# Chapter 2
# Circuit Elements

2.1 Voltage and Current Sources

2.2 Electrical Resistance (Ohm's Law)

*2.3 Construction of a Circuit Model*

2.4 Kirchhoff's Laws

2.5 Analysis of a Circuit Containing Dependent Sources

# Key points

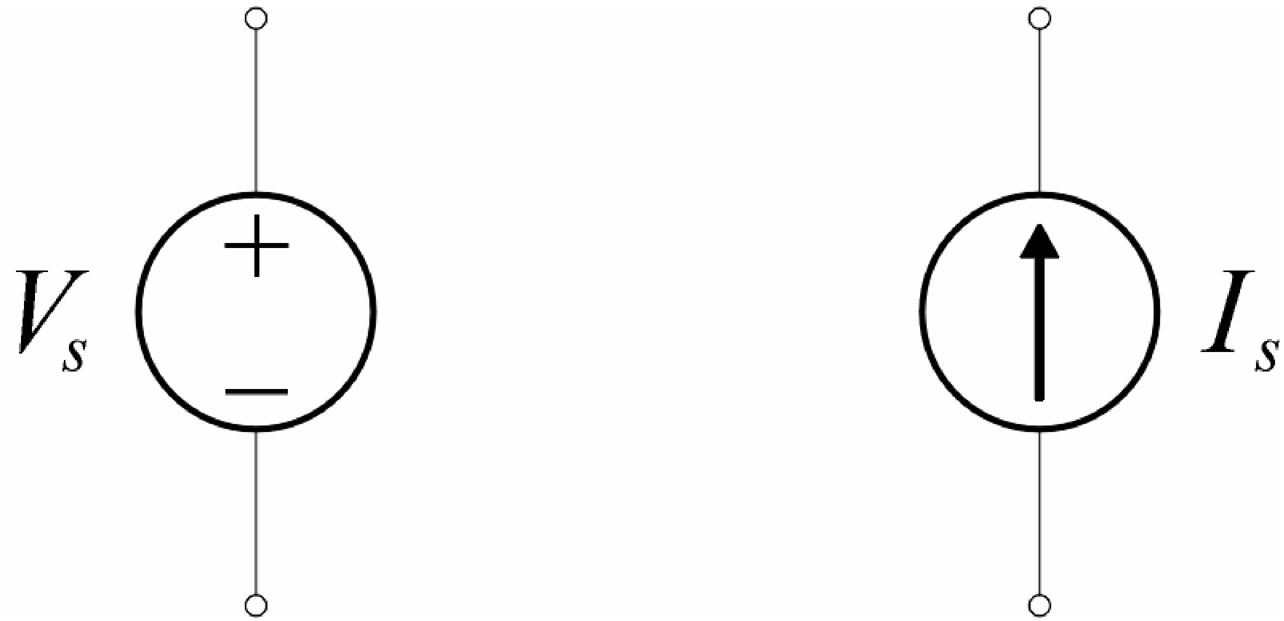
- Microscopic view of Ohm's law.
- Physics of Kirchhoff's laws.

# Section 2.1
# Voltage and Current Sources

1. Ideal sources
2. Real sources
3. Dependent sources

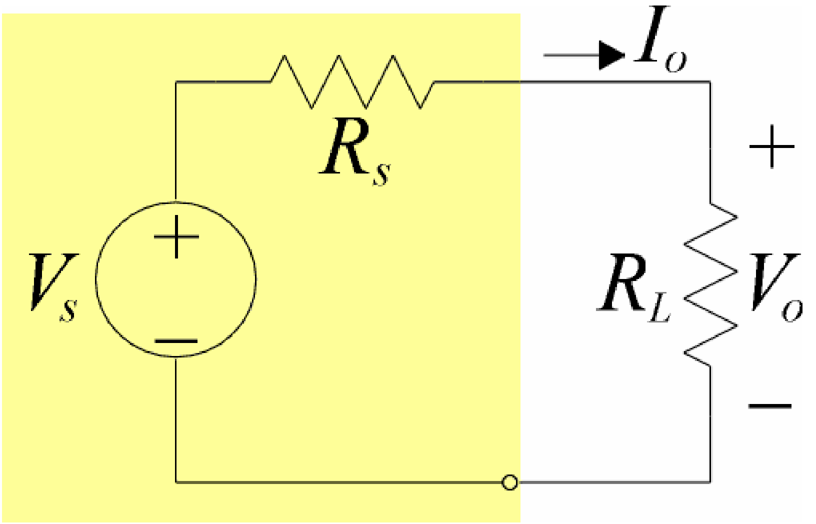
# Ideal voltage and current sources

- Ideal voltage source: maintains a prescribed voltage regardless of the current in the device.
- Ideal current source: maintains a prescribed current regardless of the voltage in the device.



