Homework Problem Set #13

(Due date: 2011/06/20)

The full score is 70 points.

1) (10 points) Show that the two divergence equations ($\nabla \cdot \vec{D} = \rho$ and $\nabla \cdot \vec{B} = 0$) can be derived by the two curl equations ($\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$) and the equation of continuity ($\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$). This means that the four Maxwell's equations are not independent of the equation of continuity.

(*Hint*: Use the vector identity $\nabla \cdot \left(\nabla \times \vec{A}\right) = 0$. Just prove $\nabla \cdot \vec{B}$ = constant. The constant must be zero because $\vec{B} = 0$ at least for $t \to \infty$.)

2) Consider a time-harmonic plane wave propagating in a lossless dielectric medium. The E-field is $\vec{E}(z,t) = \vec{a}_x 5\cos\left(2\pi \cdot 10^{10} \cdot t - 100\pi \cdot z\right) + \vec{a}_y 12\cos\left(2\pi \cdot 10^{10} \cdot t - 100\pi \cdot z - \pi/4\right)$ (V/m), where t (s) and z (m) are in SI uinits.

2a) (5 points) Find the frequency f (Hz), wavelength λ (m) of the wave, and dielectric constant ε_r of the medium, respectively.

2b) (5 points) What is the corresponding $\vec{H}(z,t)$? (*Hint*: Use vector phasors.)

2c) (5 points) Describe the state of polarization of $\vec{E}(z,t)$ by showing the parameters a, b, θ and sense of rotation of Fig. 16-2 in the lecture notes.

[*Hint*: Use eq's (16.10-12) in the lecture notes]

2d) (10 points) Write a computer program to plot the trajectory of the tip of $\vec{E}(z,t)$ projected on the xy-plane. Compare with your result of problem 2c.

2e) (5 points) Evaluate the time-average power density $\vec{P}_{av}(z)$ (W/m^2) carried by the electromagnetic wave.

2f) (10 points) The vector phasor of the e-field at $z = 0$ is $\vec{E}(0) = \vec{a}_x 5 + \vec{a}_y 12 e^{-j\pi/4}$. Let the EM wave pass through some "anisotropic" medium of length *L*. The output vector phasor becomes: $\vec{E}(z) = \vec{a}_x 5 e^{-jk_x L} + \vec{a}_y 12 e^{-j\pi/4} e^{-jk_y L}$, where $k_x = \omega\sqrt{\mu\varepsilon_x} \neq k_y = \omega\sqrt{\mu\varepsilon_y}$. If L is chosen such that $(k_y - k_x)L = \frac{\pi}{2}$ (i.e. a quarter-wave plate), what is the corresponding polarization state of the output wave?

2g) (5 points) Follow Problem 2f, what is the output polarization state if $(k_y - k_x)L = \pi$ (i.e. a half-wave plate)?

3) Consider a coaxial cable made of perfect inner and outer conductors with radii a and b, respectively. The space between the two conductors is vacuum. Supply the cable with an ac current $I = I_0 \cos(\omega t)$. Neglect the spatial dependence along the direction of propagation (*z*).

3a) (5 points) What is the magnetic field intensity $\vec{H}(r,t)$ between the two conductors ($a < r < b$)?

3b) (5 points) What is the electric field intensity $\vec{E}(r,t)$ between the two conductors?

3c) (5 points) What is the time-average power density $\vec{P}_{av}(r)$ between the two conductors?

3d) (5 points) What is the total power carried by this cable between the two conductors?