# Chapter 14
# Introduction to Frequency Selective Circuits

# Overview

- We have seen that the response of a circuit depends on the types of elements, the way the elements are connected, and the impedance of the elements that varies with frequency.
- In this chapter, we analyze the effect of varying source frequency on circuit voltages and currents. In particular, the circuits made of passive elements ($R$, $L$, $C$) that pass only a finite range of input frequencies.

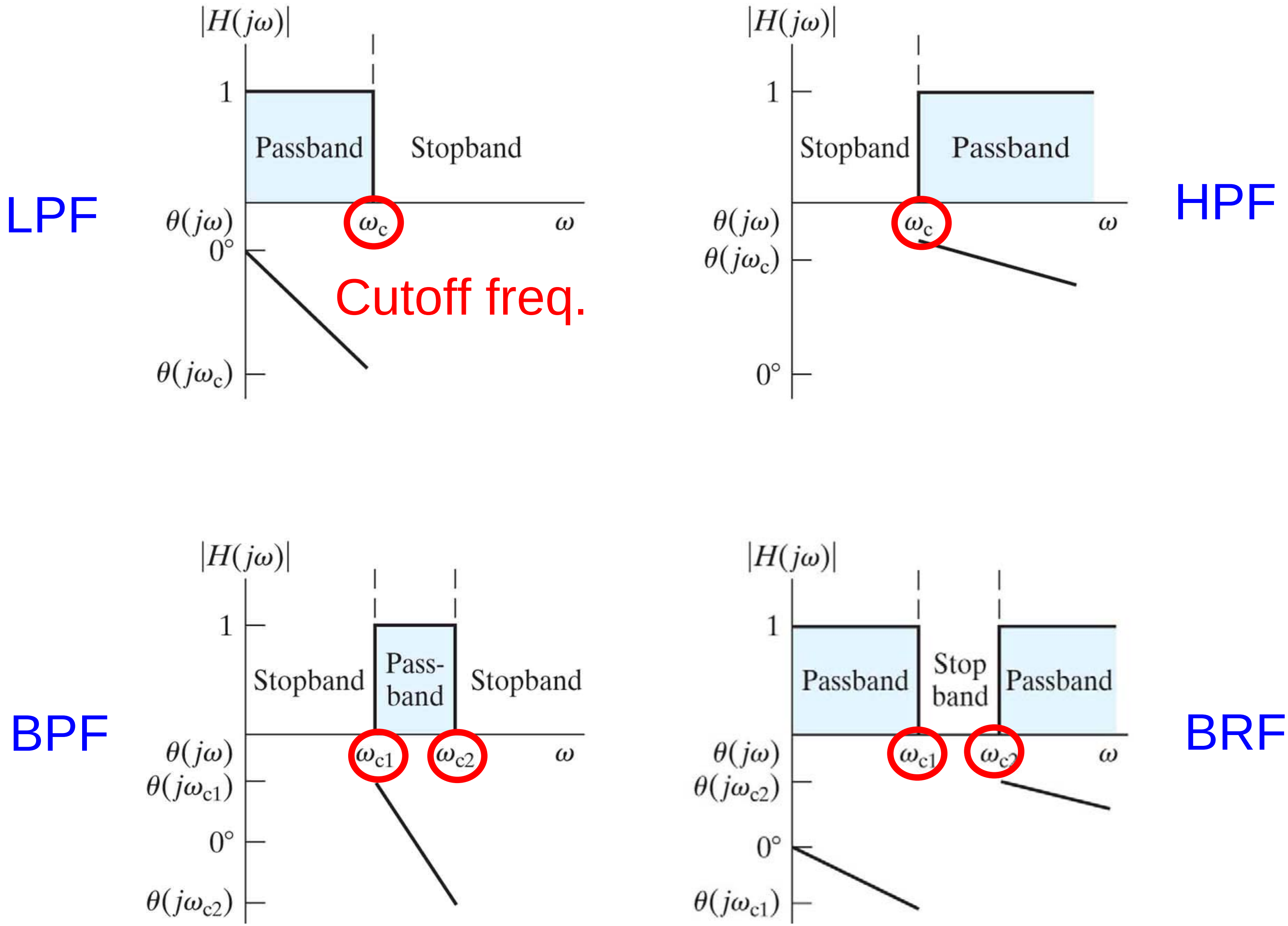
# Section 14.1
# Some Preliminaries

1. Frequency response
2. Four types of filters

# Frequency response plot

- The steady-state response due to a sinusoidal input $A\cos\omega t$ is determined by sampling the transfer function $H(s)$ along the imaginary axis, i.e. $H(j\omega)$.
- Since $H(j\omega) \in C$, a frequency response plot consists of two parts: (1) magnitude plot $|H(j\omega)|$, (2) phase angle plot $\theta(j\omega)$.