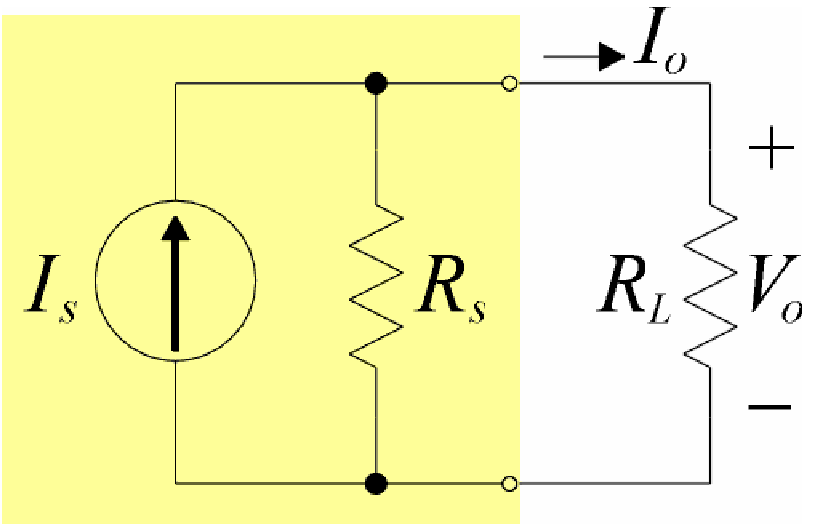
# Real voltage sources

- The output voltage $V_o$ will change with the (1) the output current $I_o$, (2) the load resistance $R_L$.
- The entire source is modeled by an ideal one plus a series resistor $R_S$.



# Real current sources

- The output current $I_o$ will change with the (1) the output voltage $V_o$, (2) the load resistance $R_L$.
- The entire source is modeled by an ideal one plus a parallel resistor $R_S$.



# Independent vs. dependent sources

- Independent sources: The output voltage (current) is not influenced by any other voltage or current in the circuit. All the cases shown above belong to this category.
- Dependent sources: The output voltage (current) is determined by some other voltage or current in the circuit. This happens when the circuit has active devices (e.g. transistors).

