Lesson 10 Steady Electric Currents

10.1 Current Density

■ Definition

Consider a group of charged particles (each has charge q) of number density N (m^{-3}), moving across an elemental surface $\vec{a}_n \Delta s$ (m^2) with velocity $\vec{u}$ (m/sec). Within a time interval Δt, the amount of charge ΔQ passing through the surface is equal to the total charge within a differential parallelepiped of volume $\Delta v = (\vec{u}\Delta t)\cdot(\vec{a}_n \Delta s)$ (Fig. 10-1):

$$\Delta Q = Nq\Delta v = \rho \Delta t (\vec{u} \cdot \vec{a}_n \Delta s),$$

where ρ (C/m^3) denotes the volume charge density. The corresponding electric current is:

$$I = \frac{\Delta Q}{\Delta t} = \rho(\vec{u} \cdot \vec{a}_n \Delta s).$$

The current I can be regarded as the "flux" of a volume current density $\vec{J}$, i.e., $I = \vec{J} \cdot (\vec{a}_n \Delta s)$. By comparing the above two relations, we have:

$$\vec{J} = \rho \vec{u} \quad (\text{A/m}^2) \tag{10.1}$$

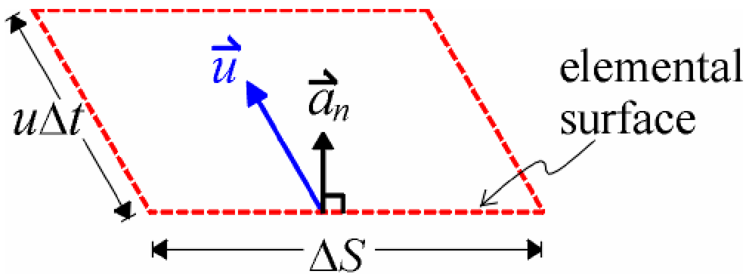

Fig. 10-1. Schematic of derivation of current density.

■ Convection currents

Convection currents result from motion of charged particles (e.g. electrons, ions) in "vacuum" (e.g. cathode ray tube), involving with mass transport but without collision.

Example 10-1 (Optional): In vacuum-tube diodes, some of the electrons boiled away from the incandescent cathode are attracted to the anode due to the external electric field, resulting in a convection current flow. Find the relation between the steady-state current density $\vec{J}$ and the bias voltage V_0. Assume the electrons leaving the cathode have zero initial velocity. This is the "space-charge limited condition", arising from the fact that a cloud of electrons (space charges) is formed near the hot cathode, repulsing most of the newly emitted electrons.

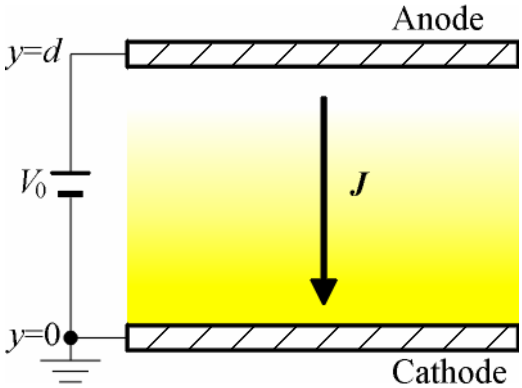

Fig. 10-2. Schematic of vacuum-tube diode.

Ans: Assume the linear dimension of cathode and anode is much larger than the length of tube d, planar symmetry leads to: (i) potential distribution is $V(y)$ (with boundary conditions: $V(0)=0$, $V(d)=V_0$), (ii) volume charge density is $\rho(y)$ (<0), (iii) charge velocity is $\vec{u}=\vec{a}_y u(y)$, respectively. Under the space-charge limited condition: (i) $u(0)=0$, (ii) the net electric field $\vec{E}=-\vec{a}_y E_y(y)$ at the cathode ($y=0$) is zero: $E_y(0)=0$, $\Rightarrow$ $V'(0)=0$.

