Homework Problem Set #5

(Due date: 2011/4/11)

This problem set covers materials of Lesson 6. The full score is 80 points.

1) Consider a ring of radius b (m) placed on the *xy*-plane and centered at the origin. The ring has a uniform line charge density ρ_l (C/m).

1a) (10%) Determine the electric field intensity $\vec{E}$ (V/m) for an arbitrary point $P(0,0,z)$ on the *z*-axis.

1b) (10%) According to the result of Problem 1a, plot the normalized magnitude of the electric field on the *z*-axis:

$$E'(z) \equiv \left|\vec{E}(0,0,z)\right| \Big/ E_0 \text{ , for } 0 \le z/b \le 5,$$

where $E_0 = \dfrac{\rho_l}{2\varepsilon_0 b}$ (V/m). Mark the value of z/b where maximum electric field occurs. For comparison, also plot $E'(z)$ for $1 \le z/b \le 5$ due to a point charge $q = \rho_l \cdot 2\pi b$ at the origin in the same figure. Discuss the two results.

2) Consider the geometry of Fig. 3-40 of the text book (p147). Shift the finite line charge in the -*x* direction such that the line ranges $-L/2 < x < L/2$.

2a) (10%) Deduce the electric potential $V(x,b)$. (You are required to show the deduction process in detail.)

2b) (5%) Plot the normalized electric potential $V'(x) \equiv V(x,b)/V(x=0,b)$ for $-1 \le x/L \le 1$ and $b = 0.05L$ (close to the line). (*Hint*: You can write a computer program to get the results by numerical integration even the analytic solution of $V(x,b)$ cannot be derived.)

2c) (5%) Plot the normalized electric potential $V'(x)$ for $-1 \le x/L \le 1$ and $b = 10L$ (far away from the line).

2d) (5%) Discuss the results of Problem 2b and 2c.

3) Consider an electric dipole shown in Fig. 6-5 of the lecture notes.

3a) (5%) According to eq. (6.18) of the lecture notes, plot the normalized electric potential:

$$V'_{dipole}(\theta) \equiv \frac{V(R_0, \theta, \phi_0)}{V(R_0, 0, \phi_0)}, \text{ for } R_0 = 10d\ ,\ \phi_0 = 0\ ,\ 0 \le \theta \le \pi\ .$$

(*Hint*: Use the *Matlab* command "polar" to plot the curve in the polar coordinate.)

3b) (15%) To test the validity of eq. (6.18) of the lecture notes, write a program to evaluate and plot $V'_{dipole}(\theta)$ for $R_0 = 10d\ ,\ \phi_0 = 0\ ,\ 0 \le \theta \le \pi$ according to the formula:

$$V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R_+} - \frac{1}{R_-}\right).$$

Show the two curves of Problem 3a and 3b in the same figure, and discuss the results.

3c) (15%) With the program, you can investigate the potential in the vicinity of the dipole. Plot two curves of $V'_{dipole}(\theta)$ for $R_0 = 1.5d\ ,\ \phi_0 = 0\ ,\ 0 \le \theta \le \pi$; obtained by eq. (6.18) of the lecture notes (like Problem 3a), and the program (like Problem 3b), respectively. Show them in the same figure, and discuss the results.