Chapter 8
Natural and Step Responses of RLC Circuits

8.1-2 The Natural Response of a Parallel *RLC* Circuit

8.3 The Step Response of a Parallel *RLC* Circuit

8.4 The Natural and Step Response of a Series *RLC* Circuit

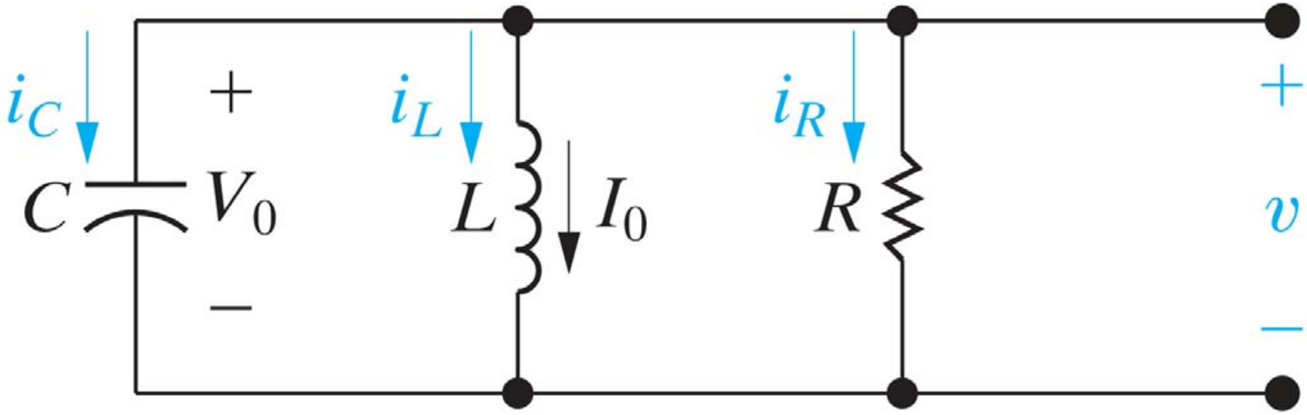
Key points

- What do the response curves of over-, under-, and critically-damped circuits look like? How to choose R, L, C values to achieve fast switching or to prevent overshooting damage?
- What are the initial conditions in an RLC circuit? How to use them to determine the expansion coefficients of the complete solution?
- Comparisons between: (1) natural & step responses, (2) parallel, series, or general RLC.

Section 8.1, 8.2
The Natural Response of a Parallel RLC Circuit

1. ODE, ICs, general solution of parallel voltage
2. Over-damped response
3. Under-damped response
4. Critically-damped response

The governing ordinary differential equation (ODE)



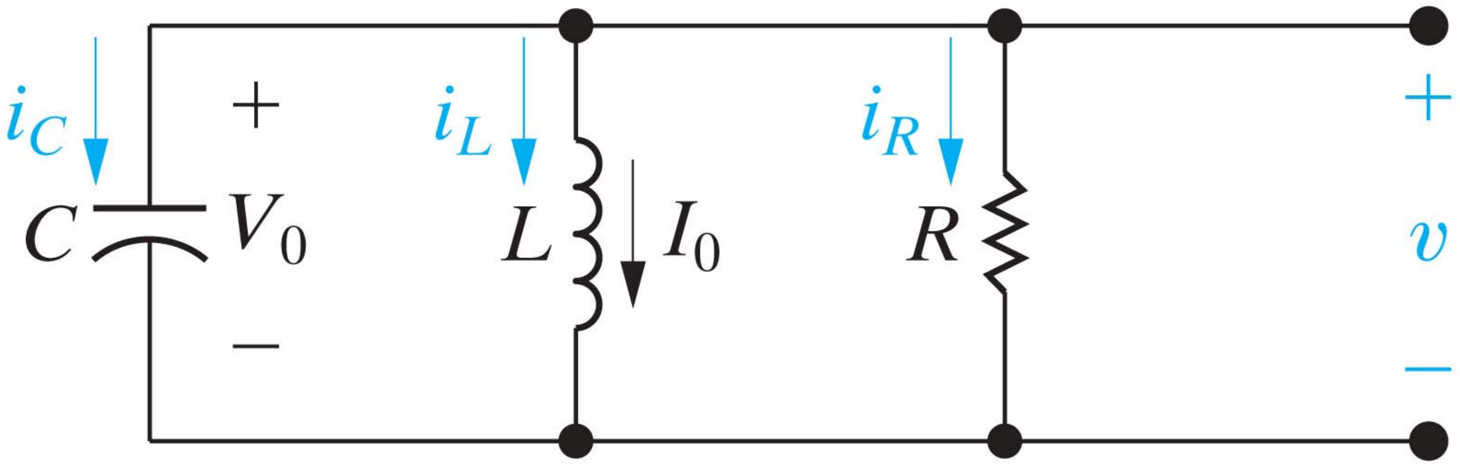
V_0, I_0, $v(t)$ must satisfy the passive sign convention.

- By KCL: $C\dfrac{dv}{dt}+\left[I_0+\dfrac{1}{L}\int_0^t v(t')dt'\right]+\dfrac{v}{R}=0.$

- Perform time derivative, we got a linear 2nd-order ODE of $v(t)$ with constant coefficients:

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0.$$

The two initial conditions (ICs)



- The capacitor voltage cannot change abruptly,

$$\Rightarrow v(0^+) = V_0 \cdots (1)$$

- The inductor current cannot change abruptly,

$$\Rightarrow i_L(0^+) = I_0,\ i_C(0^+) = -i_L(0^+) - i_R(0^+) = -I_0 - V_0/R,$$

$$\because i_C(0^+) = C\frac{dv_C}{dt}\bigg|_{t=0^+},\ \Rightarrow v_C'(0^+) = v'(0^+) = -\frac{I_0}{C} - \frac{V_0}{RC} \cdots (2)$$

General solution

- Assume the solution is $v(t) = Ae^{st}$, where A, s are unknown constants to be solved.
- Substitute into the ODE, we got an algebraic (characteristic) equation of s determined by the circuit parameters:

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0.$$

- Since the ODE is linear, $\Rightarrow$ linear combination of solutions remains a solution to the equation. The general solution of $v(t)$ must be of the form:

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t},$$

where the expansion constants A_1, A_2 will be determined by the two initial conditions.

Neper and resonance frequencies

- In general, s has two roots, which can be (1) distinct real, (2) degenerate real, or (3) complex conjugate pair.

$$s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2},$$

where

$$\alpha = \frac{1}{2RC},$$ ...neper frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$ ...resonance (natural) frequency

Three types of natural response

- How the circuit reaches its steady state depends on the relative magnitudes of α and ω_0:

The Circuit is	When	Solutions
Over-damped	$\alpha > \omega_0$	real, distinct roots s_1, s_2
Under-damped	$\alpha < \omega_0$	complex roots $s_1 = (s_2)^*$
Critically-damped	$\alpha = \omega_0$	real, equal roots $s_1 = s_2$

Over-damped response $(\alpha > \omega_0)$

- The complete solution and its derivative are of the form:

$$\begin{cases} v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}, \\ v'(t) = A_1 s_1 e^{s_1 t} + A_2 s_2 e^{s_2 t}. \end{cases}$$

where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ are distinct real.

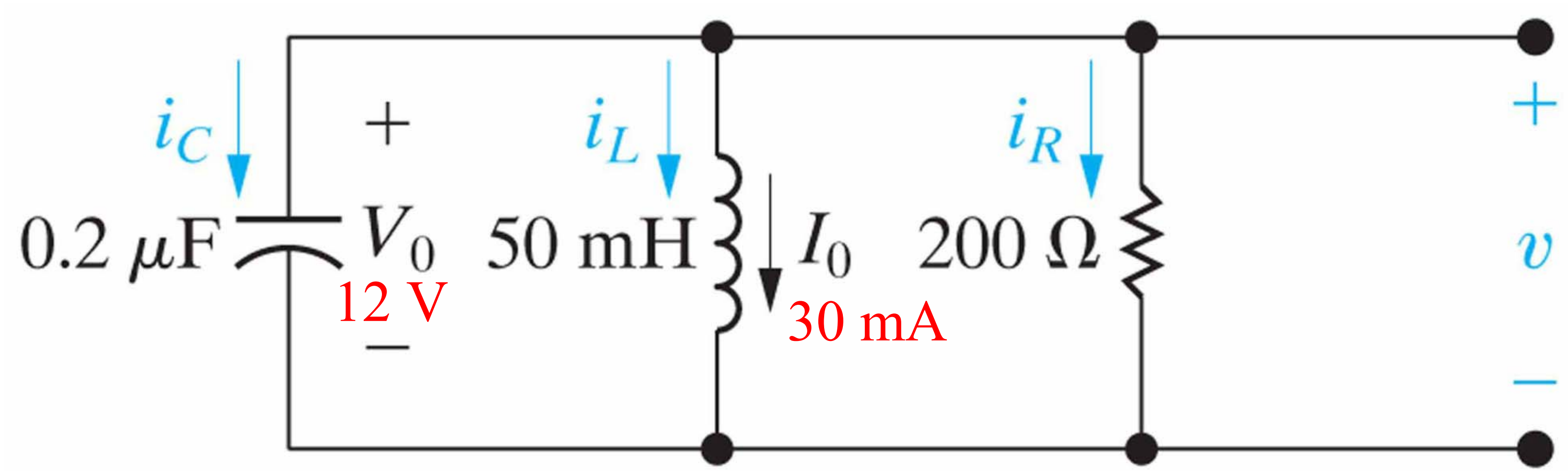
- Substitute the two ICs:

$$\begin{cases} v(0^+) = A_1 + A_2 = V_0 \cdots (1) \\ v'(0^+) = s_1 A_1 + s_2 A_2 = -\dfrac{I_0}{C} - \dfrac{V_0}{RC} \cdots (2) \end{cases}$$

$\Rightarrow$ solve A_1, A_2.