(1) In steady state, $\vec{J}=-\vec{a}_y J$ is constant. By eq. (10.1), $J=-\rho(y)u(y)$, $\Rightarrow$ $\rho(y)=-\dfrac{J}{u(y)}$.

(2) By the energy conservation: $eV(y)=\dfrac{1}{2}mu^2(y)$, where m is the mass of an electron. $\Rightarrow$

$$u(y)=\sqrt{\frac{2eV(y)}{m}}, \quad \rho(y)=-J\sqrt{\frac{m}{2eV(y)}}.$$

(3) Since there is free charge density $\rho(y)$ inside the tube, the potential $V(y)$ has to satisfy with Poisson's equation [eq. (8.1)]:

$$\frac{d^2V}{dy^2} = -\frac{\rho(y)}{\varepsilon_0} = \frac{J}{\varepsilon_0}\sqrt{\frac{m}{2eV(y)}}\ .$$

(4) It is unnecessary to solve this “nonlinear” ordinary differential equation to get $V(y)$ if only the relation of $J(V_0)$ is of interest. Instead, we multiply both sides by $2\frac{dV}{dy}$:

$$2\frac{dV}{dy}\frac{d^2V}{dy^2} = 2\frac{J}{\varepsilon_0}\frac{dV}{dy}\sqrt{\frac{m}{2eV(y)}}\ ,$$

then integrate with respect to y:

$$\int 2V'(y)\cdot V''(y)dy = \frac{2J}{\varepsilon_0}\sqrt{\frac{m}{2e}}\left(\int V^{-1/2}dV\right), \Rightarrow \ \left(\frac{dV}{dy}\right)^2 = \frac{4J}{\varepsilon_0}\sqrt{\frac{mV(y)}{2e}} + c\ .$$

By boundary conditions: $V(0) = 0$, $V'(0) = 0$, $\Rightarrow$ $c = 0$, $\Rightarrow$ $\frac{dV}{dy} = 2\sqrt{\frac{J}{\varepsilon_0}\left[\frac{mV(y)}{2e}\right]^{1/4}}$,

$$V^{-1/4}dV = 2\sqrt{\frac{J}{\varepsilon_0}\left(\frac{m}{2e}\right)^{1/4}}dy\ .$$

(5) $\int_0^{V_0} V^{-1/4}dV = 2\sqrt{\frac{J}{\varepsilon_0}\left(\frac{m}{2e}\right)^{1/4}}\left(\int_0^d dy\right), \Rightarrow$

$$J = \frac{4\varepsilon_0}{9}\sqrt{\frac{2e}{m}}\frac{V_0^{3/2}}{d^2} \tag{10.2}$$

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1) The nonlinear $I-V$ relation of vacuum-tube diode differs from the Ohm’s law ($I \propto V$), which describes the “conduction” current in conductors.
2) The $I-V$ relation of forward-biased semiconductor diodes exhibits a stronger nonlinearity: $I \propto e^{\alpha V}$.

■ Conduction (drift) currents

As discussed in Lesson 7, the electrons of conductors only partially fill the conduction band (Fig. 7-1) and can be easily released from parent nuclei as free electrons by thermal excitation at room temperatures. The velocities of individual free electrons are high in magnitude (~10^5 m/s at 300K) but random in direction, resulting in no net "drift" motion nor net current.

In the presence of static electric field $\vec{E}$, the free electrons experience: (1) electric force $-e\vec{E}$ ($e>0$) to accelerate the electrons, (2) frictional force $-\dfrac{m_n\vec{u}_d}{\tau}$ due to collisions with immobile ions, where $\vec{u}_d$ is the drift (average) velocity of electrons, m_n and τ represent the effective mass of conduction electrons and mean scattering time between collisions (considering the influence of crystal lattice), respectively. In steady state, these two forces balance with each other (Drude model), ⇒

