Homework Problem Set #3

(Due date: 2011/03/21)

This problem set covers materials of Lesson 4. The full score is 60 points.

1) Lossless transmission line with a complex load.

1a) (10%) Refer to Example 4-3 of the lecture notes. Plot $Z(z) \in C$ and $\left|V(z)/V^{+}\right|$ for $z = [-\lambda, 0]$, and find the SWR, if (i) $Z_L = Z_0(1+j) \in C$, (ii) $Z_L = Z_0(1-j) \in C$.

1b) (5%) Define $z_{\min}(>0)$ as the shortest distance from the load to the minimum of $|V(z)|$ (refer to Fig. 4-3b). Find the ranges of $z_{\min}$ if $\operatorname{Im}\{Z_L\} < 0$ and $\operatorname{Im}\{Z_L\} > 0$, respectively.

2) Determination of complex voltage phasor $V(z)$.

2a) (10%) The power absorbed by the load in Example 4-4 of the lecture notes can also be calculated by eq. (4.30), where we need to find V^{+} first. Evaluate $V^{+} \in C$ in Example 4-4.

(*Hint*: Use the two boundary conditions at the source and load ends.)

2b) (10%) Derive $V_{\max}$, $V_{\min}$, SWR, and $z_{\min}$ in Example 4-4 by the result of problem 2a and computer simulation. Are your results consistent with the formulas of eq's (4.25), (4.26)? Note that these formulas are derived by assuming resistive load ($Z_L \in R$, $\Gamma_L \in R$), while it is complex ($Z_L = 100 - j60\,\Omega$) in Example 4-4.

3) (25%) We have seen in Example 4-4 of the lecture notes that only a fraction of power can be delivered to the load if the impedance mismatch exists ($Z_L \neq Z_0$). Section 9-7.2 (page 501) of the textbook describes how to match an arbitrary load impedance to a lossless transmission line by placing a single short-circuited stub of length l in parallel with the line at a suitable distance d from the load (Fig. 1). Can you find a set of possible parameters $\frac{d}{\lambda}$ and $\frac{l}{\lambda}$ (λ is the wavelength) if $Z_0 = 50\,\Omega$ and $Z_L = 20 - j40\,\Omega$?

(*Hint*: The textbook introduces the Smith chart as a convenient tool to prevent repeated calculations of complex numbers in steady-state transmission line analysis. However, complex numbers are no longer a threat in the presence of computer, eclipsing the value of the Smith chart. Just use a program to scan variable *z* for the "inverse" of eq. (4.15) (admittance), you will be able to get the approximated answers by having $Y_i = Y_B + Y_S = \frac{1}{R_0}$. Note that there are two possible values of *d* during $d = \left[0, \frac{\lambda}{2}\right]$, and two possible values of *l* during $l = \left[0, \frac{\lambda}{2}\right]$. Finding one combination is sufficient.)

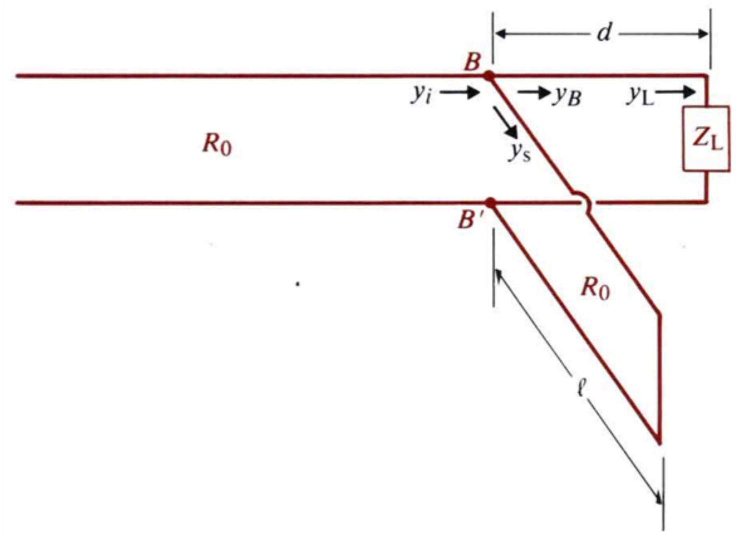

Fig. 1. Impedance matching by single-stub method (after DKC).