Homework Problem Set #8

(Due date: 2011/05/02)

This problem set covers materials of Lesson 10. The full score is 30 points.

1) (10 points) The procedures of evaluating the resistance of one piece of conducting material are summarized in the lecture notes, where we need to solve for the Laplace's equation $\nabla^2 V = 0$ under the specified boundary conditions to get the potential distribution $V(\vec{r})$. Please prove that Laplace's equation $\nabla^2 V = 0$ remains valid for steady electric currents, where free charges are actually in motion.

(*Hint*: Use laws regarding current density $\vec{J}$ and equation of continuity.)

2) (10 points) Problem **P.5–14** of the textbook. (*Hint*: $V(\vec{r}) = V(r)$)

3) Consider a "leaky" parallel-plate capacitor shown in Fig. 1. The top and bottom plates are rectangular [$0 < x < w, 0 < z < L$ (not shown)] and separated by a distance d. Two types of "imperfect" dielectric slabs with permittivities and conductivities of $\{\varepsilon_1, \sigma_1\}$, $\{\varepsilon_2, \sigma_2\}$ are filled between the two plates biased by a voltage difference of V_0 (>0).

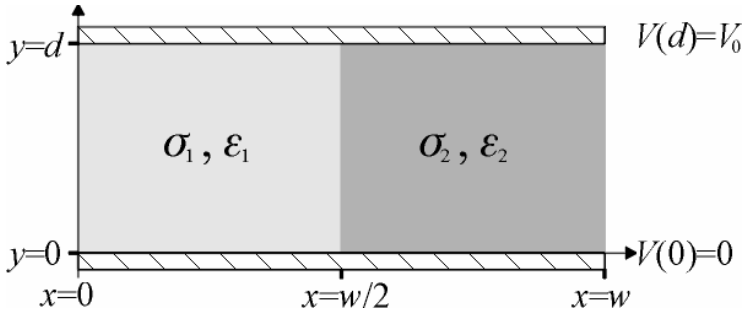

Fig. 10

3a) (5 points) Find the current densities $\vec{J}_1$, $\vec{J}_2$ in slab-1 and slab-2, respectively.

3b) (5 points) Find the total (leakage) resistance *R* of the capacitor.