiner, J. Lightw. Technol. **23**, 1979 (2005).

[3]. R. Trebino, Kluwer Academic Publishers, Boston, MA, 2000.

[4]. I. Amat-Roldán, I. G. Cormack, P. Loza-Alvarez, D. Artigas, Opt. Lett. **30**, 1063 (2005).

[5]. I. Amat-Roldan, D. Artigas, I. G. Cormack, and P. Loza-Alvarez, Opt. Eepress **14**, 4538 (2006).

[6]. S. -D. Yang, A. M. Weiner, K. R. Parameswaran, M. M. Fejer, Opt. Lett. **30**, 2164 (2005).

[7]. I. Amat-Roldan, I. G. Cormack, P. Loza-Alvarez, E. J. Gualda, D. Artigas, Opt. Eepress **12**, 1169 (2004).