Homework Problem Set #13

(Due date: 2011/06/08)

The full score is 50 points.

1) (10%) Consider a copper strip of length L on the xy-plane, where a uniform magnetic flux density $\vec{B} = \vec{a}_z B_0$ exists. One of its end is pivoted at the origin, while the entire strip is rotating counterclockwisely about the z-axis with an angular velocity ω.What is the voltage difference between the two ends (from the pivot to the tip) of the strip?

2) (5%) Consider a time-harmonic EM wave in some material with $\varepsilon = \varepsilon_0$, σ =5.70×10^7 (S/m). What is the ratio of magnitude of the conduction current density to the displacement current density at 1 GHz frequency?

3) Nonhomogeneous wave equations of fields in the frequency domain.

3a) (5%) Starting from the Maxwell's equations in the time domain, i.e. eq's (14.1), (7.8) (14.12), (11.2) in the lecture notes, write down the (phasor) Maxwell's equations of $\vec{E}$ and $\vec{H}$ in the frequency domain in a simple medium with charge and current sources.

3b) (10%) By the result of problem 3a, derive the nonhomogeneous (phasor) wave equations of $\vec{E}$ and $\vec{H}$, respectively.

4) Consider a short conducting wire of length dl carrying a spatially uniform current

$i(t) = I_0 \cos \omega t$ and placed along the *z*-axis at the origin (Fig. 1).

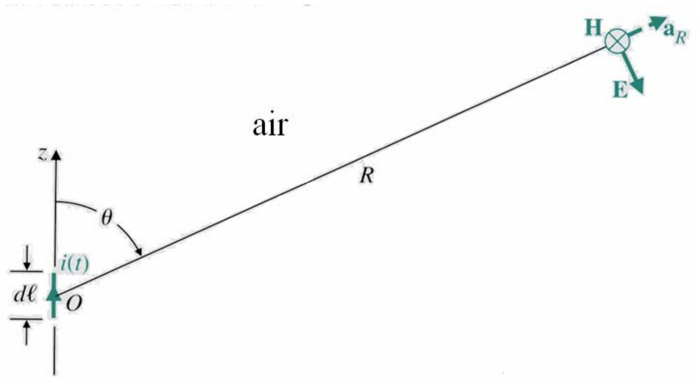

Fig. 1 A small current source creates EM fields.

4a) (5%) By eq. (15.28) in the lecture notes, find the phasor representation of resulting vector potential $\vec{A}(\vec{r})$ in spherical coordinates ($\vec{r} = \vec{a}_R R$, $R >> \lambda = 2\pi c/\omega$).

4b) (5%) By the result of Problem 4a, find the "approximated" phasor representation of magnetic field intensity $\vec{H}(\vec{r})$ by neglecting the "higher order terms". [E.g. $(kR)^{-1} + (kR)^{-2} \approx (kR)^{-1}$.]

4c) (5%) By the result of Problem 4b, find the "approximated" phasor representation of electric field intensity $\vec{E}(\vec{r})$ by neglecting the "higher order terms".

4d) (5%) Plot the EM power distribution $\left|\vec{E}(R_0, \theta, \phi_0)\right|^2$ relative to its maximum in spherical coordinates, where R_0, ϕ_0 are arbitrary constants.