Lesson 3
Transient Response of Transmission Lines

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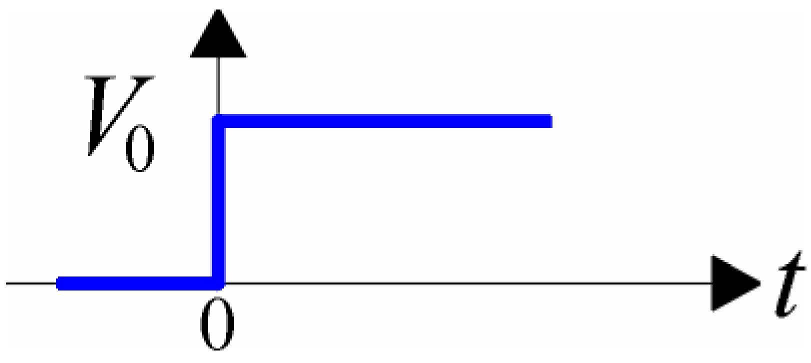
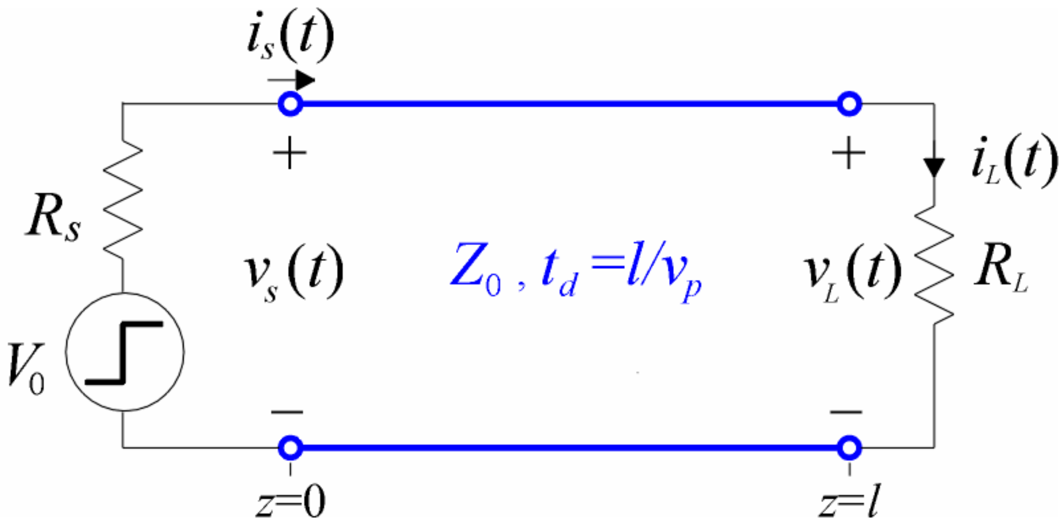
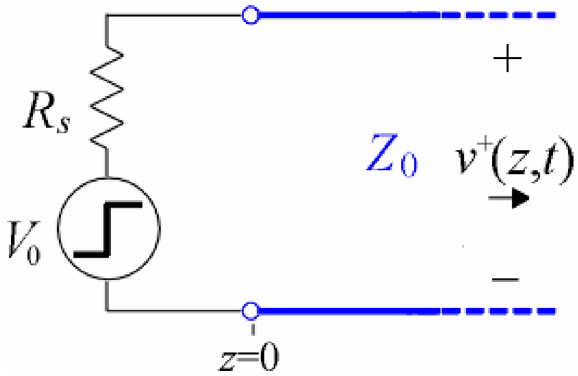
- When there is an interface between different materials (ε, μ), discontinuity exists, $\Rightarrow$ partial reflection & transmission, $\Rightarrow$ total v, i are determined by superposition (why?).
- Goal: Transient response of a terminated transmission line or cascaded lines excited by a step-like voltage source.

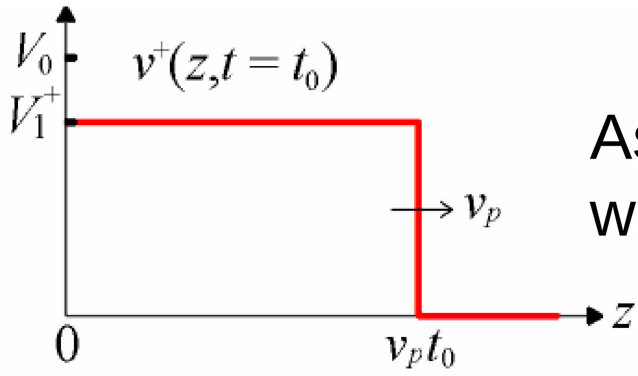


Example 3-1: A finite line with resistive load-1



(1) $t > 0$: $v_1^+(z,t) = \begin{cases} V_1^+, \text{if } z < v_p t \\ 0, \text{otherwise} \end{cases}$; $V_1^+ = \dfrac{Z_0}{Z_0 + R_S} V_0$

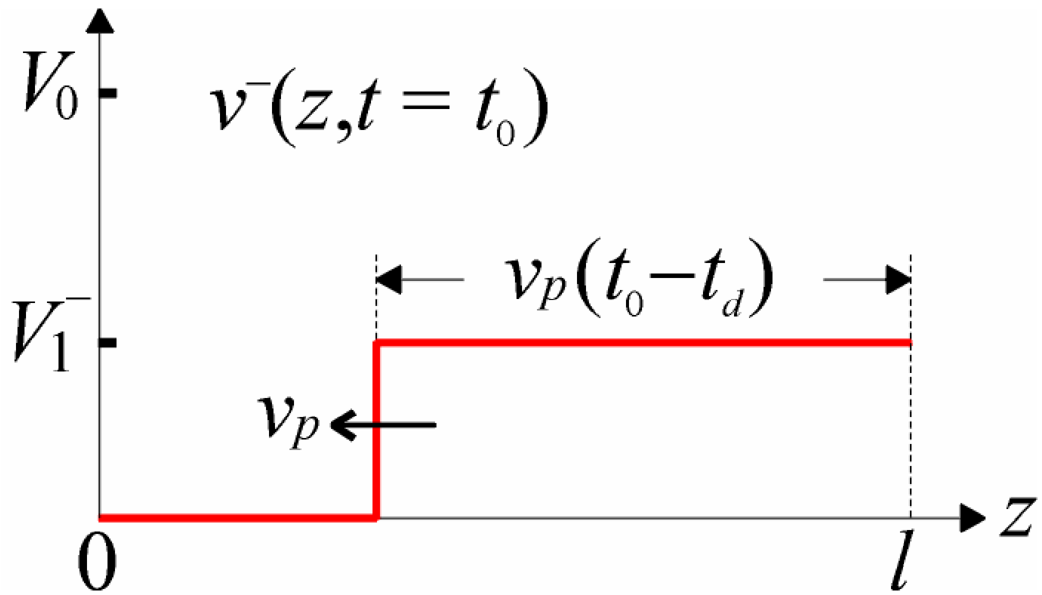




As if there were no load.

Example 3-1: A finite line with resistive load-2

$$\text{(2) } t > t_d:\ v_1^-(z,t) = \begin{cases} V_1^-, \text{if } z > l - v_p(t - t_d) \\ 0, \text{otherwise} \end{cases}$$



Reflected wave $v_1^-(z,t)$ must arise, otherwise,

$$\frac{v_L(t)}{i_L(t)} = Z_0 \neq R_L,$$

in violation with the BC.

Meanwhile, $v_1^+(z,t)$ acts as if it had kept on going to the right.

Example 3-1: A finite line with resistive load-3

$$\begin{cases} v_L(t) = v_1^+(l,t) + v_1^-(l,t) \\ i_L(t) = \dfrac{v_1^+(l,t)}{Z_0} - \dfrac{v_1^-(l,t)}{Z_0} \\ \text{BC: } v_L(t) = i_L(t) \cdot R_L(t) \end{cases}$$

$$\Rightarrow v_1^+(l,t) + v_1^-(l,t) = \frac{R_L}{Z_0}\left[v_1^+(l,t) - v_1^-(l,t)\right]$$

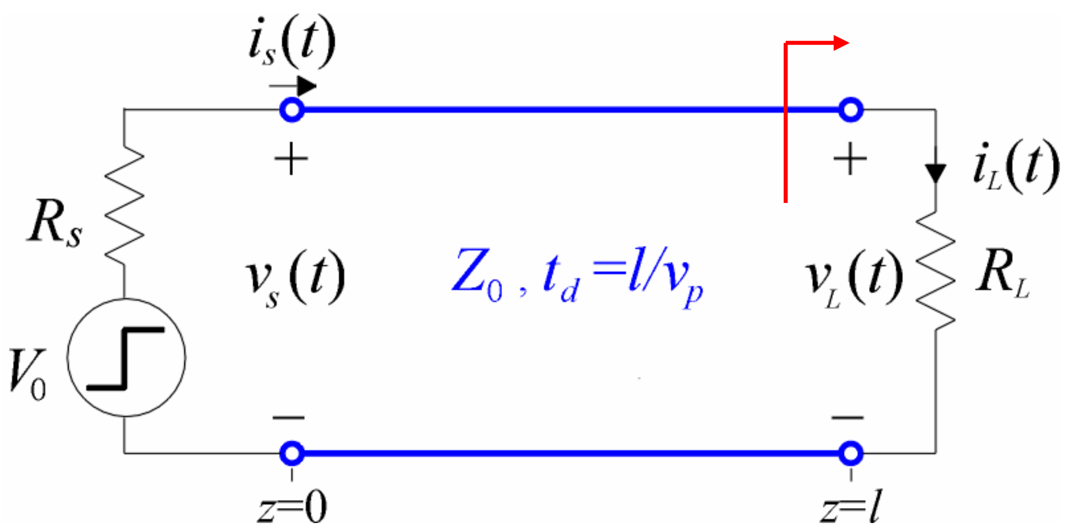
$$\Rightarrow 1 + \frac{v_1^-(l,t)}{v_1^+(l,t)} = \frac{R_L}{Z_0}\left[1 - \frac{v_1^-(l,t)}{v_1^+(l,t)}\right]$$

