

Integrated Modulator II --- Acousto-Optics

Class: Integrated Photonic Devices

Time: Fri. 8:00am ~ 11:00am.

Classroom: 資電206

Lecturer: Prof. 李明昌(Ming-Chang Lee)

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Fundamental Principles of AO effect

Mechanical strain in a solid causes a change in the index of refraction

$$\Delta n = \sqrt{n^6 p^2 10^7 P_a / (2 \rho v_a^3 A)}$$

Diagram illustrating the components of the equation for the change in refractive index (Δn):

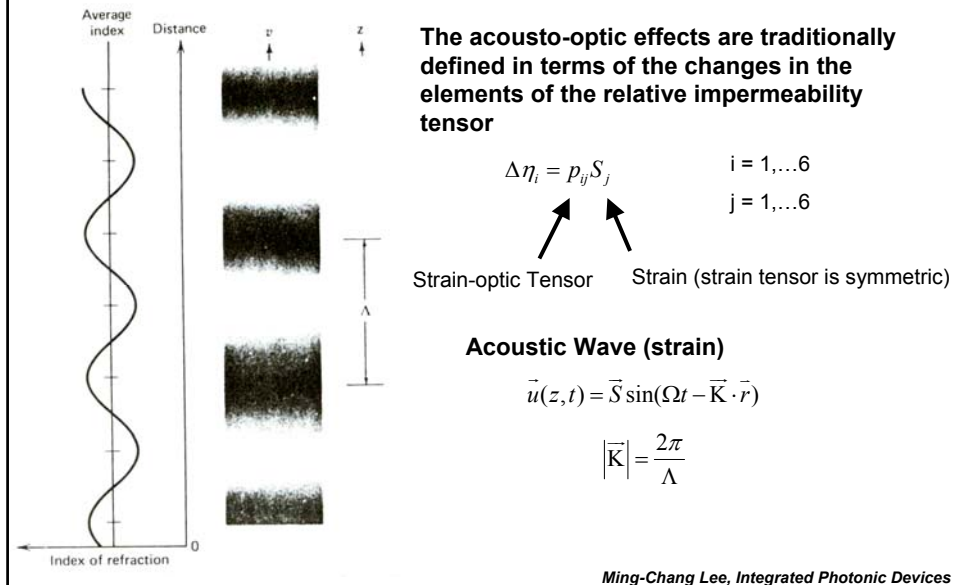
- n : Refractive Index (unstrained)
- p : Photoelastic Tensor
- P_a : Acoustic Power
- ρ : Mass Density
- v_a : Acoustic Velocity
- A : Cross-Section Area

$$\Delta n = \sqrt{M_2 10^7 P_a / (2 A)} \quad \text{where} \quad M_2 \equiv \frac{n^6 p^2}{\rho v_a^3} \quad (\text{AO figure of merit})$$

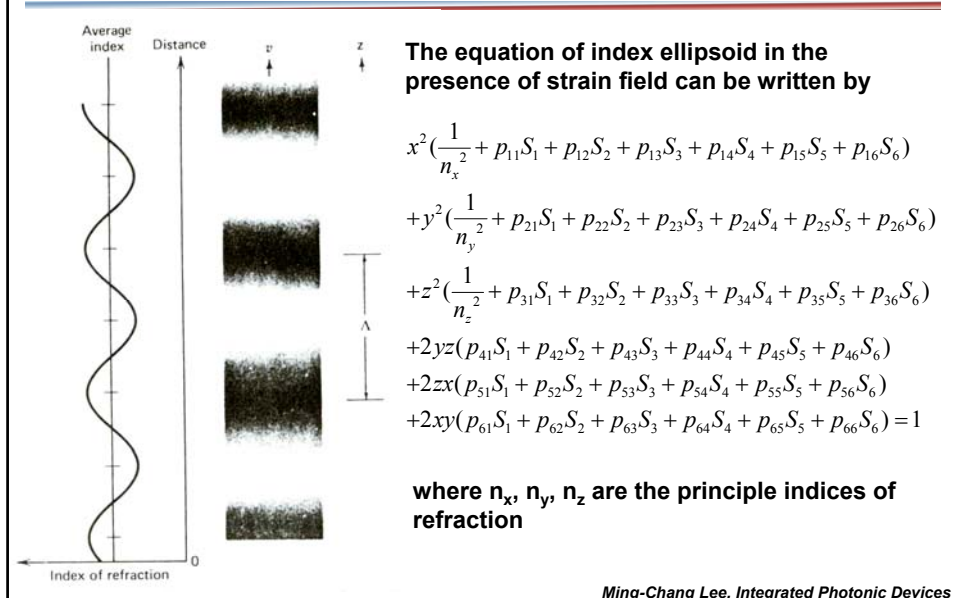
- In crystalline solids, the AO effect (photoelastic effect) depends strongly on the orientation (p).
- Overall, the AO effect is small (10^{-4}) even for an acoustic power density 100W/cm^2
- Both bulk acoustic waves and surface acoustic waves (planar waveguide) are used