# Four types of ideal filters

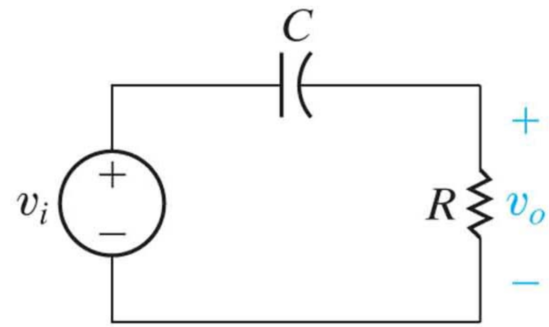


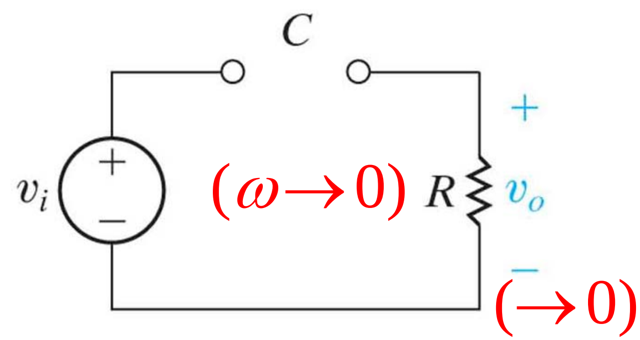
# Section 14.3
# High-pass Filters (HPFs)

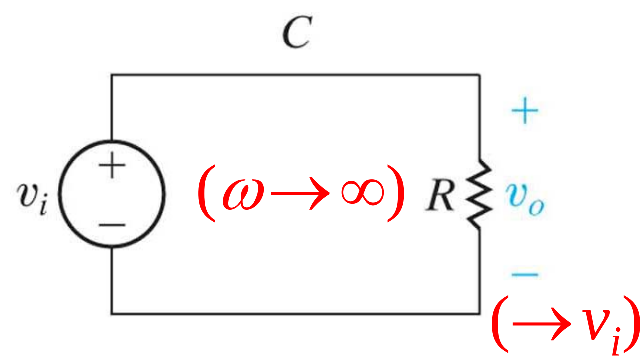
1. Frequency response of HPF
2. Effects of load

# Series RC circuit

- $Z_c = 1/(j\omega C)$.
- $Z_c \to \infty$ as $\omega \to 0$, $\Rightarrow v_o \to 0$ (no current flows through $R$).
- $Z_c \to 0$ as $\omega \to \infty$, $\Rightarrow v_o \to v_i$ (short circuit).
- The input voltage can pass through the circuit if the source frequency is high enough.



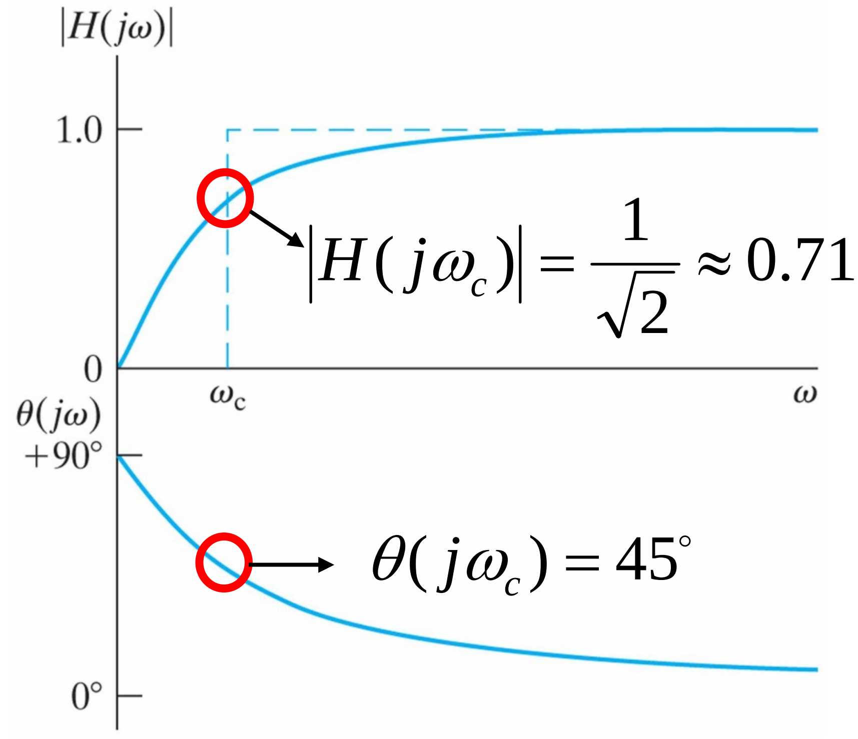
# Frequency response plot of a series RC circuit

- By the equivalent circuit in the s-domain:

$$H(s)=\frac{V_o(s)}{V_i(s)}=\frac{R}{R+(sC)^{-1}}=\frac{s}{s+\omega_c},\ \text{where } \omega_c=\frac{1}{RC}.$$

$$\Rightarrow H(j\omega)=\frac{j\omega}{j\omega+\omega_c},$$

$$\Rightarrow \begin{cases} \left|H(j\omega)\right|=\dfrac{\omega}{\sqrt{\omega^2+\omega_c^2}} \\ \theta(j\omega)=90^\circ-\tan^{-1}(\omega/\omega_c). \end{cases}$$