Example 3-1: A finite line with resistive load-4

$$\Rightarrow 1+\Gamma_L = \frac{R_L}{Z_0}\left(1-\Gamma_L\right)$$

$$\Gamma_L \equiv \frac{v_1^-(l,t)}{v_1^+(l,t)} = \frac{R_L - Z_0}{R_L + Z_0}$$

Load voltage reflection coefficient



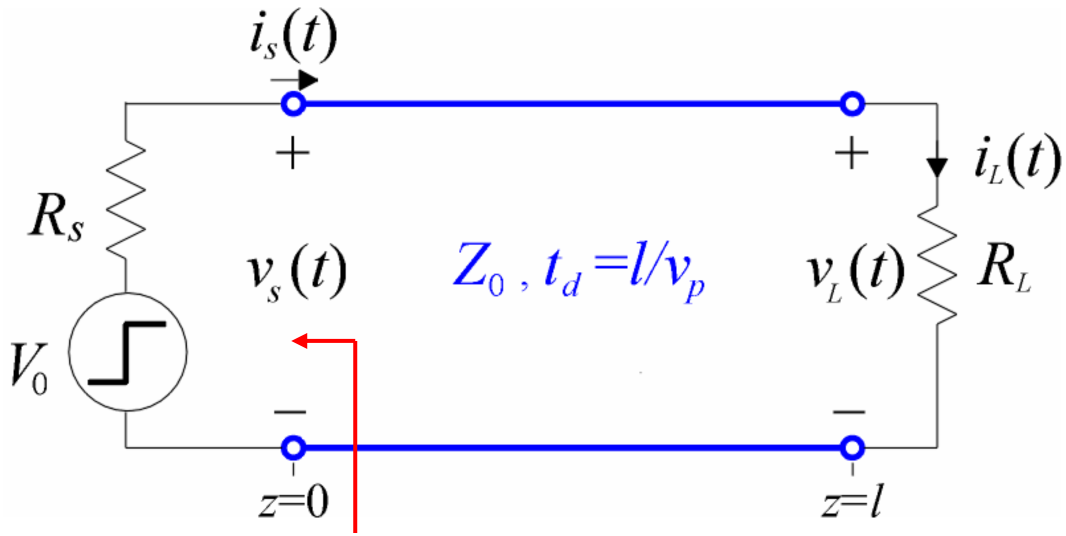
Γ_L can be used to calculate V_1^- given V_1^+ is known.

Example 3-1: A finite line with resistive load-5

(3) $t > 2t_d$: $v_2^+(z,t) = \begin{cases} V_2^+, \text{if } z < v_p(t-2t_d) \\ 0, \text{otherwise} \end{cases}$

$$\Gamma_S \equiv \frac{v_2^+(l,t)}{v_1^-(l,t)} = \frac{R_S - Z_0}{R_S + Z_0}$$

Source voltage reflection coefficient



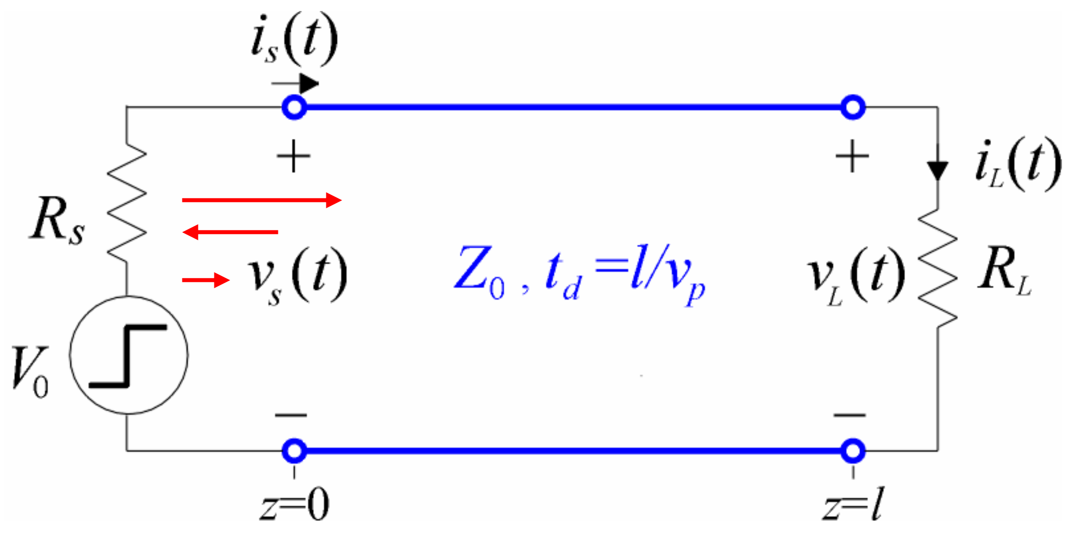
Γ_S can be used to calculate V_2^+ given V_1^- is known.

Example 3-1: A finite line with resistive load-6

At $t = 2t_d$:

$$v_S(2t_d) = v_1^+(0,2t_d) + v_1^-(0,2t_d) + v_2^+(0,2t_d) = v_1^+(0,2t_d)\left(1+\Gamma_L + \Gamma_L\Gamma_S\right)$$

$$i_S(2t_d) = \frac{v_1^+(0,2t_d)}{Z_0} - \frac{v_1^-(0,2t_d)}{Z_0} + \frac{v_2^+(0,2t_d)}{Z_0} = \frac{v_1^+(0,2t_d)}{Z_0}\left(1-\Gamma_L + \Gamma_L\Gamma_S\right)$$



Example 3-1: A finite line with resistive load-7

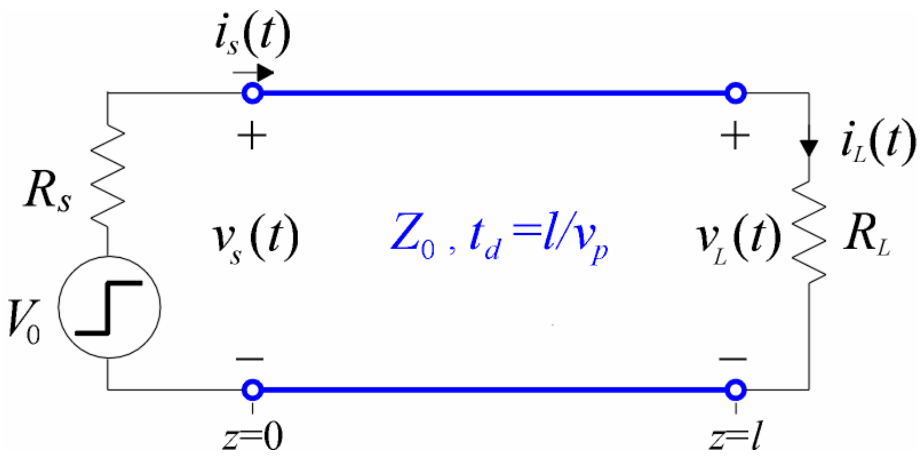
At $t \to \infty$:

$$\lim_{t\to\infty} v_S(t) = v_1^+(0, t\to\infty)\cdot\left(1+\Gamma_L + \Gamma_L\Gamma_S + \Gamma_L^2\Gamma_S + \Gamma_L^2\Gamma_S^2 + \Gamma_L^3\Gamma_S^2 + \ldots\right)$$

$$= V_1^+ \cdot\left[(1+\Gamma_L) + \Gamma_L\Gamma_S(1+\Gamma_L) + \Gamma_L^2\Gamma_S^2(1+\Gamma_L) + \ldots\right] = V_1^+ \frac{1+\Gamma_L}{1-\Gamma_L\Gamma_S}$$