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Fundamental Principles of AO effect



Fundamental Principles of AO effect



Examples of Strain-Optic Tensor

Triclinic (36)[†]

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{pmatrix}$$

Trigonal (8)
classes 3m, $\bar{3}2, \bar{3}m$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 & 0 \\ p_{12} & p_{11} & p_{13} & -p_{14} & 0 & 0 \\ p_{13} & p_{13} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & p_{41} \\ 0 & 0 & 0 & 0 & p_{14} & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$$

Monoclinic (20)

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & p_{15} & 0 \\ p_{21} & p_{22} & p_{23} & 0 & p_{25} & 0 \\ p_{31} & p_{32} & p_{33} & 0 & p_{35} & 0 \\ 0 & 0 & 0 & p_{44} & 0 & p_{46} \\ p_{51} & p_{52} & p_{53} & 0 & p_{55} & 0 \\ 0 & 0 & 0 & p_{64} & 0 & p_{66} \end{pmatrix}$$

Orthorhombic (12)

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 & 0 \\ p_{31} & p_{32} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$$

Tetragonal (10)
classes 4, $\bar{4}$, 4/m

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & p_{16} \\ p_{12} & p_{11} & p_{13} & 0 & 0 & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ p_{61} & -p_{61} & 0 & 0 & 0 & p_{66} \end{pmatrix}$$

Tetragonal (7)
classes 4mm, 422, 4/mmm

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{66} \end{pmatrix}$$

Trigonal (12)
classes 3, $\bar{3}$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{12} & p_{11} & p_{13} & -p_{14} & -p_{15} & -p_{16} \\ p_{31} & p_{31} & p_{33} & 0 & 0 & 0 \\ p_{41} & -p_{41} & 0 & p_{44} & p_{45} & -p_{51} \\ p_{51} & -p_{51} & 0 & -p_{45} & p_{44} & p_{41} \\ -p_{16} & p_{16} & 0 & -p_{15} & p_{14} & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$$

Cubic (4)
classes 23, m $\bar{3}$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{13} & p_{13} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$$

Cubic (3)
classes $\bar{4}3m, \bar{4}32, m\bar{3}m$

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{13} & 0 & 0 & 0 \\ p_{13} & p_{13} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{pmatrix}$$

A. Yariv, "Optical Waves in Crystals"

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Examples of Strain-Optic Tensor

Table 9.1. (Continued).

<p>Isotropic (2)</p>						
$\begin{pmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(p_{11} - p_{12}) \end{pmatrix}$						

[†]The number inside the parentheses indicates the number of independent coefficients.

A. Yariv, "Optical Waves in Crystals"

Examples of Strain-Optic Tensor

Table 9.2. Strain-Optic Coefficients^a [1]

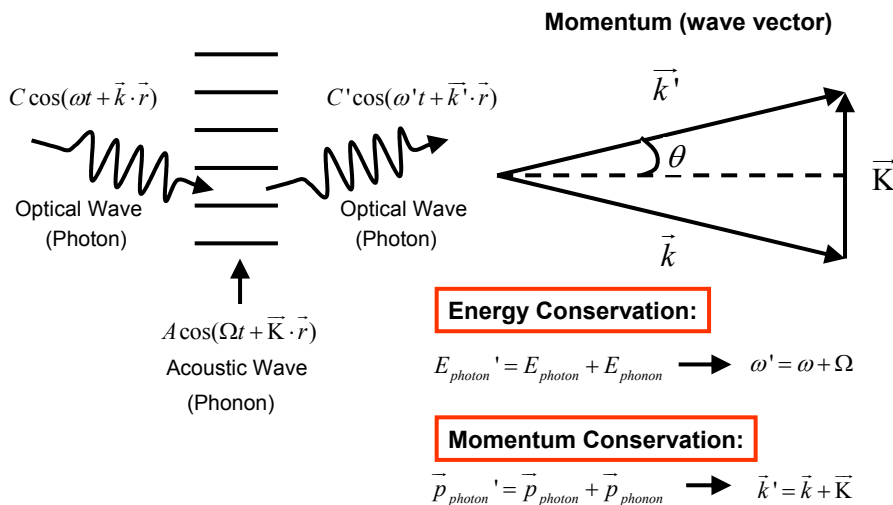
(a) Isotropic System				
Substance	Wavelength λ (μm)	P_{11}	P_{12}	
Fused silica (SiO_2)	0.63	0.121	0.270	
As_2S_3 glass	1.15	0.308	0.299	
Water	0.63	± 0.31	± 0.31	
$\text{Ge}_{33}\text{Se}_{55}\text{As}_{12}$ (glass)	1.06	± 0.21	± 0.21	
Lucite	0.63	± 0.30	± 0.28	
Polystyrene	0.63	± 0.30	± 0.31	

