Homework Problem Set #3

(Due date: 2011/03/21)

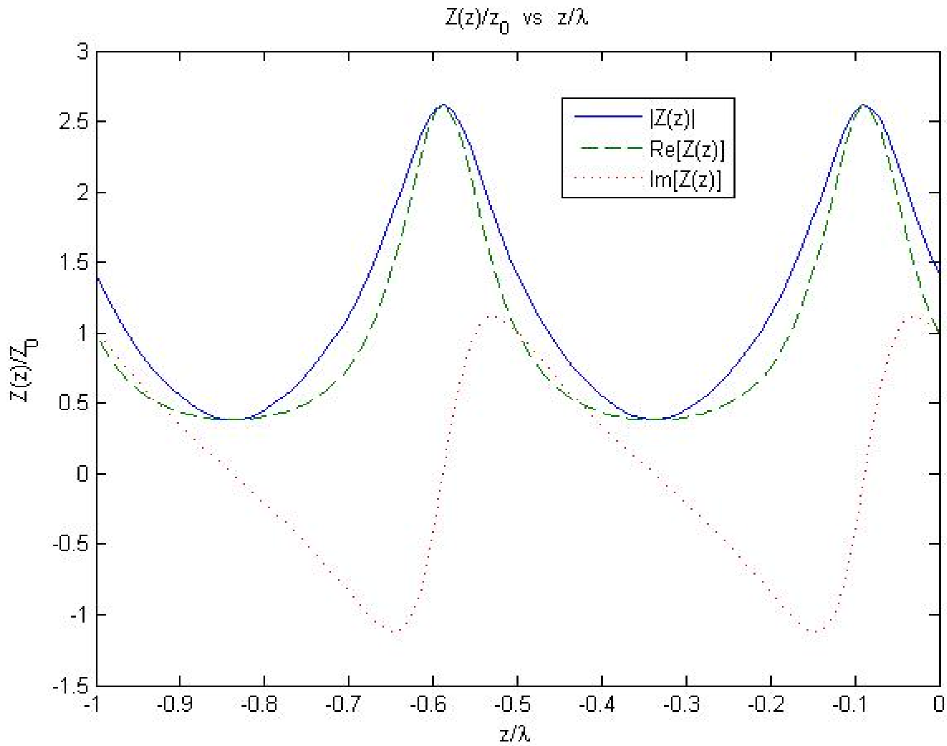
This problem set covers materials of Lesson 4. The full score is 60 points.

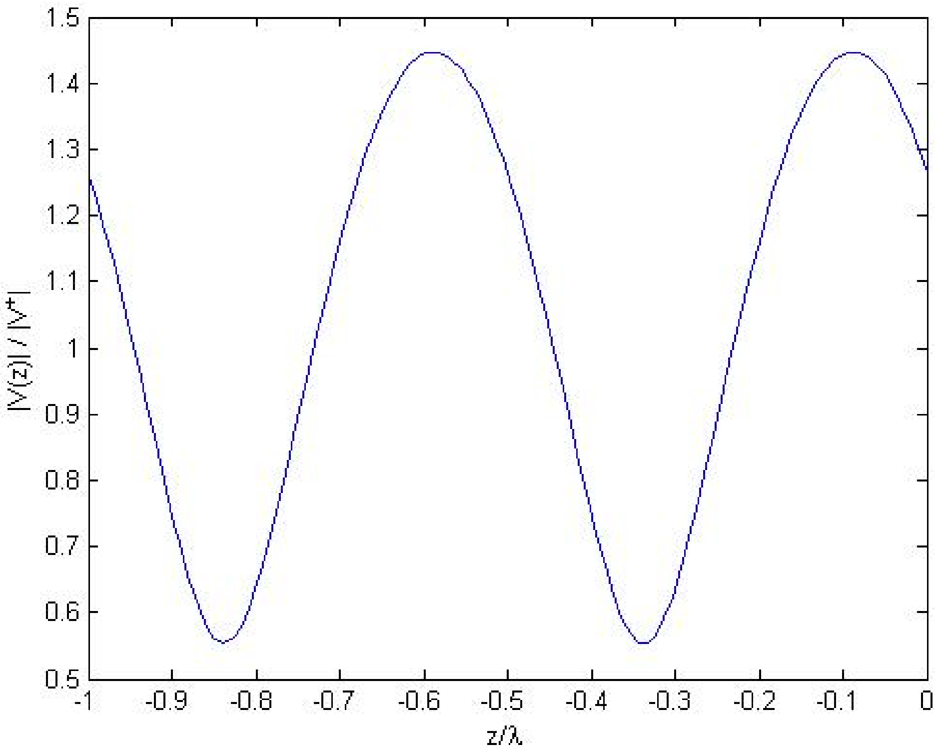
1) Lossless transmission line with a complex load.