$$V_1^+ = \frac{Z_0}{Z_0 + R_S} V_0$$

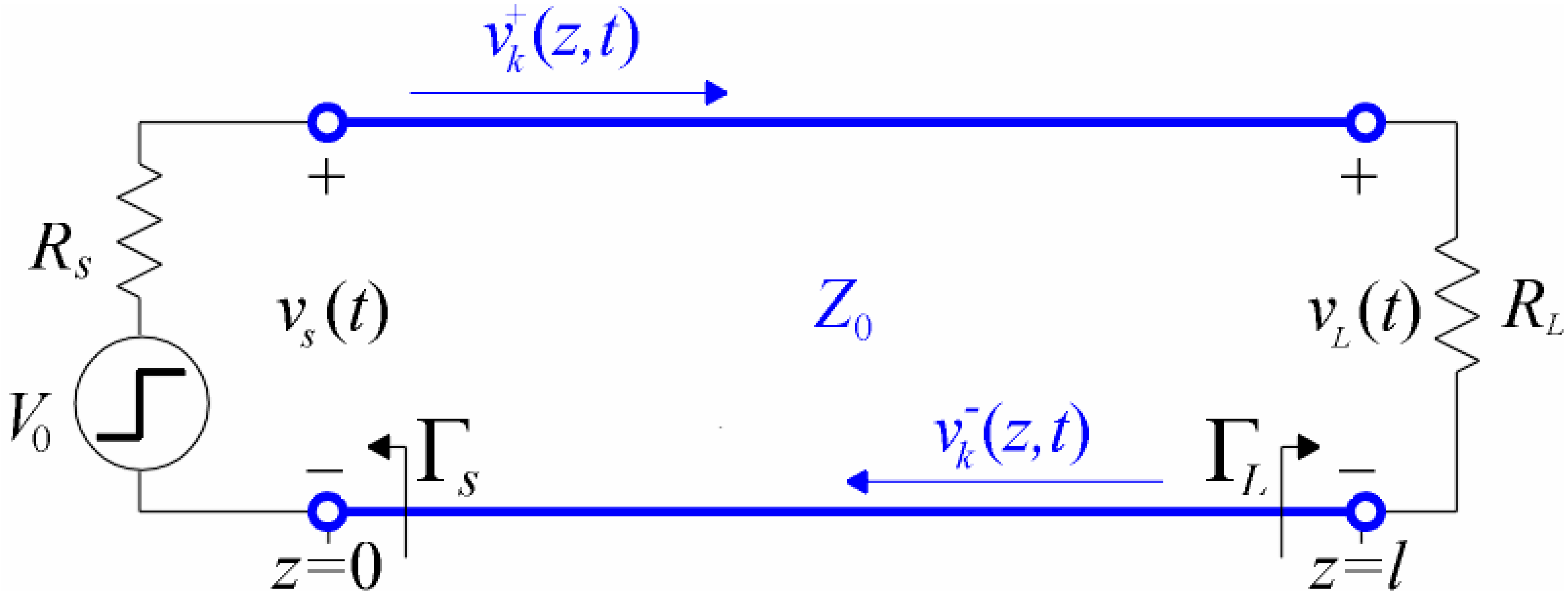
$$\Rightarrow \lim_{t\to\infty} v_S(t) = \frac{R_L}{R_S + R_L} V_0$$



Steady state response is as if there were no line.

What is the usefulness of bounce diagram?

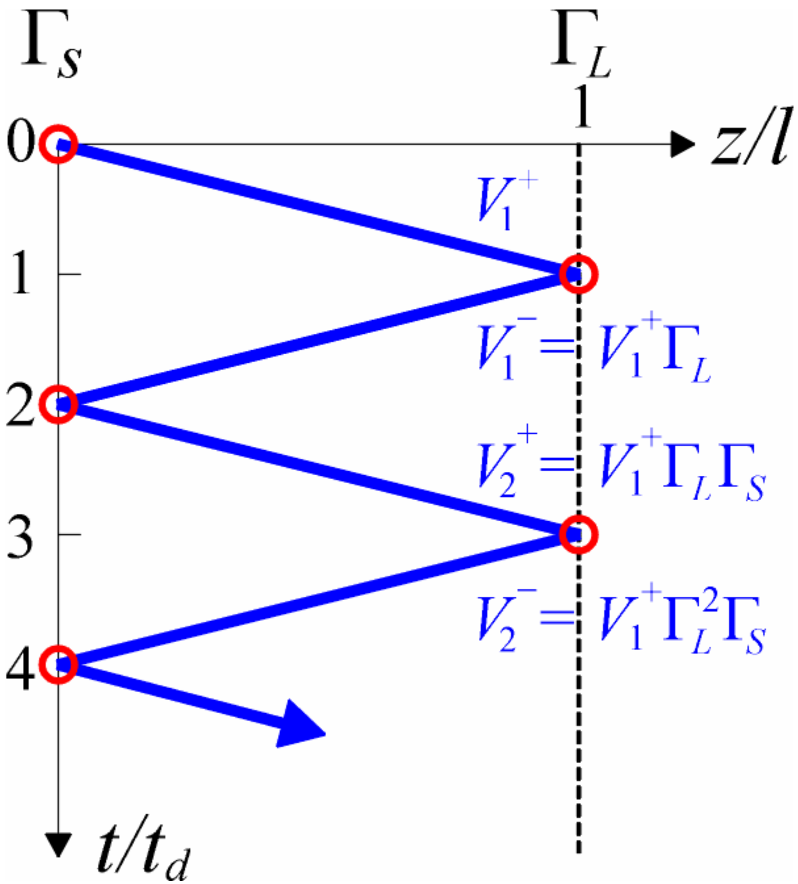
- Bounce diagram is a convenient tool to solve for transient response of a TX line driven by a step voltage and terminated by a resistor.



How to draw a bounce diagram?

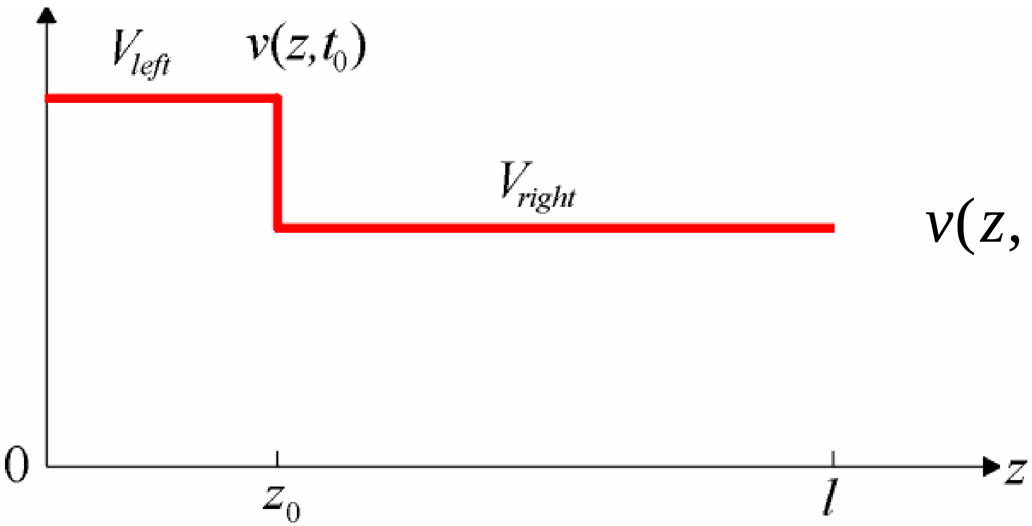
- Draw a 2D window with $0 < z/l < 1$, $t/t_d > 0$. Denote the two reflection coefficients Γ_S, Γ_L.
- Mark the points $(0,2n)$ and $(1,2n+1)$ for $n = 0, 1, 2, \ldots$
- Connecting the points by zigzag lines.
- Mark the voltage amplitude of each component wave.

$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} \qquad \Gamma_L = \frac{R_L - Z_0}{R_L + Z_0}$$



Determine the spatial voltage distribution at some time instant-1

- The spatial distribution must be of the form:

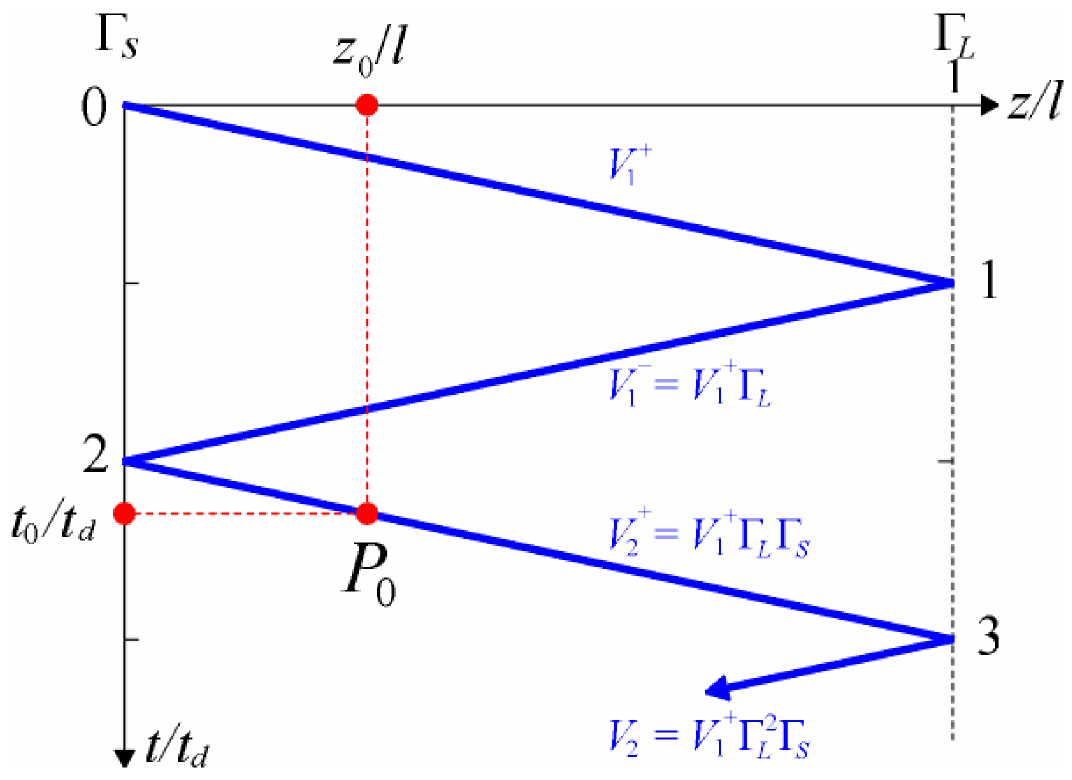


$$v(z,t_0) = \begin{cases} V_{left}, \text{ for } 0 < z < z_0 \\ V_{rightt}, \text{ for } z_0 < z < l \end{cases}$$

The problem is how to find the three key parameters: z_0, V_{left}, and V_{right}, respectively.

