Lesson 15
Wave Equations

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Outline

- Wave equations of fields (propagation)
- Wave equations of potentials (generation)
- Electromagnetic spectrum (applications)

Sec. 15-1
Wave Equations of Fields

1. Homogeneous equations in time domain
2. Homogeneous equations in frequency domain
3. EM fields in lossy media
4. Microwave ovens

Homogeneous wave equation in time domain-1

In a simple (linear, isotropic, homogeneous), charge free ($\rho = 0$), nonconducting ($\sigma = 0,\ \vec{J} = 0$) medium, Maxwell's equations reduce to:

$$\begin{cases} \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{D} = \rho \\ \nabla \times \vec{H} = \vec{J} + \dfrac{\partial \vec{D}}{\partial t} \\ \nabla \cdot \vec{B} = 0 \end{cases} \longrightarrow \begin{cases} \nabla \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} & (1) \\ \nabla \cdot \vec{E} = 0 & (2) \\ \nabla \times \vec{H} = \varepsilon \dfrac{\partial \vec{E}}{\partial t} & (3) \\ \nabla \cdot \vec{H} = 0 & (4) \end{cases}$$

Homogeneous wave equation in time domain-2

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (3)$$

$$\nabla \times \nabla \times \vec{E} = \nabla \times \left(-\mu \frac{\partial \vec{H}}{\partial t} \right) = -\mu \frac{\partial}{\partial t} \left(\nabla \times \vec{H} \right)$$

$$\nabla \cdot \vec{E} = 0 \quad (2)$$

$$= -\mu \frac{\partial}{\partial t} \left(\varepsilon \frac{\partial \vec{E}}{\partial t} \right) = -\mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad = \nabla \left(\nabla \cdot \vec{E} \right) - \nabla^2 \vec{E} = -\nabla^2 \vec{E}$$

$$\nabla^2 \vec{A} \equiv \nabla \left(\nabla \cdot \vec{A} \right) - \nabla \times \nabla \times \vec{A}$$

$$\Rightarrow \boxed{\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

Homogeneous 2nd-order PDE of vector field $\vec{E}$

Homogeneous wave equation in time domain-3

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (3)$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad (1)$$

$$\nabla \times \nabla \times \vec{H} = \nabla \times \left(\varepsilon \frac{\partial \vec{E}}{\partial t} \right) = \varepsilon \frac{\partial}{\partial t} \left(\nabla \times \vec{E} \right)$$

$$\nabla \cdot \vec{H} = 0 \quad (4)$$

$$= \varepsilon \frac{\partial}{\partial t} \left(-\mu \frac{\partial \vec{H}}{\partial t} \right) = -\mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = \nabla \left(\nabla \cdot \vec{H} \right) - \nabla^2 \vec{H} = -\nabla^2 \vec{H}$$

$$\nabla^2 \vec{A} \equiv \nabla \left(\nabla \cdot \vec{A} \right) - \nabla \times \nabla \times \vec{A}$$

$$\Rightarrow \boxed{\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0}$$

Homogeneous 2nd-order PDE of vector field $\vec{H}$

Compare with the equations of lossless TX lines:

$$\frac{\partial^2}{\partial z^2} v(z,t) = LC \frac{\partial^2}{\partial t^2} v(z,t)$$

$$\frac{\partial^2}{\partial z^2} i(z,t) = LC \frac{\partial^2}{\partial t^2} i(z,t)$$

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

Scalar waves → Vector waves

Velocity: $v_p = \frac{1}{\sqrt{LC}}$ → Velocity: $u_p = \frac{1}{\sqrt{\mu\varepsilon}}$

Comments-1

We have assumed $\rho = 0,\ \vec{J} = 0$ to derive:

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

i.e., these equations deal with how EM waves propagate, instead of how they are generated from time-varying sources $\left(\rho, \vec{J}\right)$.

Comments-2

We have assumed simple media to derive:

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \mu\varepsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

i.e., these equations (standard waves properties) have to be modified if the medium is nonlinear, anisotropic, nonhomogeneous.

Why are we interested in time-harmonic (sinusoidal) fields?

- Any periodic (aperiodic) function $\rightarrow$ superposition of discrete (continuous) sinusoidal functions by Fourier series (integral).
- Maxwell's equations are linear PDEs. $\Rightarrow$ (1) Sinusoidal sources produce sinusoidal fields of the same frequency in steady state. (2) Total field can be derived by superposition of individual sinusoidal responses.
- Easy to operate if phasors are used:

$$\frac{\partial}{\partial t} \rightarrow j\omega, \quad \int dt \rightarrow \frac{1}{j\omega}$$

From scalar phasors to vector phasors

Scalar phasors of voltages & currents are sufficient to describe steady-state response of TX lines:

$$v(z,t) = \text{Re}\left\{\underline{V(z)} \cdot e^{j\omega t}\right\}, \quad i(z,t) = \text{Re}\left\{\underline{I(z)} \cdot e^{j\omega t}\right\}$$

Vector phasors of E-field and M-field are required to describe time-harmonic EM fields:

$$\vec{E}(x,y,z,t) = \text{Re}\left\{\underline{\vec{E}(x,y,z)}e^{j\omega t}\right\}$$

$$\vec{H}(x,y,z,t) = \text{Re}\left\{\underline{\vec{H}(x,y,z)}e^{j\omega t}\right\}$$

Homogeneous wave equation in frequency domain-1

In a simple, source-free ($\rho = 0,\ \vec{J} = 0$) medium, phasors of EM fields satisfy with the equations

$$\begin{cases} \nabla \times \vec{E} = -\mu \dfrac{\partial \vec{H}}{\partial t} \\ \nabla \cdot \vec{E} = 0 \\ \nabla \times \vec{H} = \varepsilon \dfrac{\partial \vec{E}}{\partial t} \\ \nabla \cdot \vec{H} = 0 \end{cases} \xrightarrow[\frac{\partial}{\partial t} \to j\omega]{\vec{E}(\vec{r},t) \to \vec{E}(\vec{r}),\ \vec{H}(\vec{r},t) \to \vec{H}(\vec{r})} \begin{cases} \nabla \times \vec{E} = -j\omega\mu\vec{H} & (1) \\ \nabla \cdot \vec{E} = 0 & (2) \\ \nabla \times \vec{H} = j\omega\varepsilon\vec{E} & (3) \\ \nabla \cdot \vec{H} = 0 & (4) \end{cases}$$

Homogeneous wave equation in frequency domain-2

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \quad (1)$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} \quad (3)$$

$$\nabla \times \nabla \times \vec{E} = \nabla \times \left(-j\omega\mu\vec{H}\right) = -j\omega\mu\left(\nabla \times \vec{H}\right)$$

$$\nabla \cdot \vec{E} = 0 \quad (2)$$

$$= -j\omega\mu\left(j\omega\varepsilon\vec{E}\right) = \omega^2\mu\varepsilon\vec{E} = \nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2\vec{E} = -\nabla^2\vec{E}$$

$$\nabla^2\vec{A} \equiv \nabla\left(\nabla \cdot \vec{A}\right) - \nabla \times \nabla \times \vec{A}$$

$$\Rightarrow \boxed{\nabla^2\vec{E} + k^2\vec{E} = 0} \dashrightarrow \boxed{\nabla^2\vec{H} + k^2\vec{H} = 0}$$

$$k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{u_p} = \frac{2\pi}{\lambda}$$ Wavenumber (spatial angular frequency)

$$u_p = 1/\sqrt{\mu\varepsilon}$$

Homogeneous wave equation in frequency domain-3

Compare with the equations of lossless TX lines:

$$\frac{d^2}{dz^2}V(z) = -\beta^2 V(z)$$

$$\frac{d^2}{dz^2}I(z) = -\beta^2 I(z)$$

$$\beta = \omega\sqrt{LC}$$

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$

$$\nabla^2 \vec{H} + k^2 \vec{H} = 0$$

$$k = \omega\sqrt{\mu\varepsilon}$$

Scalar waves, ODEs ⟶ Vector waves, PDEs

EM fields in lossy media-complex permittivity

If the medium is conducting ($\sigma \neq 0$), the presence of $\vec{E}$ results in conduction currents $\vec{J} = \sigma\vec{E}$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t},$$ if time-harmonic, simple medium

$$\Rightarrow \nabla \times \vec{H} = \sigma\vec{E} + j\omega\varepsilon\vec{E} = j\omega\left(\frac{\sigma}{j\omega} + \varepsilon\right)\vec{E} = j\omega\varepsilon_c\vec{E},$$

where $\boxed{\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega}}$ is the complex permittivity

EM fields in lossy media-complex wavenumber

$$\varepsilon_c = \varepsilon - j\frac{\sigma}{\omega} = \varepsilon\left(1 - j\frac{\sigma}{\omega\varepsilon}\right)$$

$$k = \omega\sqrt{\mu\varepsilon}$$

$$k_c = \omega\sqrt{\mu\varepsilon_c} = \omega\sqrt{\mu\varepsilon\left(1 - j\tan\delta_c\right)}$$

Loss tangent

$$\tan\delta_c = \frac{\sigma}{\omega\varepsilon}$$

If $\tan\delta_c << 1$, $\Rightarrow k_c \approx \omega\sqrt{\mu\varepsilon} \in R$, the medium behaves like a dielectric

If $\tan\delta_c >> 1$, $\Rightarrow \mathrm{Im}\{k_c\}$ is non-negligible, the medium behaves like a conductor (TBD…)

EM fields in lossy media-propagation loss

$\mathrm{Im}\{\varepsilon_c\},\ \mathrm{Im}\{k_c\}$ are associated with the power loss when the wave propagates through the medium.