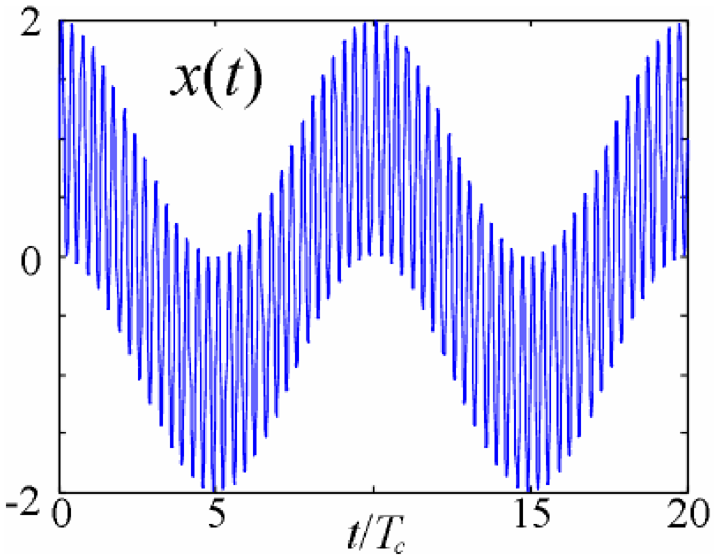
# Example: Superposition of sinusoids

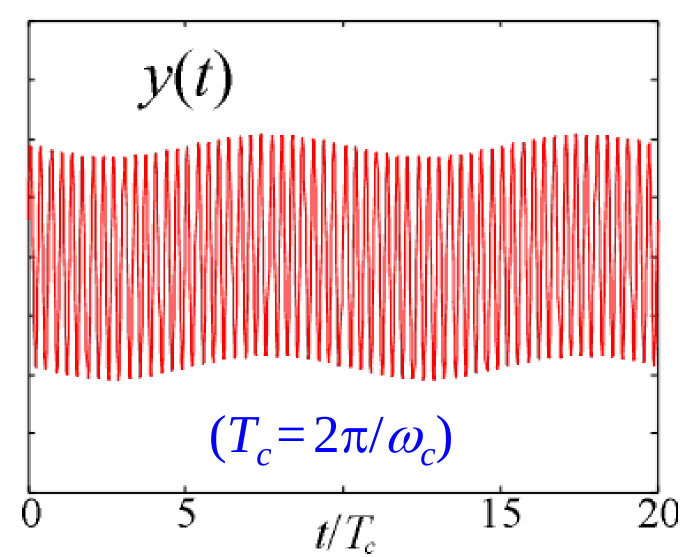
- Let $x(t) = x_1(t) + x_2(t) = \cos(\underbrace{0.1\omega_c}_{\omega_1} t) + \sin(\underbrace{3\omega_c}_{\omega_2} t).$

$$H(j\omega) = \frac{j(\omega/\omega_c)}{j(\omega/\omega_c)+1}, \Rightarrow \begin{cases} H(j\omega_1) \approx 0.10\angle 84^\circ \\ H(j\omega_2) \approx 0.95\angle 18^\circ \end{cases}$$

$$\mathbf{Y_1} \approx 1\angle 0^\circ \times 0.10\angle 84^\circ, \Rightarrow y_1(t) \approx 0.1\cos(0.1\omega_c t + 84^\circ);$$

$$\mathbf{Y_2} \approx 1\angle -90^\circ \times 0.95\angle 18^\circ, \Rightarrow y_2(t) \approx 0.95\cos(3\omega_c t - 72^\circ);$$



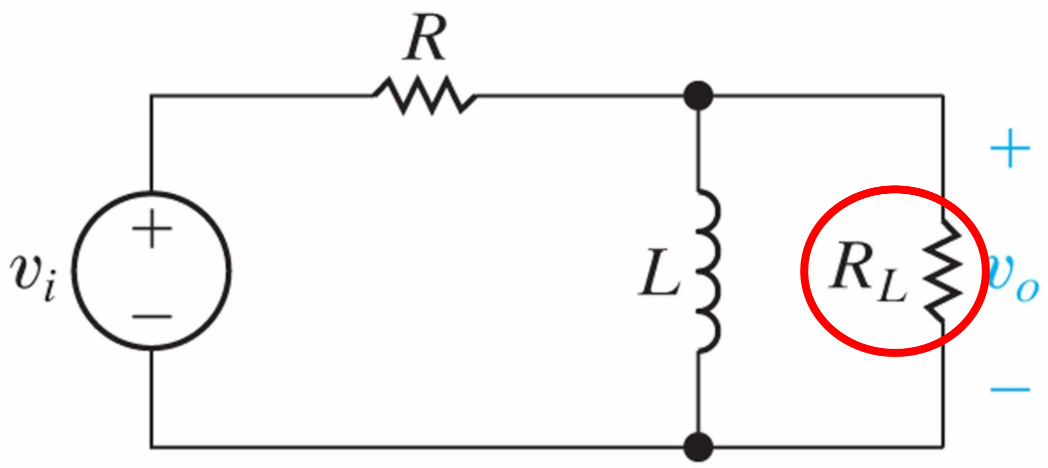
# Example: Filtering in the spatial-frequency domain

- An image is a function of $(x,y)$, which can be decomposed into many “spatial-frequency” components $(f_x, f_y)$. HPF enhances the contrast.