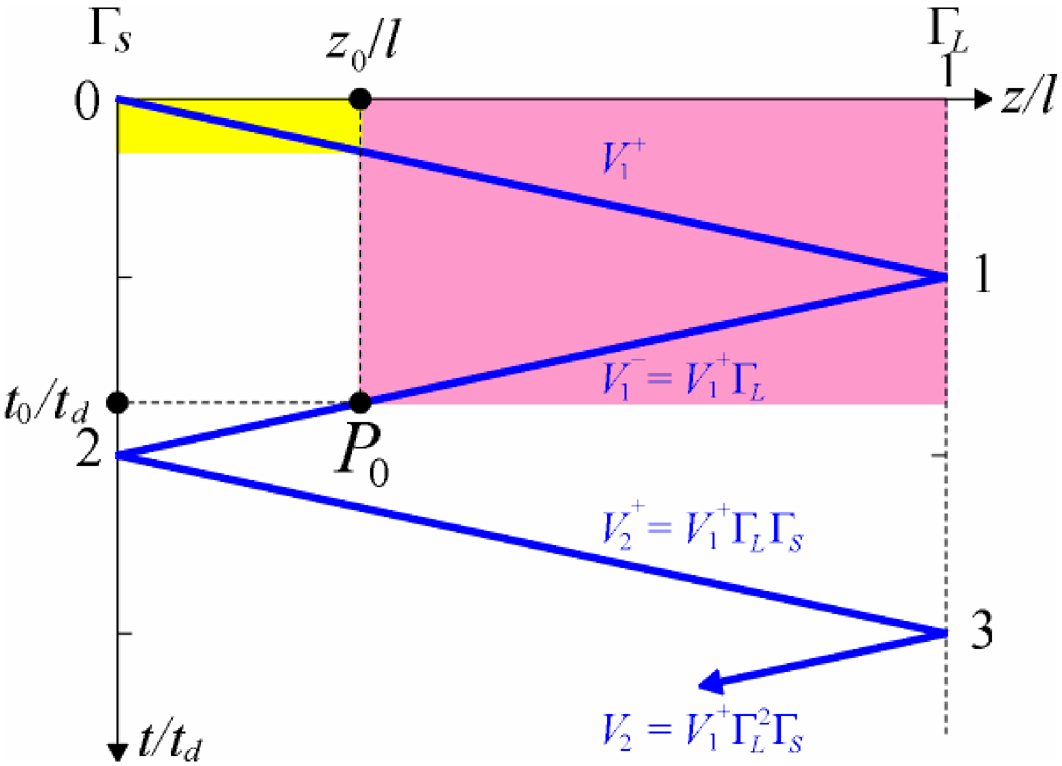
Determine the spatial voltage distribution at some time instant-2

- Draw a horizontal line $t = t_0$ (i.e. $t/t_d = t_0/t_d$), intersecting with some zigzag line at point P_0 with coordinate (z_0, t_0).



Determine the spatial voltage distribution at some time instant-3

- The determination of V_{left} and V_{right} has two cases.
- If P_0 falls on a line with positive slope:



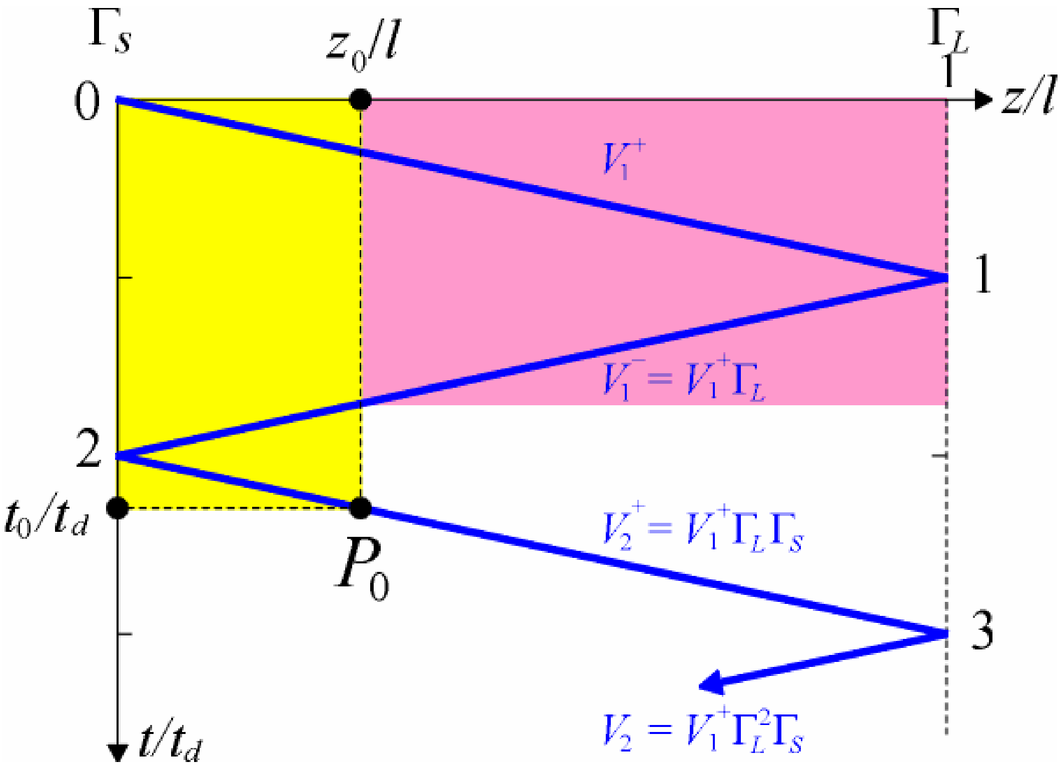
$$V_{left} = V_1^+$$

$$V_{right} = V_1^+ + V_1^-$$
$$= V_1^+ (1 + \Gamma_L)$$

V_{right} has one more component.

Determine the spatial voltage distribution at some time instant-4

- If P_0 falls on a line with negative slope:



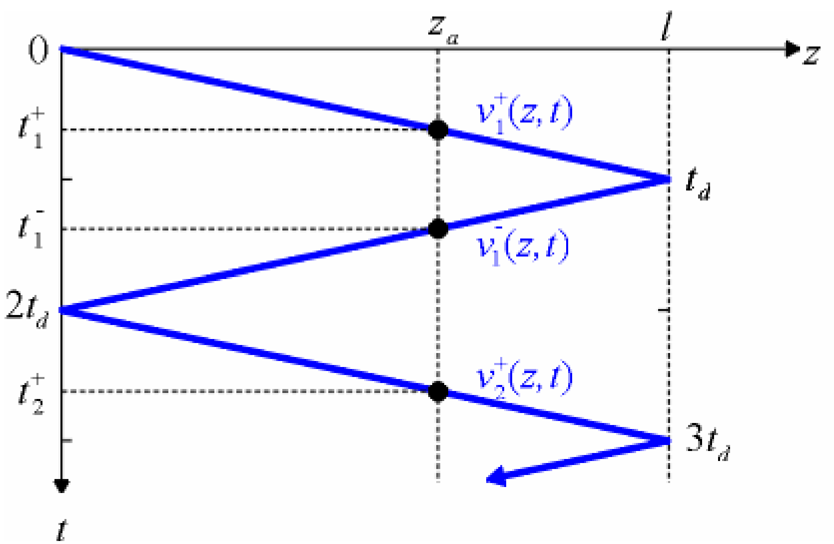
$$V_{left} = V_1^+ + V_1^- + V_2^+$$
$$= V_1^+\left(1 + \Gamma_L + \Gamma_L\Gamma_S\right)$$

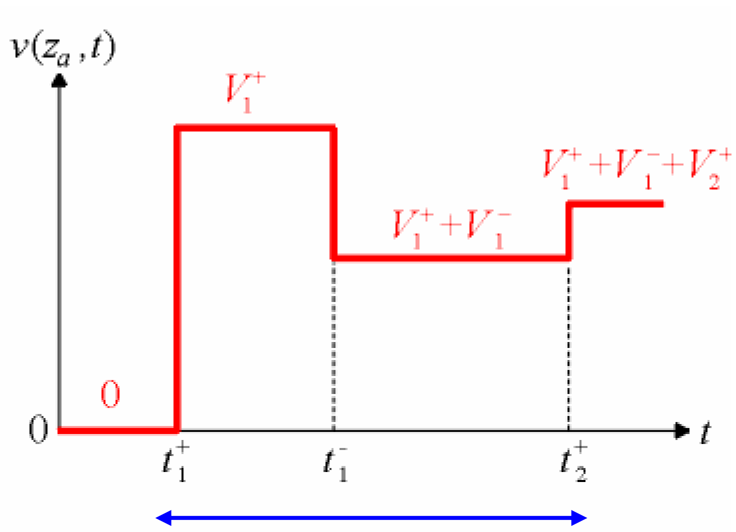
$$V_{right} = V_1^+ + V_1^-$$
$$= V_1^+\left(1 + \Gamma_L\right)$$

V_{left} has one more component.