1. $\vec{P}=\varepsilon_c\vec{E}$, $\Rightarrow$ $\mathrm{Im}\{\varepsilon_c\}$ describes $\vec{P}$ is not in phase with the driving $\vec{E}$ due to the inertia of the bound charges, $\Rightarrow$ frictional damping.

2. Ohmic power loss $P=\int_V\left(\vec{E}\cdot\vec{J}\right)dv$ due to collision among free charges and atoms.

Example 15-1: Property of a moist ground

$$\varepsilon = 10\varepsilon_0, \ \ \sigma = 10^{-2}\ (\mathrm{S/m}), \ \ \tan\delta_c = \frac{\sigma}{\omega\varepsilon}$$

At f =1 kHz, $\tan\delta_c \approx 10^4 >> 1$, like a conductor

At f =10 GHz, $\tan\delta_c \approx 10^{-3} << 1$, like a dielectric

Note: Doesn’t mean 1-kHz wave is more lossy than 10-GHz wave! The properties of EM waves in conductor & dielectric will be elucidated in Lesson 16.

Microwave oven-history

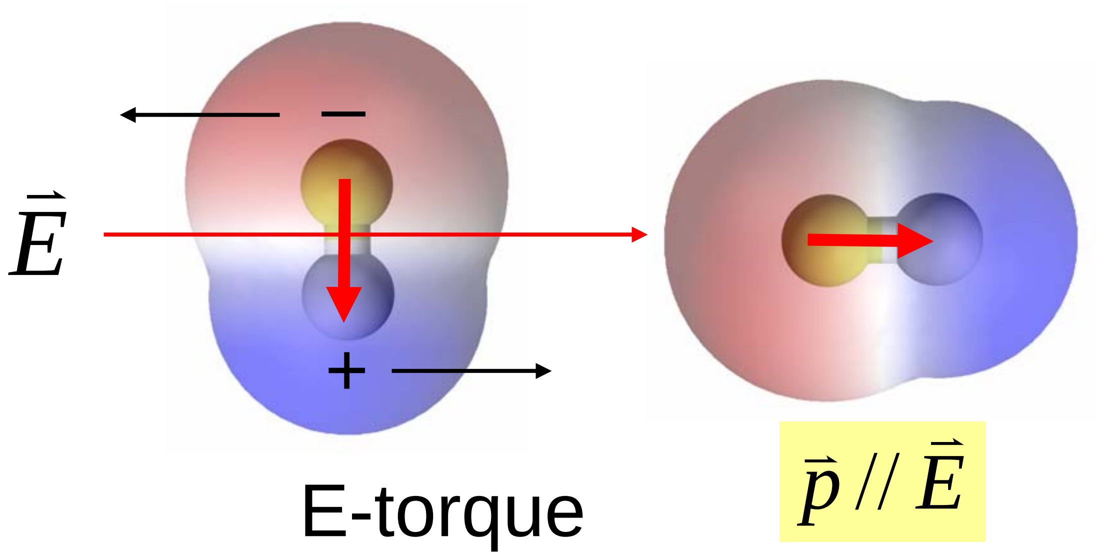
Invented by Percy Spencer (1945), chocolate bar melt when building magnetrons(磁控管) for radar sets with *Raytheon*, tested popcorn & egg.

First commercial model *Radarange* (1947).

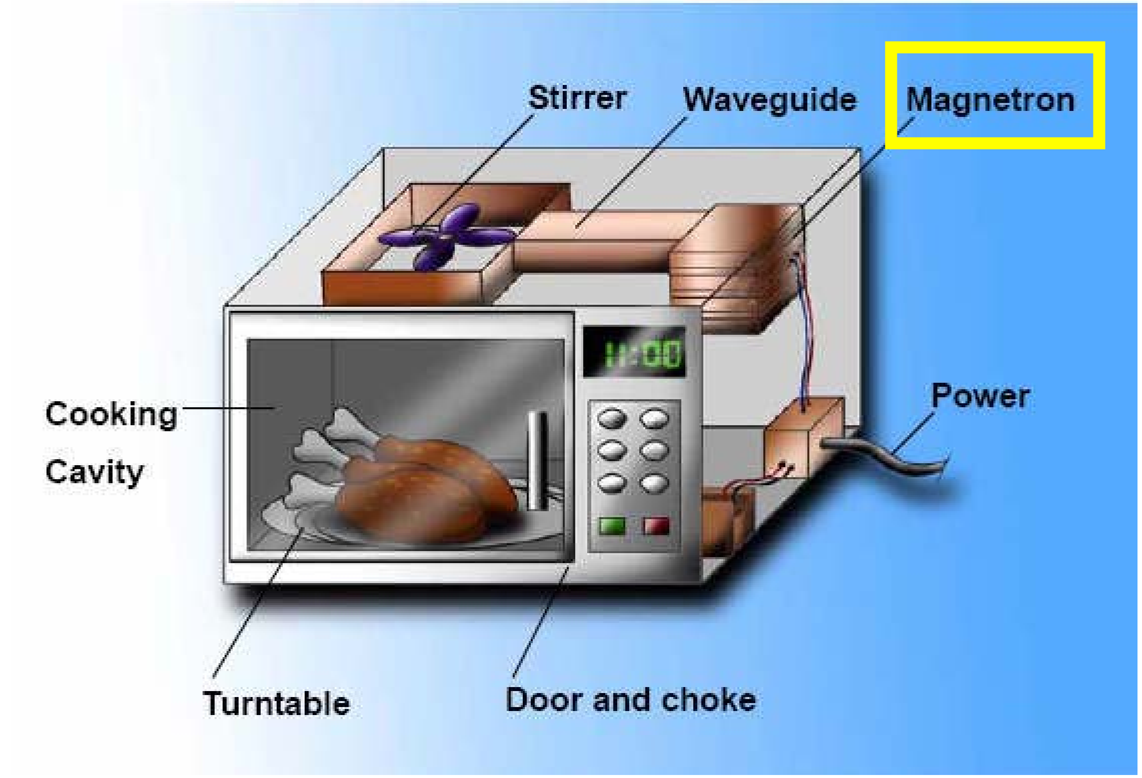
Microwave oven-working principle (1)

Pass EM wave at 2.45 GHz (λ=12.2 cm) through food. Polarized molecules (water, fat, ..) will rotate with the AC field, motion(~heat) dispersed to other non-polarized molecules by collision.

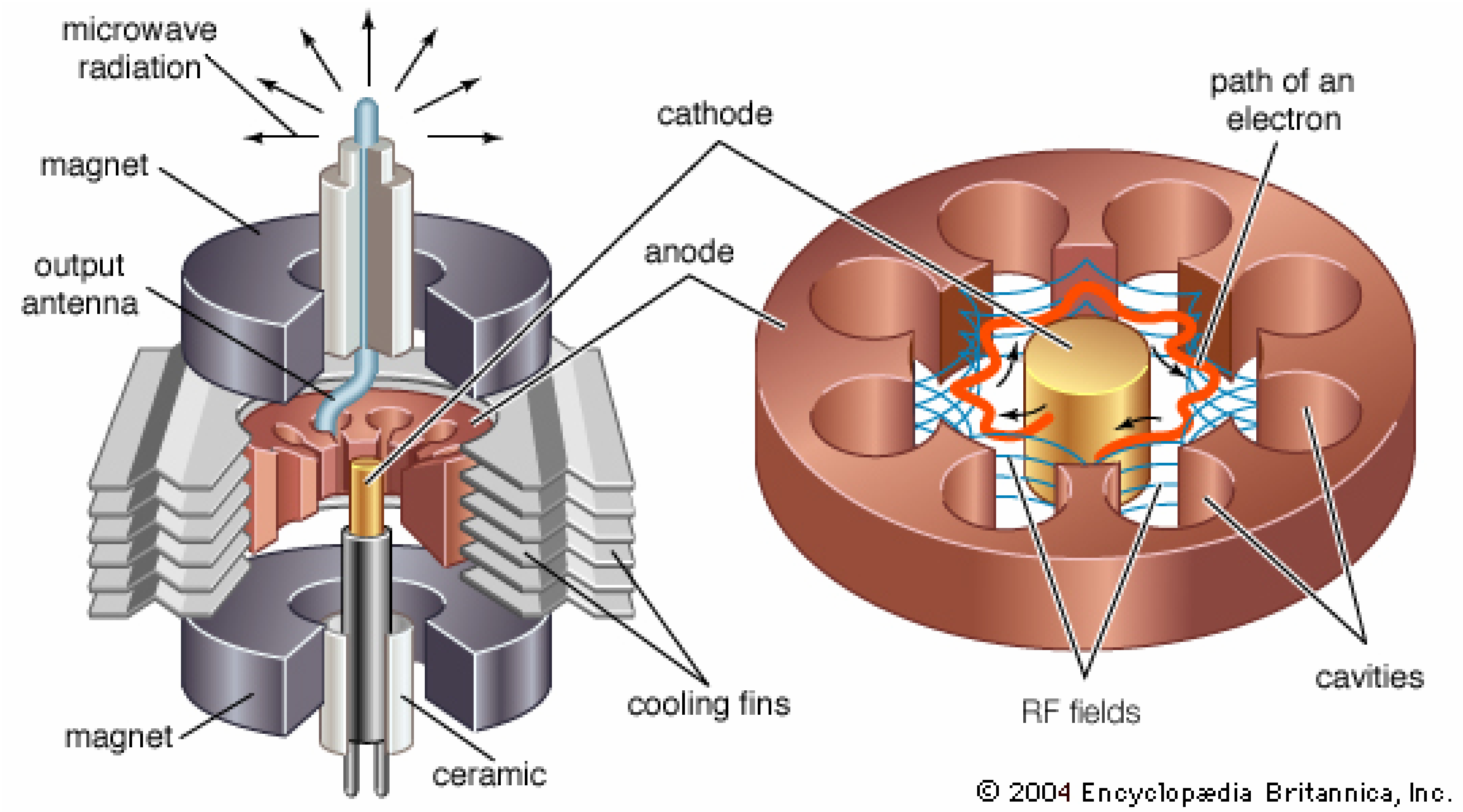
(Not due to resonance, for water vapor f_{res}=20 GHz)



Microwave oven-working principle (2)



Microwave oven-working principle (3)



Microwave oven-working principle (4)

Microwaves can penetrate dry non-conducting substances, inducing initial heat more deeply (~cm) than other methods.