Example 8.2: Discharging a parallel RLC circuit (1)

- Q: $v(t),\ i_C(t),\ i_L(t),\ i_R(t) = ?$



$$\begin{cases} \alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(2\times 10^{-7})} = 12.5\ \text{kHz}, \\ \omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(5\times 10^{-2})(2\times 10^{-7})}} = 10\ \text{kHz}. \end{cases}$$

$\Rightarrow \alpha > \omega_0$, over-damped

Example 8.2: Solving the parameters (2)

- The 2 distinct real roots of s are:

$$\begin{cases} s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -5\,\text{kHz}, & \ldots |s_1| < \alpha \text{ (slow)} \\ s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} = -20\,\text{kHz}. & \ldots |s_2| > \alpha \text{ (fast)} \end{cases}$$

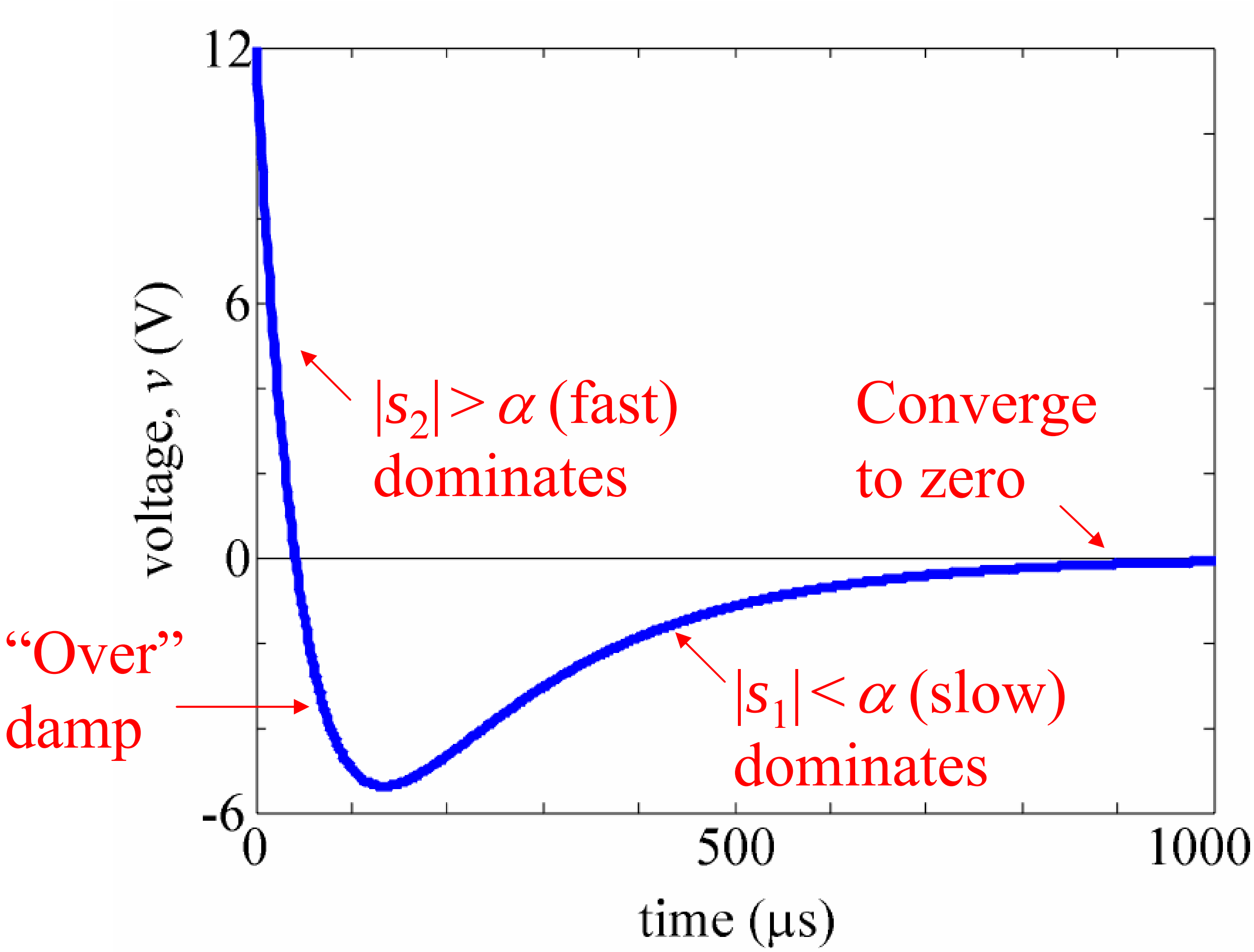
- The 2 expansion coefficients are:

$$\begin{cases} A_1 + A_2 = V_0 \\ s_1 A_1 + s_2 A_2 = -\dfrac{I_0}{C} - \dfrac{V_0}{RC} \end{cases} \Rightarrow \begin{cases} A_1 + A_2 = 12 \\ -5A_1 - 20A_2 = -450 \end{cases}$$

$$\Rightarrow A_1 = -14\,\text{V}, A_2 = 26\,\text{V}.$$

Example 8.2: The parallel voltage evolution (3)

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} = \left(-14e^{-5000t} + 26e^{-20000t}\right)\text{V}.$$



Example 8.2: The branch currents evolution (4)

- The branch current through R is:

$$i_R(t) = \frac{v(t)}{200\,\Omega} = \left(-70e^{-5000t} + 130e^{-20000t}\right)\text{mA}.$$

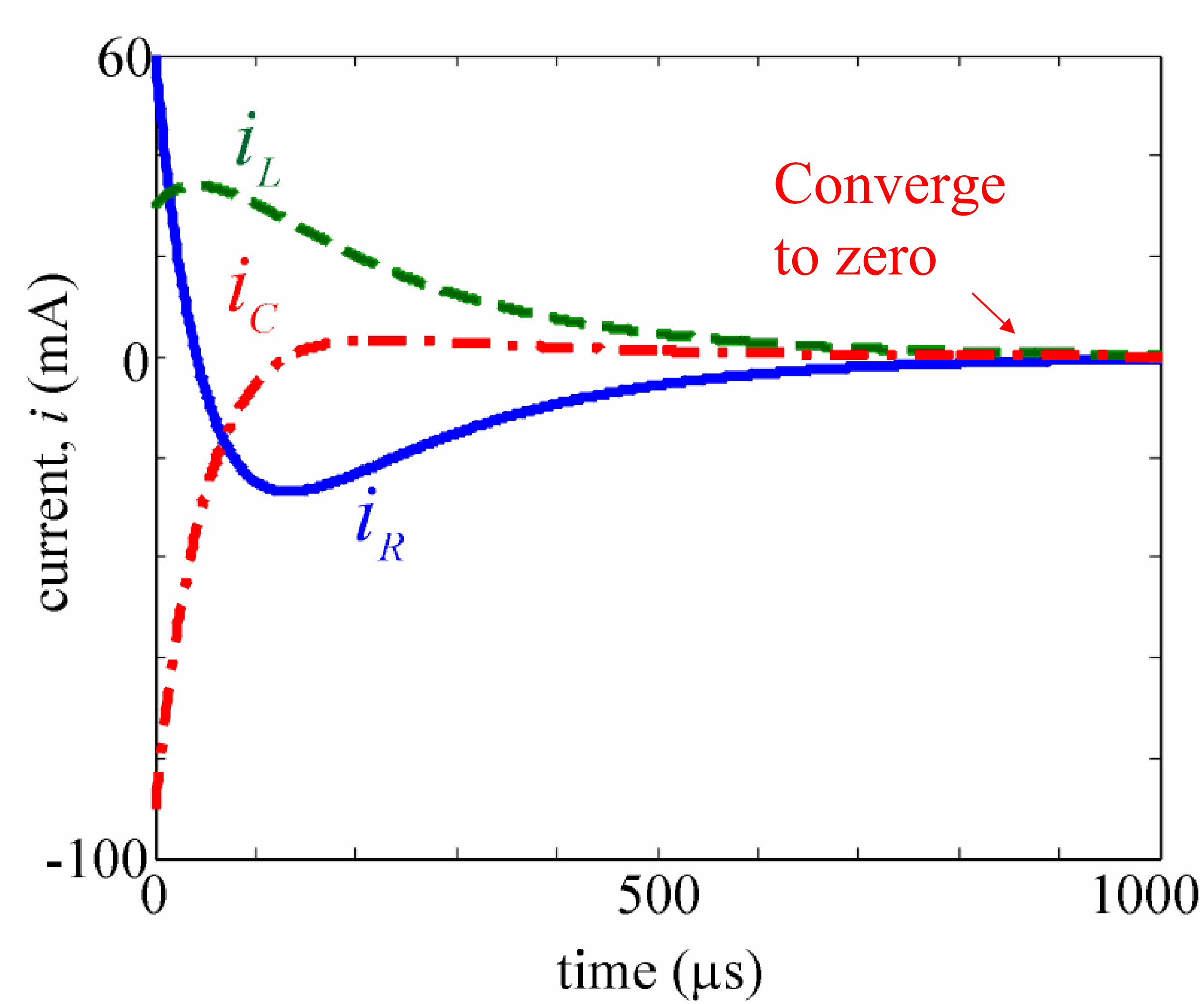
- The branch current through L is:

$$i_L(t) = 30\text{ mA} + \frac{1}{50\text{ mH}}\int_0^t v(t')dt' = \left(56e^{-5000t} - 26e^{-20000t}\right)\text{mA}.$$

- The branch current through C is:

$$i_C(t) = (0.2\ \mu\text{F})\frac{dv}{dt} = \left(14e^{-5000t} - 104e^{-20000t}\right)\text{mA}.$$

Example 8.2: The branch currents evolution (5)



General solution to under-damped response ($\alpha < \omega_0$)

- The two roots of s are complex conjugate pair:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -\alpha \pm j\omega_d ,$$

where $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$ is the damped frequency.

- The general solution is reformulated as:

$$v(t) = A_1 e^{(-\alpha + j\omega_d)t} + A_2 e^{(-\alpha - j\omega_d)t}$$
$$= e^{-\alpha t}\left[A_1\left(\cos\omega_d t + j\sin\omega_d t\right) + A_2\left(\cos\omega_d t - j\sin\omega_d t\right)\right]$$
$$= e^{-\alpha t}\left[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t\right]$$
$$= e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right).$$

Solving the expansion coefficients B_1, B_2 by ICs

- The derivative of $v(t)$ is:

$$v'(t) = B_1\left(-\alpha e^{-\alpha t}\cos\omega_d t - \omega_d e^{-\alpha t}\sin\omega_d t\right)$$
$$+ B_2\left(-\alpha e^{-\alpha t}\sin\omega_d t + \omega_d e^{-\alpha t}\cos\omega_d t\right)$$
$$= e^{-\alpha t}\left[\left(-\alpha B_1 + \omega_d B_2\right)\cos\omega_d t - \left(\alpha B_2 + \omega_d B_1\right)\sin\omega_d t\right].$$

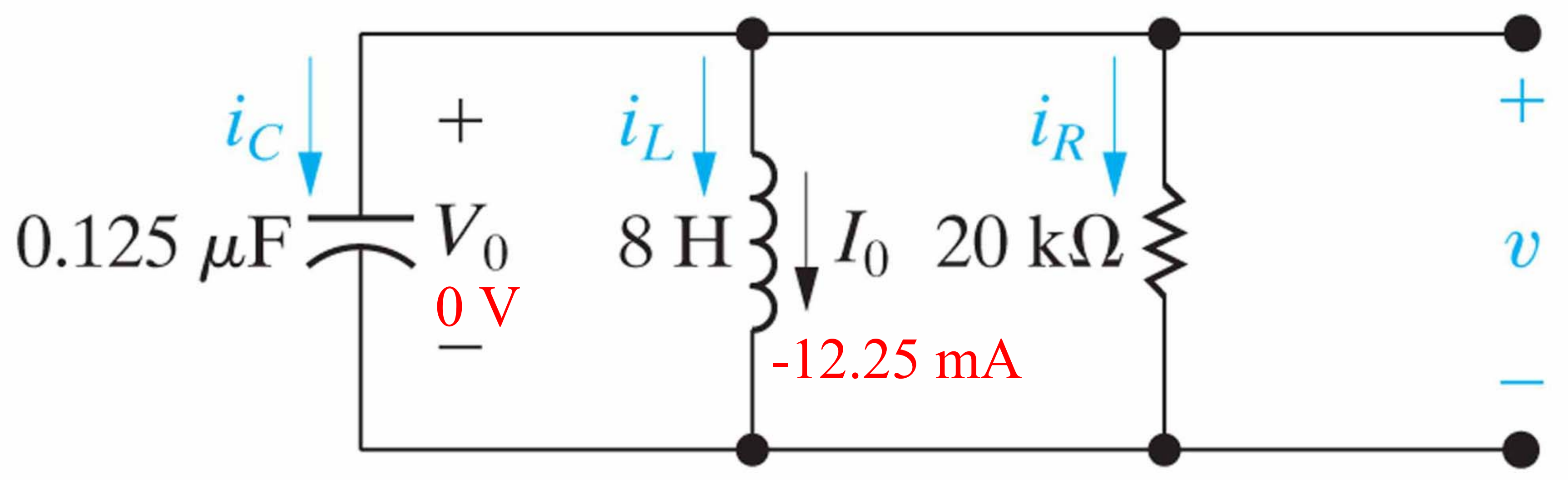
- Substitute the two ICs:

$$\begin{cases} v(0^+) = B_1 = V_0 \cdots (1) \\ v'(0^+) = -\alpha B_1 + \omega_d B_2 = -\dfrac{I_0}{C} - \dfrac{V_0}{RC} \cdots (2) \end{cases}$$

$\Rightarrow$ solve B_1, B_2.