(wikipedia.org)

# Example 14.4: RL circuit with load (1)

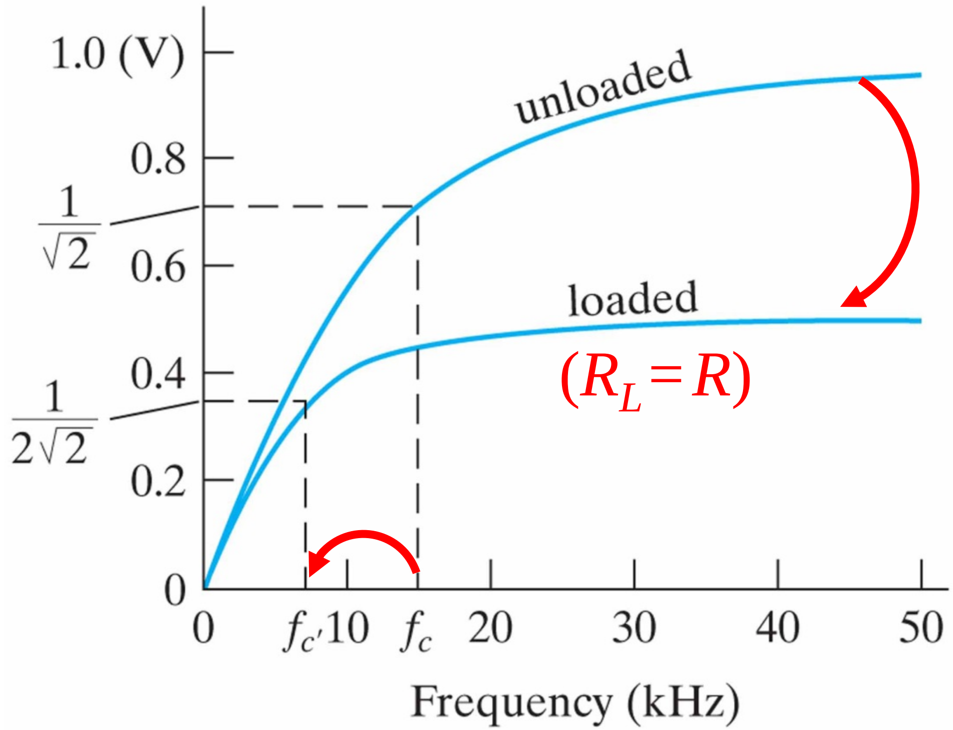
- An RL circuit can be an HPF if $v_o \equiv v_L$. (E.g. 14.3)
- When a load resistor $R_L$ is in parallel with the output inductor $L$:




$$\Rightarrow H(s) = \frac{Ks}{s + K\omega_c}, \ \text{where } \omega_c = \frac{R}{L}, \ K = \frac{1}{1 + (R/R_L)} < 1.$$

# Example 14.4: RL circuit with load (2)

- The effects of $R_L$ are: (1) reducing the passband magnitude by a factor of $K$, (2) lowering the cutoff frequency by the same factor of $K$.



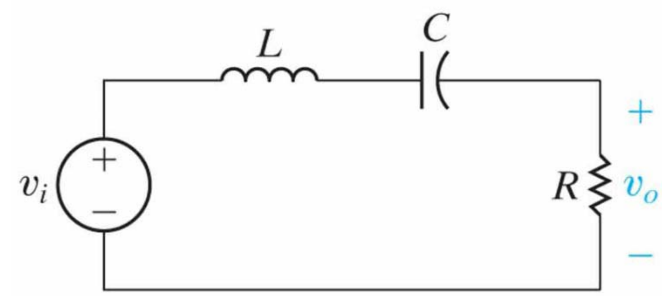

- Problem: the response varies with the load. Solution: use active filters (Ch15).

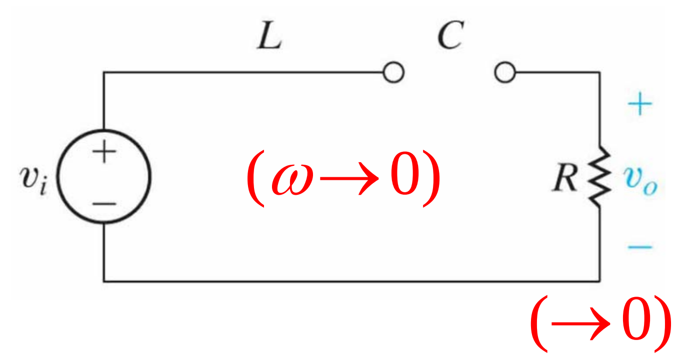
# Section 14.4
# Bandpass Filters (BPFs)

