Final Exam

(2013/6/17)

1) The Laplace transform of a function $f(t)$ is derived by an integral

$$F(s) = L\{f(t)\} \equiv \int_{0^-}^{\infty} f(t)e^{-st}dt .$$

1A) (5%) Derive $F(s)$ and indicate the valid region on the s-plane if $f(t) = e^{-\alpha t}u(t)$, where $\alpha > 0$ and $u(t)$ stands for the unit-step function.

1B) (10%) Perform partial fraction expansion for a rational function

$$F(s) = \frac{s}{[s-(-\alpha + j\beta)]\times[s-(-\alpha - j\beta)]}\times\frac{1}{s+\alpha},$$

where $\alpha,\ \beta > 0$. Represent all the complex expansion coefficients in polar form (amplitude and phase).

1C) (10%) Follow Problem 1B, carry out inverse Laplace transform to get $f(t)$.

2) Consider a circuit shown in Figure 1.

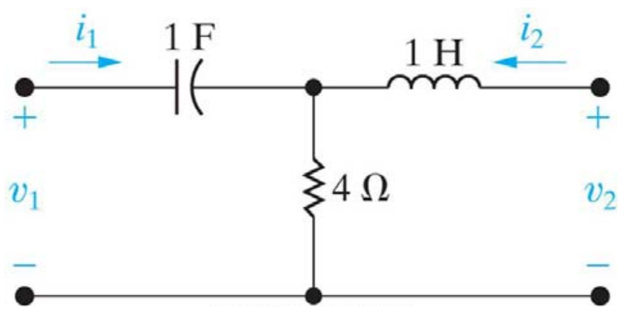

Figure 1.

2A) (5%) Let the input and output be v_1 and v_2, respectively. What type of filters does this circuit correspond to? (*Hint*: Check the v_2/v_1 values at $\omega \to 0$ and $\omega \to \infty$.)

2B) (5%) Plot the s-domain circuit if no initial energy stored in the capacitor or the inductor.

2C) (5%) Derive the transfer function of the circuit $H(s) \equiv V_2(s)/V_1(s)$.

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2D) (5%) Roughly sketch the transmission spectrum $|H(j\omega)|$. (Note that this curve has to be consistent with your answer to Problem 2A.)

2E) (10%) Derive the transfer function of the circuit $H(s) \equiv V_2(s)/V_1(s)$ if a load resistance of $R_L = \frac{1}{4}\Omega$ is connected to the output port (the two terminals of v_2).

2F) (10%) Follow Problem 2E, denote the positions of all the pole(s) and zero(es) of $H(s)$ by cross(es) "×" and open circle(s) "o", respectively. Estimate what type of filters does this loaded circuit correspond to accordingly?

2G) (5%) Follow Problem 2E, what are the values of peak transmission $T_{\max}$ and center frequency ω_c if they are defined by $|H(j\omega)| \le |H(j\omega_c)| = T_{\max}$? (*Hint*: The curve $\dfrac{\beta\omega}{\sqrt{(\omega^2-\omega_0^2)^2+(\beta\omega)^2}}$ corresponds to $T_{\max} = 1$, $\omega_c = \omega_0$.)

2H) (15%) Follow Problem 2E, derive the time-domain output voltage $v_2(t)$ if the loaded circuit is driven by an ideal voltage source of $v_g(t) = e^{-0.25t}u(t)$. (*Hint*: $v_g(t) = v_1(t)$, and $V_2(s) = H(s) \times V_g(s)$ in this case. You can borrow the results in Problems 1B and 1C with minor adjustment.)

2I) (15%) For the "unloaded" circuit shown in Figure 1, calculate the 2×2 transmission matrix [*A*] for the two-port circuit. Denote the physical units whenever exist. (*Hint*: [*A*] is defined by $\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & -a_{12} \\ a_{21} & -a_{22} \end{bmatrix} \times \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$. The matrix elements can be functions of *s*.)

2J) **(Bonus 10 points)** The configuration of Problem 2H is a special case of "terminated two-port circuit" where the source impedance $Z_g = 0$. List the equations that can be used in solving $\{V_1, I_1, V_2, I_2\}$ in the s-domain. Show that the $V_2(s)$ derived by solving the listed simultaneous equations is consistent with that obtained by $H(s) \times V_g(s)$.