$$\vec{u}_d = -\frac{e\tau}{m_n}\vec{E} = -\mu_e\vec{E},$$

where the electron mobility $\mu_e = \dfrac{e\tau}{m_n}$ (m^2/V/sec) describes how easy an external electric field can influence the motion of electrons in the conductor. For typical conductors and strength of electric fields, $|\vec{u}_d|$ is much slower (~mm/sec) than the speed of individual electrons. By eq. (10.1), the conduction current density is:

$$\vec{J} = \sigma\vec{E} \quad (\text{A/m}^2) \tag{10.3}$$

where $\sigma = -\rho_e\mu_e$ [$(\Omega\text{m})^{-1}$] ($\rho_e<0$, $\sigma>0$) denotes the electric conductivity, ρ_e means the charge density of the electrons. For semiconductors, both electrons and holes contribute to conduction currents, ⇒

$$\sigma = -\rho_e\mu_e + \rho_h\mu_h.$$

10.2 Microscopic and Macroscopic Current Laws

■ Ohm's law

Eq. (10.3) is the microscopic form of Ohm's law. Consider a piece of (imperfect) conductor of arbitrary shape and homogeneous (finite) conductivity σ (Fig. 10-3a). The potential difference between the two equipotential end faces A_1, A_2 is: $V_{12} = V_1 - V_2 = \int_L \vec{E} \cdot d\vec{l}$, where L is some path starting from A_1 and ending at A_2. The total current flowing through some surface A between A_1 and A_2 is: $I = \int_A \vec{J} \cdot d\vec{s} = \int_A \sigma \vec{E} \cdot d\vec{s}$. The resistance R of the conductor is defined as:

$$R = \frac{V_{12}}{I} = \frac{\int_L \vec{E} \cdot d\vec{l}}{\sigma \int_A \vec{E} \cdot d\vec{s}}, \tag{10.4}$$

which is a constant independent of V_{12} and I (but depending on the geometry and material of the conductor). For a conductor of "uniform" cross-sectional area S (Fig. 10-3b), eq. (10.4) gives $R = \dfrac{EL}{\sigma ES}$, $\Rightarrow$

$$R = \frac{L}{\sigma S} \tag{10.5}$$

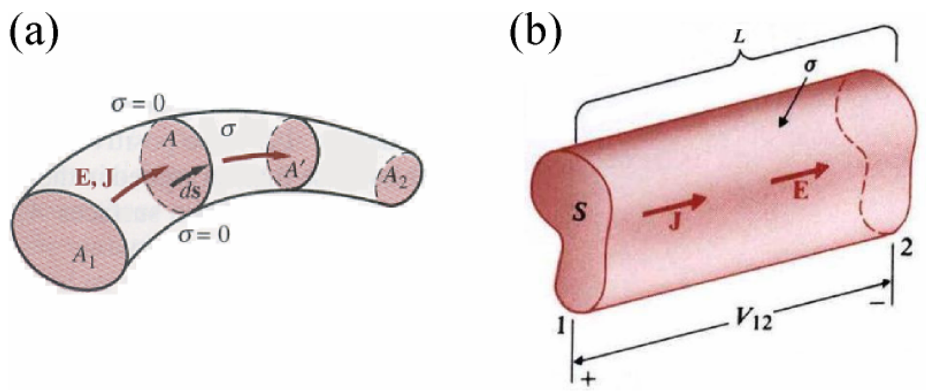

Fig. 10-3. A conductor of (a) arbitrary shape (after Inans' book), and (b) uniform cross-sectional area S and length L (after DKC).

■ Electromotive force and Kirchhoff's voltage law

If conduction current density $\vec{J}$ is driven by a conservative electric field $\vec{E}$ (created by charges) alone, $\Rightarrow$ $\vec{J} = \sigma\vec{E}$, $\oint_C \vec{E}\cdot d\vec{l} = \oint_C (\vec{J}/\sigma)\cdot d\vec{l} = 0$ [eq. (6.4)], $\Rightarrow$ no steady "loop" current exists. Therefore, a non-conservative field produced by batteries, generators …etc. is required to drive charge carriers in a closed loop.