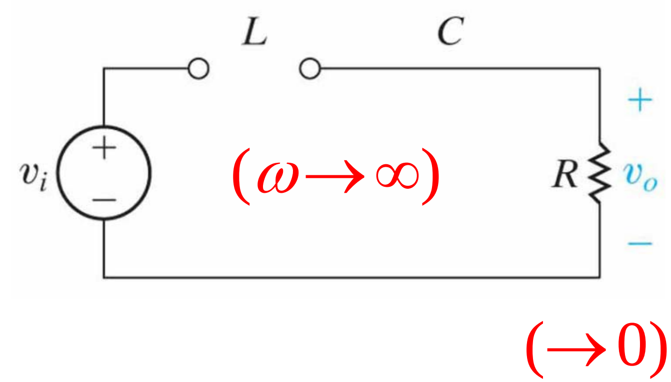
1. Frequency response of BPF
2. Relation with the poles, zeros

# Series RLC circuit

- $Z_c = 1/(j\omega C)$, $Z_L = j\omega L$.
- $Z_c \to \infty$ as $\omega \to 0$, $\Rightarrow v_o \to 0$.
- $Z_L \to \infty$ as $\omega \to \infty$, $\Rightarrow v_o \to 0$.
- At some frequency $\omega_0 \in (0, \infty)$, $Z_c + Z_L = 0$, $v_o = v_i$.
- The input voltage can pass through the circuit if the source frequency is near $\omega_0$.



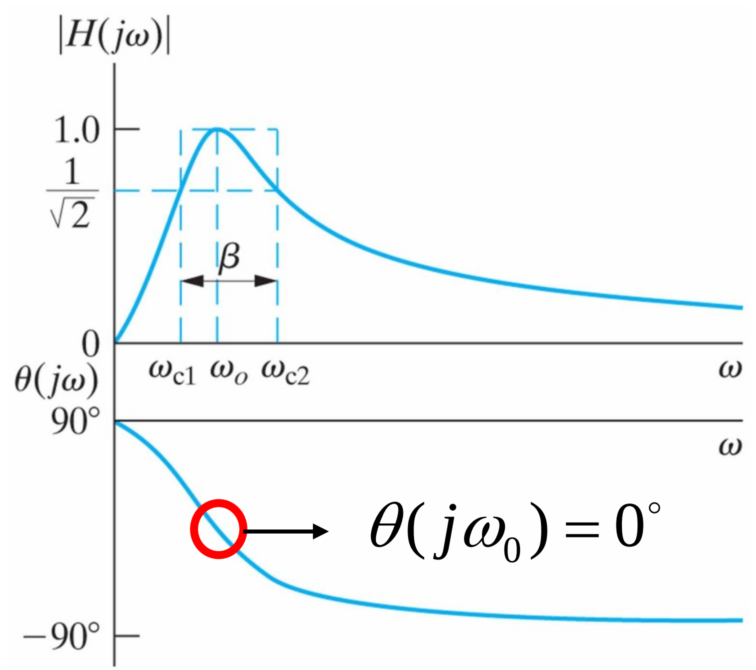



# Frequency response plot of a series RLC circuit

- By the equivalent circuit in the s-domain:

$$H(s) = \frac{R}{sL + (sC)^{-1} + R} = \frac{\beta s}{s^2 + \beta s + \omega_0^2}, \text{ where } \beta = \frac{R}{L},\ \omega_0 = \frac{1}{\sqrt{LC}}.$$

$$\Rightarrow \begin{cases} \left|H(j\omega)\right| = \dfrac{\beta\omega}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + (\beta\omega)^2}} \\ \theta(j\omega) = 90^\circ - \tan^{-1}\left(\dfrac{\beta\omega}{\omega_0^2 - \omega^2}\right). \end{cases}$$



# Calculations of parameters

- Center (resonance) frequency is derived by:

$$j\omega L + \frac{1}{j\omega C} = 0, \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}.$$

- Cutoff frequencies are derived by:

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{2}}, \Rightarrow \omega_{c1,c2} = \sqrt{\left(\frac{\beta}{2}\right)^2 + \omega_0^2} \mp \frac{\beta}{2}.$$