(Not cook from the inside out)

The oven door usually has a window with a layer of conductive mesh for shielding. The grid size is << λ(=12.2 cm), most of the microwave radiation cannot pass through the door, while visible light can (for observation).

Example 15-2: Microwave oven

A microwave oven generates an AC E-field of:

$$E(t) = \underline{250} \cdot \cos(2\pi \cdot \underline{2.45\times 10^9} \cdot t)\ (\mathrm{V/m})$$

to cook a beef steak of $\varepsilon = 40\varepsilon_0$, and $\tan\delta_\mathrm{c} = 0.35$

$$\tan\delta_c = \frac{\sigma}{\omega\varepsilon},\ \Rightarrow\ \ \sigma = \omega\varepsilon\tan\delta_c =$$

$$(2\pi \cdot 2.45\times 10^9)\cdot\left(40\frac{10^{-9}}{36\pi}\right)\cdot(0.35) = 1.9\,(\mathrm{S/m})$$

$$p(t) = \vec{E}(t)\cdot\vec{J}(t) = \sigma\left|\vec{E}(t)\right|^2, \Rightarrow P_{avg} = \frac{1}{2}\sigma E_0^2 \approx 60\left(\mathrm{mW/cm^3}\right)$$

Sec. 15-2
Wave Equations of Potentials

1. Non-homogeneous equations in time domain
2. Solutions to homogeneous equations
3. Non-homogeneous equations in frequency domain

Potentials in time-varying cases (1)

In the presence of time-varying fields:

$\nabla \cdot \vec{B} = 0$ remains, $\Rightarrow$ $\vec{B} = \nabla \times \vec{A}$ remains valid.

M-field vector potential

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \neq 0, \Rightarrow \vec{E} \neq -\nabla V$$

Instead, $\nabla \times \vec{E} = -\frac{\partial}{\partial t}\left(\nabla \times \vec{A}\right)$, $\nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t}\right) = 0$, $-\nabla V$

$$\Rightarrow \quad \vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

Potentials in time-varying cases (2)

scalar potential,

conservative component,

~charge distribution

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

vector potential,

nonconservative component,

~time-varying current

Nonhomogeneous wave equations of vector potential in time domain-1

In simple media:

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \longrightarrow \nabla \times \vec{B} = \mu \vec{J} + \mu\varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \nabla \times \vec{B} = \nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$$

$$\frac{\partial \vec{E}}{\partial t} = -\nabla\left(\frac{\partial V}{\partial t}\right) - \frac{\partial^2 \vec{A}}{\partial t^2}$$

$$\Rightarrow \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A} = \mu \vec{J} - \mu\varepsilon \nabla\left(\frac{\partial V}{\partial t}\right) - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2}$$

Nonhomogeneous wave equations of vector potential in time domain-2

$$\Rightarrow \quad \nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J} + \nabla\left(\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t}\right)$$

By Lorentz gauge: $\boxed{\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0}$

$\vec{A}$ is decoupled with V,

$$\Rightarrow \quad \boxed{\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J}}$$

Nonhomogeneous wave equation

Nonhomogeneous wave equations of scalar potential in time domain

In simple media:

$$\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho \longrightarrow \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

Lorentz gauge

$$\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0$$

$$\Rightarrow \nabla \cdot \vec{E} = -\nabla \cdot (\nabla V) - \frac{\partial}{\partial t}\left(\nabla \cdot \vec{A}\right) = -\nabla^2 V - \frac{\partial}{\partial t}\left(\nabla \cdot \vec{A}\right)$$

$$\Rightarrow \nabla \cdot \vec{E} = -\nabla^2 V + \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = \frac{\rho}{\varepsilon}$$

$$\Rightarrow \boxed{\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}}$$

Nonhomogeneous wave equation

Comments-1

Given charge and current distributions $\rho(\vec{r},t)$, $\vec{J}(\vec{r},t)$

Solve

$$\begin{cases} \nabla^2 V - \mu\varepsilon \dfrac{\partial^2 V}{\partial t^2} = -\dfrac{\rho}{\varepsilon} \\ \nabla^2 \vec{A} - \mu\varepsilon \dfrac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J} \end{cases} \xrightarrow{\text{TBD...}} \begin{cases} V(\vec{r},t) \\ \vec{A}(\vec{r},t) \end{cases}$$

Derive fields by:

$$\begin{cases} \vec{E} = -\nabla V - \dfrac{\partial \vec{A}}{\partial t} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

Comments-2

In static cases:

Lorentz gauge

$$\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0$$

→

Coulomb's gauge

$$\nabla \cdot \vec{A} = 0$$

Nonhomogeneous wave equations

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J}$$

→

Poisson's equations

$$\nabla^2 V = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 \vec{A} = -\mu\vec{J}$$

Solutions to nonhomogeneous wave equations in time domain (1)

Consider a point charge at origin (spherical symmetry)

$$\left[\nabla^2 V(R)\right]_{R\phi\theta}$$

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon} \longrightarrow \frac{1}{R^2}\frac{\partial}{\partial R}R^2\left(\frac{\partial V}{\partial R}\right) - \mu\varepsilon\frac{\partial^2 V}{\partial t^2} = 0$$

(except for the origin)

Define $U(R,t) = R \cdot V(R,t)$

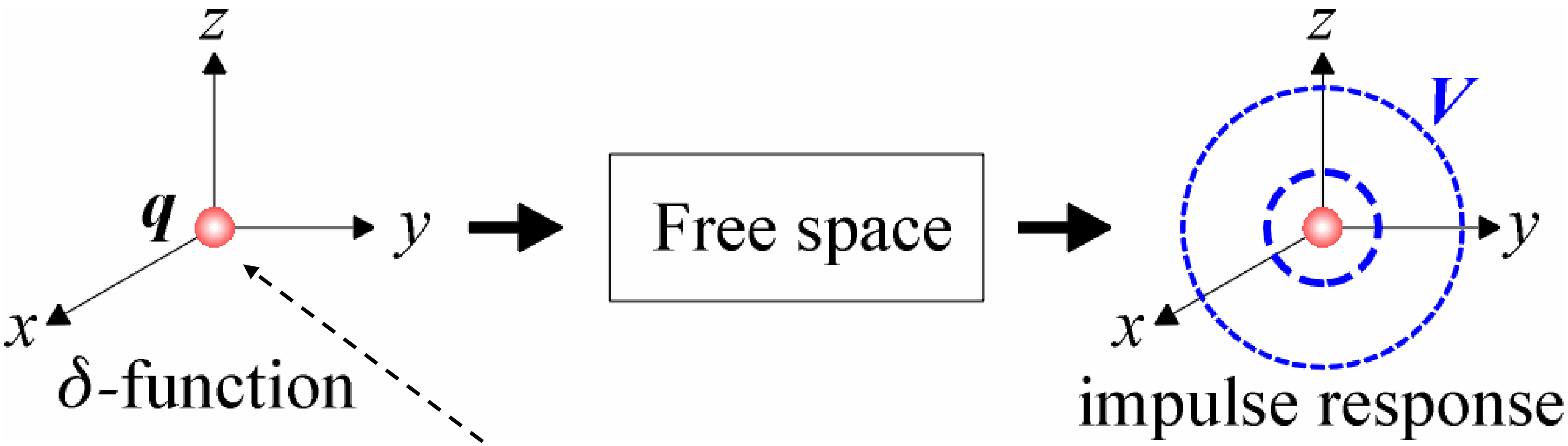
$$\Rightarrow \frac{\partial^2 U}{\partial R^2} - \mu\varepsilon\frac{\partial^2 U}{\partial t^2} = 0$$ …standard wave equation

$$\Rightarrow \underline{U(R,t) = f(\tau)},\quad V(R,t) = \frac{f(\tau)}{R},\quad \begin{cases} \tau = t - R/u_p, \\ u_p = 1/\sqrt{\mu\varepsilon} \end{cases}$$

Solutions to nonhomogeneous wave equations in time domain (2)

In static cases:

$$V(R)=\frac{q}{4\pi\varepsilon\cdot R}$$



In time-varying ($\rho(t)dv'$ at origin) cases:

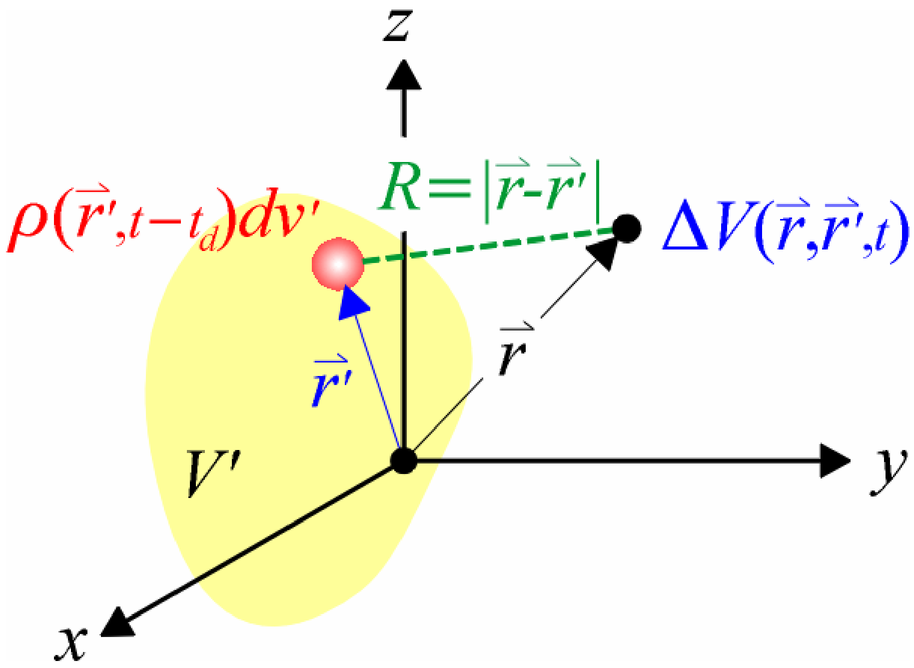
$$\Delta V(R,t)=\frac{\rho(t-R/u_p)dv'}{4\pi\varepsilon\cdot R}$$

$f(\tau)$

Solutions to nonhomogeneous wave equations in time domain (3)

Since $\nabla^2 V - \mu\varepsilon \dfrac{\partial^2 V}{\partial t^2} = -\dfrac{\rho}{\varepsilon}$ is linear,

the potential due to $\rho(\vec{r}\,',t)$ over a volume V' is:



$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho(\vec{r}\,',t-t_d)}{R}\,dv',$$

$$t_d = R/u_p$$

Solutions to nonhomogeneous wave equations in time domain (4)

Similarly, the vector potential due to $\vec{J}(\vec{r}\,',t)$ over volume V' is:

$$t_d = R/u_p$$

$$\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\int_{V'} \frac{\vec{J}(\vec{r}\,',t - t_d)}{R}\,dv'$$

The potentials $V(\vec{r},t)$, $\vec{A}(\vec{r},t)$ are determined by the source at $\vec{r}\,'$ at an earlier time $t - R/u_p$

$\Rightarrow$ Potential (& field) propagates with finite speed u_p

Nonhomogeneous wave equations of potentials in frequency domain

For time-harmonic waves:

$$\nabla^2 V - \mu\varepsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 \vec{A} - \mu\varepsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J}$$

$$\nabla \cdot \vec{A} + \mu\varepsilon \frac{\partial V}{\partial t} = 0$$

$\longrightarrow$

$$\nabla^2 V + k^2 V = -\frac{\rho}{\varepsilon}$$

$$\nabla^2 \vec{A} + k^2 \vec{A} = -\mu\vec{J}$$

$$\nabla \cdot \vec{A} + j\omega\mu\varepsilon V = 0$$

phasors,

$$k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{u_p} = \frac{2\pi}{\lambda}$$

Solutions to nonhomogeneous wave equations of potentials in frequency domain

For time-harmonic waves:

time retardation → phase shift

$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho(\vec{r}',t - R/u_p)}{R} dv' \longrightarrow V(\vec{r}) = \frac{1}{4\pi\varepsilon}\int_{V'} \frac{\rho(\vec{r}')e^{-jkR}}{R} dv'$$

$$\vec{A}(\vec{r},t) = \frac{\mu}{4\pi}\int_{V'} \frac{\vec{J}(\vec{r}',t - R/u_p)}{R} dv' \longrightarrow \vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int_{V'} \frac{\vec{J}(\vec{r}')e^{-jkR}}{R} dv'$$

Justification

$$V_0 \cos(\omega t + \phi) \rightarrow V_0 e^{j\phi},$$

$$V_0 \cos[\omega(t - t_d) + \phi] =$$

$$V_0 \cos[\omega t + \phi - \omega \cdot t_d] \rightarrow V_0 e^{j(\phi - \omega \cdot t_d)}$$

phasor

$$\longrightarrow \omega \cdot t_d = \omega \frac{R}{u_p} = \omega \frac{R}{\lambda \cdot f}$$

$$= 2\pi \frac{R}{\lambda} = kR$$

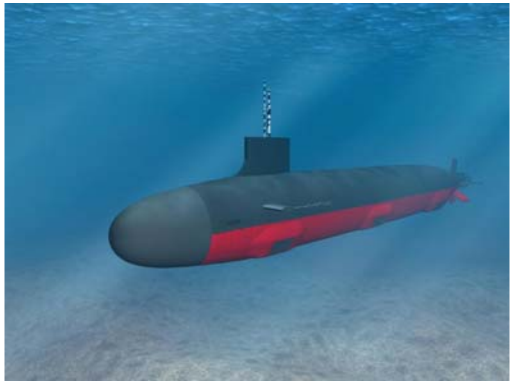
Sec. 15-3
Electromagnetic Spectrum

Extremely low frequency (ELF): 0-300 Hz (1)

Global communications with deeply submerged submarines.

Why using ELF: attenuation of EM waves in sea water is lower for lower frequencies (TBD…).

Difficulties: (1) low data rate, (2) huge antenna size (in principle $\lambda/2$), only support unidirectional communications.



What's the problem?

Crimson Tide(赤色風暴, 1995)

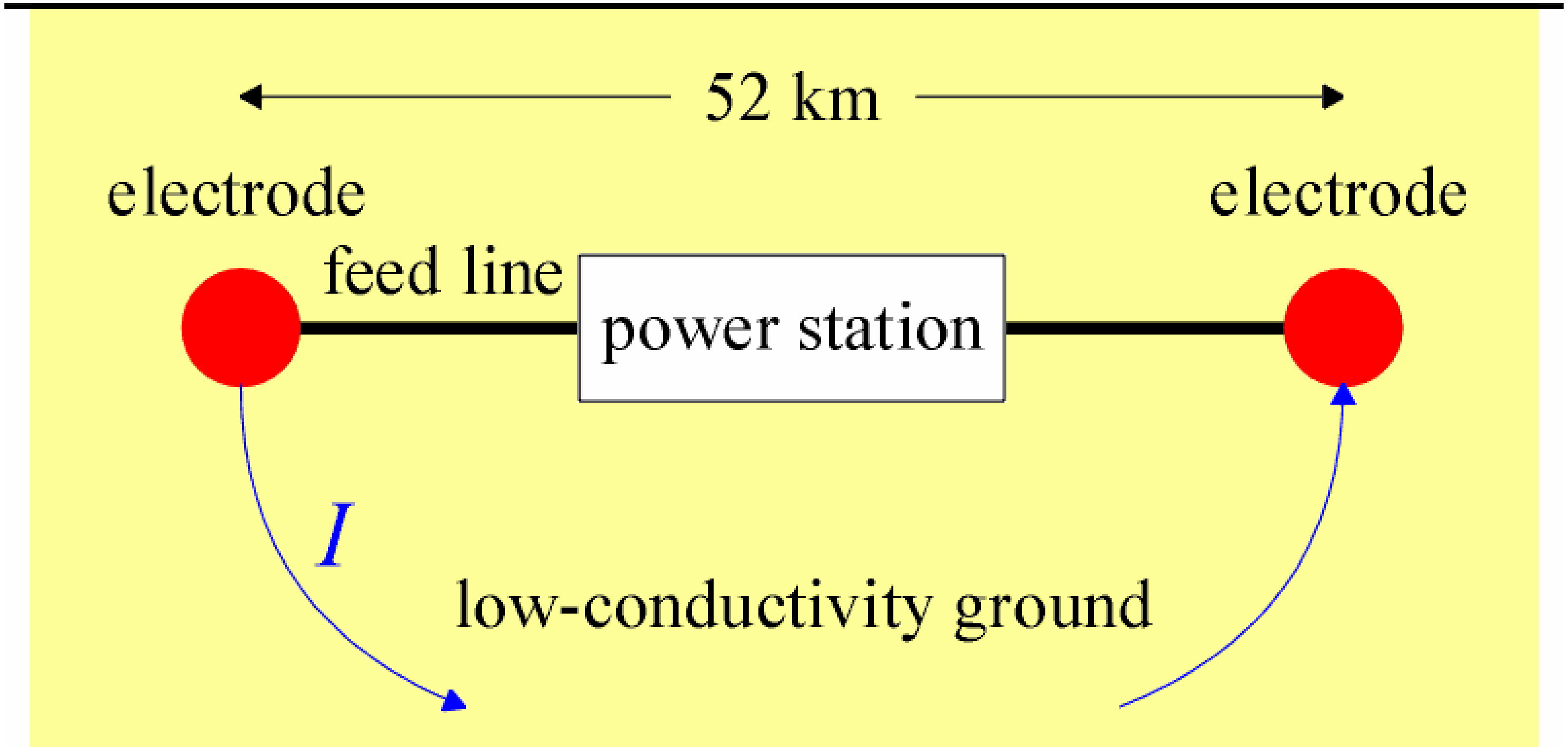
Not confirmed yet…

Launch nuke missiles now!