# Symbols of dependent sources

- Voltage-controlled voltage source

$v_s = \mu v_x$

voltage gain

- Voltage-controlled current source

$i_s = \alpha v_x$

trans-conductance

- Current-controlled voltage source

$v_s = \rho i_x$

trans-resistance

- Current-controlled current source

$i_s = \beta i_x$

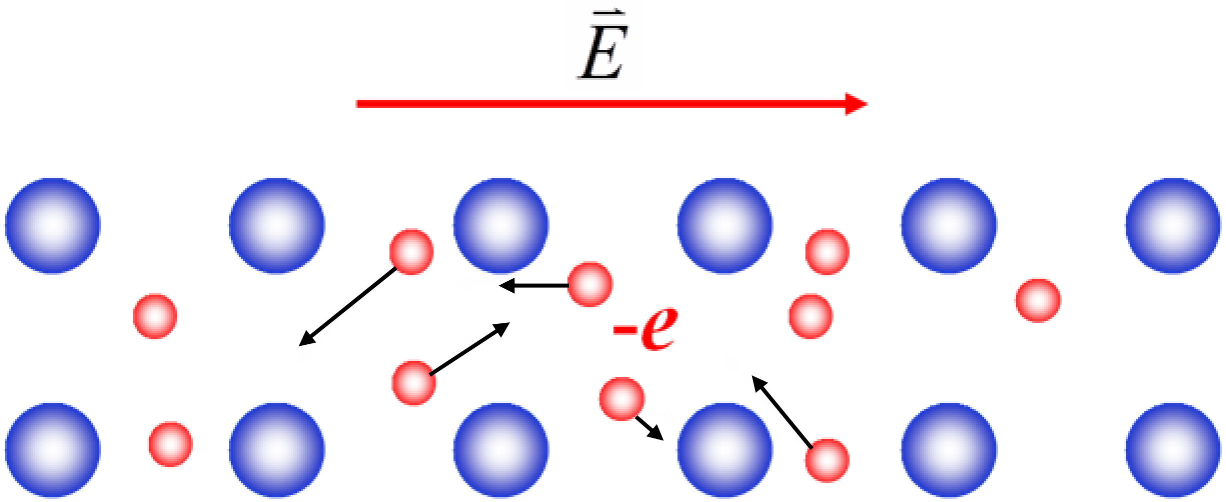
current gain

# Section 2.2
# Electrical Resistance
# (Ohm's Law)

1. Physics
2. Electric power

# Microscopic view of conductors

- A conductor is made of (1) immobile ions, and (2) free electrons that move fast (Fermi speed, $v_F \sim 10^6$ m/s) and randomly.
- When an external E-field is applied, electrons have slow (~mm/s) drift velocity $v_d$ statistically.

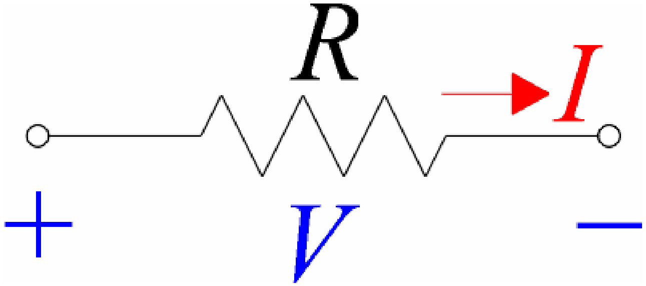


# Ohm's law

- The drift velocity $v_d$ ($\propto$ net current $i$) is proportional to the applied electric field $E$ ($\propto$ applied voltage $v$), resulting in:

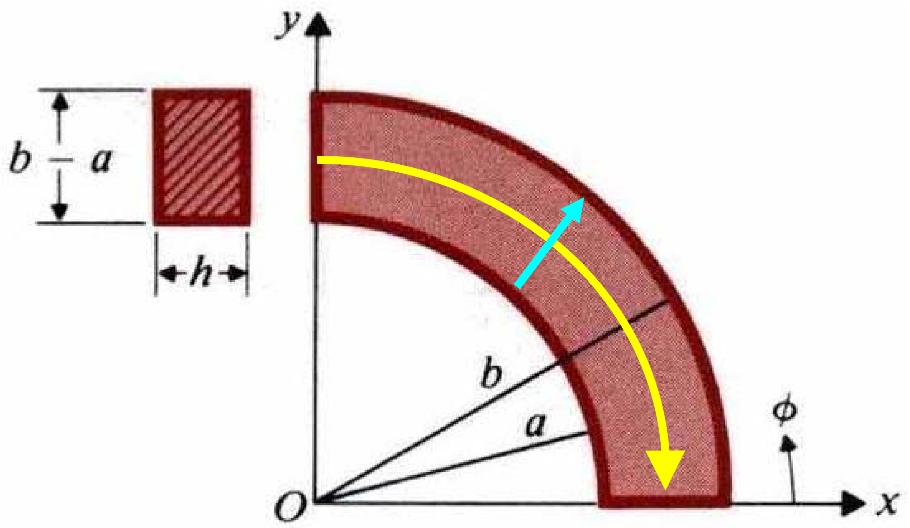
$$v = iR$$

if passive sign convention is held:



## Comments

- The Ohm's law is valid for good conductors, instead of every material. Counter examples: vacuum tubes ($I \propto V^{1.5}$), transistors ($I \propto e^{\alpha V}$).
- Resistance depend on material, shape, and the way of connection. E.g. washer resistor:



## Electric power consumption

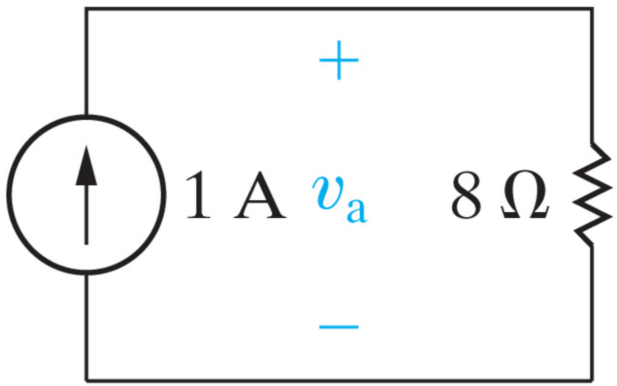
- Microscopically, frequent collisions between free electrons and immobile ions transfer energy from electric field to thermal vibration.
- Power in a resistor in terms of current:

$$p = vi = (iR)i = i^2R$$

- Power in a resistor in terms of voltage:

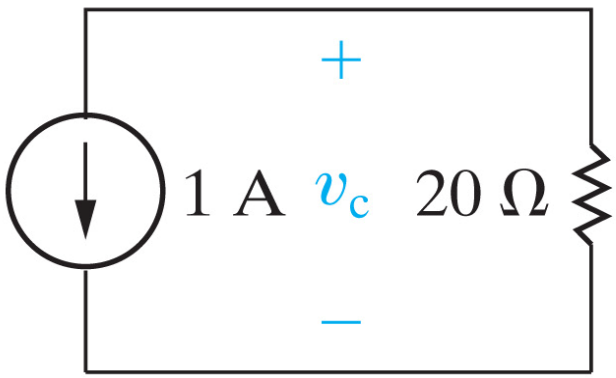
$$p = \frac{v^2}{R}$$

# Example 2.3




$$v_a = 1 \times 8 = 8 \text{ V}$$

$$p = vi = 8 \times 1 = 8 \text{ W}$$




$$v_c = iR = (-1) \times 20 = -20 \text{ V}$$

$$p = vi = (-20) \times (-1) = 20 \text{ W}$$

- $vi > 0$, power is always dissipated in a resistor.