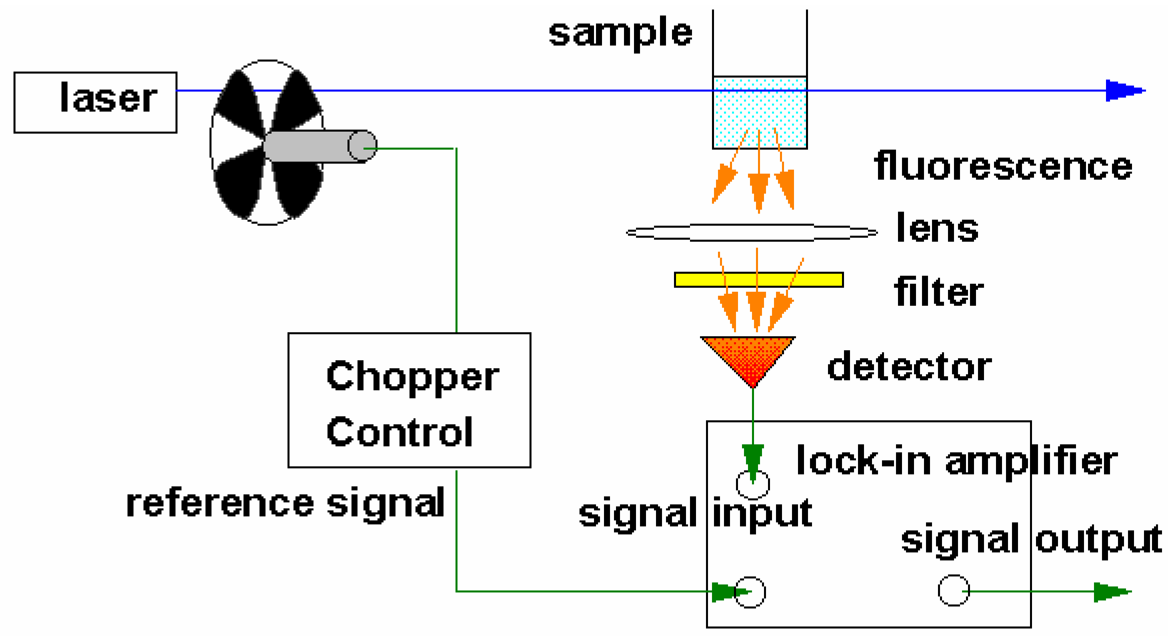
$\Rightarrow$ Bandwidth $\omega_{c2} - \omega_{c1} = \beta = \dfrac{R}{L}; \quad \omega_0 = \sqrt{\omega_{c1}\omega_{c2}} \neq \dfrac{\omega_{c1} + \omega_{c2}}{2}.$

- Quality factor (~ inversed relative bandwidth) is:

$$\boxed{Q \equiv \frac{\omega_0}{\beta}} = \frac{\sqrt{L/C}}{R}.$$

## Example: White-noise rejection

- If a signal of specific frequency $f_1$ is buried in strong "white" noise, a BPF centered at $f_1$ can be used to extract the signal.



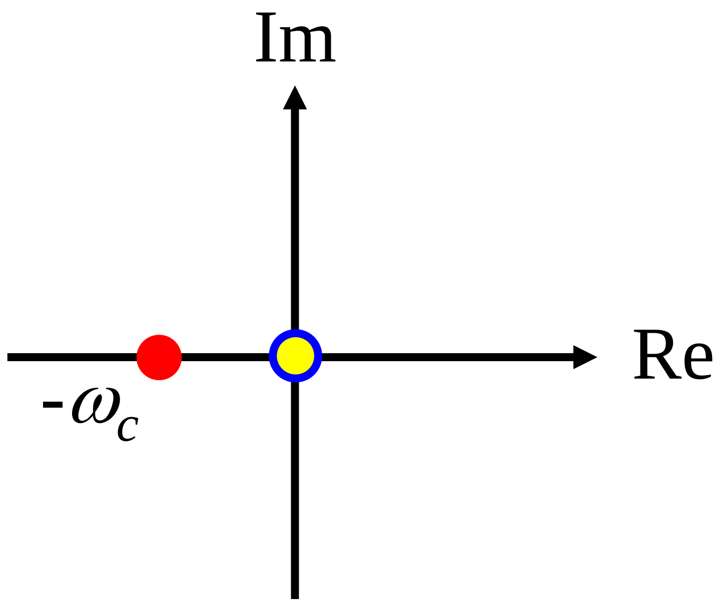

(www.chem.vt.edu)

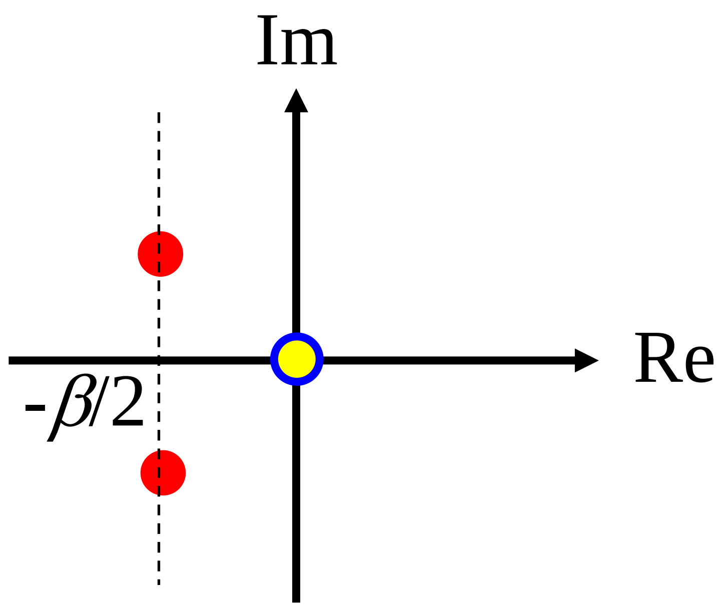
# Frequency response visualized by poles, zeros

- The poles and zeros of HPF and BPF are:

$$H_{HPF}(s) = \frac{s}{s+\omega_c}, \Rightarrow p = -\omega_c, z = 0.$$

$$H_{BPF}(s) = \frac{\beta s}{s^2+\beta s+\omega_0^2}, \Rightarrow p = \frac{-\beta \pm \sqrt{\beta^2 - 4\omega_0^2}}{2}, \ z = 0.$$



# Practical Perspective
# Pushbutton Telephone Circuits

# Application of BPFs

- Q: How to tell which button was pushed? How to tell the difference between the button tones and the normal vocal sounds (both are within 300-3000 Hz) or ringing bell tones (20 Hz)?

