Homework Problem Set #4

(Due date: 2011/03/28)

This problem set covers materials of Lesson 5. The full score is 40 points.

1) Please evaluate:

1a) (5%) $\vec{a}_\phi \times \vec{a}_z$ (Represent the result in Cartesian coordinate system.)

1b) (5%) $\frac{\partial}{\partial \phi}\vec{a}_\phi$ (Represent the result in Spherical coordinate system.)

2) (10%) For a scalar function V and a vector function $\vec{A}$, prove that:

$$\nabla \times \left(V\vec{A}\right) = V\left(\nabla \times \vec{A}\right) + \left(\nabla V\right) \times \vec{A}.$$

Note that this is the chain rule of outer product.

3) (10%) The position vector of a point $P(x, y, z)$ is $\vec{R} = \vec{a}_R R$, where $R = \left|\vec{R}\right| = \sqrt{x^2 + y^2 + z^2}$ is a scalar field. Evaluate $\nabla\left(R^{-1}\right)$, and represent the result by spherical coordinate system.

4) Consider a vector field $\vec{A}(R, \phi) = \vec{a}_R \frac{\sin^2 \phi}{R^3}$ existing in a volume V bounded by two concentric spherical shells with $R = 2$, $R = 3$ (center at the origin).

4a) (5%) Evaluate the net outward flux $\oint_S \vec{A} \cdot d\vec{s}$, where S is the surface enclosing V.

4b) (5%) Evaluate $\int_V \left(\nabla \cdot \vec{A}\right) dv$.