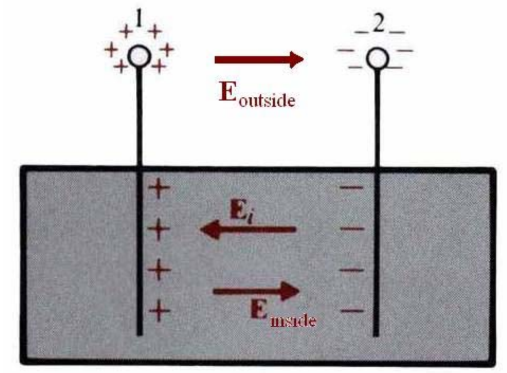

Fig. 10-4. Electric fields inside an electric battery (after DKC).

Consider an open-circuited battery, where some positive and negative charges are accumulated in electrodes 1 and 2 due to chemical reaction (Fig. 10-4). Inside the battery, an impressed field $\vec{E}_i$ (not an electric field, but a "force") produced by chemical reaction balances the electrostatic field $\vec{E}_{\text{inside}}$ arising from the accumulated charges, preventing charges from further movement. The electromotive force (emf), defined as the line integral of $\vec{E}_i$ from electrode 2 to electrode 1:

$$\mathcal{V} \equiv \int_2^1 \vec{E}_i \cdot d\vec{l} ,$$

describes the strength of the non-conservative source. By $\vec{E}_{\text{inside}} = -\vec{E}_i$ and eq. (6.4), we have: $\oint_C \vec{E}\cdot d\vec{l} = \int_1^2 \vec{E}_{\text{outside}}\cdot d\vec{l} + \int_2^1 \vec{E}_{\text{inside}}\cdot d\vec{l} = \int_1^2 \vec{E}_{\text{outside}}\cdot d\vec{l} - \int_2^1 \vec{E}_i\cdot d\vec{l} = 0, \Rightarrow$

$$\mathcal{V} = \int_1^2 \vec{E}_{\text{outside}}\cdot d\vec{l} = V_1 - V_2 \ \ \text{(Volt)} \tag{10.6}$$

If the two terminals are connected by a uniform conducting wire of resistance $R = \dfrac{L}{\sigma S}$, the total field:

$$\vec{E} + \vec{E}_i = \begin{cases} \vec{E}_{\text{inside}} + \vec{E}_i = 0, \text{inside the battery} \\ \vec{E}_{\text{outside}}, \text{outside the battery} \end{cases}$$

drives a loop current I of volume density $\vec{J}$ (where $J = I/S$). In the conducting wire, $\vec{J} = \sigma\vec{E}_{\text{outside}}$, $\mathcal{V} = \int_1^2 \vec{E}_{\text{outside}} \cdot d\vec{l} = \oint_C \left(\frac{\vec{J}}{\sigma}\right) \cdot d\vec{l} = \frac{IL}{\sigma S}$, $\Rightarrow$ $\mathcal{V} = RI$. For a closed path with multiple sources and resistors, we get the Kirchhoff's voltage law:

$$\sum_j \mathcal{V}_j = \sum_k R_k I_k \tag{10.7}$$

■ Equation of continuity and Kirchhoff's current law

Consider a net charge Q confined in a volume V bounded by a closed surface S. Based on the principle of conservation of charge (a fundamental postulate of physics), a net current I flowing out of V must result in decrease of the enclosed charge:

$$I = -\frac{dQ}{dt}, \Rightarrow \oint_S \vec{J} \cdot d\vec{s} = -\frac{d}{dt}\int_V \rho dv .$$

By the divergence theorem [eq. (5.24)] and assuming that the volume V is stationary (does not moving with time), $\Rightarrow$ $\int_V (\nabla \cdot \vec{J}) dv = -\int_V \frac{\partial \rho}{\partial t} dv$, leading to the equation of continuity:

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \tag{10.8}$$

For "steady currents", $\frac{\partial \rho}{\partial t} = 0$, eq. (10.8) is reduced to:

$$\nabla \cdot \vec{J} = 0 \tag{10.9}$$

This means there is no steady current source/sink, and the field lines of $\vec{J}$ always close upon themselves. The total current flowing out of a circuit junction enclosed by surface S becomes:

$$\sum_j I_j = \oint_S \vec{J} \cdot d\vec{s} = \int_V (\nabla \cdot \vec{J}) dv = 0, \Rightarrow$$

$$\sum_j I_j = 0 \tag{10.10}$$

The Kirchhoff's current law eq. (10.10) is the macroscopic form of eq. (10.8) in steady state.

Example 10-2: Show the dynamics (time dependence) of free charge density ρ inside a homogeneous conductor with constant electric conductivity σ and permittivity ε.

Ans: By eq's (10.3), (10.8),

$$\nabla\cdot\vec{J} = \nabla\cdot\left(\sigma\vec{E}\right) = \sigma\left(\nabla\cdot\vec{E}\right) = -\frac{\partial\rho}{\partial t}.$$

By eq's (7.8), (7.12),

$$\nabla\cdot\vec{D} = \nabla\cdot\left(\varepsilon\vec{E}\right) = \varepsilon\left(\nabla\cdot\vec{E}\right) = \rho.$$

$\Rightarrow \dfrac{\partial\rho}{\partial t} + \dfrac{\sigma}{\varepsilon}\rho = 0$, $\rho(t) = \rho_0 e^{-t/\tau}$, where

$$\tau = \frac{\varepsilon}{\sigma} \tag{10.11}$$

represents the life time of free charges inside the conductor (for the initial charge density ρ_0 will decay to its $1/e$ within a time interval of τ. For a good conductor like copper, $\tau \approx 10^{-19}$ sec. Therefore, the dynamics is too fast to be considered.

■ Joule's law

In the presence of an electric field $\vec{E}$, free electrons in a conductor have a drift (average) velocity $\vec{u}_d$. Collisions among free electrons and immobile atoms transfer energy from the electric field to thermal vibration. Quantitatively, the work done by $\vec{E}$ in moving an amount of charge Q for a differential "drift" displacement $\Delta\vec{l}_d$ is: $\Delta w = Q\vec{E}\cdot\Delta\vec{l}_d$, corresponding to a power dissipation of: $p = \lim\limits_{\Delta t\to 0}\dfrac{\Delta w}{\Delta t} = Q\vec{E}\cdot\vec{u}_d$. For a conductor of free charge density ρ where an electric field $\vec{E}$ exists, the power dissipation in a differential volume dv

($Q=\rho dv$) becomes: $\Delta p = \rho dv(\vec{E}\cdot\vec{u}_d) = (\vec{E}\cdot\rho\vec{u}_d)dv = (\vec{E}\cdot\vec{J})dv$. This means that $\vec{E}\cdot\vec{J}$ (W/m^3) represents the (ohmic) power density, and the total power dissipated in a volume V with inhomogeneous $\vec{E}(\vec{r})$ and $\sigma(\vec{r})$ is described by the Joule's law:

$$P = \int_V (\vec{E}\cdot\vec{J})dv \quad \text{(W)} \tag{10.12}$$

If we apply a voltage difference V_{12} across a homogeneous conductor of uniform cross-sectional area S and length L (Fig. 10-3b), ⇒

$$P = \int_V (\vec{E}\cdot\vec{J})dv = \int_L (\vec{E}\cdot\vec{J})Sdl = \int_L \vec{E}\cdot I d\vec{l} = V_{12} I \,.$$

10.3 Boundary Conditions

■ Derivation

As in electrostatics, we can derive the boundary conditions for (conduction) current density $\vec{J}$ across an interface between two media with different conductivities σ_1, σ_2 by:

1) Deduce the divergence and curl relations of $\vec{J}$: (i) For steady currents, $\nabla\cdot\vec{J}=0$ [eq. (10.9)]. (ii) By eq's (10.3) and (6.2), $\nabla\times\vec{E} = \nabla\times(\vec{J}/\sigma) = 0$.
2) Transform the equations into their integral forms: (i) $\oint_S \vec{J}\cdot d\vec{s} = 0$. (ii) $\oint_C \frac{\vec{J}}{\sigma}\cdot d\vec{l} = 0$.
3) Apply the integral relations to a differential (i) pillbox, and (ii) rectangular loop across the interface, respectively. We can arrive at (i) normal, and (ii) tangential components of boundary conditions:

$$J_{1n} = J_{2n} \tag{10.13}$$

$$\frac{J_{1t}}{\sigma_1} = \frac{J_{2t}}{\sigma_2} \tag{10.14}$$

<u>Example 10-3</u>: Represent the surface charge density ρ_s on the interface between two lossy

media with permittivities ε_1, ε_2 and conductivities σ_1, σ_2 by either of the two normal components of $\vec{D}$.

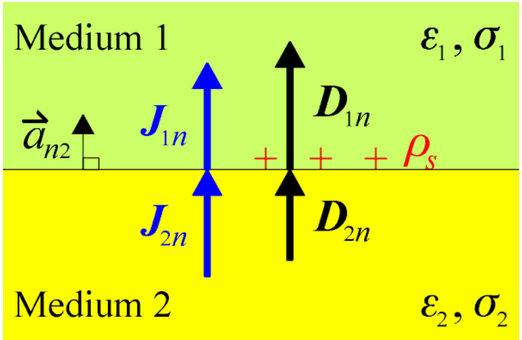

Fig. 10-5. Interface between two lossy media.

Ans: Use the normal boundary conditions of $\vec{E}$ and $\vec{J}$: (1) By eq's (7.12), (7.15), $\Rightarrow$

$$\rho_s = D_{1n} - D_{2n} = \varepsilon_1 E_{1n} - \varepsilon_2 E_{2n},$$

where D_{in}, E_{in} $(i = 1, 2)$ denote the projections of $\vec{D}_i$, $\vec{E}_i$ along $\vec{a}_{n2}$, respectively. (2) By eq's (10.3) and (10.13), $\Rightarrow$ $J_{1n} = J_{2n}$, $\sigma_1 E_{1n} = \sigma_2 E_{2n}$, $\Rightarrow$ $E_{1n} = \dfrac{\sigma_2}{\sigma_1} E_{2n}$, or $E_{2n} = \dfrac{\sigma_1}{\sigma_2} E_{1n}$. $\Rightarrow$

$$\rho_s = \left(\frac{\sigma_2 \varepsilon_1}{\sigma_1 \varepsilon_2} - 1\right) D_{2n} = \left(1 - \frac{\sigma_1 \varepsilon_2}{\sigma_2 \varepsilon_1}\right) D_{1n} \tag{10.15}$$

Eq. (10.15) means that $\rho_s = 0$ only if (1) $\dfrac{\sigma_1}{\varepsilon_1} = \dfrac{\sigma_2}{\varepsilon_2}$, which is rare in reality; (2) $\sigma_1 = \sigma_2 = 0$ (both media are lossless, no free charge exists), where eq. (10.15) fails. For air-conductor interface (Fig. 7-1), $\sigma_2 \to \infty$, $\rho_s = D_{1n}$, consistent with eq. (7.4).

<u>Example 10-4</u>: Find $\vec{J}$, $\vec{E}$ in the two lossy media between two parallel conducting plates biased by a dc voltage V_0 (Fig. 10-6). Also find the surface charge densities on the two conducting plates and on the interface between the two lossy media, respectively.

