Homework Problem Set #3

(Due by 2008/04/07)

This problem set covers the content of Lessons 5−6 or EK 12.11, 12.8−9.

1) (10%) Solve the initial-value problem (problem 2 in HW2) by Fourier transform:

PDE: $u_t = \alpha^2 u_{xx}$, $\{-\infty<x<\infty,\ t>0\}$

IC: $u(x,0)=e^{-(x/L)^2}$

2) (10%) Problem **12.11.4**. You can find that integral transform is applicable in solving nonhomogeneous PDE with nonhomogeneous BCs.

3a) (10%) Solve the semi-infinite heat diffusion problem by Laplace transform:

PDE: $u_t = \alpha^2 u_{xx}$, $\{0<x<\infty,\ 0<t<\infty\}$

BC: $u(0,t) = \delta(t)$

IC: $u(x,0)=0$

3b) (5%) Plot three curves of $u(x,t)$ for $\alpha=1$, $t=1, 2, 4$, respectively.

4) (10%) Solve the semi-infinite wave propagation problem by Laplace transform:

PDE: $u_{tt} = c^2 u_{xx}$, $\{0<x<\infty,\ 0<t<\infty\}$

BC: $u(0,t) = \delta(t)$

ICs: $u(x,0)=0$, $u_t(x,0)=0$

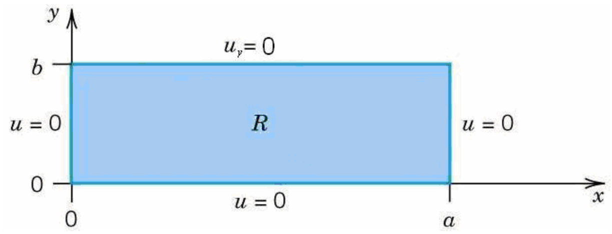
You may sense the differences of behavior between heat flow and wave propagation by problems 2 and 3!

Edited by: Shang-Da Yang

5) Solve 2-D heat diffusion problem in a rectangular plate:

PDE: $u_t = \alpha^2(u_{xx} + u_{yy})$ {$0<x<a$, $0<y<b$, $0<t<\infty$}

BCs: $u_y=0$ (heat insulated) on the upper side, $u=0$ (zero temperature) for the remaining three boundaries (see the figure)



5a) (5%) What are the three ODEs if we perform separation of variables: $u(x,y,t)= X(x)Y(y)T(t)$?

5b) (10%) Solve the eigenfunctions and eigenvalues by the four homogeneous BCs.

5c) (5%) Write down the fundamental mode according to (3b). Note that $u(x,y,t)$ will approach this form as $t\to\infty$ regardless of the ICs.

5d) (10%) Solve the exact solution $u(x,y,t)$ if IC is: $u(x,y,t=0)=100$.

6) (10%) Problem **12.8.18**.

7) Consider a circular membrane with fixed rim governed by:

PDE: $u_{tt} = c^2\left(u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}\right)$, {$0<r<\rho$, $t>0$}

BC: $u(r=\rho,\theta,t)=0$

The (m,n) normal mode $u_{mn}(r,\theta,t)$ is a function of r, θ, t.

7a) (10%) Plot nodal lines of mode u_{22} if $c=1$, $\rho=1$.

7b) (5%) Evaluate the ratio of resonance frequencies $\frac{\nu_{mn}}{\nu_{01}}$ for (m,n)=(0,2), (1,1), (1,2).