Example 8.4: Discharging a parallel RLC circuit (1)

- Q: $v(t)$, $i_C(t)$, $i_L(t)$, $i_R(t) = ?$



$$\begin{cases} \alpha = \dfrac{1}{2RC} = \dfrac{1}{2(2\times 10^4)(1.25\times 10^{-7})} = 0.2\text{ kHz}, \\ \omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(8)(1.25\times 10^{-7})}} = 1\text{ kHz}. \end{cases}$$

$\Rightarrow \alpha < \omega_0$, under-damped

Example 8.4: Solving the parameters (2)

- The damped frequency is:

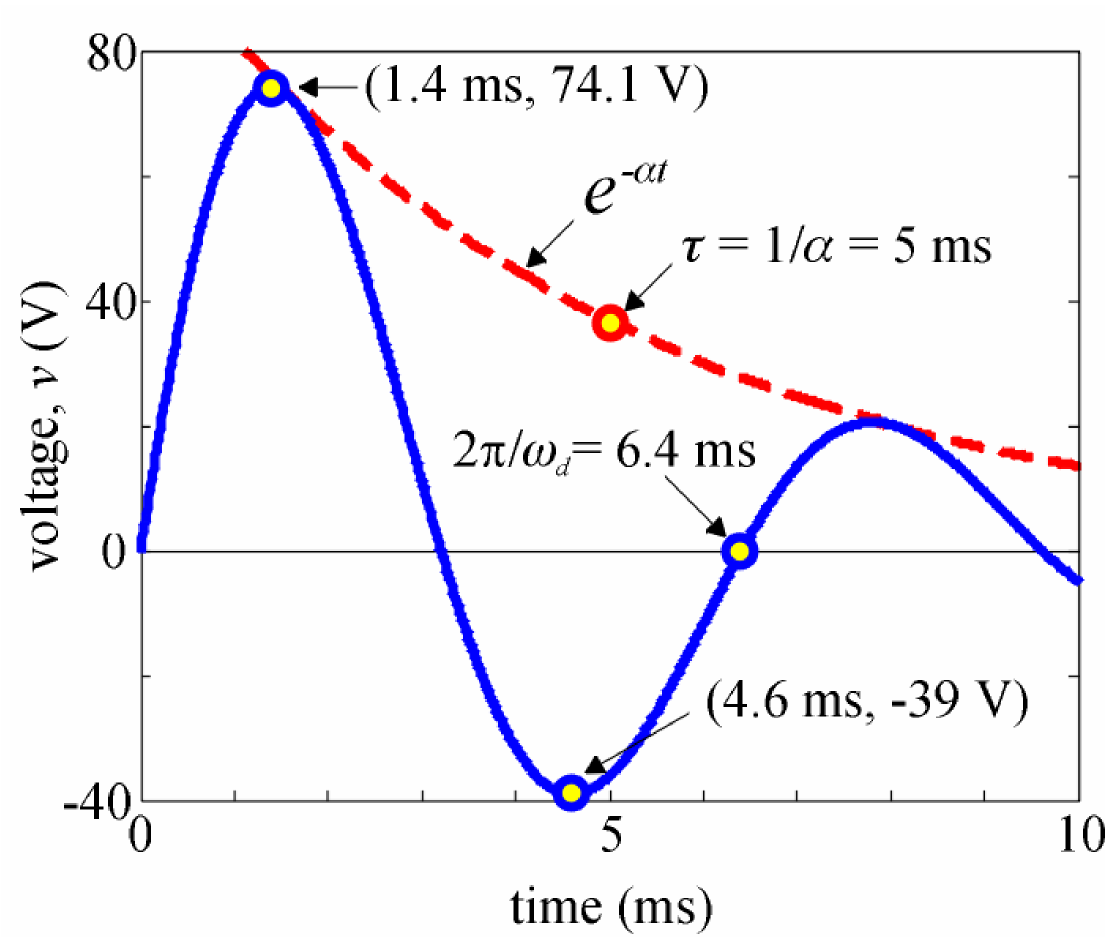
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{1^2 - 0.2^2} \approx 0.98\,\text{kHz}.$$

- The 2 expansion coefficients are:

$$\begin{cases} B_1 = V_0 = 0 \cdots (1) \\ -\alpha B_1 + \omega_d B_2 = -\dfrac{I_0}{C} - \dfrac{V_0}{RC} \cdots (2) \end{cases} \Rightarrow \begin{cases} B_1 = 0, \\ B_2 \approx 100\text{ V} \end{cases}$$

Example 8.4: The parallel voltage evolution (3)

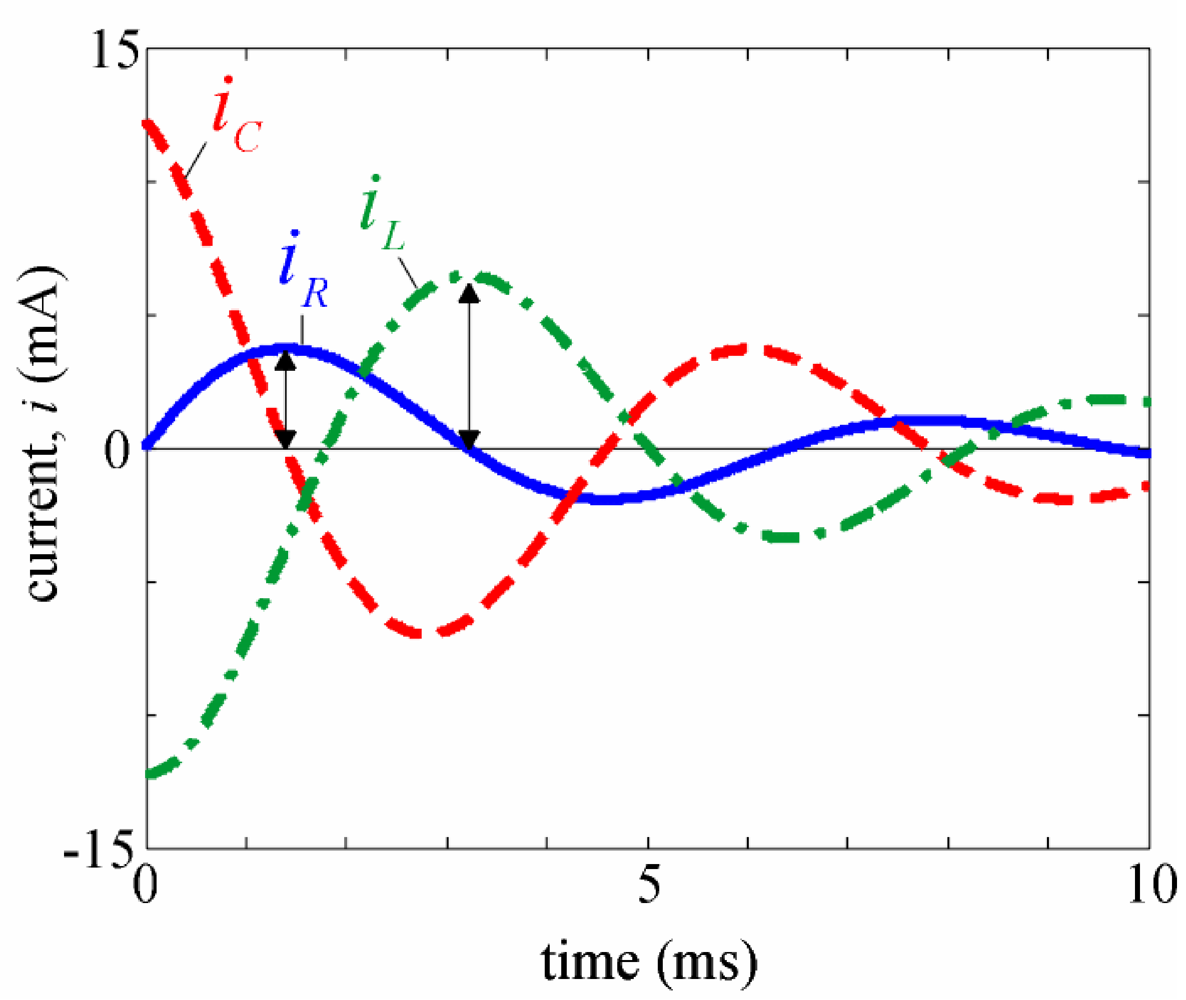
$$v(t) = B_1 e^{-\alpha t}\cos\omega_d t + B_2 e^{-\alpha t}\sin\omega_d t \approx \left(100e^{-200t}\sin 980t\right)\ \text{V}.$$



- The voltage oscillates ($\sim\omega_d$) and approaches the final value ($\sim\alpha$), different from the over-damped case (no oscillation, 2 decay constants).

Example 8.4: The branch currents evolutions (4)

- The three branch currents are:



Rules for circuit designers

- If one desires the circuit reaches the final value as fast as possible while the minor oscillation is of less concern, choosing R, L, C values to satisfy under-damped condition.
- If one concerns that the response not exceed its final value to prevent potential damage, designing the system to be over-damped at the cost of slower response.

General solution to critically-damped response ($\alpha = \omega_0$)

- Two identical real roots of s make

$$v(t) = A_1 e^{st} + A_2 e^{st} = (A_1 + A_2)e^{st} = A_0 e^{st},$$

not possible to satisfy 2 independent ICs (V_0, I_0) with a single expansion constant A_0.

