Homework Solutions #10

(Due date: 2011/05/25)

The full score is 50 points.

1) Uniformly magnetized sphere.

1a) (5 points) Problem **P.6–26(a)** of the textbook.

Answer:

$$\vec{J}_m = \vec{\nabla} \times \vec{M} = 0$$

$$\vec{J}_{ms} = (\vec{a}_R \cos\theta - \vec{a}_\theta \sin\theta) M \times \vec{a}_R = \vec{a}_\phi M_0 \sin\theta$$

1b) (5 points) Problem **P.6–26(b)** of the textbook.

Answer:

Apply Eq.(6-38) to a loop of radius $b\sin\theta$ carrying a current $J_{ms} b d\theta$:

$$d\vec{B} = \vec{a}_z \frac{\mu_0 (J_{ms} b d\theta)(b\sin\theta)^2}{2(b^2)^{3/2}} = \vec{a}_z \frac{\mu_0 M_0}{2} \sin^3\theta$$

$$\vec{B} = \int d\vec{B} = \vec{a}_z \frac{\mu_0 M_0}{2} \int_0^\pi \sin^3\theta d\theta = \vec{a}_z \frac{2}{3}\mu_0 M_0 = \frac{2}{3}\mu_0 \vec{M}.$$

1c) (10 points) Follow Problem **P.6–26** of the textbook, plot the magnitude of on-axis magnetic flux density normalized to that at the spherical center $\left|\frac{\vec{B}(0,0,z)}{\vec{B}(0,0,0)}\right|$, where $\vec{B} = \vec{B}(x,y,z)$ is shown in Cartesian coordinates.

Answer:

The z-component of magnetic flux density at position (0,0,z) due to an infinitesimal ring of width $bd\theta$ at azimuthal angle θ (the center of the ring is located at

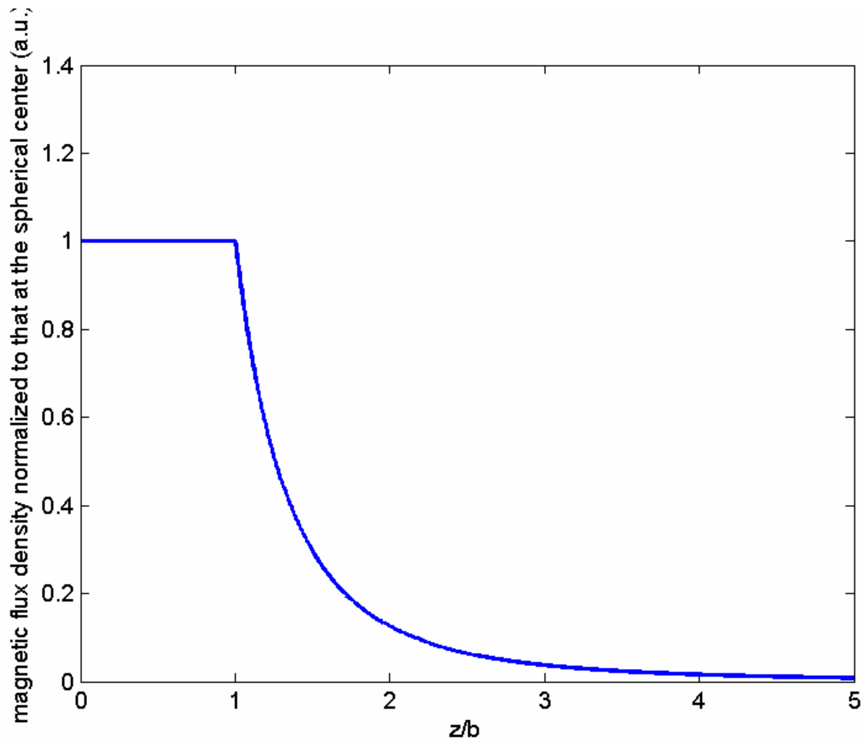
(0,0,z')) is:

$$d\vec{B} = \vec{a}_z \frac{\mu_0 (J_{ms} b d\theta)(b\sin\theta)^2}{2\left[(z-z')^2 + (b\sin\theta)^2\right]^{3/2}} = \vec{a}_z \frac{\mu_0 J_{ms} b(b^2 \sin^2\theta)}{2\left[(z-b\cos\theta)^2 + (b\sin\theta)^2\right]^{3/2}} d\theta,$$

where

$$z' = b\cos\theta$$

The total magnetic flux density at (0,0,z) can be derived by integration of dB with respect to θ from 0 to π.