1a) (10%) Refer to Example 4-3 of the lecture notes. Plot $Z(z) \in C$ and $\left| V(z)/V^{+} \right|$ for $z = [-\lambda, 0]$, and find the SWR, if (i) $Z_L = Z_0(1+j) \in C$, (ii) $Z_L = Z_0(1-j) \in C$.

Answer:

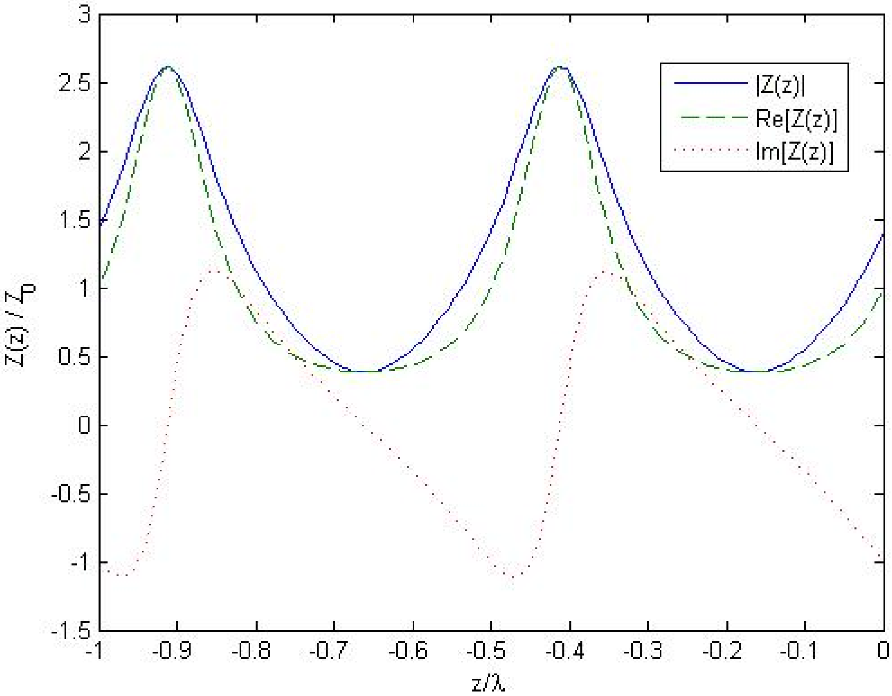
(i)