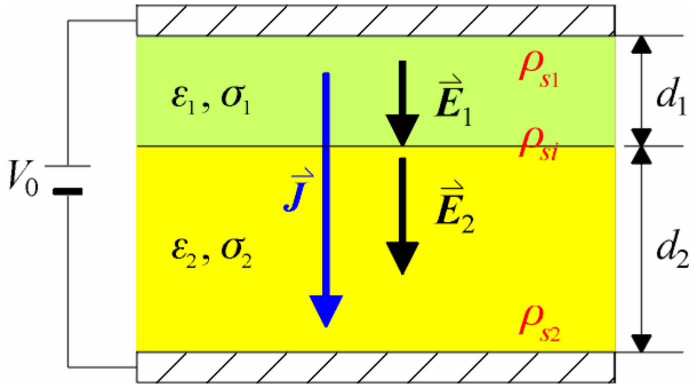

Fig. 10-6. Parallel-plate capacitor filled with two lossy media stacked in series.

Ans: (1) By planar symmetry and eq. (10.13), there is a constant current density $\vec{J} = -\vec{a}_y J$ between the plates. By eq. (10.3), $\Rightarrow$ $\vec{E}_i = \dfrac{\vec{J}}{\sigma_i} = -\vec{a}_y E_i$, where $E_i = \dfrac{J}{\sigma_i}$. Since $V_0 = \sum_{i=1,2} E_i d_i$ $= J\sum_{i=1,2}\dfrac{d_i}{\sigma_i}$, $\Rightarrow$ $J = \dfrac{V_0}{(d_1/\sigma_1)+(d_2/\sigma_2)}$.

(2) $E_1 = \dfrac{\sigma_2 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$, $E_2 = \dfrac{\sigma_1 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$.

(3) By eq. (7.15), $\rho_{s1} = D_1 = \varepsilon_1 E_1 = \dfrac{\varepsilon_1 \sigma_2 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$; $\rho_{s2} = -D_2 = -\varepsilon_2 E_2 = -\dfrac{\varepsilon_2 \sigma_1 V_0}{\sigma_2 d_1 + \sigma_1 d_2}$.

(4) By eq. (10.15), the surface charge density between the two lossy media is:

$$\rho_{si} = \left(1 - \frac{\sigma_1 \varepsilon_2}{\sigma_2 \varepsilon_1}\right)\varepsilon_1(-E_1) = \frac{(\varepsilon_2\sigma_1 - \varepsilon_1\sigma_2)V_0}{\sigma_2 d_1 + \sigma_1 d_2}.$$

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1) $|\rho_{s1}| \neq |\rho_{s2}|$, but $\rho_{s1} + \rho_{s2} + \rho_{si} = 0$. By Gauss's law, there is no electric field outside the parallel-plate capacitor.

2) In this example, static charge and steady current (i.e., static magnetic field) coexist, causing "electromagnetostatic" field. However, the magnetic field is a consequence and does not enter into the calculation of the electric field.

10.4 Evaluation of Resistance

■ Resistance of single imperfect conductor

The resistance R of a piece of homogeneous lossy medium of finite conductivity σ can be evaluated by: (1) Assume a potential difference V_0 for the two "selected" end faces. (2) Find the potential distribution $V(\vec{r})$ by solving boundary-value problem. (3) Find $\vec{E}$ by $\vec{E} = -\nabla V$. (4) Find the total current by $I = \int_S \vec{J} \cdot d\vec{s} = \int_S \sigma \vec{E} \cdot d\vec{s}$. (5) By eq. (10.4), $R = \dfrac{V_0}{I}$.

<u>Example 10-5</u>: Consider a quarter-circular washer of rectangular cross section (Fig. 10-7) and finite conductivity σ. Find the resistance if the two electrodes are located at $\phi = 0$ and $\phi = \pi/2$.