Determine the temporal voltage distribution at some position

- $v(z_a,t)$ must have constant voltages switched at a series of time instants.
- Draw a vertical line $z = z_a$, intersecting with the zigzag lines at $t = t_1^+, t_1^-, t_2^+, ...$



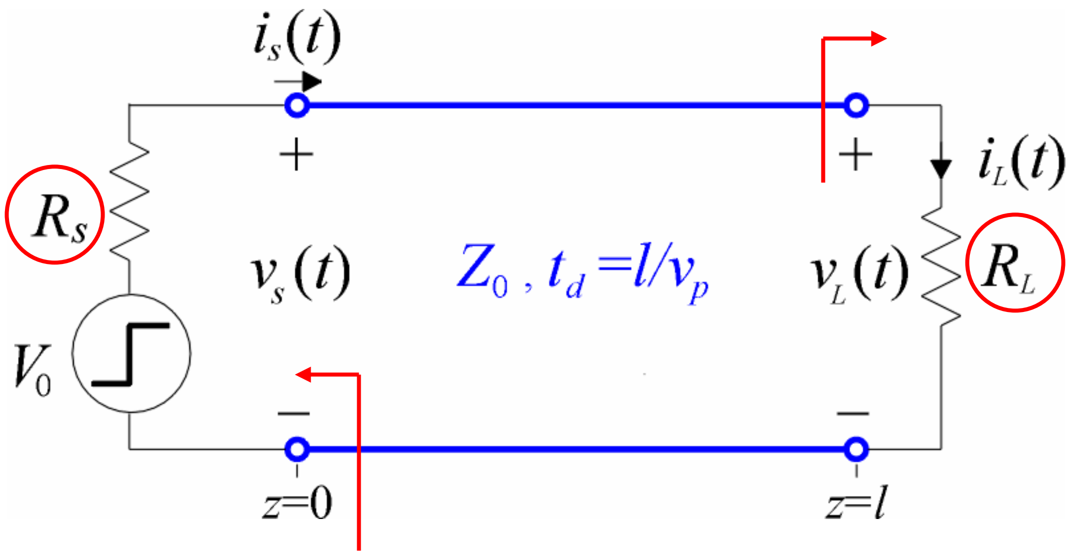


$$t_{i+1}^+ - t_i^+ = t_{i+1}^- - t_i^+ = 2t_d$$

Example 3-2: Single TX line with open termination-1

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 1$$

$R_s = 0.25Z_0$



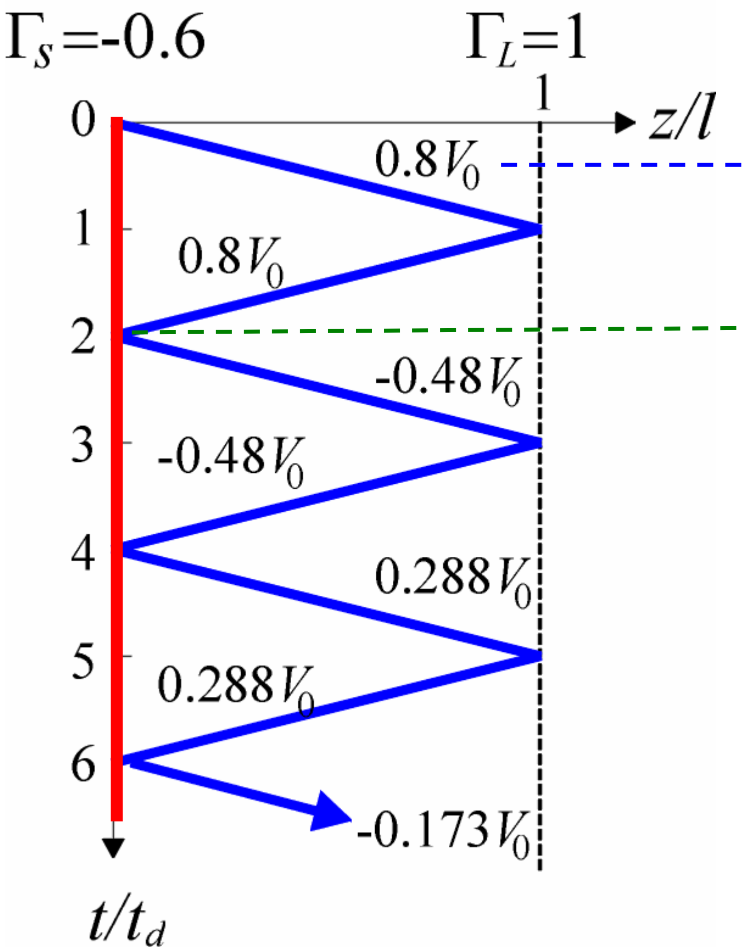
$R_L = \infty$

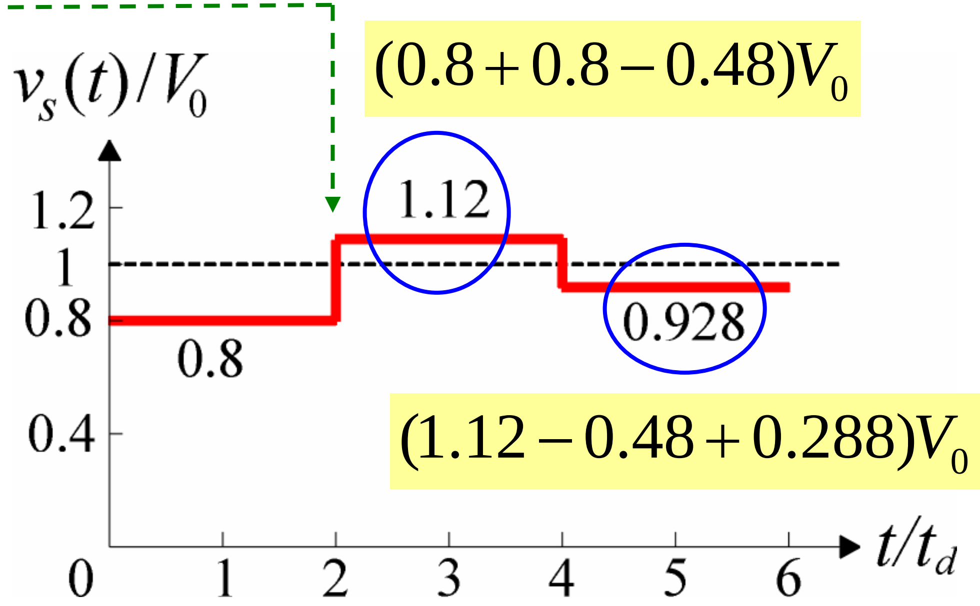
$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = -0.6$$