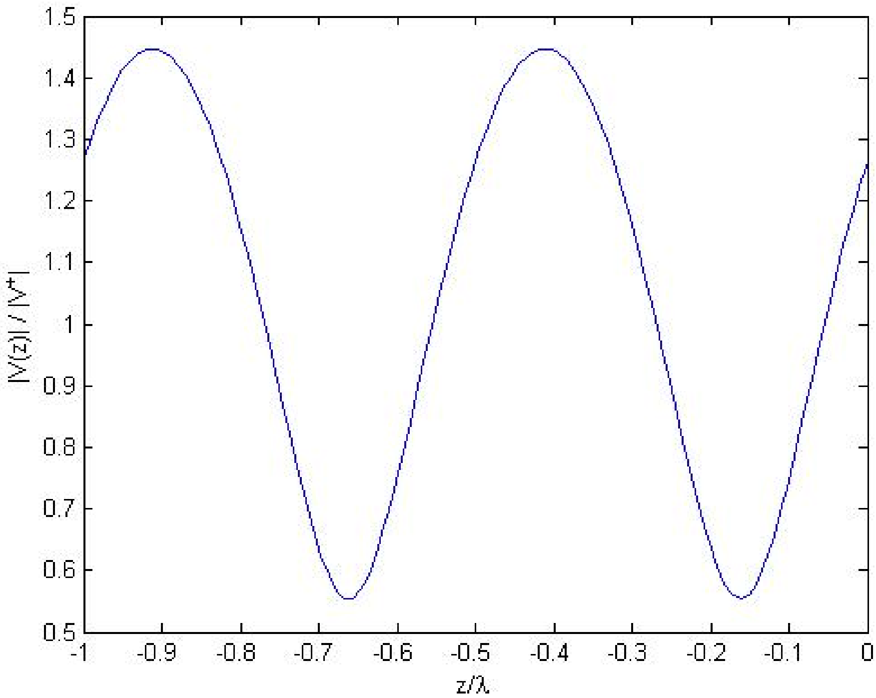




$$S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+\left|\dfrac{j}{2+j}\right|}{1-\left|\dfrac{j}{2+j}\right|} = \frac{1.4472}{0.5528} = 2.6179$$

(ii)





$$S=\frac{1+\left|\Gamma_L\right|}{1-\left|\Gamma_L\right|}=\frac{1+\left|\frac{-j}{2-j}\right|}{1-\left|\frac{-j}{2-j}\right|}=\frac{1.4472}{0.5528}=2.6179$$

1b) (5%) Define $z_{\min}(>0)$ as the shortest distance from the load to the minimum of $|V(z)|$ (refer to Fig. 4-3b). Find the ranges of $z_{\min}$ if $\operatorname{Im}\{Z_L\}<0$ and $\operatorname{Im}\{Z_L\}>0$, respectively.

Answer:

$\lambda/4<z_{\min}<\lambda/2$, if $\operatorname{Im}\{Z_L\}>0$,

$0<z_{\min}<\lambda/4$, if $\operatorname{Im}\{Z_L\}<0$.

2) Determination of complex voltage phasor $V(z)$.

2a) (10%) The power absorbed by the load in Example 4-4 of the lecture notes can also be calculated by eq. (4.30), where we need to find V^+ first. Evaluate $V^+ \in C$ in Example 4-4.

(*Hint*: Use the two boundary conditions at the source and load ends.)

Answer:

$$\Gamma_L = \frac{(100 - j60) - 50}{(100 - j60) + 50} = 0.483 e^{-j0.496}$$

$$V_s = V(-l) = V^+ e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right) = \frac{Z_{in}}{Z_S + Z_{in}} V_0$$

$$\Rightarrow V^+ = \frac{Z_{in}}{(Z_S + Z_{in}) e^{j\beta l}\left(1 + \Gamma_L e^{-j2\beta l}\right)} V_0 = 50 e^{j2.36} \quad \text{(V)}$$

2b) (10%) Derive $V_{\max}$, $V_{\min}$, SWR, and $z_{\min}$ in Example 4-4 by the result of problem 2a and computer simulation. Are your results consistent with the formulas of eq's (4.25), (4.26)? Note that these formulas are derived by assuming resistive load ($Z_L \in R$, $\Gamma_L \in R$), while it is complex ($Z_L = 100 - j60\,\Omega$) in Example 4-4.

Answer:

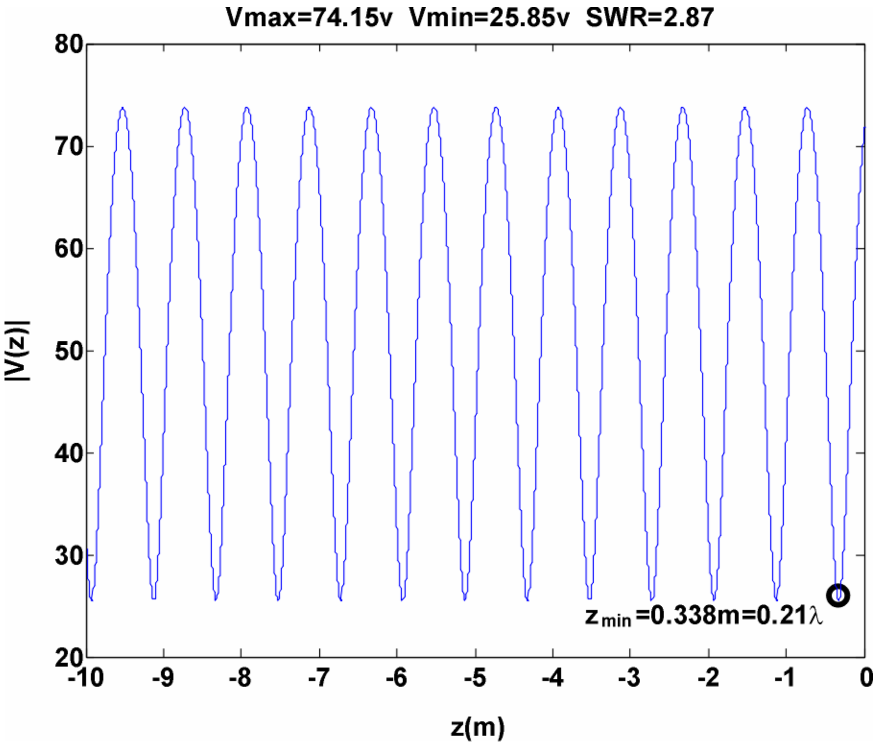
Derived from formulas:

$$V_{\max} = \left|V^+\right|\left(1 + \left|\Gamma_L\right|\right) = 50 \cdot 1.483 = 74.15$$

$$V_{\min} = \left|V^+\right|\left(1 - \left|\Gamma_L\right|\right) = 50 \cdot 0.571 = 25.85$$

$$\text{SWR} = \frac{V_{\max}}{V_{\min}} = \frac{1 + \left|\Gamma_L\right|}{1 - \left|\Gamma_L\right|} = \frac{74.15}{25.85} = 2.87$$

Computer simulation:



The values derived from the formulas and the simulations are the same.

3) (25%) We have seen in Example 4-4 of the lecture notes that only a fraction of power can be delivered to the load if the impedance mismatch exists ($Z_L \neq Z_0$). Section 9-7.2 (page 501) of the textbook describes how to match an arbitrary load impedance to a lossless transmission line by placing a single short-circuited stub of length l in parallel with the line at a suitable distance d from the load (Fig. 1). Can you find a set of possible parameters $\frac{d}{\lambda}$ and $\frac{l}{\lambda}$ (λ is the wavelength) if $Z_0 = 50\,\Omega$ and $Z_L = 20 - j40\,\Omega$?

(*Hint*: The textbook introduces the Smith chart as a convenient tool to prevent repeated calculations of complex numbers in steady-state transmission line analysis. However, complex numbers are no longer a threat in the presence of computer, eclipsing the value of the Smith chart. Just use a program to scan variable *z* for the "inverse" of eq. (4.15) (admittance), you will be able to get the approximated answers by having $Y_i = Y_B + Y_S = \frac{1}{R_0}$. Note that there are two possible values of *d* during $d = \left[0, \frac{\lambda}{2}\right]$, and two possible values of *l* during $l = \left[0, \frac{\lambda}{2}\right]$. Finding one combination is sufficient.)