(b) Cubic System: Classes $\bar{4}3m$, 432 , and $m\bar{3}m$					
Substance	Wavelength λ (μm)	P_{11}	P_{12}	P_{44}	$P_{11} - P_{12}$
CdTe	10.60	-0.152	-0.017	-0.057	-0.135
GaAs	1.15	-0.165	-0.140	-0.072	-0.025
GaP	0.633	-0.151	-0.082	-0.074	-0.069
Ge	2.0-2.2	-0.063	-0.0535	-0.074	-0.0095
	10.60	0.27	0.235	0.125	
NaCl	0.55-0.65	0.115	0.159	-0.011	-0.042

A. Yariv, "Optical Waves in Crystals"

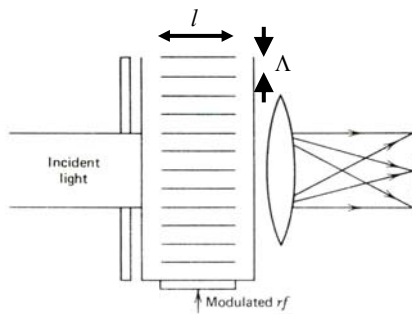
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Particle Picture of Acousto-optic Interaction



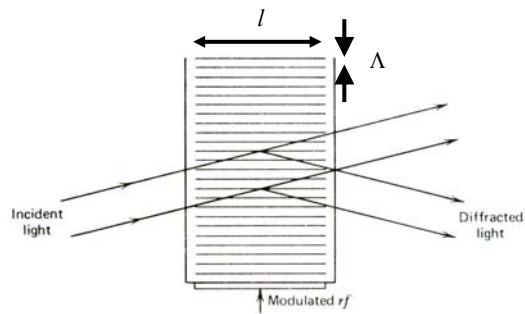
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Raman-Nath-Type vs. Bragg-Type Modulators



Raman-Nath-Type Modulator

$$l \ll \frac{\Lambda^2}{\lambda}$$

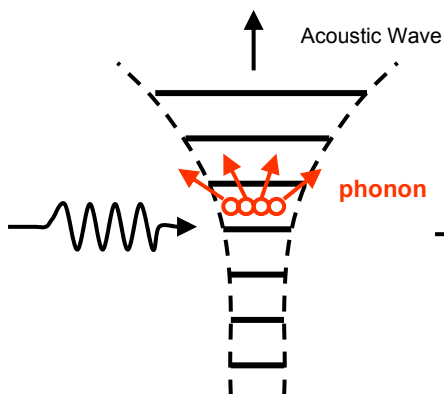


Bragg-Type Modulator

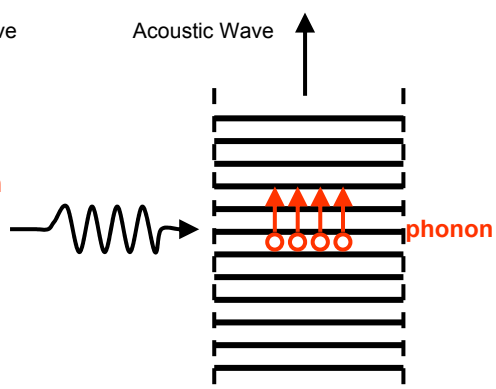
$$l \gg \frac{\Lambda^2}{\lambda}$$

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Momentum Figures of Raman-Nath Scattering and Bragg Scattering