- The general solution is reformulated as:

$$v(t) = e^{-\alpha t}\left(D_1 t + D_2\right).$$

- You can prove the validity of this form by substituting it into the ODE:

$$v''(t) + (RC)^{-1} v'(t) + (LC)^{-1} v(t) = 0.$$

Solving the expansion coefficients D_1, D_2 by ICs

- The derivative of $v(t)$ is:

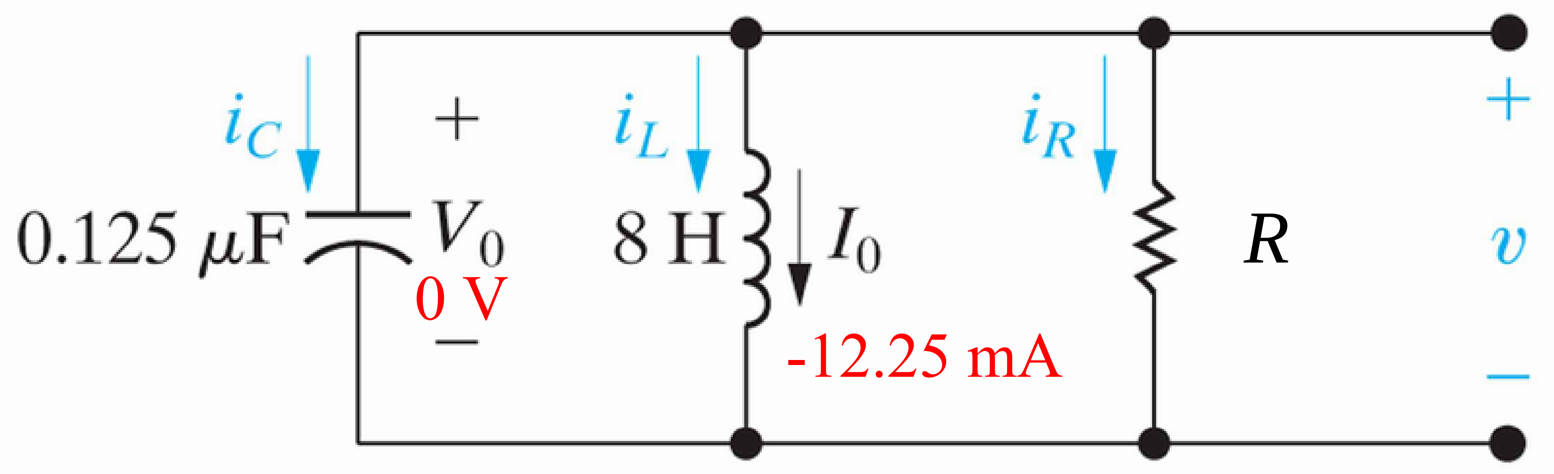
$$v'(t) = D_1\left(e^{-\alpha t} - \alpha t e^{-\alpha t}\right) - \alpha D_2 e^{-\alpha t} = \left[\left(D_1 - \alpha D_2\right) - \alpha D_1 t\right] e^{-\alpha t}.$$

- Substitute the two ICs:

$$\begin{cases} v(0^+) = D_2 = V_0 \cdots (1) \\ v'(0^+) = D_1 - \alpha D_2 = -\dfrac{I_0}{C} - \dfrac{V_0}{RC} \cdots (2) \end{cases} \Rightarrow \text{solve } D_1,\ D_2.$$

Example 8.5: Discharging a parallel RLC circuit (1)

- Q: What is R such that the circuit is critically-damped? Plot the corresponding $v(t)$.



$$\alpha = \omega_0, \ \frac{1}{2RC} = \frac{1}{\sqrt{LC}}, \ R = \frac{1}{2}\sqrt{\frac{L}{C}} = \frac{1}{2}\sqrt{\frac{8}{1.25\times10^{-7}}} = 4\ \mathrm{k}\Omega.$$

- Increasing R tends to bring the circuit from over- to critically- and even under-damped.

Example 8.5: Solving the parameters (2)

- The neper frequency is:

$$\alpha = \frac{1}{2RC} = \frac{1}{2(4\times 10^3)(1.25\times 10^{-7})} = 1\,\text{kHz},$$

$$\Rightarrow \tau = \frac{1}{\alpha} = 1\,\text{ms}.$$

- The 2 expansion coefficients are:

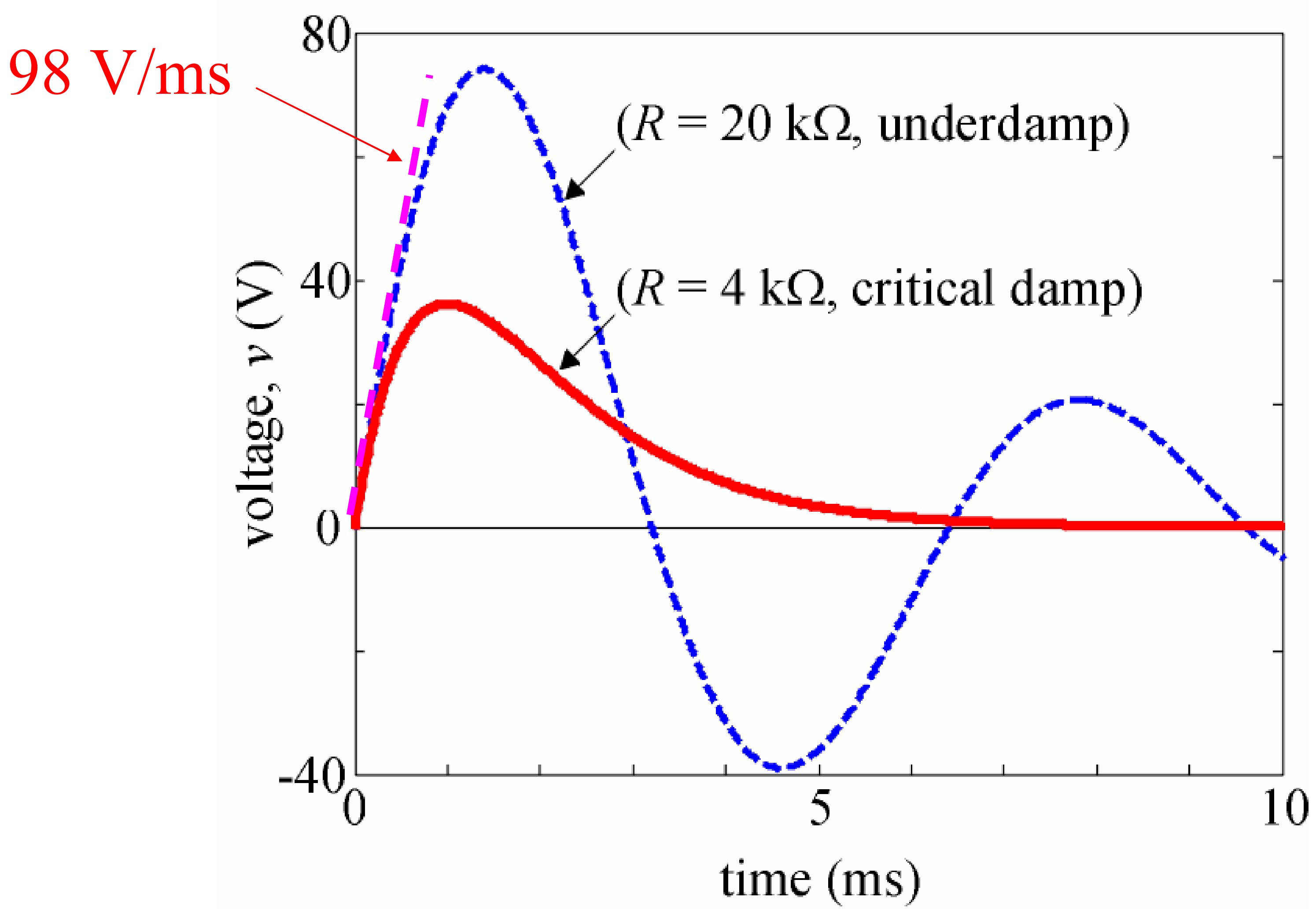
$$\begin{cases} D_2 = V_0 = 0 \cdots (1) \\ D_1 - \alpha D_2 = -\frac{I_0}{C} - \frac{V_0}{RC} \cdots (2) \end{cases} \Rightarrow \begin{cases} D_1 = 98\,\text{kV/s} \\ D_2 = 0 \end{cases}$$

-12.25 mA

0.125 μF

Example 8.5: The parallel voltage evolution (3)

$$v(t) = D_1 te^{-\alpha t} + D_2 e^{-\alpha t} = \left(98{,}000te^{-1000t}\right)\text{V}.$$



Procedures of solving nature response of parallel RLC

- Calculate parameters $\alpha = (2RC)^{-1}$ and $\omega_0 = 1/\sqrt{LC}$.
- Write the form of $v(t)$ by comparing α and ω_0:

$$v(t) = \begin{cases} A_1 e^{s_1 t} + A_2 e^{s_2 t},\ s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2},\ \text{if } \alpha > \omega_0, \\ e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right),\ \omega_d = \sqrt{\omega_0^2 - \alpha^2},\ \text{if } \alpha < \omega_0, \\ e^{-\alpha t}\left(D_1 t + D_2\right),\ \text{if } \alpha = \omega_0. \end{cases}$$

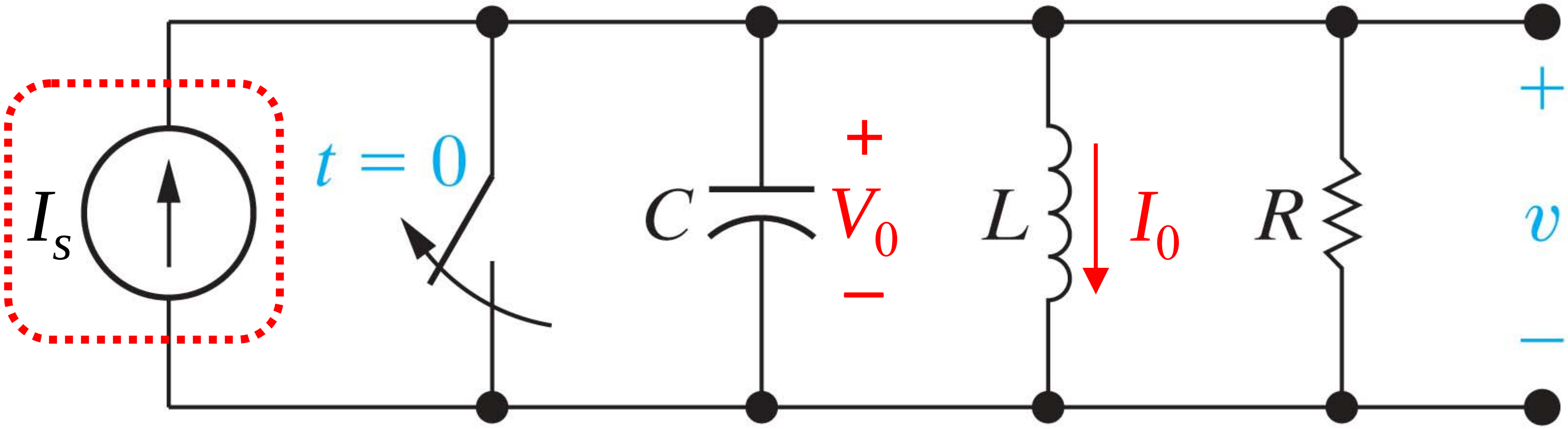
- Find the expansion constants (A_1, A_2), (B_1, B_2), or (D_1, D_2) by two ICs:

$$\begin{cases} v(0^+) = V_0 \cdots (1), \\ v'(0^+) = -\dfrac{I_0}{C} - \dfrac{V_0}{RC} \cdots (2) \end{cases}$$