Example 3-2: Single TX line with open termination-2

(1) $z=0$: $t_1^+ = 0,\ t_1^- = t_2^+ = 2t_d,\ t_2^- = t_3^+ = 4t_d, \ldots$

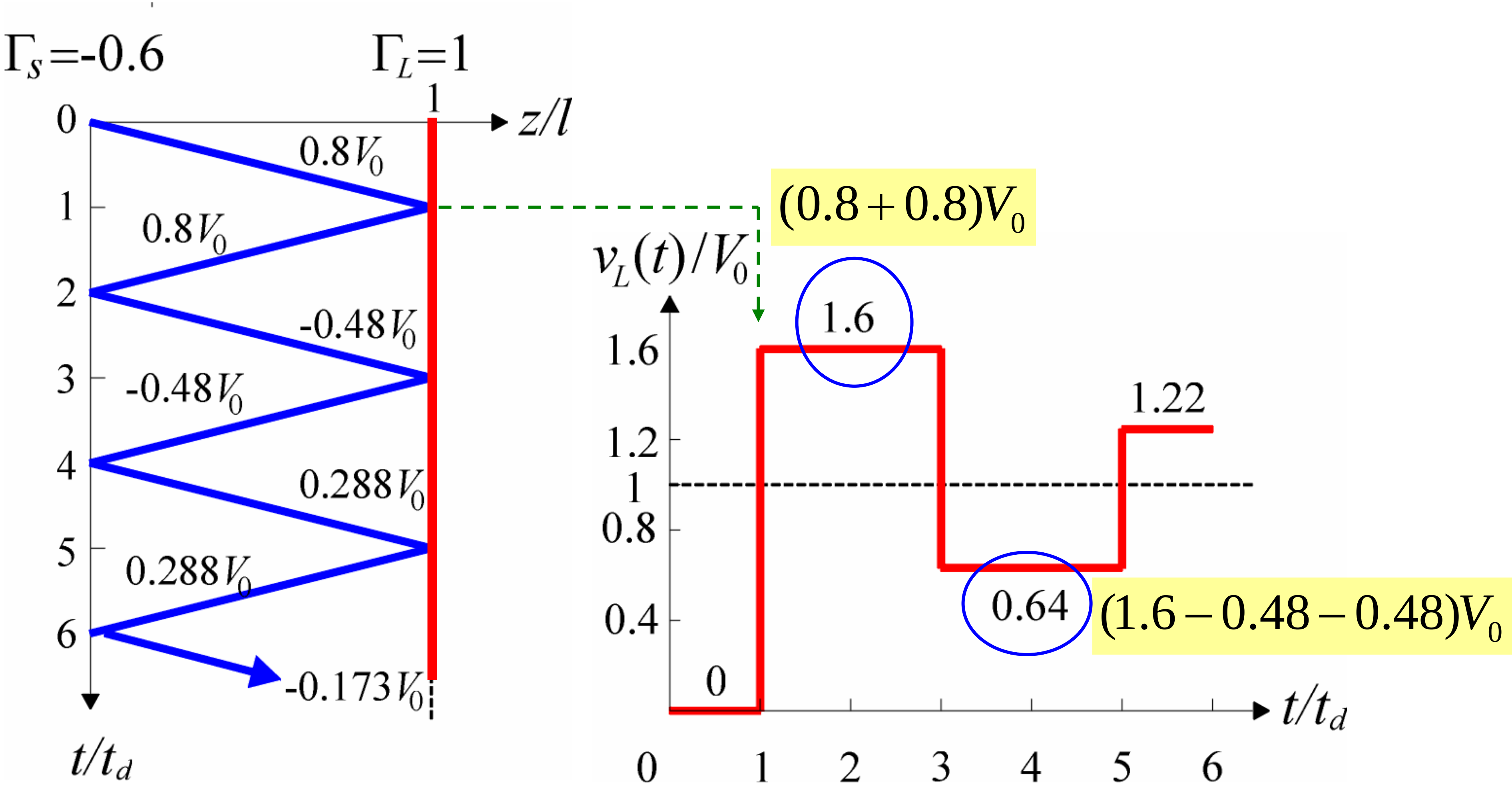
$$V_1^+ = \frac{Z_0}{Z_0 + R_S} V_0 = 0.8V_0$$





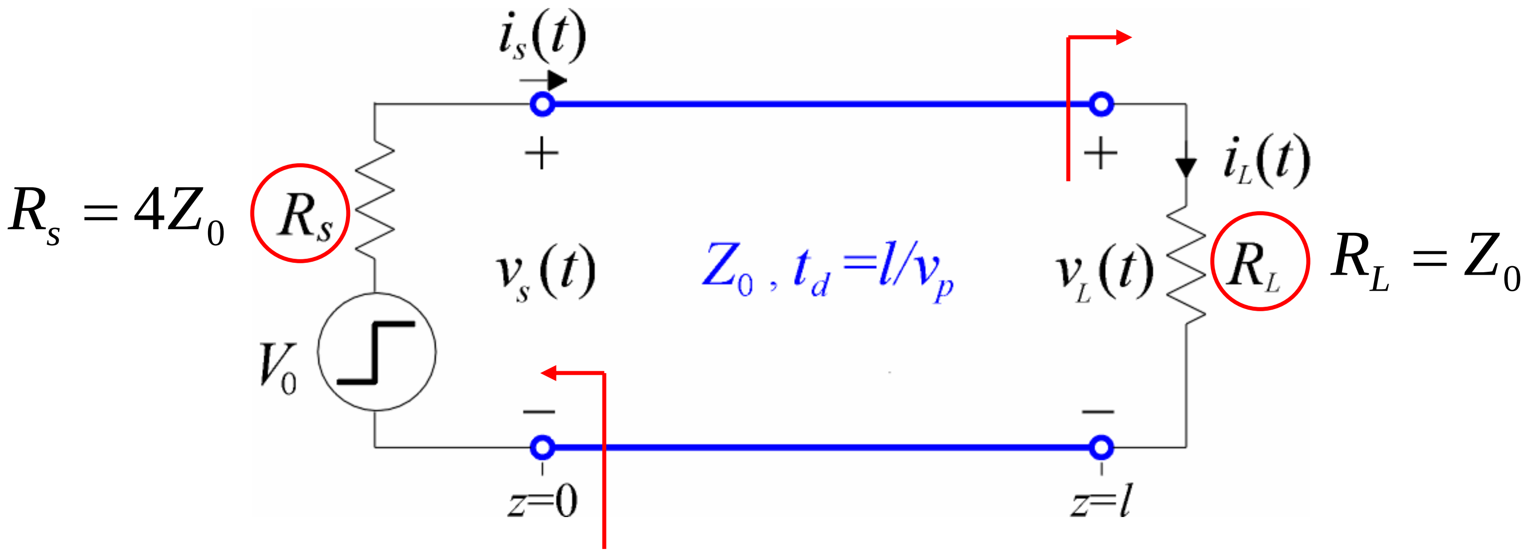
Example 3-2: Single TX line with open termination-3

(2) $z = l$: $t_1^+ = t_1^- = t_d, \; t_k^+ = t_k^- = (2k-1)t_d, ...$



Example 3-3: Single TX line with matched termination-1

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 0$$

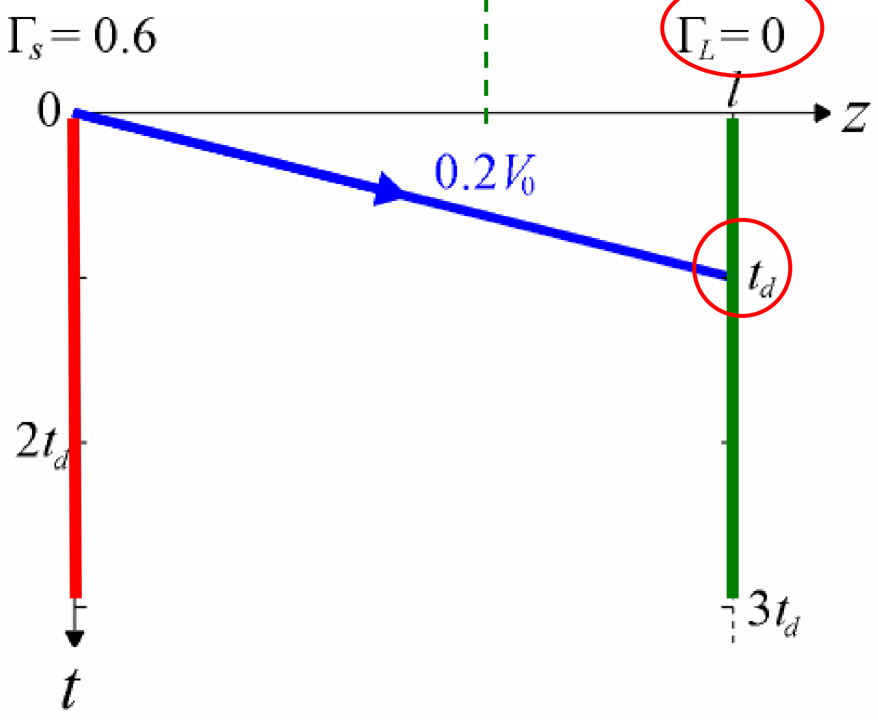


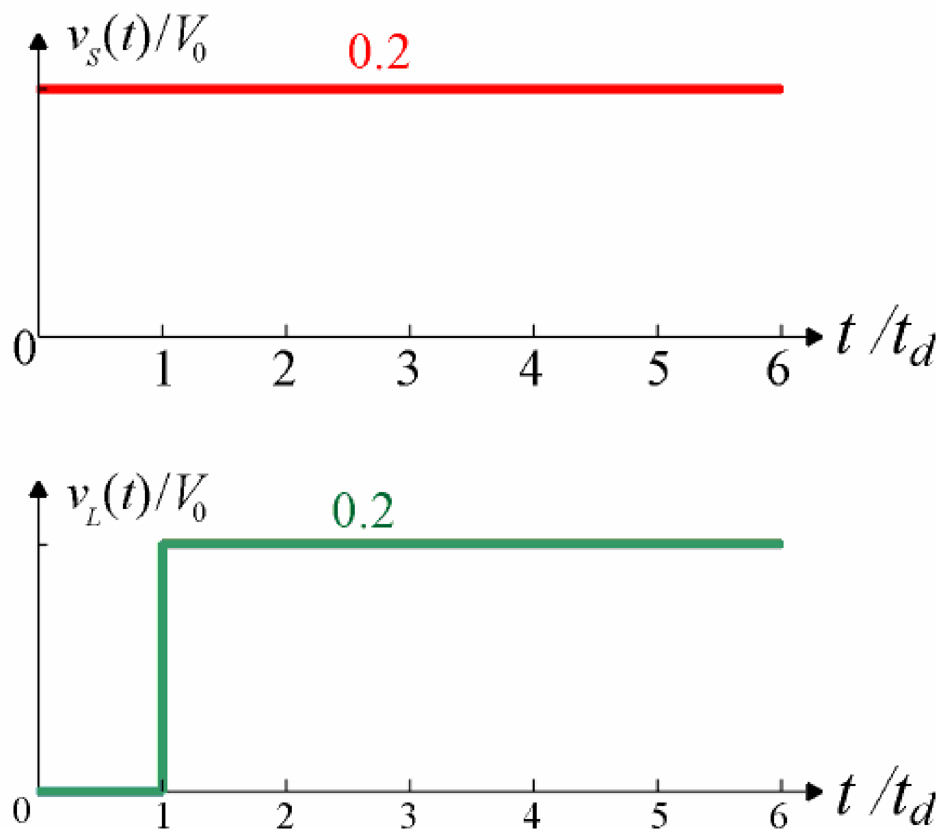
$$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = 0.6$$

