Homework Solutions #11

(Due date: 2010/06/01)

The full score is 50 points.

1) (10 points) Problem **P.6–36**.

Answer:

$$\oint_C \vec{B}\cdot d\vec{l} = \mu_0 \int_S \vec{J}\cdot d\vec{s}$$

Inside the inner conductor:

$$\oint_{C_1} \vec{B}_1\cdot d\vec{l} = \int_0^{2\pi} \vec{a}_\phi B_{\phi 1}\cdot \vec{a}_\phi r_1 d\phi = 2\pi r_1 B_{\phi 1},\ \ I_1 = \left(\frac{r_1}{a}\right)^2 I,\ \ \vec{B}_1 = \vec{a}_\phi \frac{\mu_0 r_1 I}{2\pi a^2},\ \ r_1 \le a$$

Between the inner and outer conductors:

$$\oint_{C_2} \vec{B}_2\cdot d\vec{l} = \int_0^{2\pi} \vec{a}_\phi B_{\phi 2}\cdot \vec{a}_\phi r_2 d\phi = 2\pi r_2 B_{\phi 2},\ \ I_2 = I,\ \ \vec{B}_2 = \vec{a}_\phi \frac{\mu_0 I}{2\pi r_2},\ \ a \le r_2 \le b$$

Inside the outer conductor:

$$\oint_{C_3} \vec{B}_3\cdot d\vec{l} = \int_0^{2\pi} \vec{a}_\phi B_{\phi 3}\cdot \vec{a}_\phi r_3 d\phi = 2\pi r_3 B_{\phi 3},\ \ I_3 = I - \left[\frac{r_3^2 - b^2}{(b+d)^2 - b^2}\right] I = \left[\frac{(b+d)^2 - r_3^2}{(b+d)^2 - b^2}\right] I,$$

$$\vec{B}_3 = \vec{a}_\varphi \frac{\mu_0 I}{2\pi r_3}\left[\frac{(b+d)^2 - r_3^2}{(b+d)^2 - b^2}\right],\ \ b \le r_3 \le b+d$$

$$W_m = \frac{1}{2\mu_0}\int_0^d B^2 2\pi r dr = \frac{1}{2\mu_0}\left[\int_0^a B_1^2 2\pi r dr + \int_a^b B_2^2 2\pi r dr + \int_b^{b+d} B_3^2 2\pi r dr\right]$$

$$= \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi}\right)^2 2\pi \left[\int_0^a \frac{r^2}{a^4} r dr + \int_a^b \frac{1}{r^2} r dr + \int_b^{b+d} \frac{1}{r^2}\left[\frac{(b+d)^2 - r^2}{(b+d)^2 - b^2}\right]^2 r dr\right]$$

$$= \frac{\mu_0}{4\pi} I^2 \left\{\frac{1}{4} + \ln\left(\frac{b}{a}\right) + \left[\frac{1}{(b+d)^2 - b^2}\right]^2 \left[(b+d)^4 \ln\left(\frac{b+d}{b}\right) - (b+d)^4 + b^2 (b+d)^2 + \frac{(b+d)^4 - b^4}{4}\right]\right\}$$

With the Eq.(6-163) of the text book, $L = \dfrac{2W_m}{I^2}$

$$L = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) + \frac{\mu_0}{2\pi}\left[\frac{(b+d)^2}{(b+d)^2 - b^2}\right]^2 \ln\left(\frac{b+d}{b}\right) + \frac{\mu_0}{8\pi}\frac{b^2 - 3(b+d)^2}{(b+d)^2 - b^2} \quad \text{(H/m)}$$

2) (10 points) Problem **P.6–39**.

Answer:

Assume a current I.

B at $P(r, \theta)$ is $\bar{a}_\phi \dfrac{\mu_0 I}{2\pi(d + r\cos\theta)}$

$$\Lambda_{12} = \frac{\mu_0 I}{2\pi}\int_0^b \int_0^{2\pi} \frac{r dr d\theta}{d + r\cos\theta} = \frac{\mu_0 I}{2\pi}\int_0^b \frac{2\pi r dr}{\sqrt{d^2 - r^2}} = \mu_0 I(d - \sqrt{d^2 - b^2}).$$

$$L_{12} = \mu_0(d - \sqrt{d^2 - b^2}).$$

3) (10 points) Problem **P.6–45**. Instead of deriving analytic solution to the force, you just need to specify the direction of force exerted on the circular loop. Justify your answer.

Answer:

By all the concepts learned from Static Magnetic Fields, we can find that the magnetic flux density $\vec{B}_{12}$ induced by current I_1 on the infinitely straight line will produce a magnetic force $d\vec{F} = I_2 d\vec{l} \times \vec{B}_{12}$. The direction of the magnetic force is pointed toward the center of the circular loop. After taking the integral form of $d\vec{F}$ the component in vertical direction will be cancelled and then only the component in horizontal direction will be left.

From equation $\vec{B}_{\phi}(R) = \vec{a}_{\phi} \dfrac{\mu_0 I_1}{2\pi R}$, it can be derived that shorter the value of R, larger the value of $\vec{B}_{\phi}(R)$ as well as $d\vec{F}$ and then the component in horizontal direction contributed on the left hand side of the circular loop is larger than that on the right hand side.

Therefore, the result of the net force induced by current I_1 contributes to a right direction force on the circular loop.

4) (10 points) Explain how to distinguish an *n*-type from a *p*-type semiconductor slab by using the Hall effect.

(*Hint*: Refer to Sec. 6-13 of the textbook.)

Answer:

In the case of a *p*-type semiconductor slab (referring to Figure 6-28 of the textbook), the charge carrier is electron holes and move in positive *y*-direction and we also assume the center of this slab is located at origin. The uniform magnetic field is in positive *z*-direction, so the magnetic force tends to move the electron holes in the positive *x*-direction. That is, the voltage at the surface located at $x = \frac{d}{2}$ will be positive.

Similarly, in the case of a *n*-type semiconductor slab, the charge carrier is electrons and move in positive *y*-direction. The uniform magnetic field is also in positive *z*-direction, so the magnetic force tends to move the electrons in the positive *x*-direction. That is, the voltage at the surface located at $x = \frac{d}{2}$ will be negative.

5) (10 points) As shown in Fig. 1, a loud speaker consists of a permanent magnet and a coil wound around a cylinder between the poles. The coil-cylinder assembly is attached to the apex of a movable cone. Explain the working principle of this loud speaker.

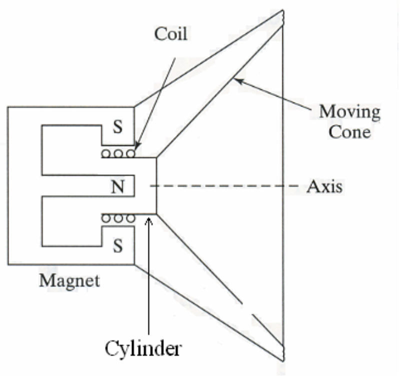

Fig. 1. Cross-sectional view of a loud speaker.

Answer:

When the electric signal is applied to the coil, a magnetic field is created by the electric current which thus becomes an electromagnet field. This electromagnet and the permanent magnet's magnetic field interacts, generating a force which causes the coil, and so the attached moving cone, to move back and forth with the same frequency as the input signal. The moving cone will quake the air and produce sound, under the control of the applied electrical signal coming from the amplifier.