Extremely low frequency: 0-300 Hz (2)

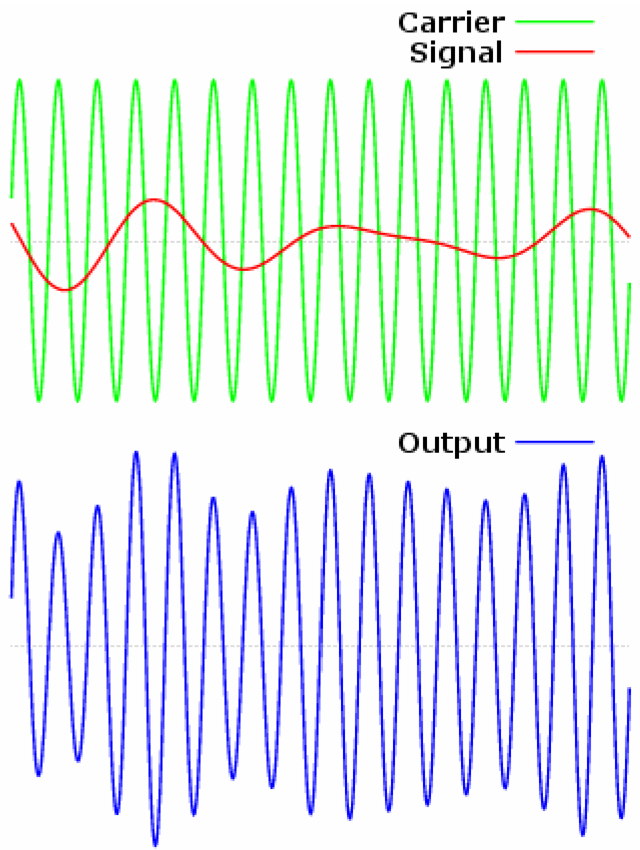
US Navy Seafarer system: 76 Hz ($\lambda \approx$3,900 km).

Low-conductivity ground, $\Rightarrow$ deeper penetration of current (use a part of the globe as antenna).



Medium frequency (MF): 0.3-3 MHz

Amplitude modulation (AM) broadcast: 0.53-1.61 MHz, antenna length $\lambda/2 \approx 150$ m.



1906: 1st experiment (Canada).

Max audio BW: 10.2 kHz (channel spacing: 20.4 kHz).

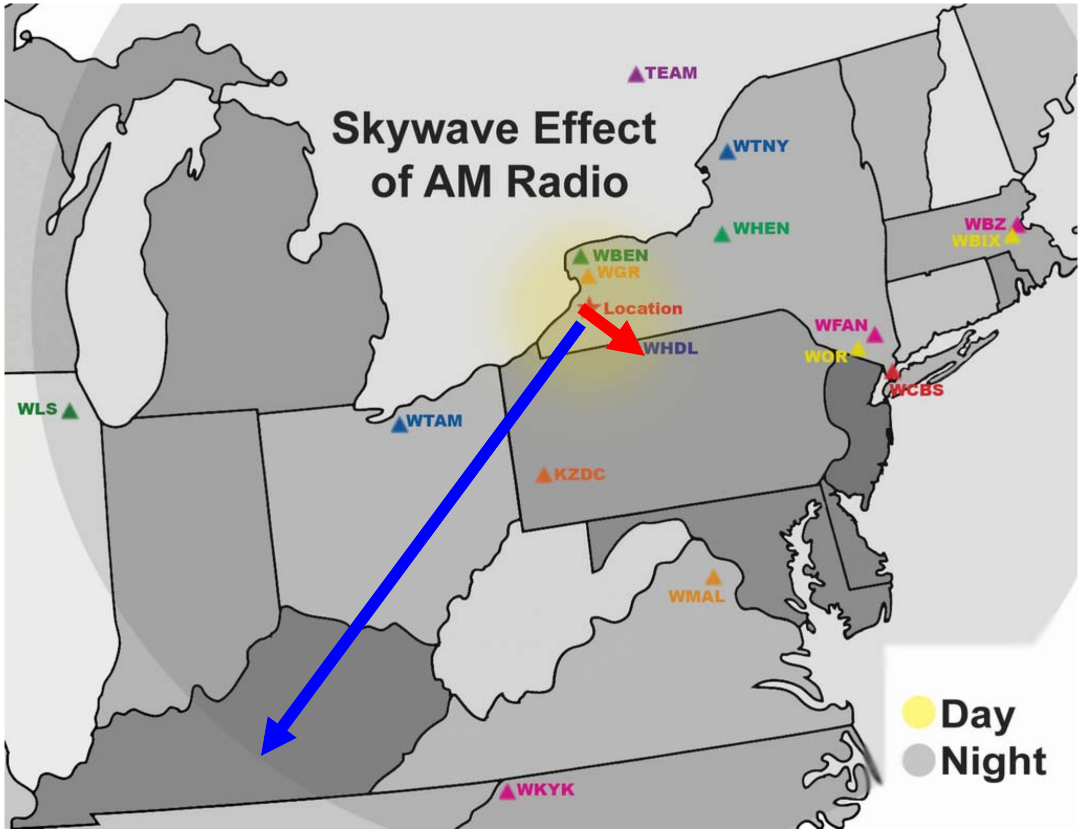
(commons.wikimedia.org)

Signal range of AM broadcast

Day time: groundwave, diffracting around the curve of the earth, ~100 km.

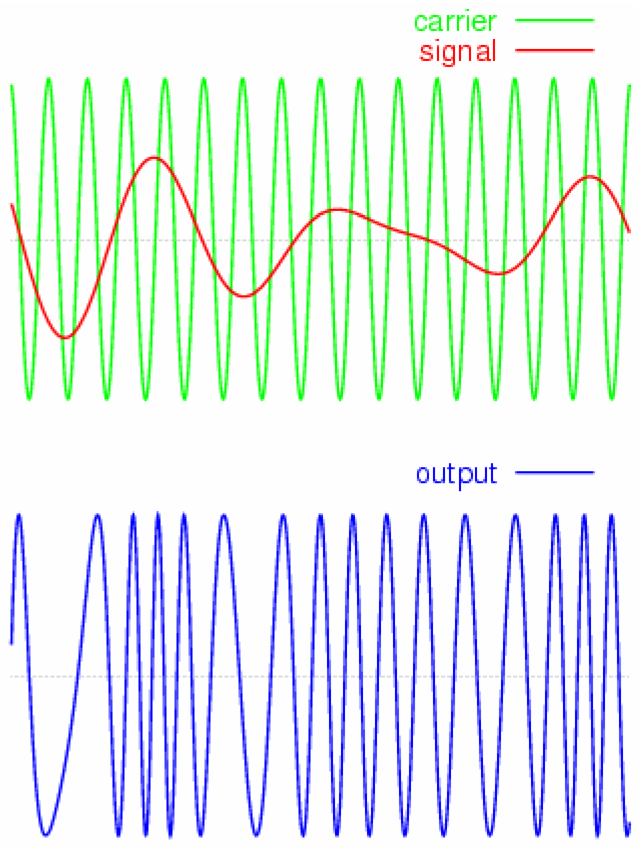
(wikipedia.org)

Night: skywave (ionsphere reflection), much longer.



Very high frequency (VHF): 30-300 MHz

Frequency modulation (FM) broadcast: 88-108 MHz, antenna length $\lambda/2 \approx 1.5$ m.



1933: invented.

In US, 101 channels, from 87.9 MHz to 107.9 MHz (spacing 200 kHz).

No ionsphere reflection, limited to line-of-sight range (~100 km).

(commons.wikimedia.org)

Ultra high frequency (UHF): 300 MHz-3 GHz

TV broadcast: 530-596 MHz. (Audio:174-216 MHz)

Cell phone: Global System for Mobile communications (GSM), 2G: (900 MHz, 1.8 GHz); 3G: (850/880 MHz, 1.9, 2.0, 2.1 GHz).

Global positioning system (GPS): all satellites broadcast 1.58 GHz (L1), 1.23 GHz (L2).

2.45 GHz: Wi-Fi, Bluetooth, microwave oven.

Super high frequency (SHF): 3-30 GHz

$\lambda \approx$ centimeters.

Radar: L(1-2G), S(2-4G), C(4-8G, airborne weather), X(8-12G, missile guidance), Ku(12-18G).

Wireless Local Area Network (WLAN): provide access point to the internet.

Terahertz: 10^{12} Hz

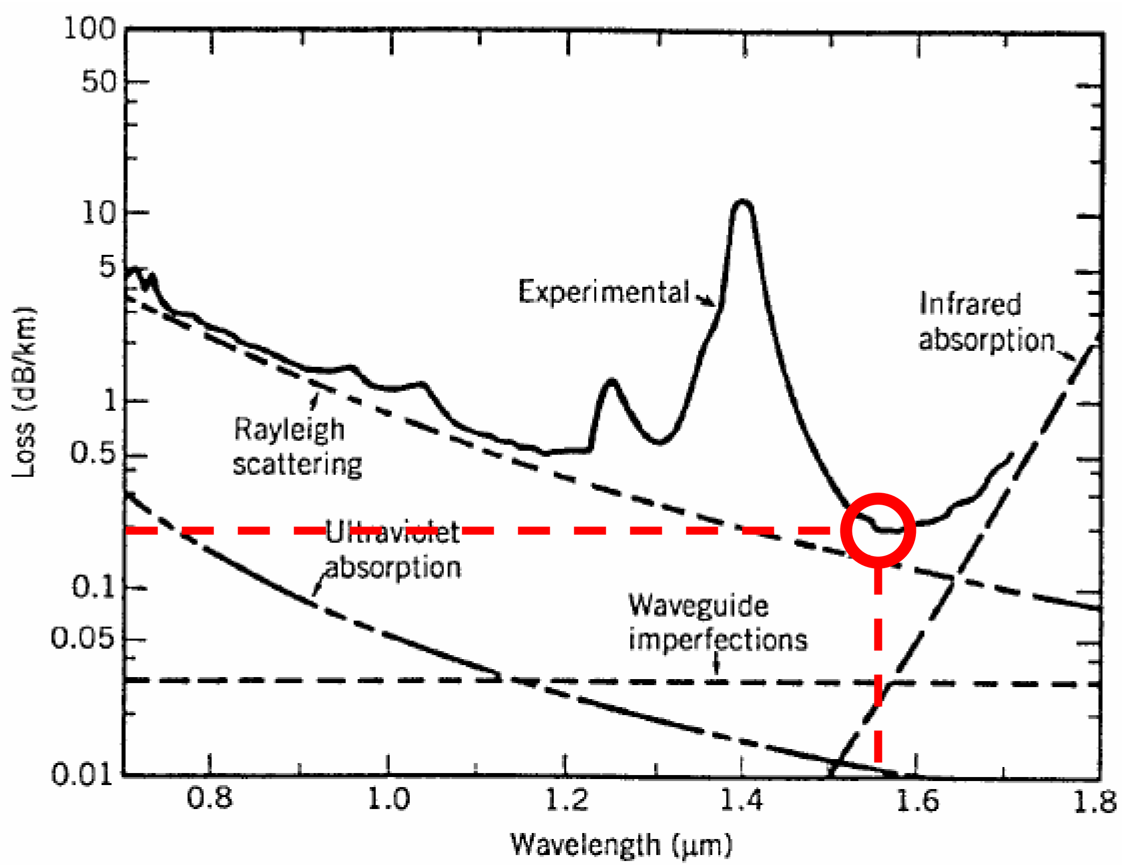
Difficult to be generated/detected by conventional electronic/optical means.