(jouielovesyou.wordpress.com)


(wikipedia.org)

# Dual-tone-multiple-frequency (DTMF)

- Each bottom is encoded with a unique pair of sinusoidal tones ($f_L$, $f_H$), where the relative timing and amplitudes must be close enough.

"High Group" frequencies [Hz]

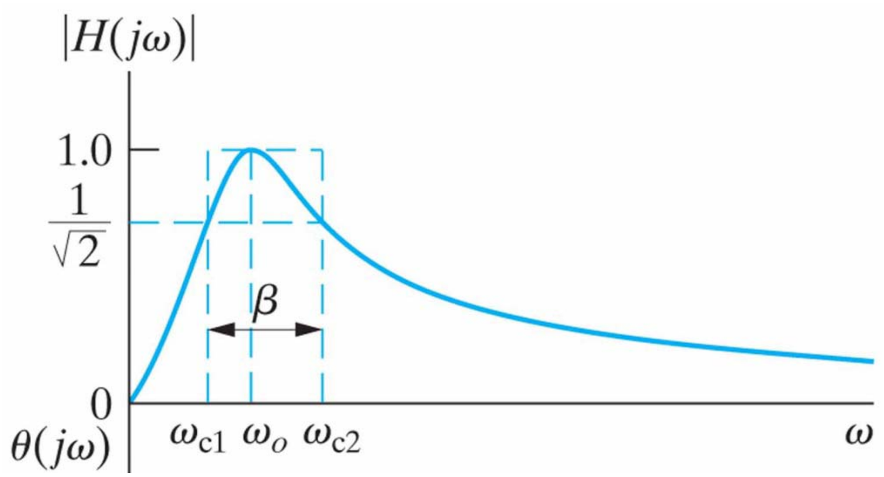
| "Low Group" frequencies [Hz] | 1209 | 1336 | 1477 | 1633 | |
|---|---|---|---|---|---|
| 697 | 1 | 2 | 3 | A | (Row 1) |
| 770 | 4 | 5 | 6 | B | (Row 2) |
| 852 | 7 | 8 | 9 | C | (Row 3) |
| 941 | * | 0 | # | D | (Row 4) |
| | (Column 1) | (Column 2) | (Column 3) | (Column 4) | |

(sohilp.blogspot.com)

- BPFs: (1) identify whether both freq. groups are present, (2) select among possible tones.

# BPF design for the low-frequency group

- The transmission spectrum of a series RLC is:




$$|H(j\omega)| = \frac{\beta\omega}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+(\beta\omega)^2}},$$

$$\text{where } \beta = \frac{R}{L},\ \ \omega_0 = \frac{1}{\sqrt{LC}}.$$

- For the low-freq. group, $\omega_{c1} = 2\pi(697) = 4379$ rad/s, $\omega_{c2} = 2\pi(941) = 5912$ rad/s, $\Rightarrow \beta = \omega_{c2} - \omega_{c1} = 1533$ rad/s.

- $\beta = R/L, \ \Rightarrow L = R/\beta = (600\,\Omega)/1533 = 0.39$ H.

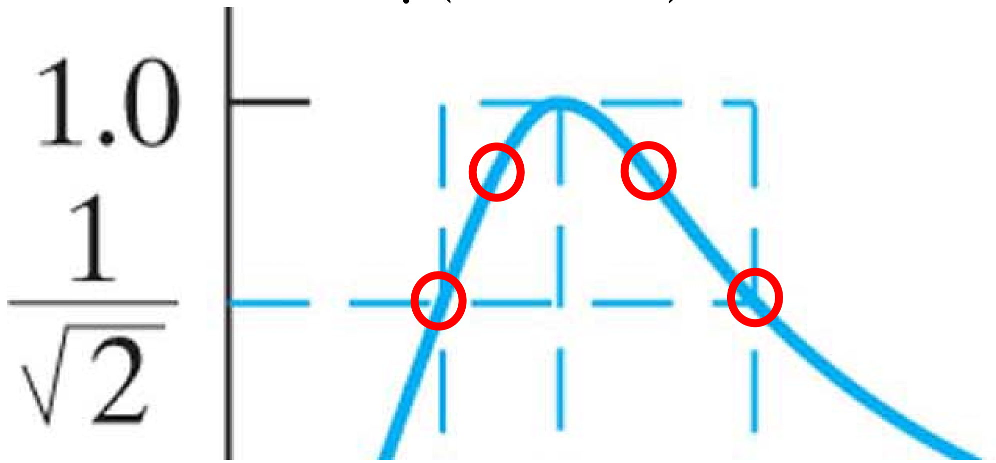
  $\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = 1/\sqrt{LC}, \Rightarrow C = 1/(L\omega_{c1}\omega_{c2}) = 100$ nF.