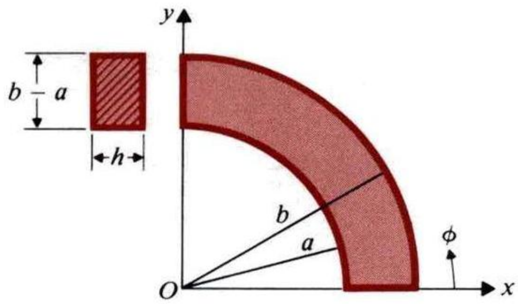

Fig. 10-7. A quarter-circular conducting washer of rectangular cross section (after DKC).

Ans: Assume: (1) $V(\phi = 0) = 0$, $V(\phi = \pi/2) = V_0$, (2) the current flow and electric field are only in $\vec{a}_\phi$-direction, $\Rightarrow$ $V(r, \phi, z) = V(\phi)$. Laplace's equation [eq. (8.2)] in cylindrical coordinates is simplified as: $V''(\phi) = 0$, $\Rightarrow$ $V(\phi) = c_1\phi + c_2$. By the two boundary conditions,

$$\Rightarrow \ V(\phi) = \frac{2V_0}{\pi}\phi, \ \Rightarrow \ \vec{E} = -\nabla V = -\vec{a}_\phi \frac{2V_0}{\pi}\frac{1}{r}\left(\propto \frac{1}{r}\right), \ \vec{J} = \sigma\vec{E} = -\vec{a}_\phi \frac{2V_0\sigma}{\pi}\frac{1}{r}, \ I = \int_S \vec{J} \cdot d\vec{s}$$

$$= \int_a^b \frac{2V_0\sigma}{\pi r} \cdot h dr = \frac{2V_0\sigma h}{\pi}\ln\left(\frac{b}{a}\right), \ R = \frac{\pi}{2\sigma h \ln(b/a)}.$$

■ Resistance between two perfect conductors

The resistance R between two perfect conductors immersed in some homogeneous lossy medium with permittivity ε and conductivity σ can be derived by evaluating the corresponding capacitance C and applying the relation:

$$RC = \frac{\varepsilon}{\sigma} \tag{10.16}$$

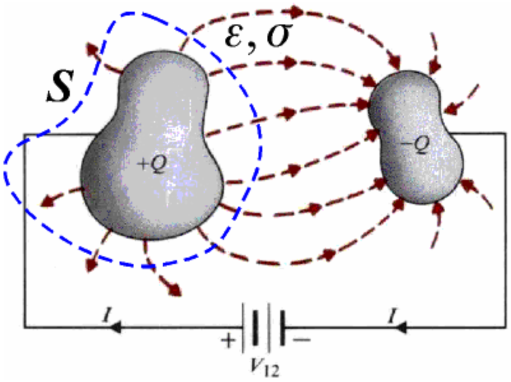

Fig. 10-8. Evaluation of resistance by RC constant (after DKC).

Proof: By definitions, $RC = \frac{V}{I} \cdot \frac{Q}{V} = \frac{Q}{I}$. By eq's (7.10), (10.3), (7.12), $\Rightarrow \frac{Q}{I} = \frac{\oint_S \vec{D} \cdot d\vec{s}}{\oint_S \vec{J} \cdot d\vec{s}} = \frac{\varepsilon}{\sigma}$,

as long as ε and σ have the same spatial dependence.

Example 10-6: Find the leakage resistance between the inner conductor (of radius a) and outer conductor (of inner radius b) of a coaxial cable of length L (Fig. 9-4), where a lossy dielectric medium of conductivity σ is filled between the conductors.

Ans: By eq. (9.4), $C = \frac{2\pi\varepsilon L}{\ln(b/a)}$. By eq. (10.16), $R = \frac{\varepsilon}{\sigma C} = \frac{\ln(b/a)}{2\pi\sigma L}$.

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Since the range of dielectric constant of available materials is very limited (1-100), electric flux usually cannot be well confined within a dielectric slab. The fringing flux around the edges of conductors makes the computation of capacitance less accurate. In contrast, conductivity of available materials differ a lot (10^{-17}–10^{7} S/m), and fringing effect is normally negligible.

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