Aircraft-to-satellite communications (low water vapor environment).

Security

Infrared (IR): 10^{13} -10^{14} Hz, λ=0.7-100 μm (1)

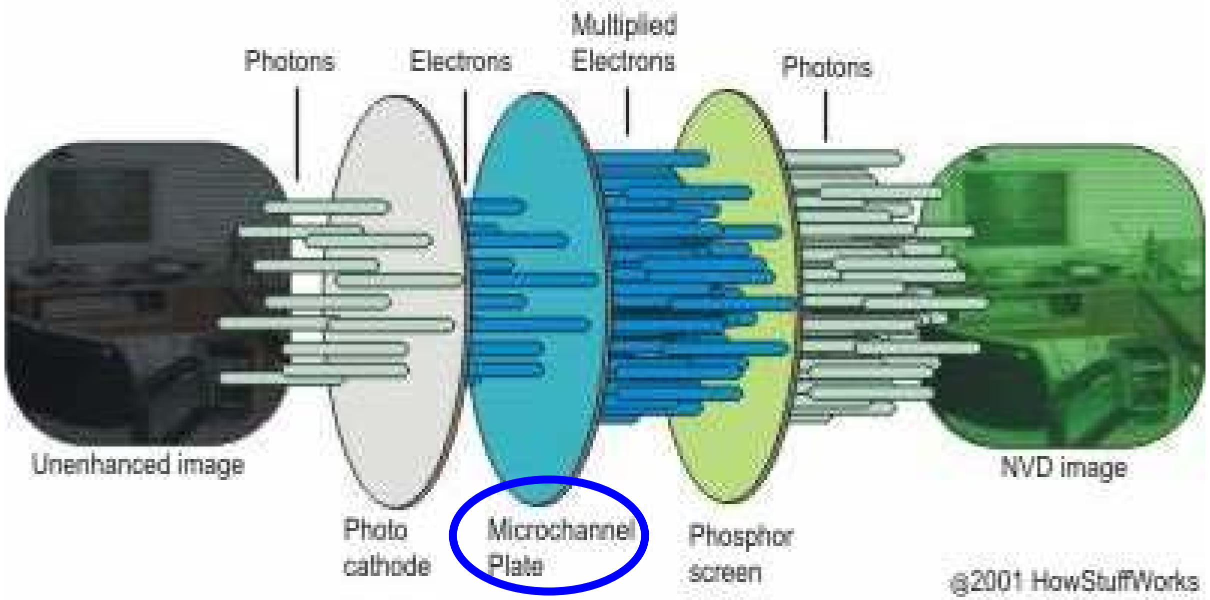
Fiber communications: $\lambda \approx$1.55 μm (Near IR).



Corning SMF-28, 0.2 dB/km, $\Rightarrow$ 50% power after 15 km propagation.

Infrared (IR): 10^{13} -10^{14} Hz, λ=0.7-100 μm (2)

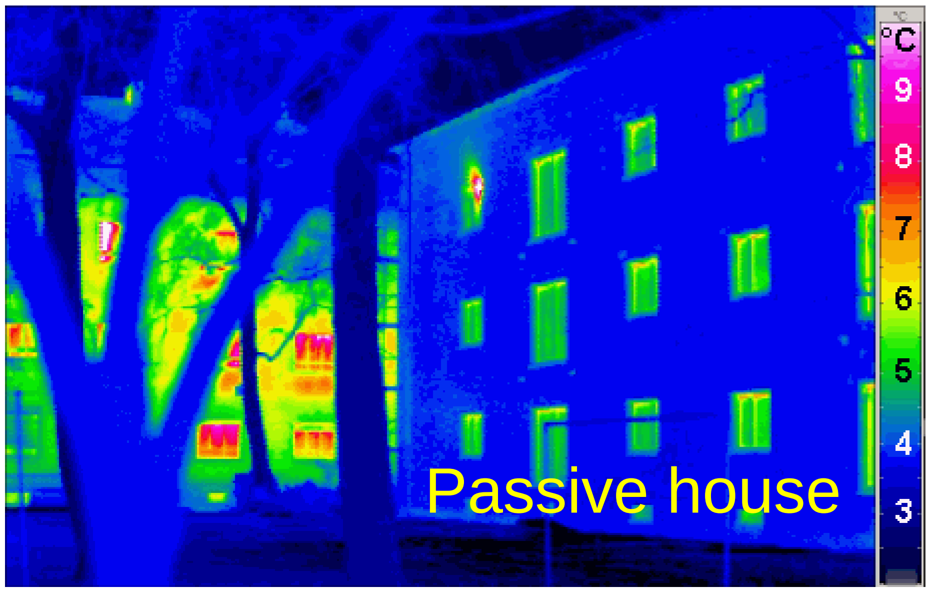
Night vision devices (image intensifier):
Photocathode converts weak visible & near IR photons into electrons, amplified by MCP, converted to visible photons by phosphor.



Infrared (IR): 10^{13} -10^{14} Hz, λ=0.7-100 μm (3)

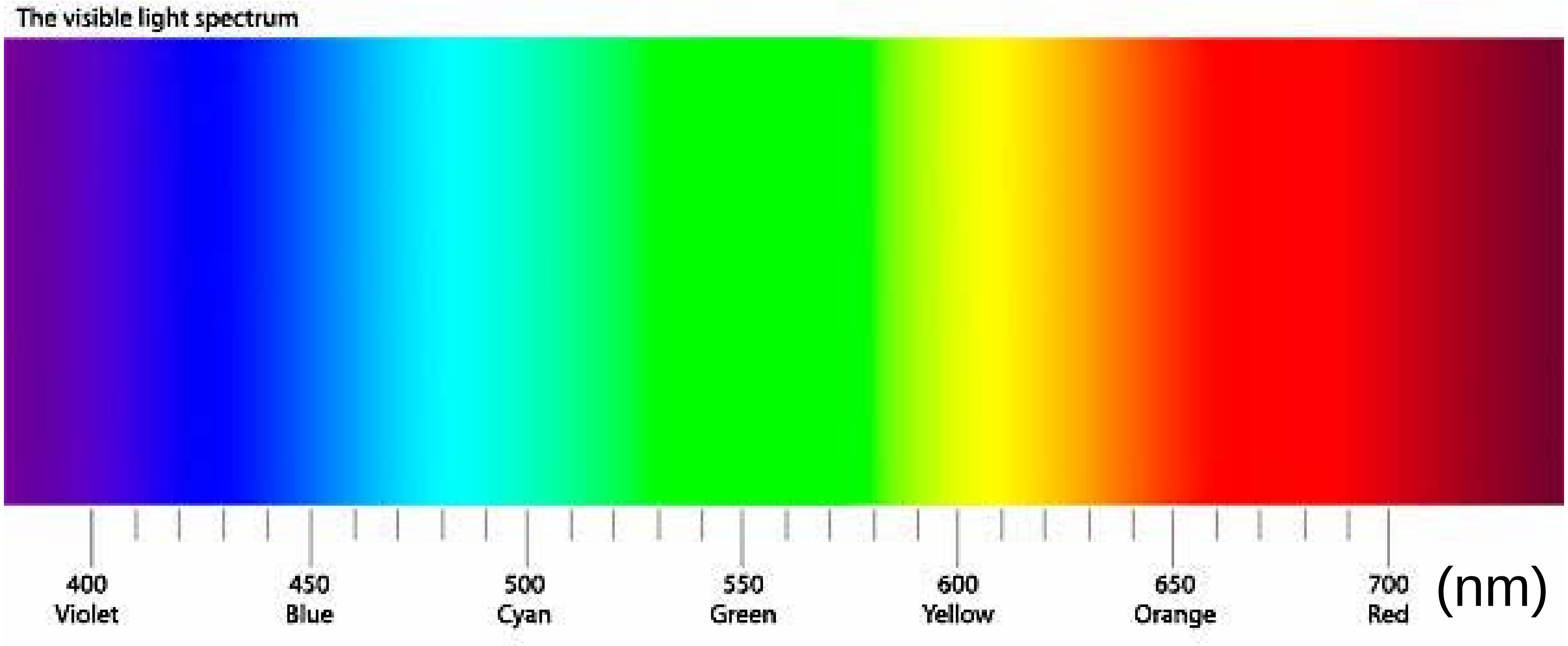
Thermal imaging: universal black body radiation (live human λ=9.5 μm; missile λ=3–5 μm; mid-IR), sense temperature variation.

