Homework 1 Solutions

(Due date: 2011/3/2)

This problem set covers materials of Lesson 1–2. The full score is 40 points.

1) (10%) In the NTHU EE teaching laboratories, what is the linear dimension (roughly) of the electronic circuits you built on the breadboard? What is the maximum signal frequency of the function generator? Based on these two numbers, which theory (lumped or distributed circuit) will be more suitable to analyze the behavior of your circuits? Why?

Answer:

The linear diagonal of the electronic circuits on the breadboard is 25 cm, the maximum signal frequency of the function generator 25 MHz, corresponding to a minimum wavelength of 12 m (>> 25 cm). So, lumped circuit will be sufficient to analyze the behavior of your circuits.

2) Equations of lossy transmission lines.

2a) (10%) In the lecture notes, eq's (2.1), (2.2) are derived by assuming $R=0$, $G=0$ in Fig. 2-5. Please modify them if the line is lossy ($R\neq 0$, $G\neq 0$).

Answer:

$$v(z,t)-v(z+\Delta z,t)=i(z,t)\cdot R\Delta z+L\Delta z\cdot\frac{\partial}{\partial t}i(z,t)$$

When $\Delta z\to 0$,

$$\to\frac{\partial}{\partial z}v(z,t)=-i(z,t)\cdot R-L\cdot\frac{\partial}{\partial t}i(z,t) \quad (1)$$

$$i(z,t)=i(z+\Delta z,t)+C\Delta z\cdot\frac{\partial}{\partial t}v(z+\Delta z,t)+v(z+\Delta,t)\cdot G\Delta z$$

When $\Delta z\to 0$, and assume $v(z,t)\approx v(z+\Delta z,t)$

$$\to\frac{\partial}{\partial z}i(z,t)=-v(z,t)\cdot G-C\cdot\frac{\partial}{\partial t}v(z,t) \qquad (2)$$

2b) (10%) Please modify eq's (2.3), (2.4) if $R\neq 0$, $G\neq 0$.

Answer:

By taking $\frac{\partial}{\partial z}$ for both sides of eq. (1) in Problem 2a

$$\to\frac{\partial^2}{\partial z^2}v(z,t)=-\frac{\partial}{\partial z}i(z,t)\cdot R-L\cdot\frac{\partial}{\partial t}\frac{\partial}{\partial z}i(z,t).$$

Then replace $i(z,t)$ by eq. (2)

$$\to\frac{\partial^2}{\partial z^2}v(z,t)=\left[v(z,t)\cdot G+\frac{\partial}{\partial t}v(z,t)\cdot C\right]\cdot R+L\cdot\left[\frac{\partial}{\partial t}v(z,t)\cdot G+\frac{\partial^2}{\partial t^2}v(z,t)\cdot C\right]$$

$$\to\frac{\partial^2}{\partial z^2}v(z,t)=GR\cdot v(z,t)+(CR+LG)\cdot\frac{\partial}{\partial t}v(z,t)+LC\cdot\frac{\partial^2}{\partial t^2}v(z,t)$$

By taking $\frac{\partial}{\partial z}$ for both sides of eq. (2) in Problem 2a

$$\to\frac{\partial^2}{\partial z^2}i(z,t)=-\frac{\partial}{\partial z}v(z,t)\cdot G-C\cdot\frac{\partial}{\partial t}\frac{\partial}{\partial z}v(z,t)$$

Then replace $v(z,t)$ by eq. (1)

$$\to\frac{\partial^2}{\partial z^2}i(z,t)=\left[i(z,t)\cdot R+\frac{\partial}{\partial t}i(z,t)\cdot L\right]\cdot G+C\cdot\left[\frac{\partial}{\partial t}i(z,t)\cdot R+\frac{\partial}{\partial t^2}i(z,t)\cdot L\right]$$

$$\to\frac{\partial^2}{\partial z^2}i(z,t)=RG\cdot i(z,t)+(LG+RC)\cdot\frac{\partial}{\partial t}i(z,t)+LC\cdot\frac{\partial^2}{\partial t^2}i(z,t)$$

3) (10%) The cross-sectional geometry of a widely used coaxial able RG58/U is shown in Fig. 1(below), where the parameters are: $a = 0.45$ mm, $b = 1.47$ mm, permittivity of vacuum $\varepsilon_0 = \frac{1}{36\pi}\times 10^{-9}$ (F/m), permeability of vacuum $\mu_0 = 4\pi\times 10^{-7}$ (H/m). By Table 9-2 of the textbook (p447), the inductance and capacitance per unit length of a coaxial cable are formulated as: $L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right)$, $C = \frac{2\pi\varepsilon}{\ln(b/a)}$. Evaluate L, C, v_p, Z_0 of a RG58/U cable, respectively. (The numbers will remind you of the typical ranges of these parameters in the real world.)

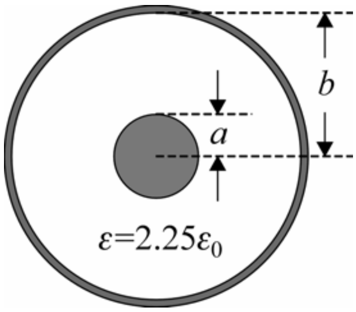

Fig. 1. Cross-section of RG58/U coaxial cable.

Answer:

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) = \frac{4\pi\times 10^{-7}}{2\pi}\ln\left(\frac{1.47}{0.45}\right) = 2.37\times 10^{-7}\ \text{(H/m)} = 0.237\ (\mu\text{H/m})$$

$$C = \frac{2\pi\varepsilon}{\ln(b/a)} = \frac{2\pi\times 2.25\times\frac{1}{36\pi}\times 10^{-9}}{\ln\left(\frac{1.47}{0.45}\right)} = 1.06\times 10^{-10}\ \text{(F/m)} = 106\ \text{(pF/m)}$$

$$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.37\times 10^{-7}\times 1.06\times 10^{-10}}} = 1.995\times 10^{8}\ \text{(m/s)} \approx 20\ \text{(cm/ns)}$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{2.37\times 10^{-7}}{1.06\times 10^{-10}}} \approx 47.3\ (\Omega)$$