Section 8.3 The Step Response of a Parallel *RLC* Circuit

1. Inhomogeneous ODE, ICs, and general solution

The homogeneous ODE

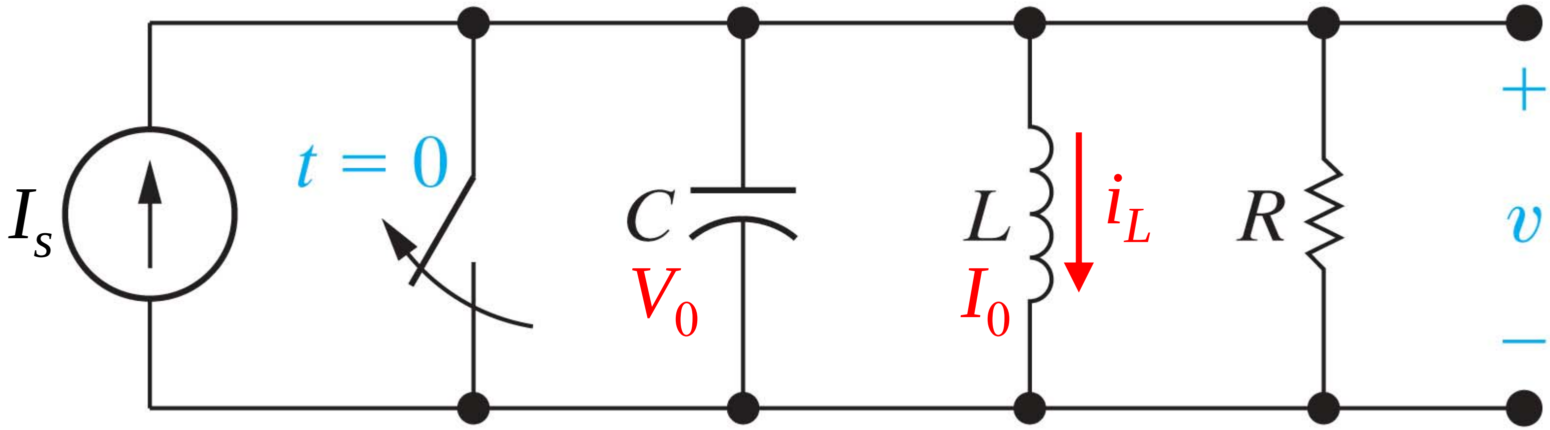


- By KCL: $C\frac{dv}{dt}+\left[I_0+\frac{1}{L}\int_0^t v(t')dt'\right]+\frac{v}{R}=I_s.$

- Perform time derivative, we got a homogeneous ODE of $v(t)$ independent of the source current I_s:

$$\frac{d^2v}{dt^2}+\frac{1}{RC}\frac{dv}{dt}+\frac{v}{LC}=0.$$

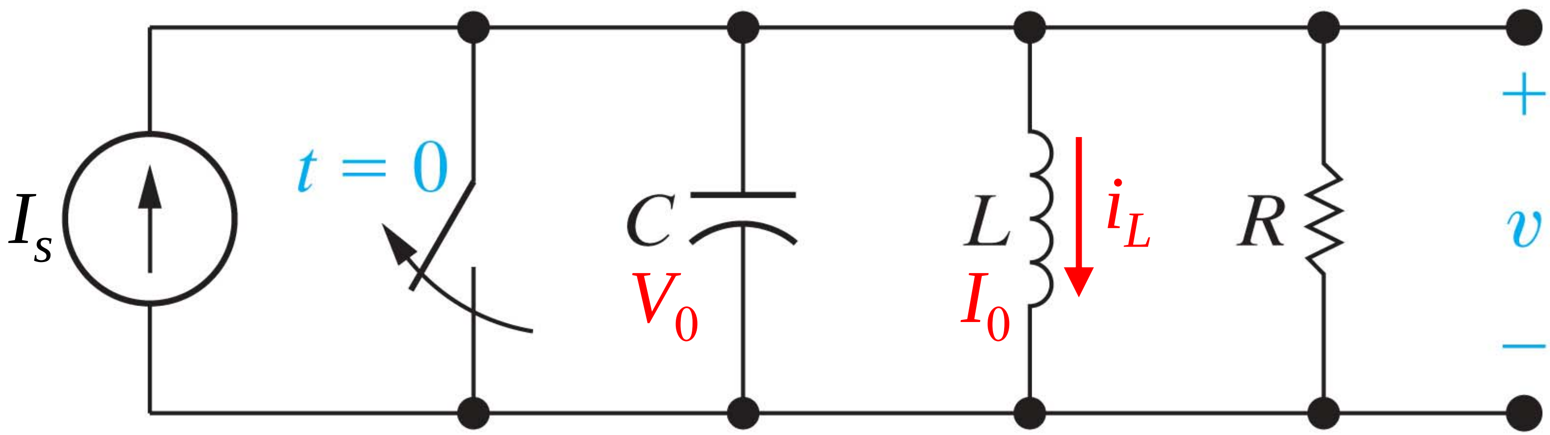
The inhomogeneous ODE



- Change the unknown to the inductor current $i_L(t)$:

$$\begin{cases} C\dfrac{dv}{dt} + i_L + \dfrac{v}{R} = I_s, \\ v = L\dfrac{di_L}{dt}, \end{cases} \Rightarrow \frac{d^2 i_L}{dt^2} + \frac{1}{RC}\frac{di_L}{dt} + \frac{i_L}{LC} = \frac{I_s}{LC}$$

The two initial conditions (ICs)



- The inductor current cannot change abruptly,

$$\Rightarrow i_L(0^+) = I_0 \cdots (1)$$

- The capacitor voltage cannot change abruptly,

$$\Rightarrow v_C(0^+) = V_0 = v_L(0^+),$$

$$\because v_L(0^+) = L\left.\frac{di_L}{dt}\right|_{t=0^+}, \Rightarrow i_L'(0^+) = \frac{V_0}{L} \cdots (2)$$

General solution

- The solution is the sum of final current $I_f = I_s$ and the nature response $i_{L,nature}(t)$:

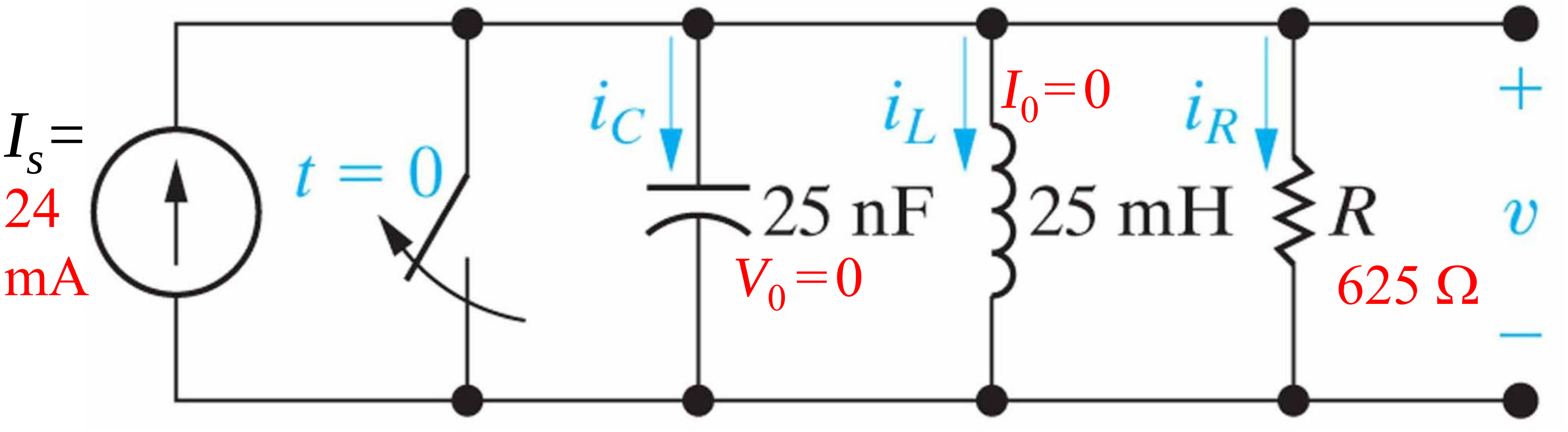
$$i_L(t) = I_f + i_{L,nature}(t),$$

where the three types of nature responses were elucidated in Section 8.2:

$$i_L(t) = \begin{cases} I_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}, \text{if } \alpha > \omega_0, \\ I_f + e^{-\alpha t}\left(B_1' \cos \omega_d t + B_2' \sin \omega_d t\right), \text{if } \alpha < \omega_0, \\ I_f + e^{-\alpha t}\left(D_1' t + D_2'\right), \text{if } \alpha = \omega_0. \end{cases}$$

Example 8.7: Charging a parallel RLC circuit (1)

- Q: $i_L(t) = ?$



$$\begin{cases} \alpha = \dfrac{1}{2RC} = \dfrac{1}{2(625)(2.5\times10^{-8})} = 32\,\text{kHz}, \\ \omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.5\times10^{-2})(2.5\times10^{-8})}} = 40\,\text{kHz}. \end{cases}$$

$\Rightarrow \alpha < \omega_0$, under-damped

Example 8.7: Solving the parameters (2)

- The complete solution is of the form:

$$i_L(t) = I_s + e^{-\alpha t}\left(B_1' \cos \omega_d t + B_2' \sin \omega_d t\right),$$