Detectors: InSb(銻化銦, III-V semiconductor, 0.17 eV, sensitive to 1-5 μm), bolometer (測輻射熱計, sensitive to all λ's)



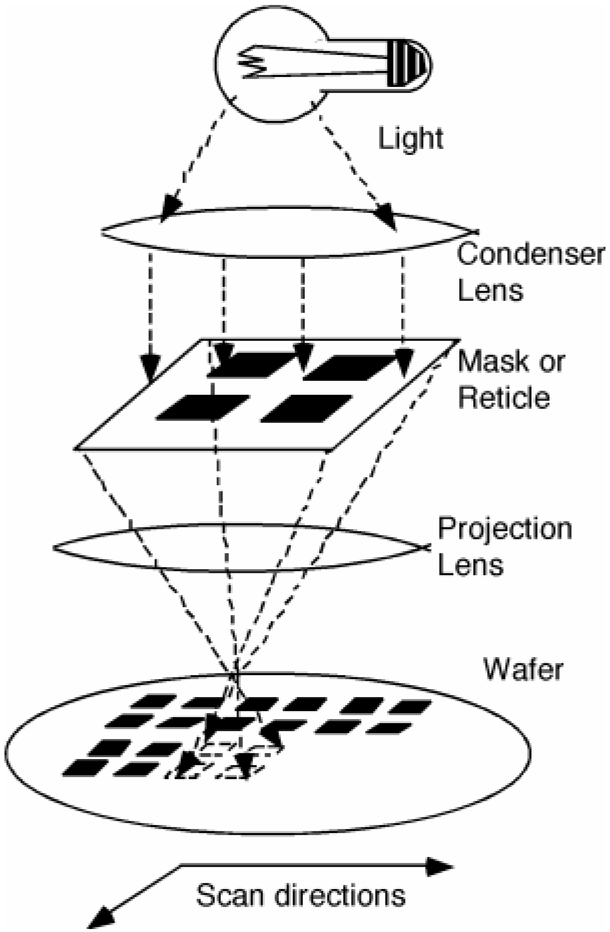
(wikipedia.org)

Visible: $(4\text{-}7)\times10^{14}$ Hz, λ=0.4-0.7 μm



Ultraviolet (UV): λ=10-100 nm

Photolithography



(cnx.org)

Light sources:

Mercury vapor lamp + filter:

λ=365 nm.

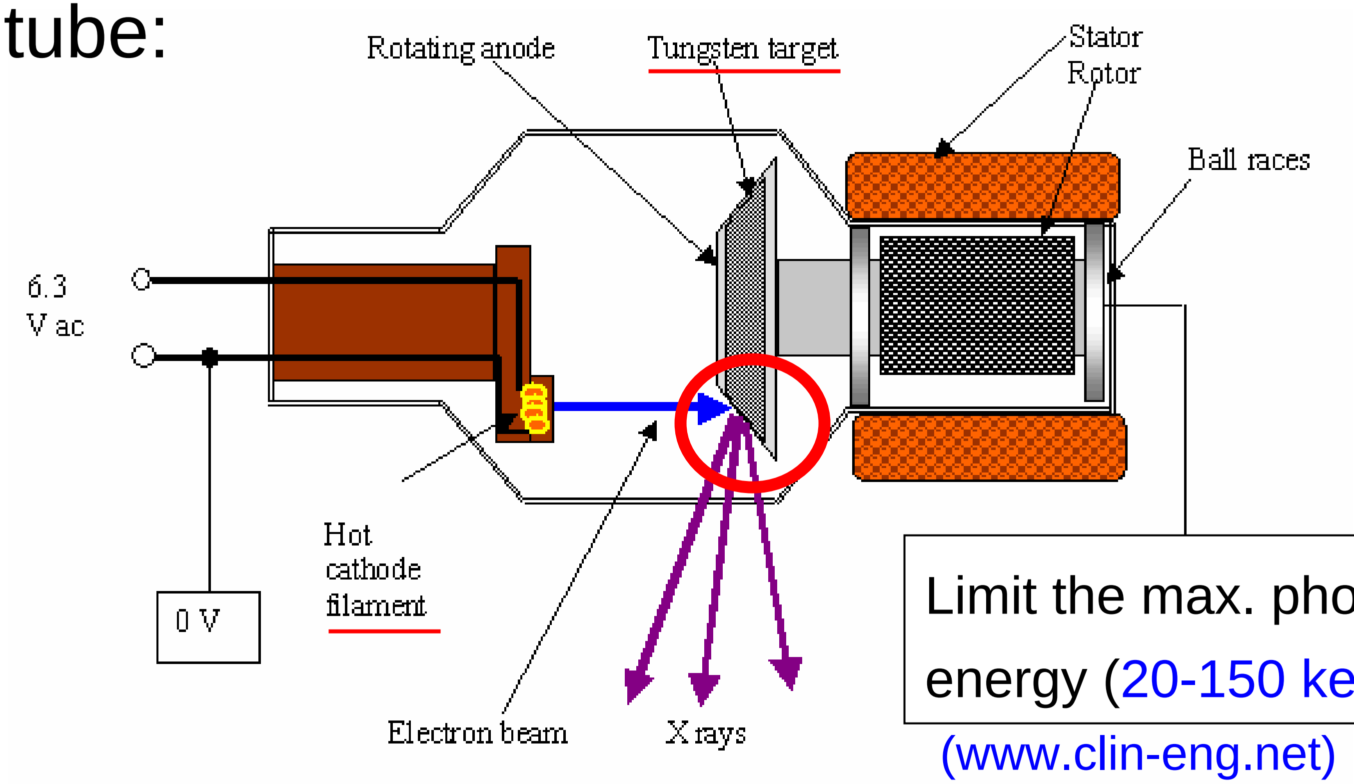
Excimer lasers(准分子雷射):

KrF λ=248 nm, ArF λ=193 nm

(enable 30 nm feature size).

X-rays: λ=0.01-10 nm, E~keV (1)

X-ray tube:



Limit the max. photon energy (20-150 keV)

(www.clin-eng.net)

1. X-ray fluorescence: knock out inner shell e^-, $\Rightarrow$ e^- at higher energy levels fall (discrete lines).
2. Bremsstrahlung: e^- deflected by nucleus (cont. lines).

X-rays: λ=0.01-10 nm, E~keV (2)

Medical diagnostics:

Soft X-rays (λ>0.1 nm, E<12 keV) will be absorbed by the body, $\Rightarrow$ filtered by thin Al sheet over X-ray tube.

Bones (higher e^- density) absorb X-ray photons by photoelectric process, cause white on the film.

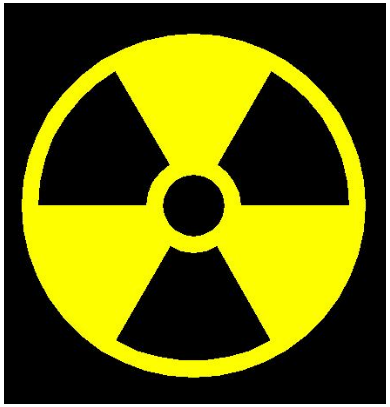
Wilhelm C. Röntgen (1895)

Computed tomography

γ-rays: $\lambda<0.01$ nm, E~MeV

$\lambda <$ atom size, $\Rightarrow$ created by sub-atomic particle interaction.

Ionizing radiation, $\Rightarrow$ hazardous.



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(audiokarma.org)