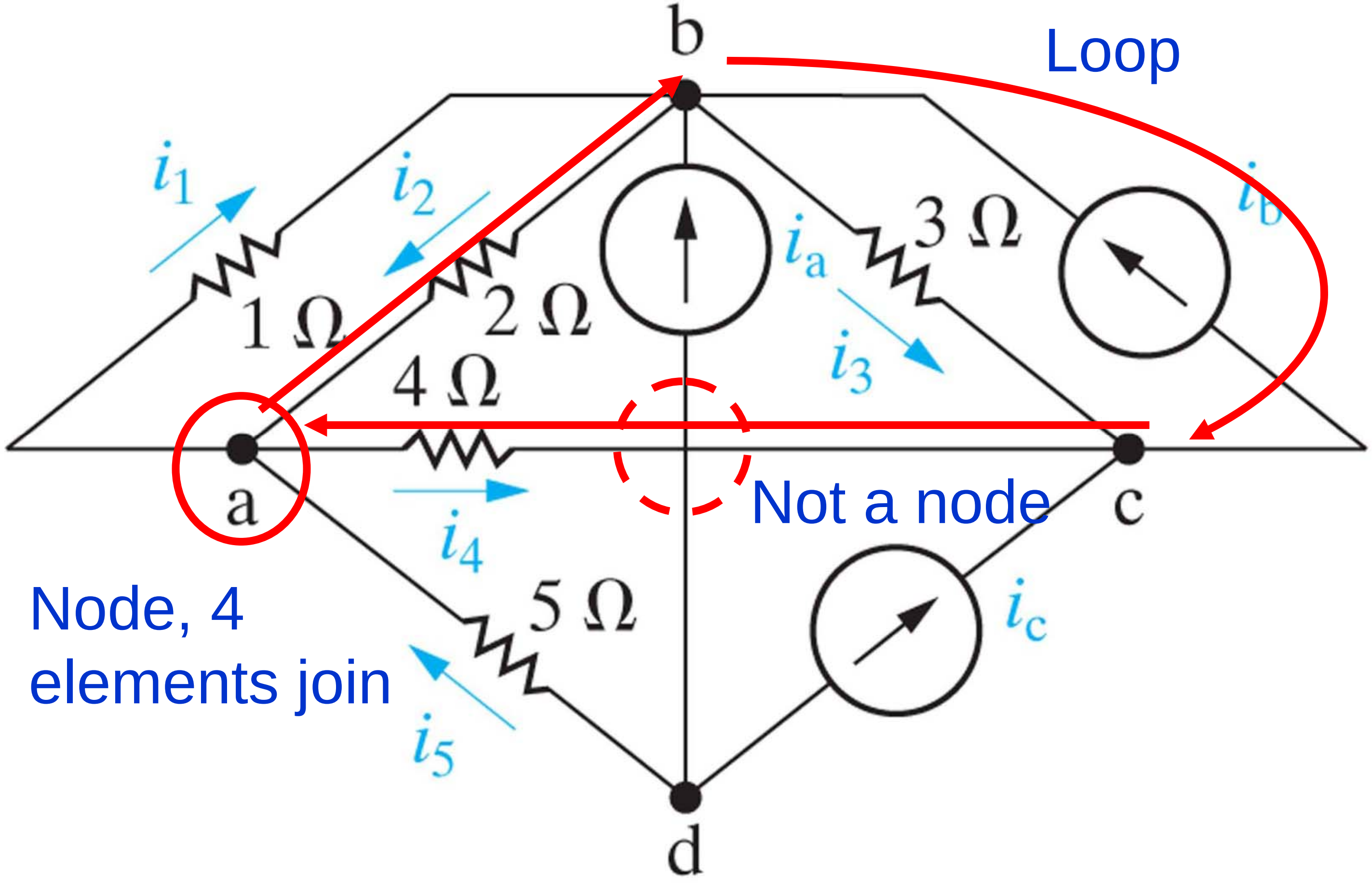
# Section 2.4
# Kirchhoff's Laws

1. Nodes & loops
2. Kirchhoff's laws & the physics

## Nodes and loops

- Electric circuits consist of connected basic circuit elements.
- A node is a point where two or more circuit elements join.
- A loop is a closed path, starting and ending at the same node without passing through any intermediate node more than once.

# Example:



b
Loop
$i_1$
$i_2$
$i_b$
$i_a$
3 Ω
1 Ω
2 Ω
4 Ω
$i_3$
a
Not a node
c
$i_4$
Node, 4
elements join
5 Ω
$i_c$
$i_5$
d

# Kirchhoff’s current law (KCL)

- The algebraic sum of all the currents at any node in a circuit equals zero.
- Microscopic equivalent: Conservation of charges.

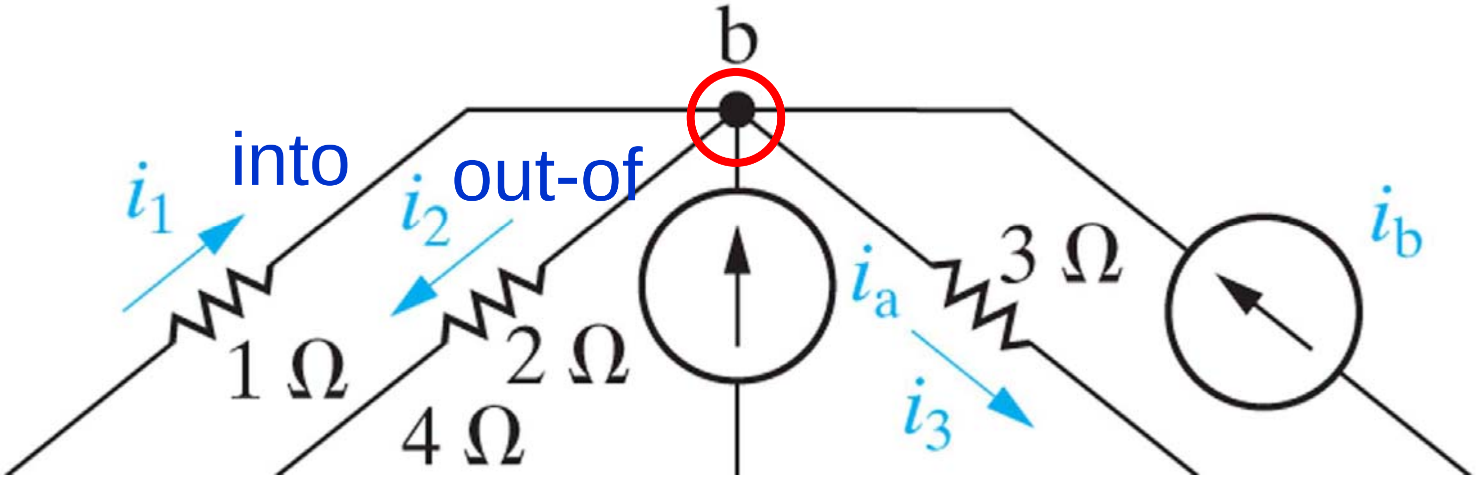
# Kirchhoff’s voltage law (KVL)