Example 3-3: Single TX line with matched termination-2

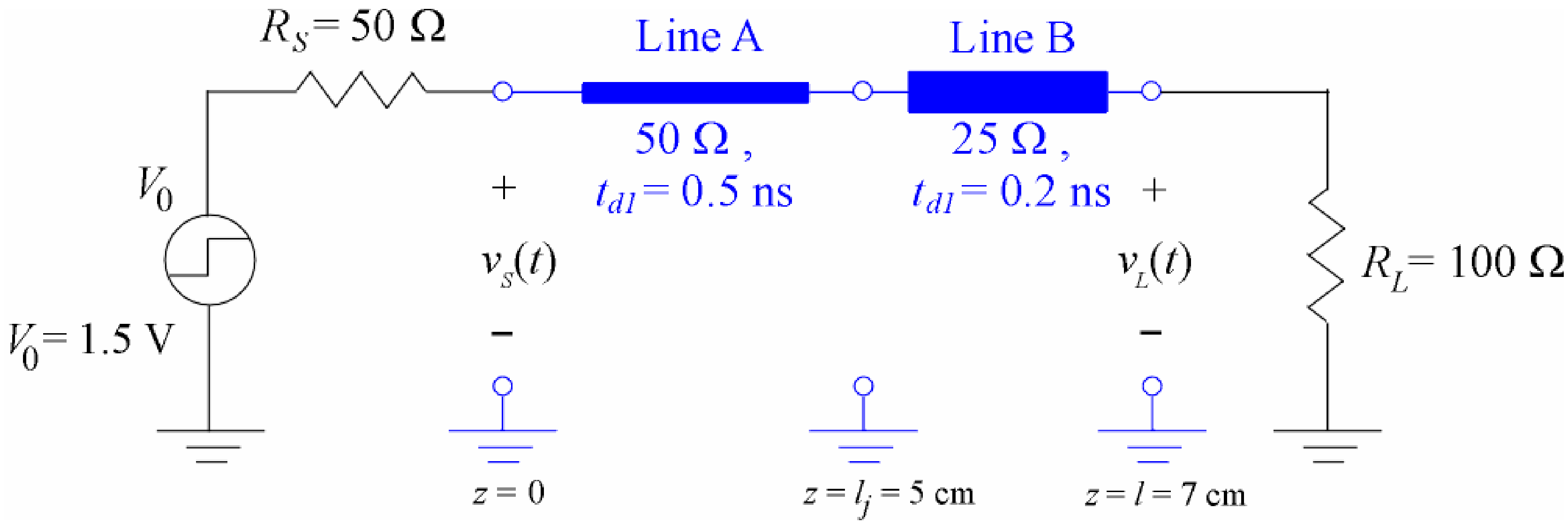
$$V_1^+ = \frac{Z_0}{Z_0 + R_S} V_0 = 0.2V_0$$

No overshooting & ringing (good!)



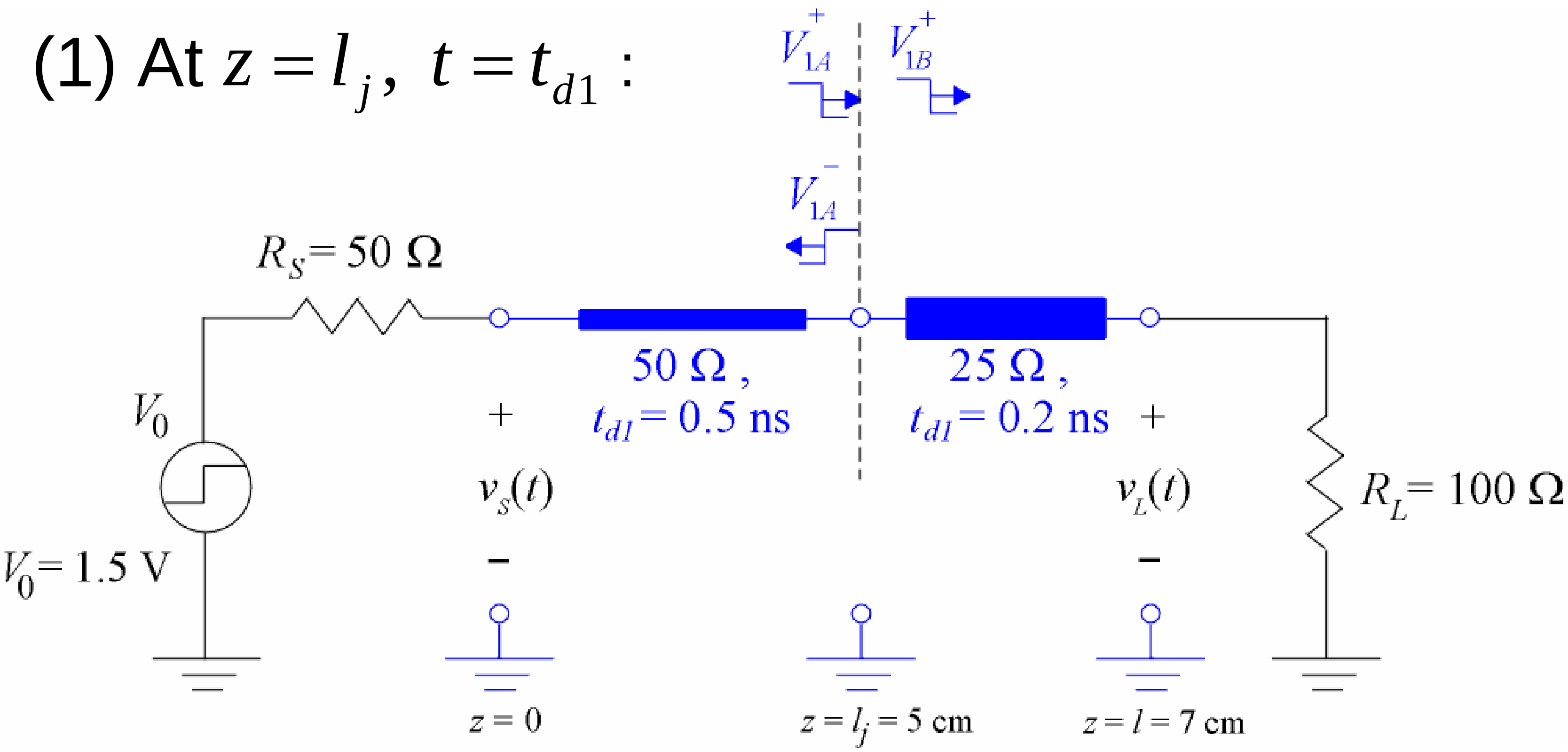


Example 3-4: Cascaded TX line-1



Example 3-4: Cascaded TX line-2

(1) At $z = l_j,\ t = t_{d1}$:



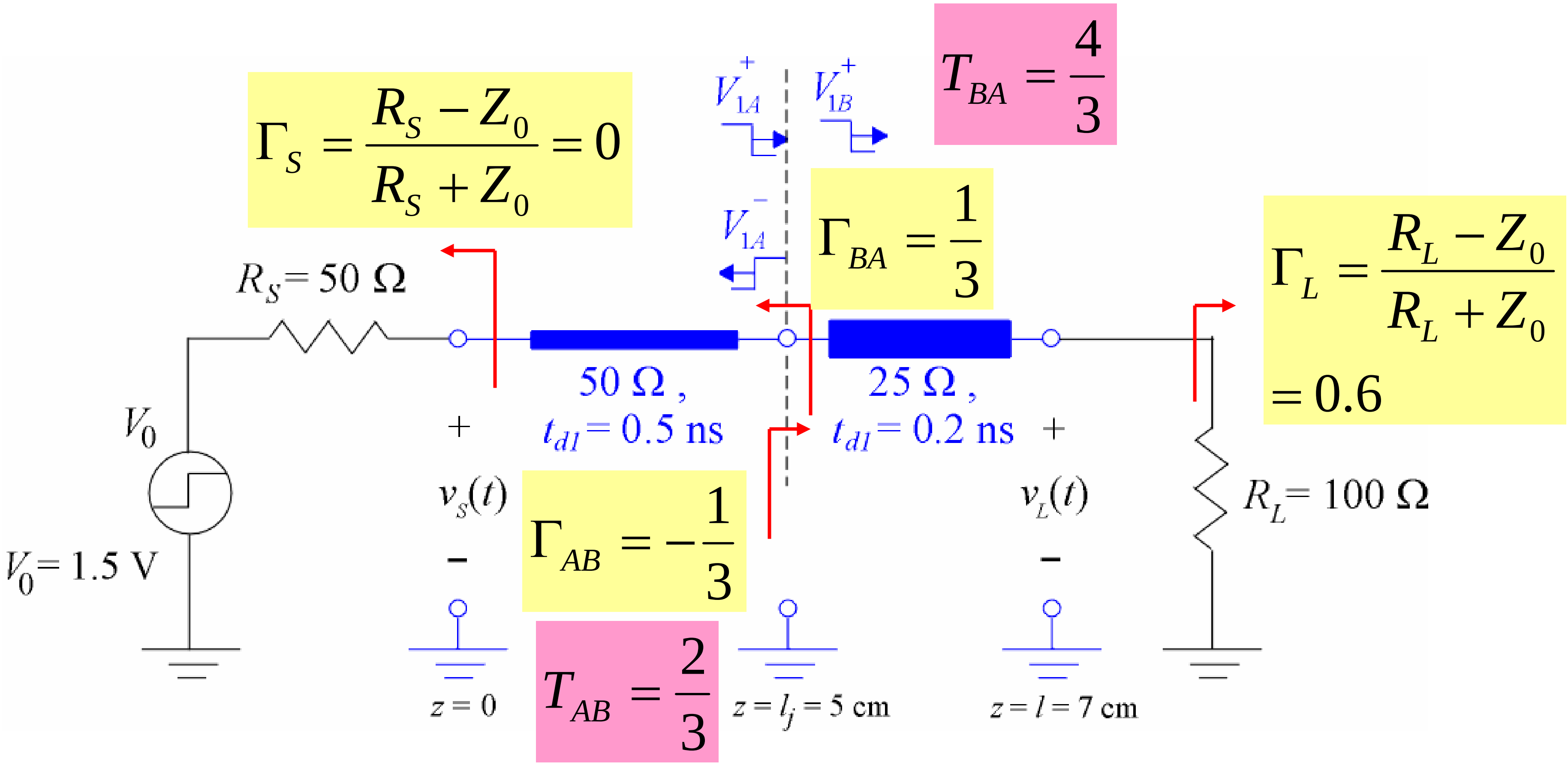
BC: $v_{1A}^+(l_j, t_{d1}) + v_{1A}^-(l_j, t_{d1}) = v_{1B}^+(l_j, t_{d1})$

Example 3-4: Cascaded TX line-3

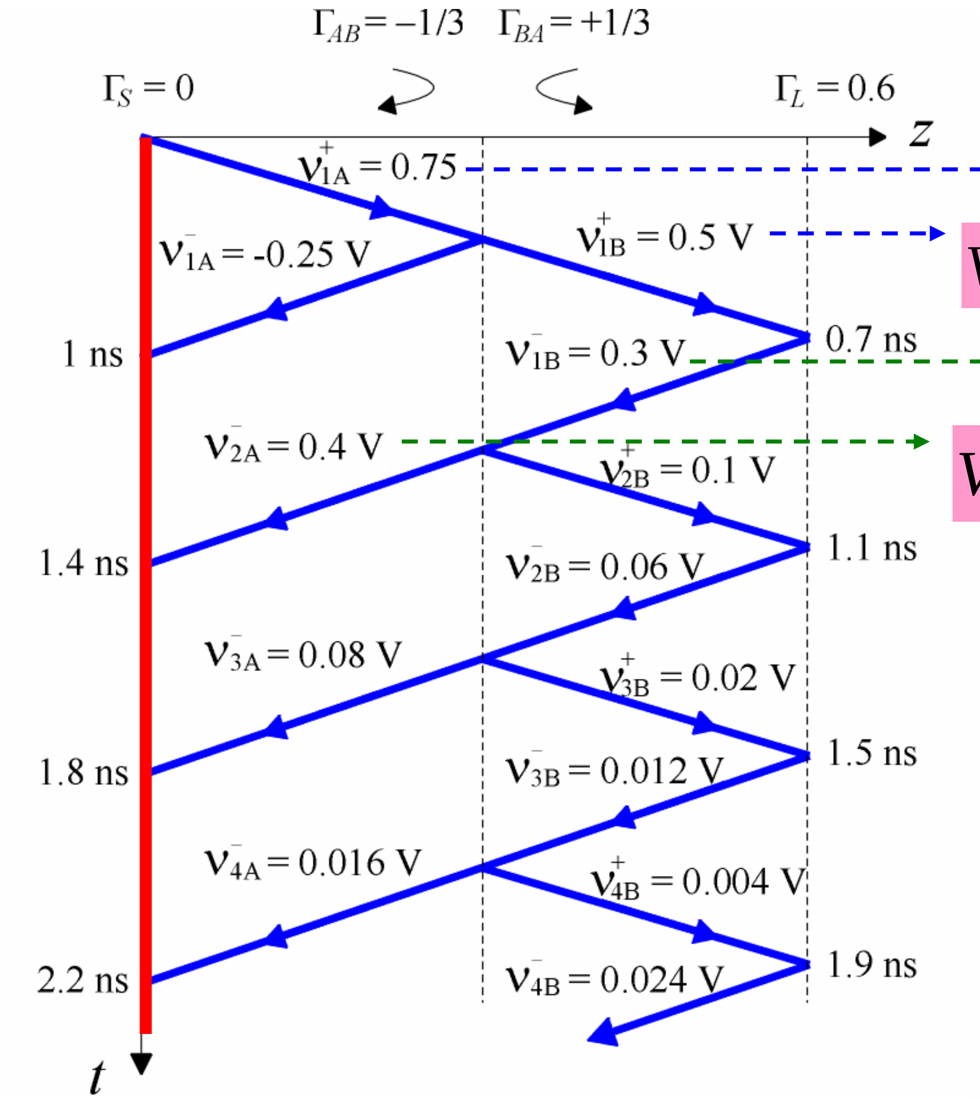
$$v_{1A}^{+}(l_j, t_{d1}) + v_{1A}^{-}(l_j, t_{d1}) = v_{1B}^{+}(l_j, t_{d1})$$

$$\Rightarrow 1 + \frac{v_{1A}^{-}(l_j, t_{d1})}{v_{1A}^{+}(l_j, t_{d1})} = \frac{v_{1B}^{+}(l_j, t_{d1})}{v_{1A}^{+}(l_j, t_{d1})},$$

$$\Rightarrow 1 + \Gamma_{AB} = T_{AB}$$ …Transmission coefficient


$\Gamma_S = \frac{R_S - Z_0}{R_S + Z_0} = 0$
V_{1A}^{+}
V_{1B}^{+}
$T_{BA} = \frac{4}{3}$
V_{1A}^{-}
$\Gamma_{BA} = \frac{1}{3}$
$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} = 0.6$
$R_S = 50\ \Omega$
$50\ \Omega$, $t_{d1} = 0.5$ ns
$25\ \Omega$, $t_{d1} = 0.2$ ns
V_0
$V_0 = 1.5$ V
$v_S(t)$
$v_L(t)$
$R_L = 100\ \Omega$
$\Gamma_{AB} = -\frac{1}{3}$
$T_{AB} = \frac{2}{3}$
$z = 0$
$z = l_j = 5$ cm
$z = l = 7$ cm

Example 3-4: Cascaded TX line-5

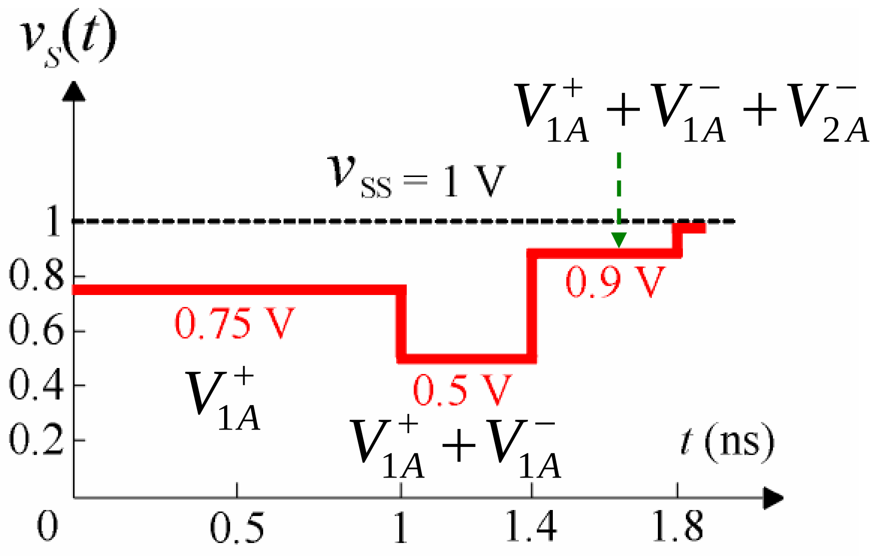


$$V_{1A}^{+} = \frac{Z_{0A}}{Z_{0A} + R_S} V_0 = 0.5V_0 = 0.75\text{ V}$$

$$V_{1B}^{+} = (0.75\text{ V}) \cdot T_{AB}$$

$$V_{1B}^{-} = (0.5\text{ V}) \cdot \Gamma_L$$

$$V_{2A}^{+} = (0.3\text{ V}) \cdot T_{BA}$$



Example 3-4: Cascaded TX line-6