Raman-Nath Scattering



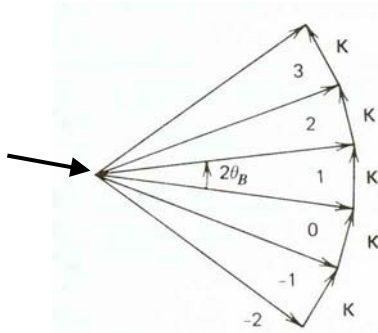
Bragg Scattering

- In Raman-Nath cases, the direction of phonon momentum or the wavevector of acoustic wave is diverged

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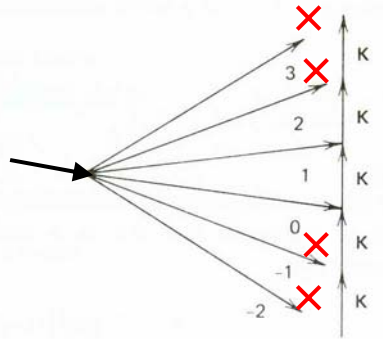
Momentum Figures of Raman-Nath Scattering and Bragg Scattering

Multiple Beam Diffraction



Multiple Raman-Nath Scattering

Bragg Diffraction



Multiple Bragg Scattering

- In Bragg scattering cases, since the phonon momentum is constant, multiple scattering is difficult due to momentum non-conservative (phase mismatched)

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Intensity Modulation of Raman-Nath AO Modulator

To evaluate the diffraction efficiencies associated with each diffraction order, we consider the periodically perturbed sheet acting as a phase grating

$$\Delta n(x, y, z, t) = \begin{cases} \Delta n_0 \sin(\Omega t - \vec{K} \cdot \vec{r}) & 0 < z < L \\ 0 & \text{Otherwise} \end{cases}$$

An incident optical wave

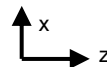
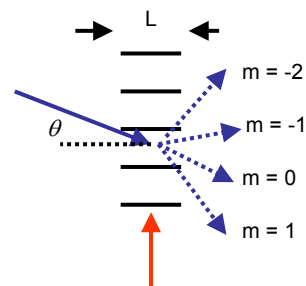
$$E = E_0 \exp[j(\omega t - \vec{k} \cdot \vec{r})]$$

Thus, the transmitted wave can be written

$$E_t = E_0 \exp[-j\phi + j(\omega t - \vec{k} \cdot \vec{r})]$$

where

$$\phi = \int \frac{\omega}{c} \Delta n ds \quad s: \text{interaction path}$$



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Intensity Modulation of Raman-Nath AO Modulator

If L is small enough, the integration can simply be given by

$$\phi = \frac{\omega}{c} \Delta n_0 \frac{L}{\cos \theta} \sin(\Omega t - \vec{K} \cdot \vec{r}) \equiv \delta \sin(\Omega t - \vec{K} \cdot \vec{r})$$

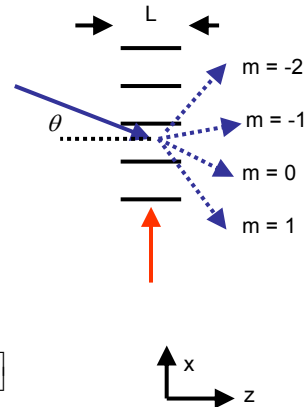
Then the transmitted wave

$$E_t = E_0 \exp \left[-j\delta \sin(\Omega t - \vec{K} \cdot \vec{r}) + j(\omega t - \vec{k} \cdot \vec{r}) \right]$$

By using the identity for Bessel functions

$$e^{-j\delta \sin x} = \sum_{m=-\infty}^{\infty} J_m(\delta) e^{-jmx}$$

$$E_t = E_0 \sum_{m=-\infty}^{\infty} J_m(\delta) \exp \left[j(\omega - m\Omega)t - j(\vec{k} - m\vec{K}) \cdot \vec{r} \right]$$



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Intensity Modulation of Raman-Nath AO Modulator