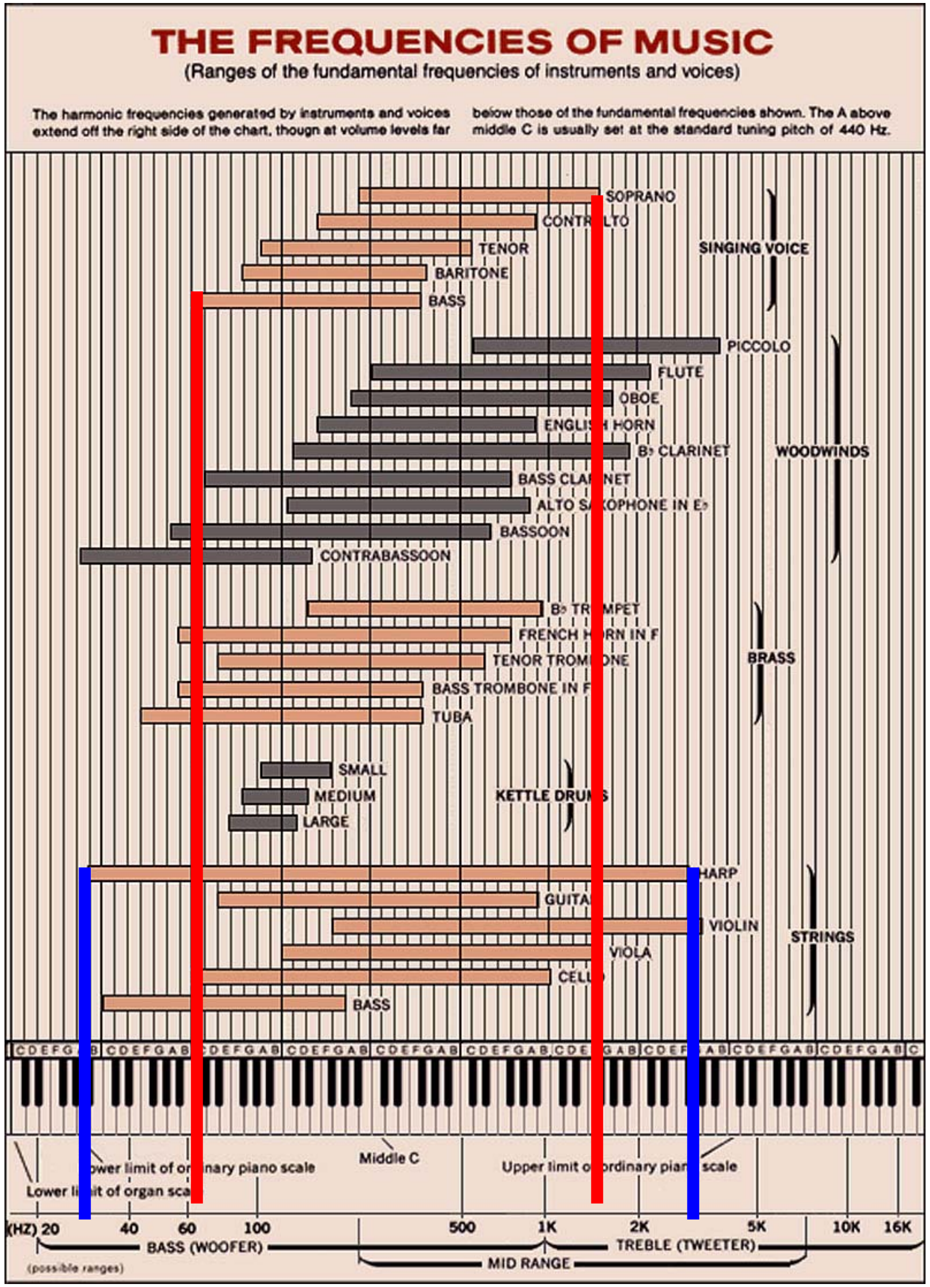
Appendix: Frequency chart

Human voice/cello: 60 Hz-1.5 kHz

Harp: 30 Hz-3 kHz

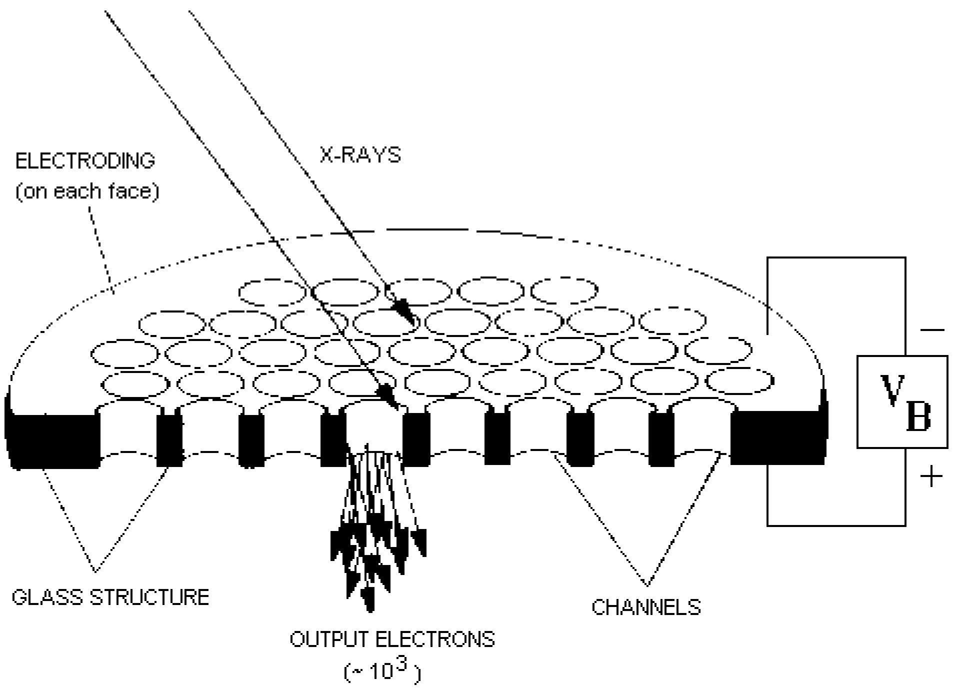
Middle C: 440 Hz

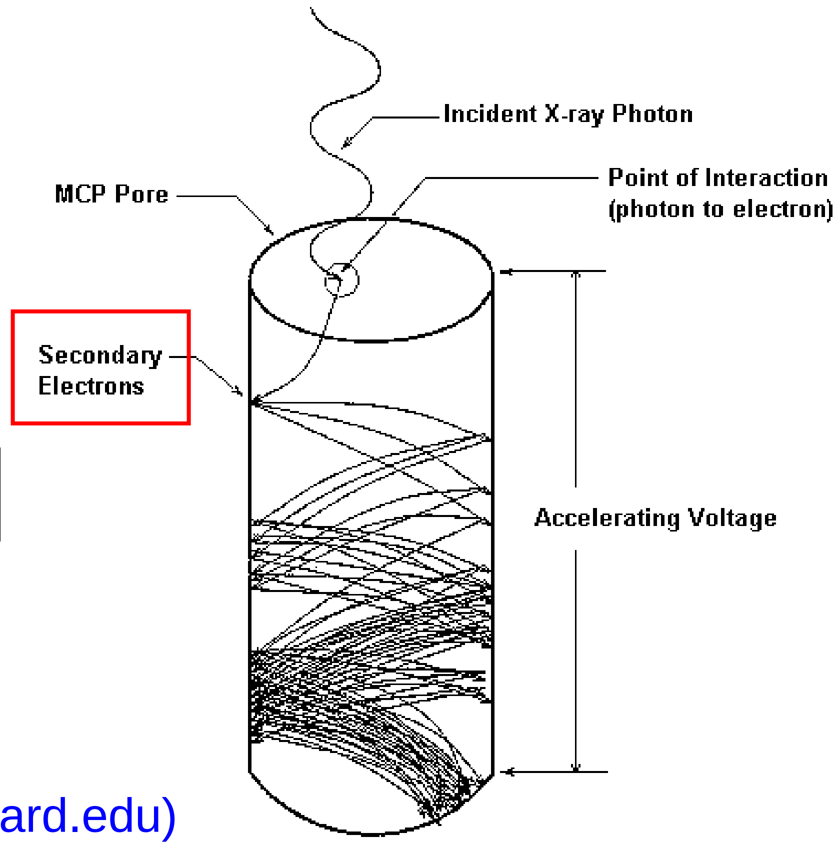
Human hearing: 20 Hz-20 kHz (2-4 kHz most sensitive)



Appendix: Microchannel plate (MCP, 微通道面板)

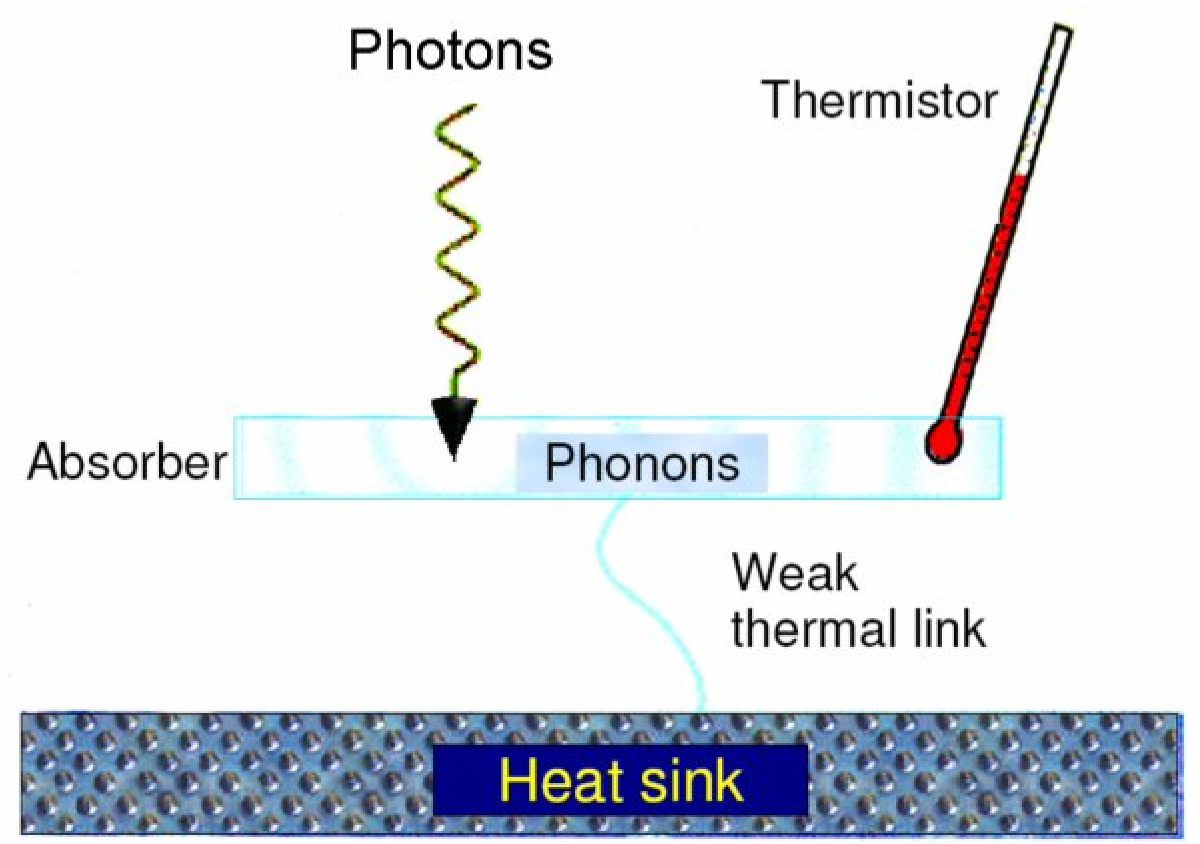
For detecting weak signals of ions or photons





(hea-www.harvard.edu)

Appendix: Bolometer(測輻射熱計)



Best choice for λ=200 μm - 1 mm (far-IR).