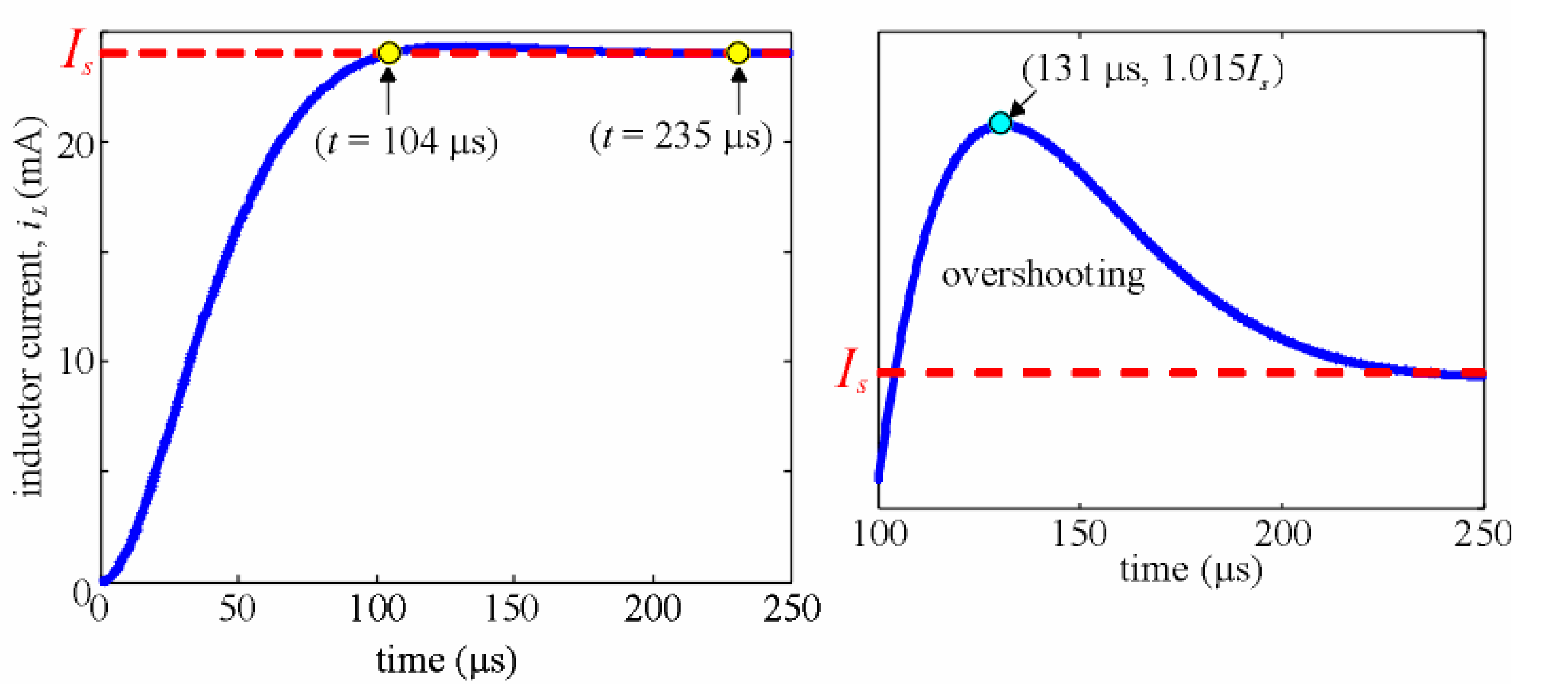
where $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{40^2 - 32^2} = 24 \text{ kHz}.$

- The 2 expansion coefficients are:

$$\begin{cases} I_s + B_1' = I_0 = 0 \cdots (1) \\ -\alpha B_1' + \omega_d B_2' = \dfrac{V_0}{L} = 0 \cdots (2) \end{cases} \Rightarrow \begin{cases} B_1' = -24 \text{ mA}, \\ B_2' \approx -32 \text{ mA} \end{cases}$$

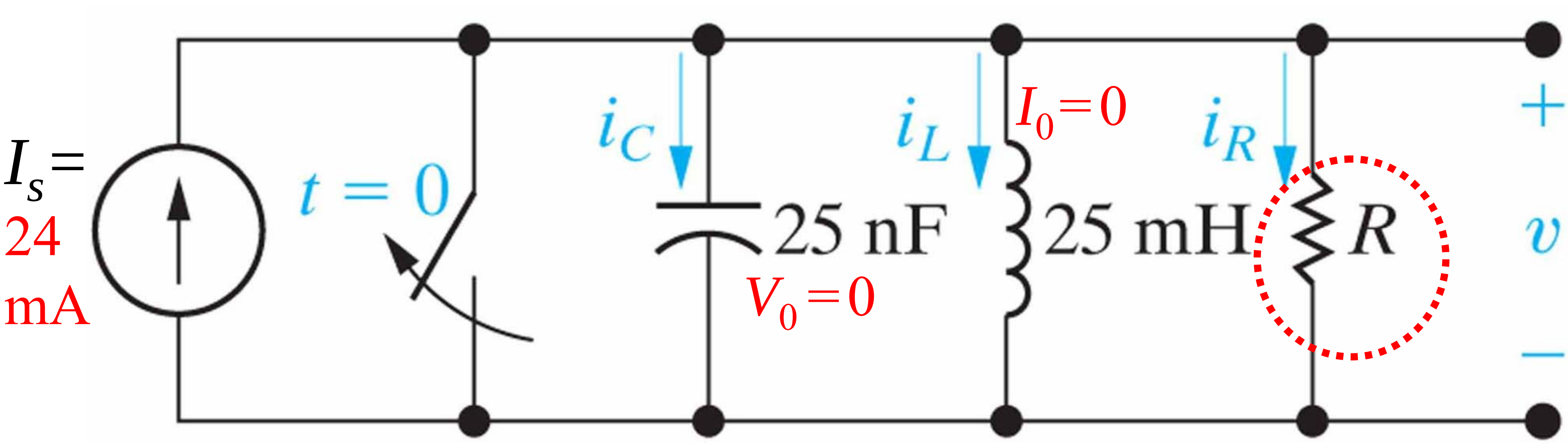
Example 8.7: Inductor current evolution (3)

$$i_L(t) = \left[24 - 24e^{-32{,}000t}\cos(24{,}000t) - 32e^{-32{,}000t}\sin(24{,}000t)\right] \text{ mA.}$$



Example 8.9: Charging of parallel RLC circuits (1)

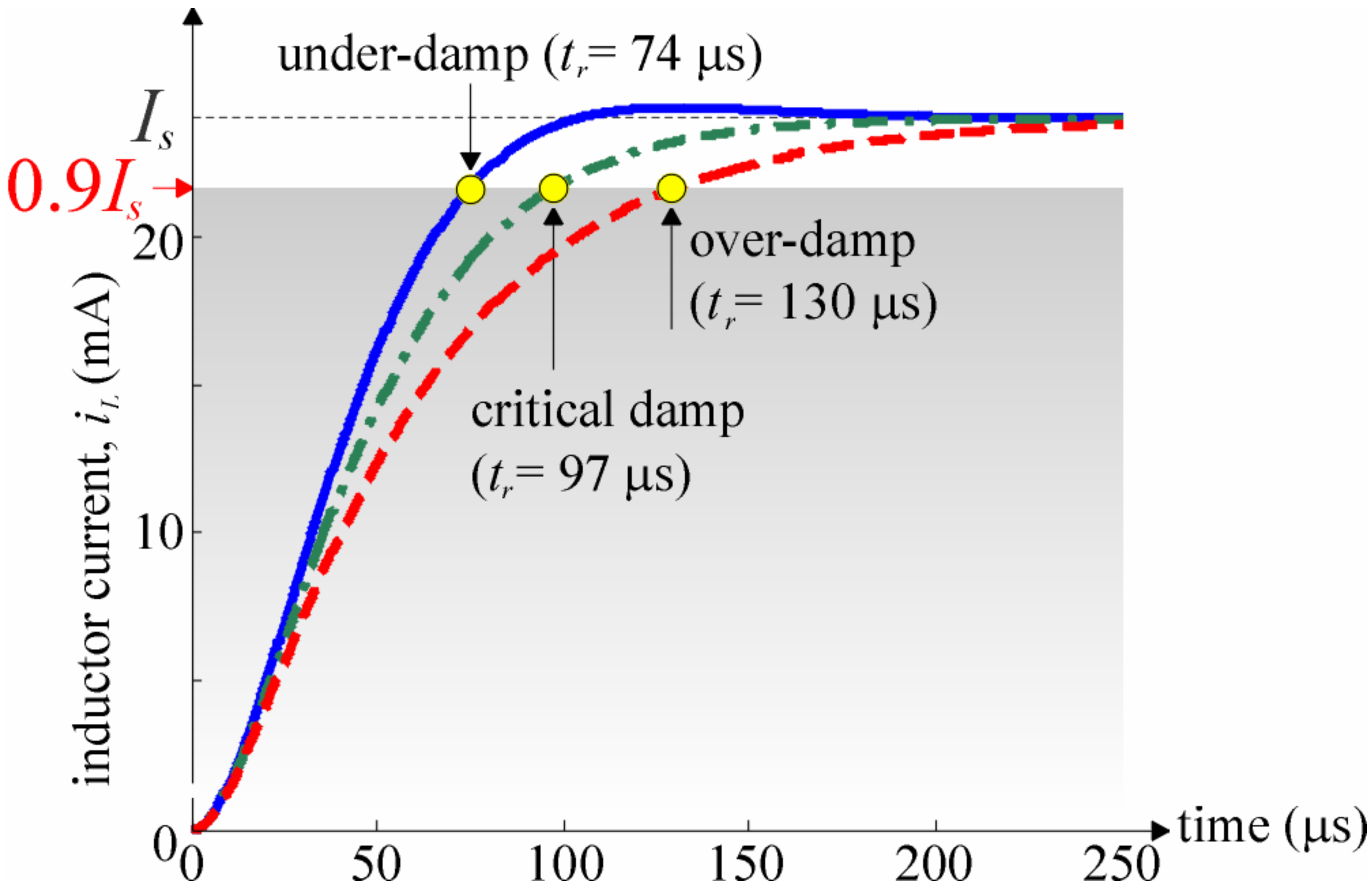
- Q: Compare $i_L(t)$ when the resistance $R = 625\ \Omega$ (under-damp), $500\ \Omega$ (critical damp), $400\ \Omega$ (over-damp), respectively.



- Initial & final conditions remain: $i_L(0^+)=0$, $i'_L(0^+)=0$, $I_f=24$ mA. Different R's give different functional forms and expansion constants.

Example 8.9: Comparison of rise times (2)

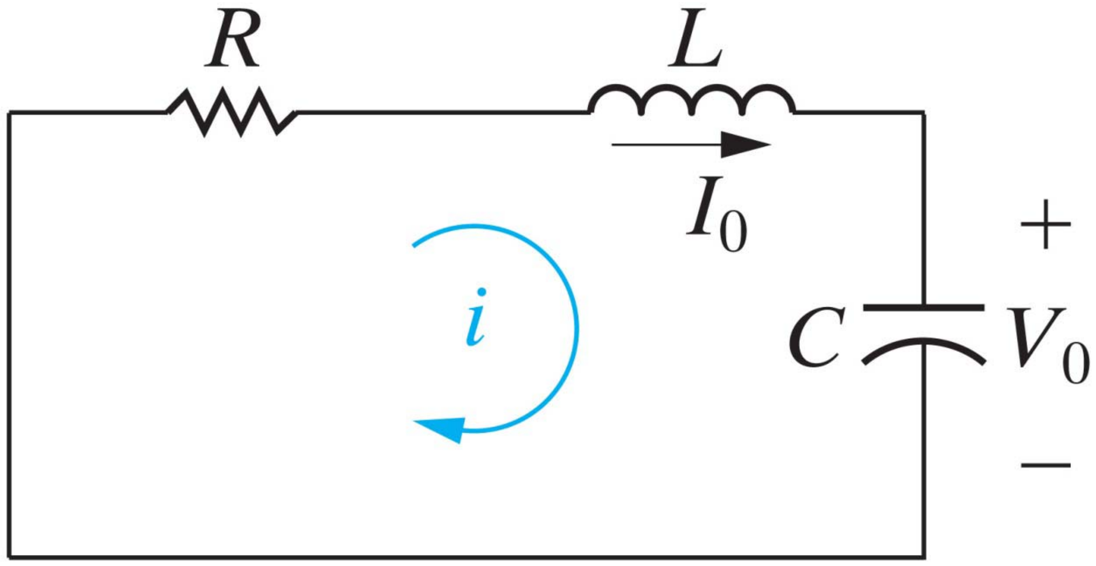
- The current of an under-damped circuit rises faster than that of its over-damped counterpart.