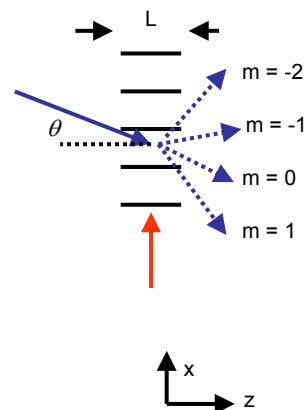
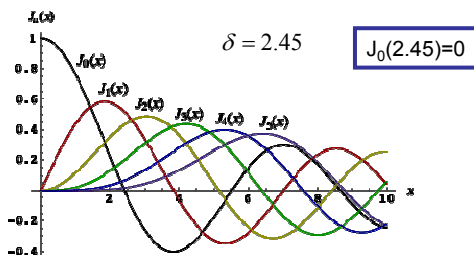
The diffraction efficiency for the m th-order Raman-Nath diffraction is

$$\eta_m = J_m^2(\delta) = J_m^2 \left(\frac{2\pi L \Delta n_0}{\lambda \cos \theta} \right)$$

The diffraction efficiency for the order of $m = 1$ or -1 is maximum when the modulation index

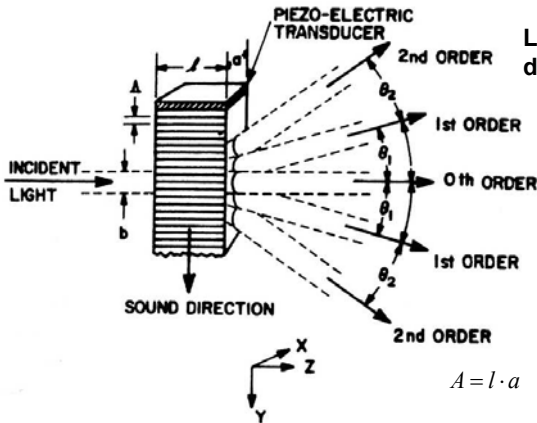
$$\delta = 1.85$$

The zeroth order is completely quenched as



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Raman-Nath-Type Modulators



Light passing through in the z-direction undergoes a phase shift

$$\Delta\varphi = \frac{\Delta n 2\pi l}{\lambda_0} \sin\left(\frac{2\pi y}{\Lambda}\right)$$

Δn : index change (AO)

l : interaction length

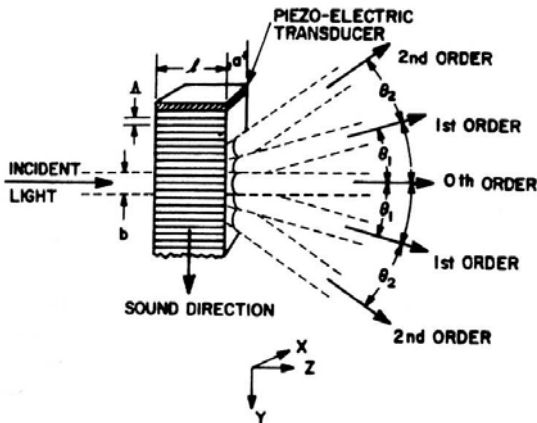
Λ : acoustic wavelength

$$A = l \cdot a \quad \Delta\varphi = \frac{2\pi}{\lambda_0} \sqrt{\frac{M_2 10^7 P_a l}{2a}} \sin\left(\frac{2\pi y}{\Lambda}\right)$$

- The optical beam is incident transversely to the acoustic beam.
- The optical waves undergo a simple phase grating diffraction (short acoustic beam width)

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Raman-Nath-Type Modulators



For Raman-Nath type diffraction, the interaction length should be small

$$l \ll \frac{\Lambda^2}{\lambda}$$

The diffraction angle (mode)

$$\sin \theta = \frac{m\lambda_0}{\Lambda}, \quad m = 0, \pm 1, \pm 2, \dots$$

The intensity of these order modes

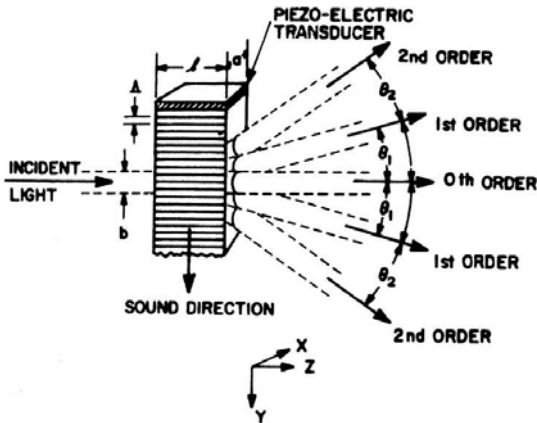
$$\frac{I}{I_0} = \begin{cases} [J_m(\Delta\varphi')]^2 / 2, & |m| > 0 \\ [J_0(\Delta\varphi')]^2, & |m| = 0 \end{cases}$$

$$\text{where } \Delta\varphi' = \max\{\Delta\varphi\} = \frac{\Delta n 2\pi l}{\lambda_0} = \frac{2\pi}{\lambda_0} \sqrt{\frac{M_2 10^7 P_a l}{2a}}$$