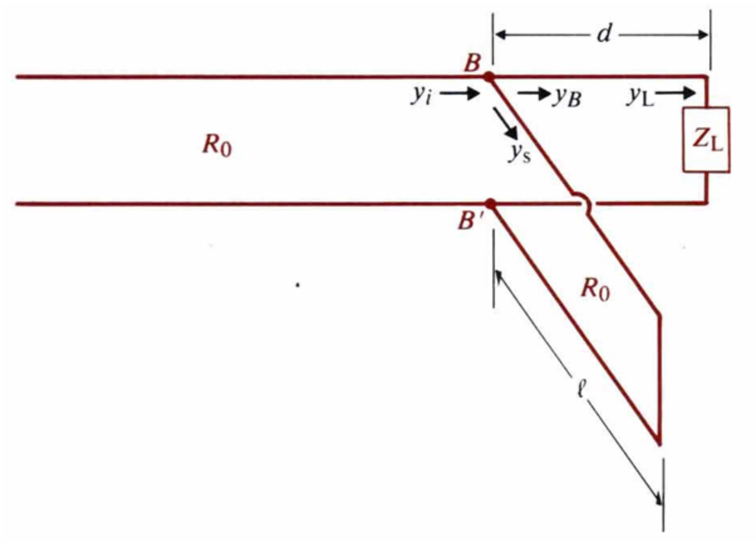


Fig. 1. Impedance matching by single-stub method (after DKC).

Answer:

From the hint, our purpose is to find out the suitable Y_B and Y_S and make $Y_B + Y_S = Y_0 \equiv$

$1/R_0$. By eq. (4.15), we can get the input impedance from BB' toward Z_L, $\frac{1}{Y_B} = Z_0 \frac{Z_L - jZ_0 \tan(\beta d)}{Z_0 - jZ_L \tan(\beta d)}$, where d is a variable which we have to use program to scan. First, in order to find out d, the minimum value of the difference between the **real** part of Y_B and Y_0 has to be found out, and the value d can be determined as shown in Fig. 2.

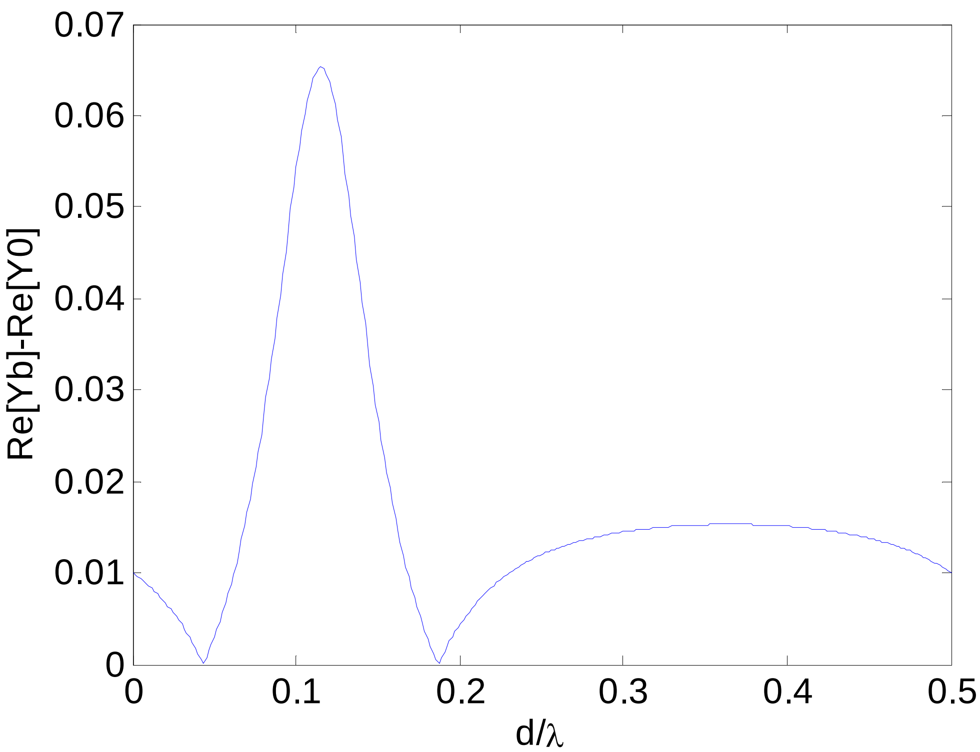


Fig. 2 Finding the suitable d.

The values of d are **0.043λ** and **0.187λ** respectively (for d= [0, λ/2]).

Second, following the similar procedure of finding d, find out the minimum value of the difference between the **imaginary** part of Y_B and Y_S, and the value ℓ can be gotten.

ℓ = **0.410λ**, **0.090λ** respectively (for ℓ = [0, λ/2]).