Section 8.4 The Natural and Step Response of a Series *RLC* Circuit

1. Modifications of time constant, neper frequency

ODE of nature response

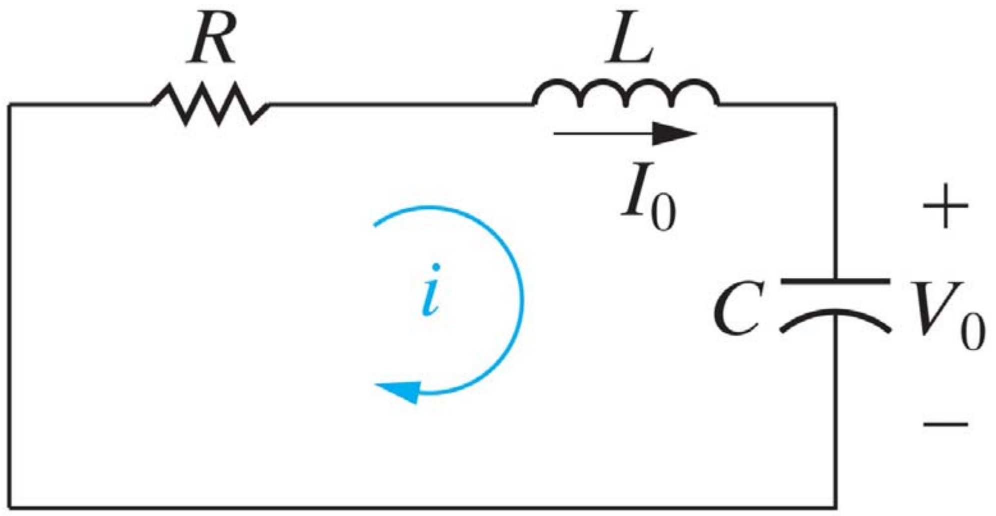


V_0, I_0, $i(t)$ must satisfy the passive sign convention.

- By KVL: $Ri + L\frac{di}{dt} + \left[V_0 + \frac{1}{C}\int_0^t i(t')dt'\right] = 0.$
- By derivative: $\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0.$

$\frac{1}{RC}$ in parallel RLC

The two initial conditions (ICs)



- The inductor cannot change abruptly,

$$\Rightarrow i(0^+) = I_0 \cdots (1)$$

- The capacitor voltage cannot change abruptly,

$$\Rightarrow v_C(0^+) = V_0,\ v_L(0^+) = -v_C(0^+) - v_R(0^+) = -V_0 - I_0 R,$$

$$\because v_L(0^+) = L\frac{di_L}{dt}\bigg|_{t=0^+},\ \Rightarrow i_L'(0^+) = i'(0^+) = -\frac{V_0 + I_0 R}{L} \cdots (2)$$

General solution

- Substitute $i(t) = Ae^{st}$ into the ODE, we got a different characteristic equation of s:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0. \Rightarrow s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}.$$

- The form of $s_{1,2}$ determines the form of general solution:

$$i(t) = \begin{cases} A_1 e^{s_1 t} + A_2 e^{s_2 t}, \text{ if } \alpha > \omega_0 \\ e^{-\alpha t}\left(B_1 \cos \omega_d t + B_2 \sin \omega_d t\right), \text{ if } \alpha < \omega_0 \\ e^{-\alpha t}\left(D_1 t + D_2\right), \text{ if } \alpha = \omega_0 \end{cases}$$

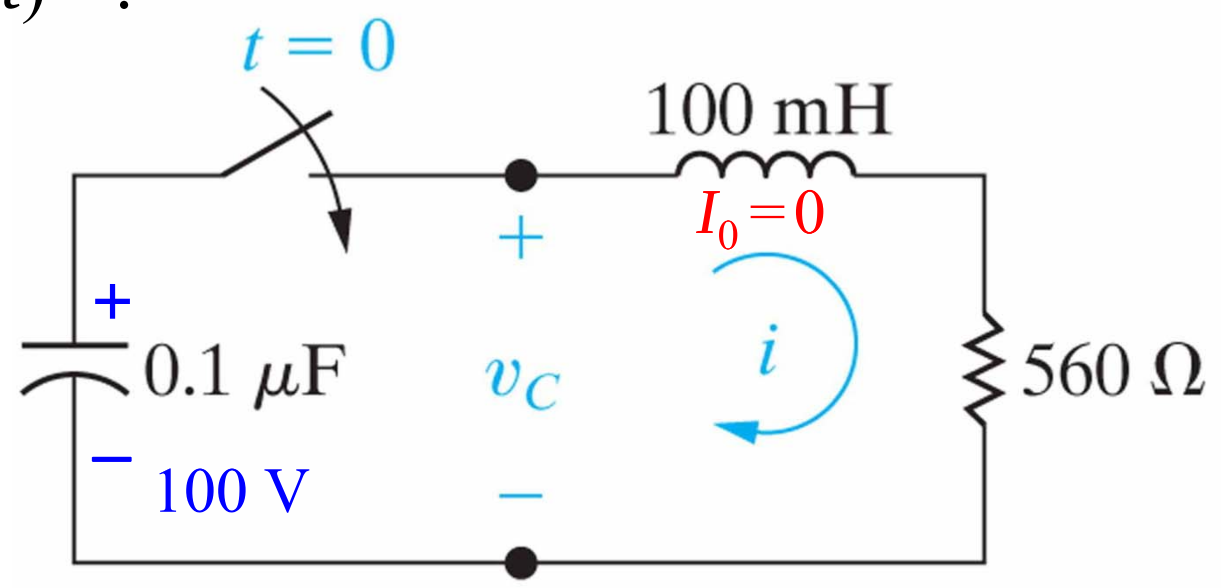
where $\alpha = \frac{R}{2L}$, $\omega_0 = \frac{1}{\sqrt{LC}}$, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$.

$(2RC)^{-1}$ in parallel RLC

Example 8.11: Discharging a series RLC circuit (1)

- Q: $i(t), v_C(t) = ?$

$v_C(0^-) = 100$ V,
$\Rightarrow V_0 = -100$ V



$$\begin{cases} \alpha = \dfrac{R}{2L} = \dfrac{560}{2(0.1)} = 2.8\,\text{kHz}, \\ \omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.1)(1\times 10^{-7})}} = 10\,\text{kHz}. \end{cases}$$

$\Rightarrow \alpha < \omega_0$, under-damped

Example 8.11: Solving the parameters (2)

- The damped frequency is:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{10^2 - 2.8^2} = 9.6\,\text{kHz}.$$

- The 2 expansion coefficients are:

$$\begin{cases} B_1 = I_0 = 0 \cdots (1) \\ -\alpha B_1 + \omega_d B_2 = -\dfrac{V_0 + I_0 R}{L} \cdots (2) \end{cases} \Rightarrow \begin{cases} B_1 = 0, \\ B_2 \approx 104.2\,\text{mA} \end{cases}$$