2) (15 points) Problem **P.6–29(a-b)** of the textbook.

Answer:

(a) The magnetic flux density B has increased from 0 to B_f when t increases from 0 to t_f.

$$W_1 = \frac{1}{S}\int_0^{t_f} P_1 dt = \frac{1}{S}\int_0^{t_f} -\upsilon_1 I dt = \frac{1}{S}\int_0^{t_f} -\left(-n\frac{d\Phi}{dt}\right)\frac{H}{n}dt = \int_0^{B_f} HdB$$

(b) In the hysteresis loop of p.259 of the text book, the magnetic field intensity function of the curve P_3P_3' is $H_1(B)$, and that of the curve $P_3'P_3$ is $H_2(B)$.

The work done per unit volume for a cycle of change, $W_2 = \frac{1}{S}\left[\int_{B_f}^{-B_f} H_1 dB + \int_{-B_f}^{B_f} H_2 dB\right]$

$$= \frac{1}{S}\left[\int_{B_f}^{B_r} H_1 dB + \int_{B_r}^{0} H_1 dB + \int_0^{-B_f} H_1 dB + \int_{-B_f}^{-B_r} H_2 dB + \int_{-B_r'}^{0} H_2 dB + \int_0^{B_f} H_2 dB\right]$$

$$= \frac{1}{S}\left\{\int_{B_f}^{B_r} H_1 dB + \int_{B_r}^{0} H_1 dB + \int_0^{-B_f} H_1 dB + \int_{-B_f}^{-B_r} H_2 dB + \int_{-B_r'}^{0} H_2 dB + \int_0^{B_f} H_2 dB\right\}$$

$$= \frac{1}{S}\left\{\left[\int_{B_f}^{B_r} H_1 dB + \int_0^{B_f} H_2 dB\right] + \int_{B_r}^{0} H_1 dB + \int_{-B_r'}^{0} H_2 dB + \left[\int_0^{-B_f} H_1 dB + \int_{-B_f}^{-B_r} H_2 dB\right]\right\}$$

$$= \mathrm{A}_1, \text{ Area of the hysteresis loop}$$

3a) (5 points) Find the typical magnitude B_{earth} of the earth's magnetic flux density $\vec{B}$ (in Tesla).

Answer:

It is ~ 31 μT at equator; ~ 58 μT at 50° latitude.

3b) (5 points) Consider a hollow solenoid (Fig. 11-2 of the lecture notes) made by copper wire of radius $a = 1$ mm. The solenoid has 5 turns per centimeter, and a circular cross-section of radius $b = 2$ cm. What is the current needed to flow along the wire to generate a magnetic field as strong as the earth's?

Answer:

$$31\times10^{-6}T = \mu_0 NI = 4\pi\times10^{-7}\times500\times I\,,\;\; I = 50\left(mA\right),$$

where $\mu_0 = 4\pi\times10^{-7}\left(H/m\right)$ and $N = 500\left(\text{turns}/m\right)$

3c) (5 points) Follow Problem 3b, what is the corresponding ohmic power consumption?

Answer:

The conductivity for copper is $1.68\times10^{-8}\left(\Omega\cdot m\right)$ at 20 ℃.

A is the cross section area, which equals to $10^{-6}\pi$.

l is the wire length per meter, which equals to 20π .

Because the resistance is defined as: $R = \rho\dfrac{l}{A}$,

$$R = 1.68\times10^{-8}\times\frac{20\pi}{10^{-6}\pi} = 0.336\,\Omega.$$

The power consumptions is $P = I^2 R = 0.84\,mW$.