I_0 : the intensity of the transmitted optical beam in the absent of acoustic waves

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Raman-Nath-Type Modulators



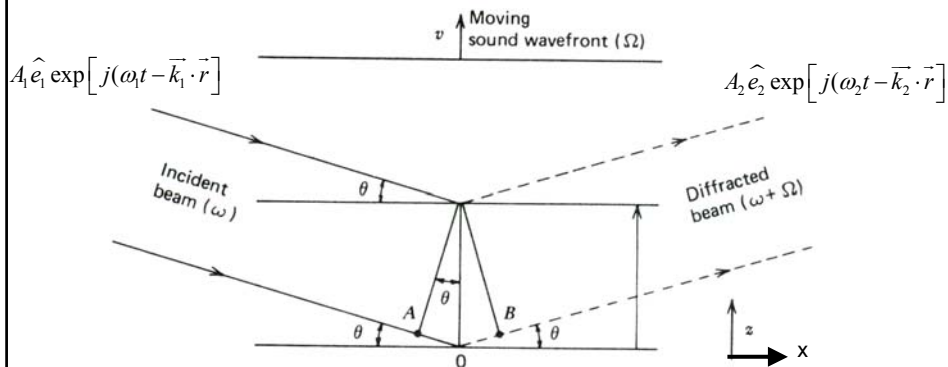
The output channel is usually taken to be the zeroth-order mode. The modulation index is defined by

$$\eta_{RN} = \frac{[I_0 - I(m=0)]}{I_0} = 1 - \left[J_0(\Delta\phi) \right]^2$$

- Raman-Nath modulators generally have a smaller modulation index
- They can not be used as switches because the light is distributed over many orders

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Intensity Modulation of Bragg-type AO Modulator



$$\begin{aligned} \vec{E} &= A_1 \hat{e}_1 \exp[j(\omega_1 t - \vec{k}_1 \cdot \vec{r})] + A_2 \hat{e}_2 \exp[j(\omega_2 t - \vec{k}_2 \cdot \vec{r})] \\ &= A_1 \hat{e}_1 \exp[j(\omega_1 t - \alpha_1 x - \beta_1 z)] + A_2 \hat{e}_2 \exp[j(\omega_2 t - \alpha_2 x - \beta_2 z)] \end{aligned}$$

x-component z-component

all the wavevectors are on x-z plane

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Intensity Modulation of Bragg-type AO Modulator

The electric field E must satisfy the following wave equation:

$$(\nabla^2 + \omega^2 \mu \varepsilon + \omega^2 \mu \Delta \varepsilon) E = 0$$

↑
Photoelastic perturbation

According to the expression of electric field in the previous slide

$$\begin{aligned} & \sum_{m=1,2} \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - 2j\beta_m \frac{\partial}{\partial z} - 2j\alpha_m \frac{\partial}{\partial x} \right] A_m \hat{e}_m \exp[j(\omega_m t - \alpha_m x - \beta_m z)] \\ &= -\omega^2 \mu \sum_{i=1,2} \Delta \varepsilon A_i \hat{e}_i \exp[j(\omega_i t - \alpha_i x - \beta_i z)] \\ & \quad \nwarrow \varepsilon_i \{ \exp[j(\Omega t - Kz)] + \exp[-j(\Omega t - Kz)] \} \end{aligned}$$

Suppose the second derivatives are neglected and only the first derivative remains

$$\frac{\partial^2}{\partial x^2} = 0 \quad \frac{\partial^2}{\partial z^2} = 0$$

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Intensity Modulation of Bragg-type AO Modulator

Suppose A_1 and A_2 only vary in x direction

$$\begin{aligned} & -2j\alpha_1 \frac{dA_1}{dx} \hat{e}_1 \exp[j(\omega_1 t - \alpha_1 x - \beta_1 z)] - 2j\alpha_2 \frac{dA_2}{dx} \hat{e}_2 \exp[j(\omega_2 t - \alpha_2 x - \beta_2 z)] \\ &= -\omega^2 \mu \{ \varepsilon_1 \exp[j(\Omega t - Kz)] + \varepsilon_1 \exp[-j(\Omega t - Kz)] \} \\ & \times \{ A_1 \hat{e}_1 \exp[j(\omega_1 t - \alpha_1 x - \beta_1 z)] + A_2 \hat{e}_2 \exp[j(\omega_2 t - \alpha_2 x - \beta_2 z)] \} \end{aligned}$$