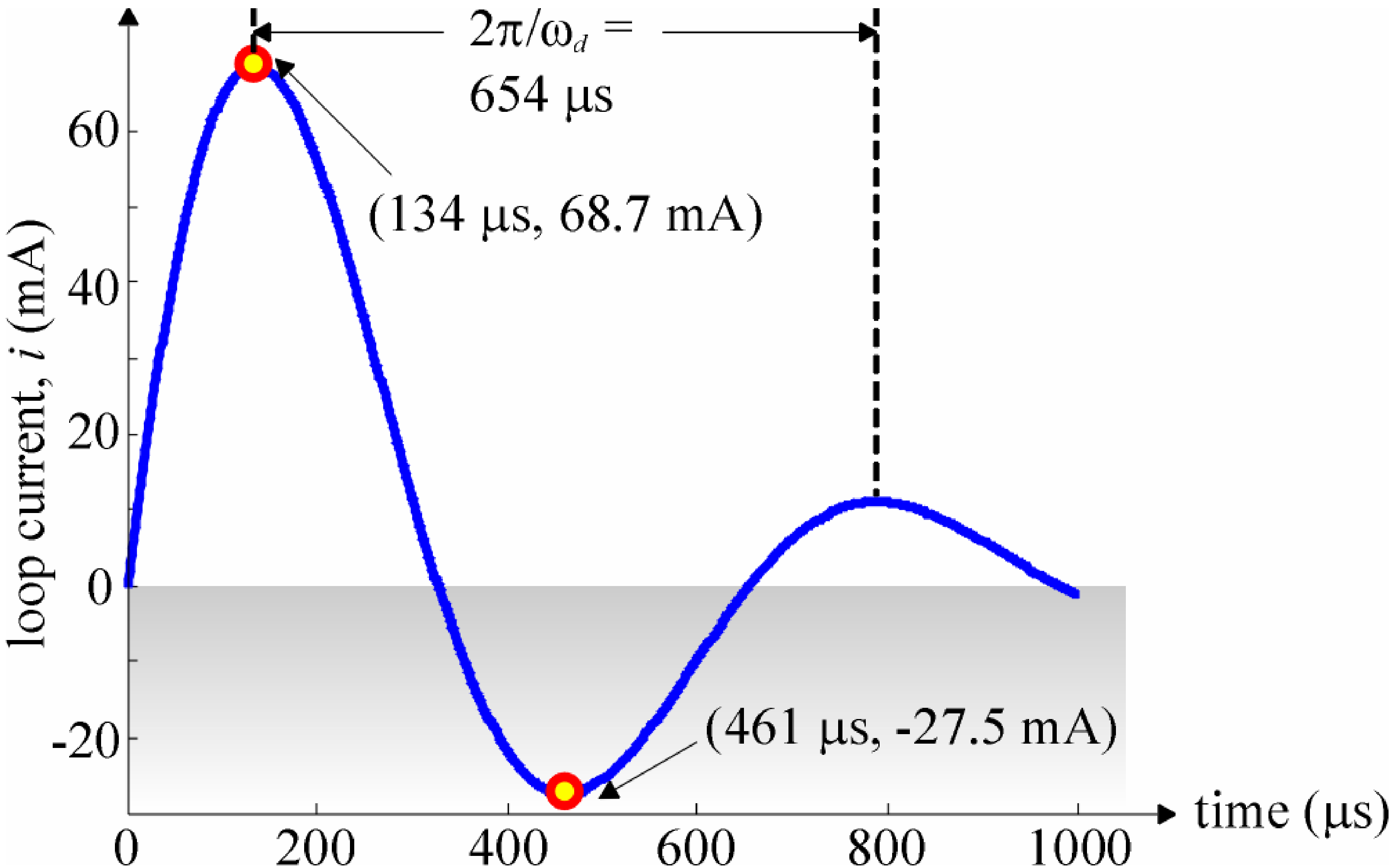
-100 V

9.6 kHz

100 mH

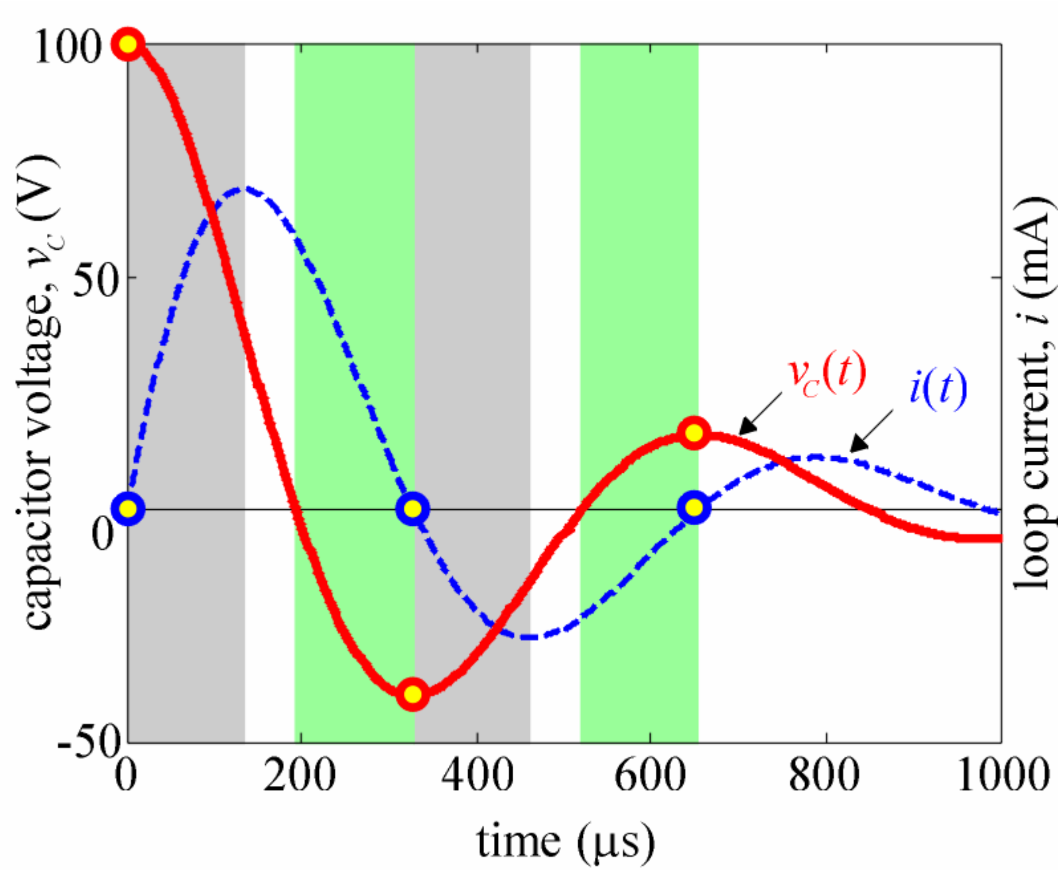
Example 8.11: Loop current evolution (3)

$$i(t) = e^{-\alpha t}\left(B_1 \cos\omega_d t + B_2 \sin\omega_d t\right) = \left(104.2e^{-2,800t} \sin 9,600t\right) \text{ mA.}$$



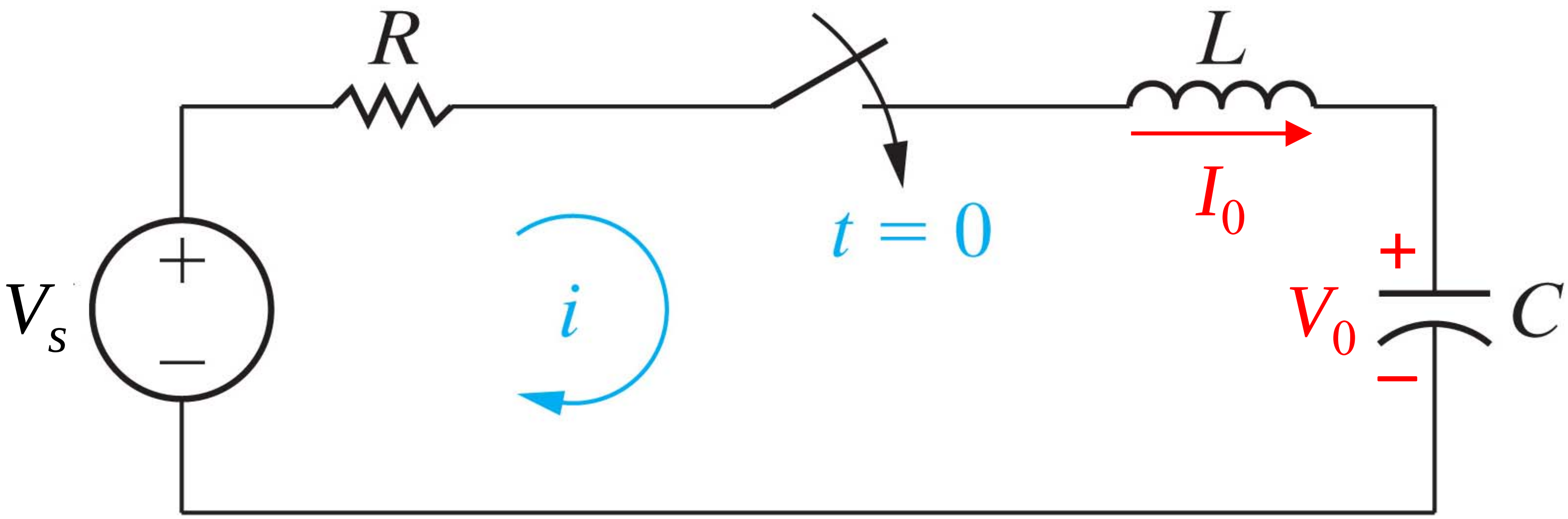
Example 8.11: Capacitor voltage evolution (4)

$$v_c(t) = Ri(t) + Li'(t) =$$
$$e^{-2,800t}\left(100\cos 9{,}600t + 29.17\sin 9{,}600t\right)\text{ V.}$$



- When the capacitor energy starts to decrease, the inductor energy starts to increase.
- Inductor energy starts to decrease before capacitor energy decays to 0.

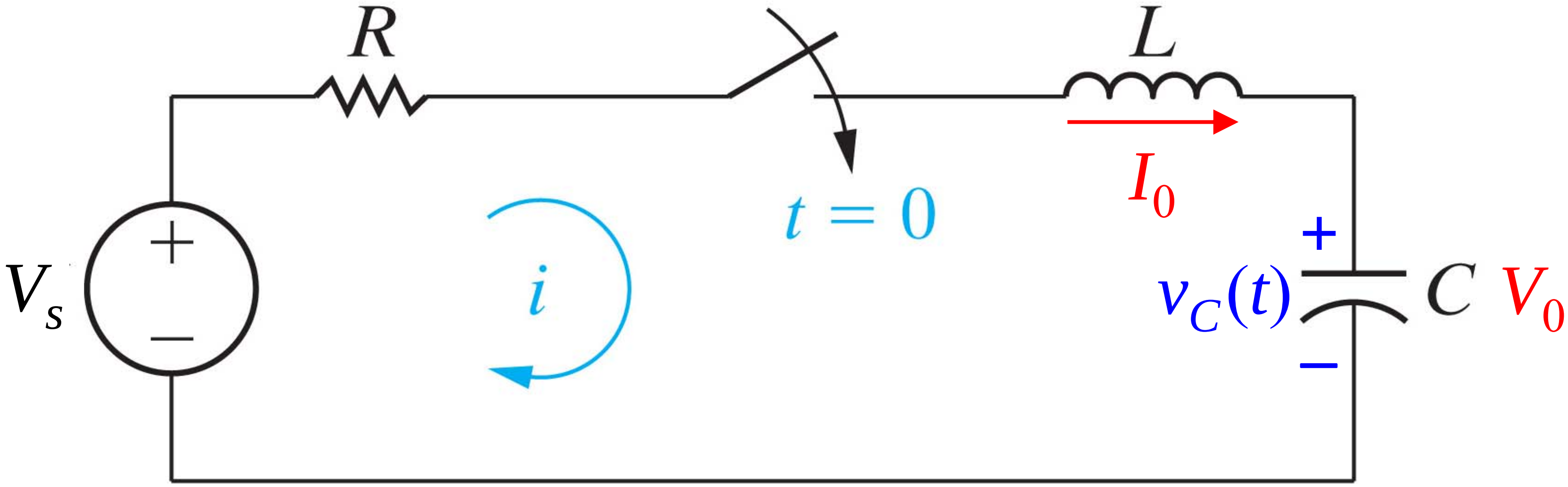
ODEs of step response



- By KVL: $Ri + L\dfrac{di}{dt} + \left[V_0 + \dfrac{1}{C}\int_0^t i(t')dt'\right] = V_s.$
- The homogeneous and inhomogeneous ODEs of $i(t)$ and $v_C(t)$ are:

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0, \text{ and } \frac{d^2 v_C}{dt^2} + \frac{R}{L}\frac{dv_C}{dt} + \frac{v_C}{LC} = \frac{V_s}{LC}.$$

The two initial conditions (ICs)



- The capacitor voltage cannot change abruptly,

$$\Rightarrow v_C(0^+) = V_0 \cdots (1)$$

- The inductor current cannot change abruptly,

$$\Rightarrow i_L(0^+) = I_0 = i_C(0^+),$$

$$\because i_C(0^+) = C\left.\frac{dv_C}{dt}\right|_{t=0^+}, \Rightarrow v_C'(0^+) = \frac{I_0}{C} \cdots (2)$$

General solution

- The solution is the sum of final voltage $V_f = V_s$ and the nature response $v_{C,nature}(t)$:

$$v_C(t) = V_f + v_{C,nature}(t),$$

where the three types of nature responses were elucidated in Section 8.4.

$$v_C(t) = \begin{cases} V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}, \text{ if } \alpha > \omega_0, \\ V_f + e^{-\alpha t}\left(B_1' \cos\omega_d t + B_2' \sin\omega_d t\right), \text{ if } \alpha < \omega_0, \\ V_f + e^{-\alpha t}\left(D_1' t + D_2'\right), \text{ if } \alpha = \omega_0. \end{cases}$$

Key points

- What do the response curves of over-, under-, and critically-damped circuits look like? How to choose R, L, C values to achieve fast switching or to prevent overshooting damage?
- What are the initial conditions in an RLC circuit? How to use them to determine the expansion coefficients of the complete solution?
- Comparisons between: (1) natural & step responses, (2) parallel, series, or general RLC.