- The algebraic sum of all the voltages around any loop in a circuit equals zero.
- Microscopic equivalent: Static electric field is conservative

$$\oint \vec{E} \cdot d\vec{l} = 0,$$

which is not true in time-varying cases!

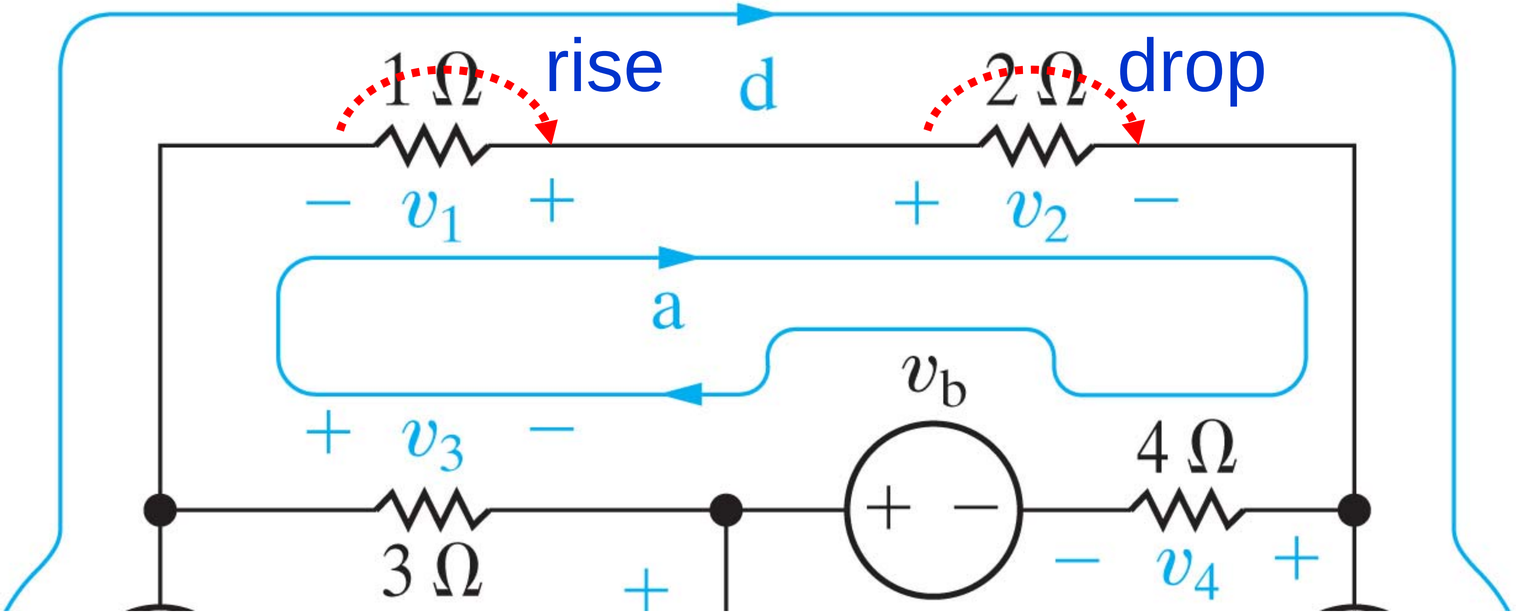
# Example 2.6: KCL




Apply KCL over Node b:

$$(-i_1)+i_2+(-i_a)+i_3+(-i_b)=0$$

# Example 2.7: KVL

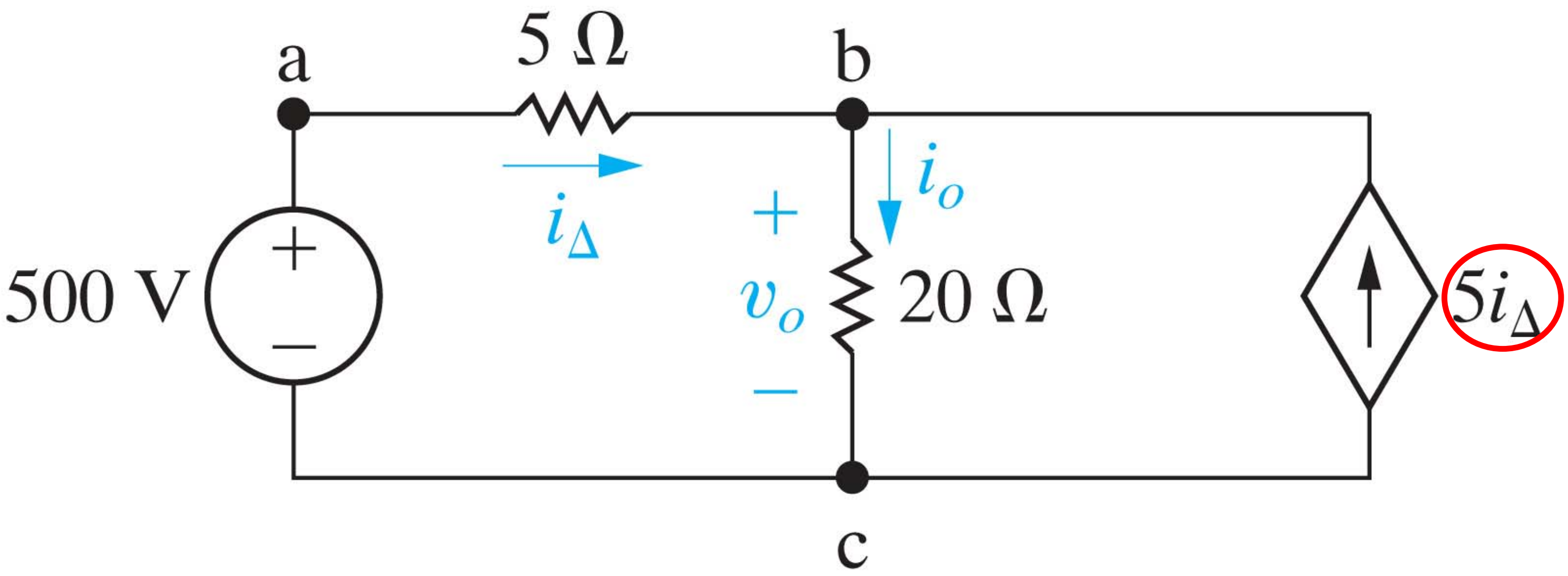



Apply KVL over Loop a:

$$(-v_1) + v_2 + v_4 + (-v_b) + (-v_3) = 0$$

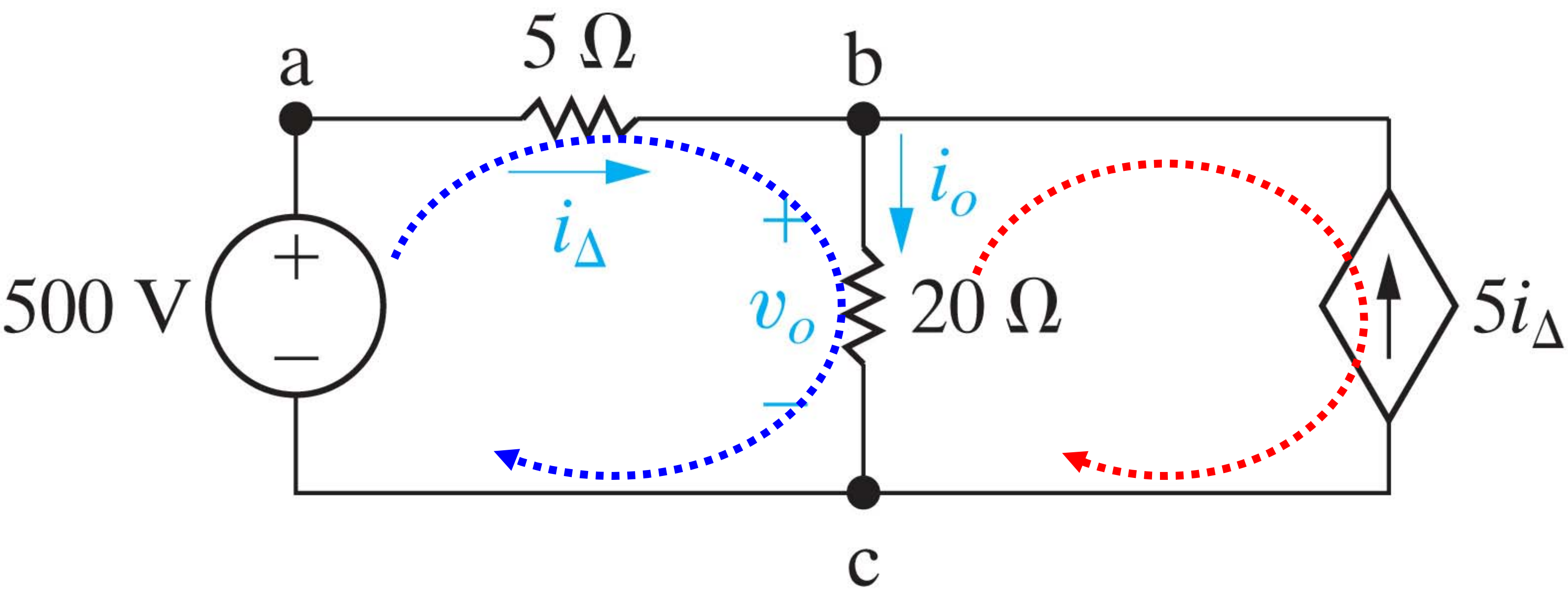
# Section 2.5
# Analysis of a Circuit Containing Dependent Sources

# Step 1: Specify unknowns



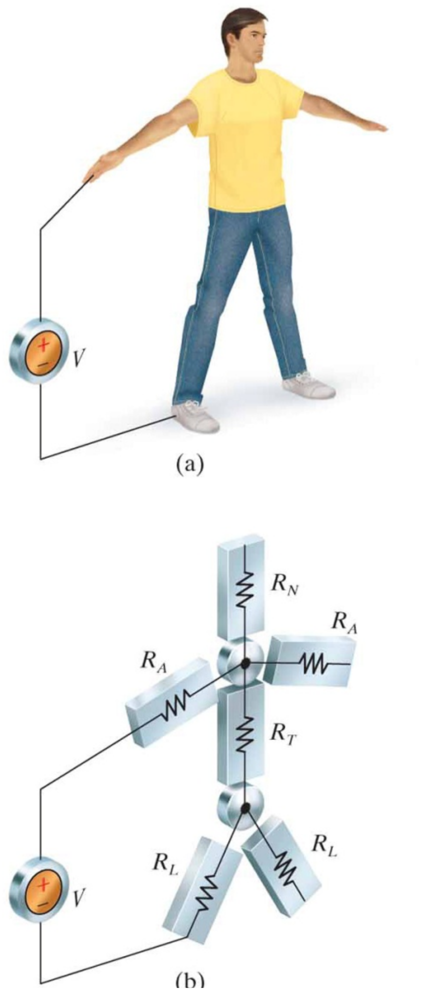

3 branch currents: $\{i_{ab} = i_\Delta, i_{bc} = i_o, i_{cb} = 5i_\Delta\}$, $\Rightarrow$ only 2 independent unknowns, $\Rightarrow$ need 2 independent equations.

## Step 2: Apply KVL/KCL




- KVL for Loop cabc: $500 = 5i_\Delta + 20i_o$ ...(1)
- KVL for Loop cbc: $-20i_o + (??) = 0$, failed for voltage across a current source is undetermined.
- KCL for Node b: $i_\Delta + 5i_\Delta = i_o$ …(2) $\Rightarrow$ $i_\Delta = 4$ A, $i_o = 24$ A.

# Practical Perspective: Electrical Safety




- Burns is not the major electrical injury.
- Current can disturb the electrochemical signals of nerves that control oxygen supply to the brain or regulate heartbeat.
- Barely perceptible: 5 mA.
- Heart stoppage: 500 mA.

# Key points

- Microscopic view of Ohm's law.
- Physics of Kirchhoff's laws.