# BPF performance (1)

- The transmission of the 4 frequencies in the low-frequency group are imperfect (<1):

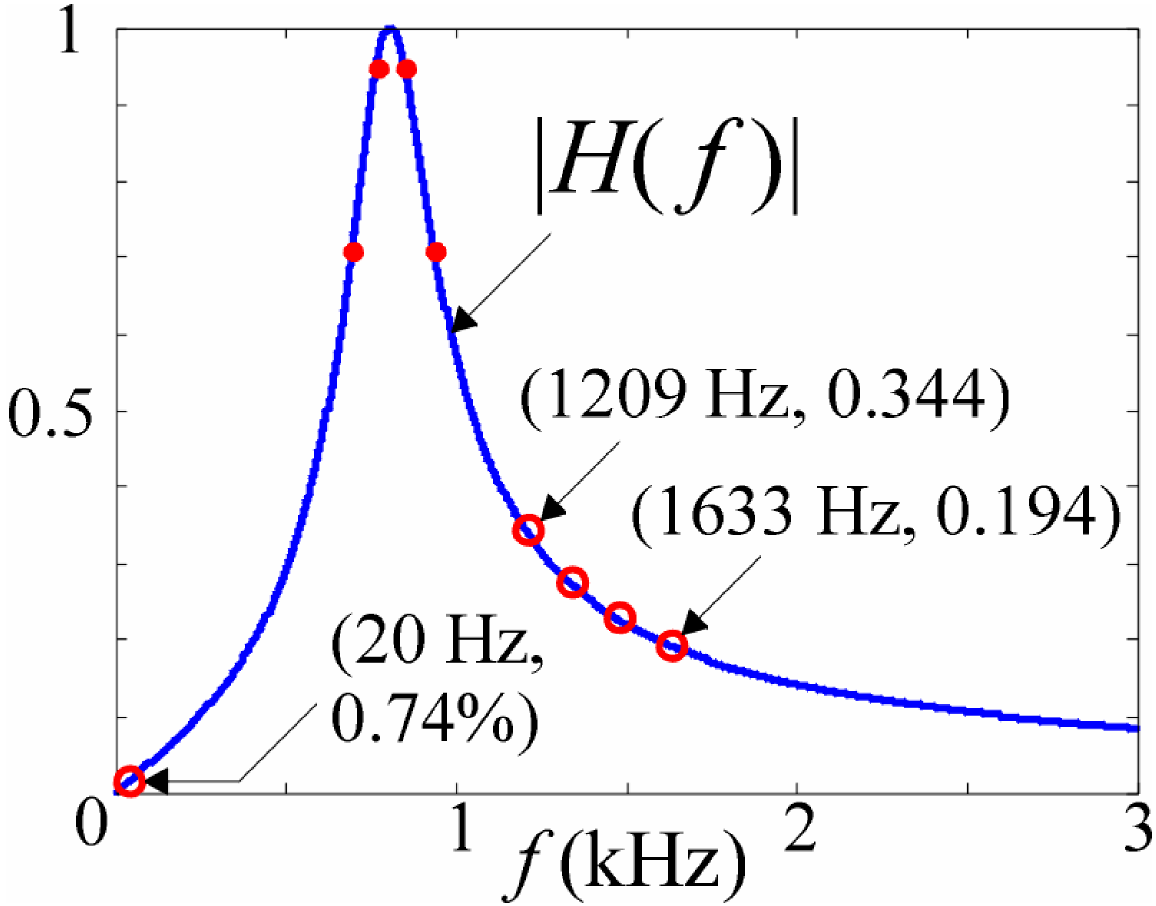
$$\left|H_{697\mathrm{Hz}}\right| = \left|H_{941\mathrm{Hz}}\right| = \frac{1}{\sqrt{2}} = 0.707,$$

$$\left|H_{770\mathrm{Hz}}\right| = \left|H_{852\mathrm{Hz}}\right| = \frac{\beta\omega}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+(\beta\omega)^2}} = 0.948.$$



# BPF performance (2)

- The transmission of the high-frequency group tones and the 20 Hz ringing tone are imperfect (>0):



# BPF design for the high-frequency group

- The transmission spectrum of a series RLC is:

$$\left|H(j\omega)\right| = \frac{\beta\omega}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + (\beta\omega)^2}}, \quad \beta = \frac{R}{L}, \ \omega_0 = \frac{1}{\sqrt{LC}}.$$

- For the high-freq. group, $\omega_{c1} = 2\pi(1209) = 7596$ rad/s, $\omega_{c2} = 2\pi(1633) = 10260$ rad/s, $\Rightarrow \beta = \omega_{c2} - \omega_{c1} =$ 2664 rad/s.

- Both L and C values are smaller:

$$\beta = R/L, \ \Rightarrow L = R/\beta = (600\,\Omega)/2664 = 0.26\text{ H}.$$

$$\omega_0 = \sqrt{\omega_{c1}\omega_{c2}} = 1/\sqrt{LC}, \Rightarrow C = 1/(L\omega_{c1}\omega_{c2}) = 57\text{ nF}.$$

# BPF performance

- The transmission of the low-frequency group tones and the 20 Hz ringing tone are imperfect (>0):