By scalar multiplication $\hat{e}_i^* \exp[-j(\omega_i t - \alpha_i x - \beta_i z)] \quad i=1,2$

$$\frac{dA_1}{dx} = -j\kappa_{12} A_2 \exp(j\Delta \alpha x)$$

$$\frac{dA_2}{dx} = -j\kappa_{12}^* A_1 \exp(-j\Delta \alpha x)$$

supposing

$$\begin{aligned} \beta_2 &= \beta_1 \pm K \\ \omega_2 &= \omega_1 \pm \Omega \end{aligned}$$

$$\theta_B = \sin^{-1} \left(\frac{K}{2k} \right) = \sin^{-1} \left(\frac{\lambda}{2\Lambda n} \right)$$

(+: phonon absorbed
-: phonon generated)

where

$$\kappa_{12} = \frac{\sqrt{\omega_1 \omega_2}}{4\mu} \left[\hat{e}_1^* \cdot \varepsilon_1 \cdot \hat{e}_2 \right]$$

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Intensity Modulation of Bragg-type AO Modulator

Suppose the x-direction momentum matched

$$\Delta\alpha = 0$$

The coupled equations become

$$\begin{aligned}\frac{dA_1}{dx} &= -j\kappa_{12}A_2 \\ \frac{dA_2}{dx} &= -j\kappa_{12}^*A_1\end{aligned}$$

The solution is

$$\begin{aligned}A_1(x) &= A_1(0)\cos\kappa x - j\frac{\kappa_{12}}{\kappa}A_2(0)\sin\kappa x \\ A_2(x) &= A_2(0)\cos\kappa x - j\frac{\kappa_{12}^*}{\kappa}A_1(0)\sin\kappa x\end{aligned}$$

where $\kappa = |\kappa_{12}|$

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Intensity Modulation of Bragg-type AO Modulator

In the special case of a single wave incident at $x = 0$, the solution is

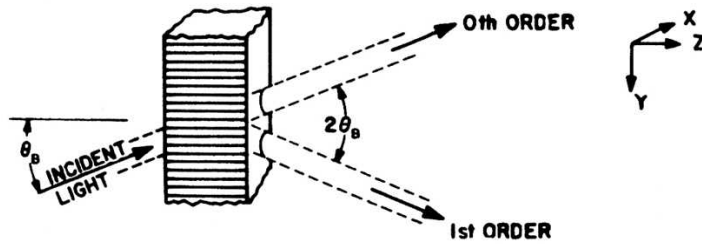
$$\begin{aligned}A_1(x) &= A_1(0)\cos\kappa x \\ A_2(x) &= -j\frac{\kappa_{12}^*}{\kappa}A_1(0)\sin\kappa x\end{aligned}$$

The fraction of the power of the incident beam transferred in a distance L into the diffracted beam

$$\frac{I_{\text{diffracted}}}{I_{\text{incident}}} = \frac{|A_2(L)|^2}{|A_1(0)|^2} = \sin^2\kappa L$$

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Bragg-Type Modulators



For Bragg-type diffraction, the interaction length between the optical and acoustic beams must be relatively long. Therefore, the multiple diffraction can occur

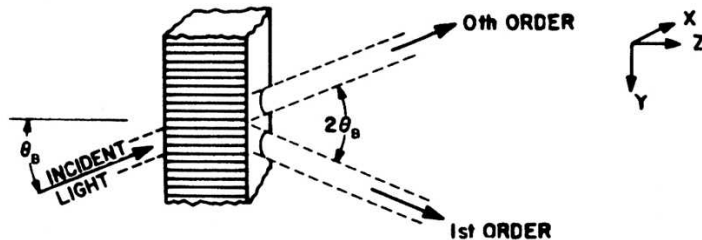
$$l \gg \frac{\Lambda^2}{\lambda} \quad \leftarrow \text{effective wavelength}$$

In the case of Bragg-type modulators, the input angle of incident beam should be the Bragg angle

$$\sin \theta_B = \frac{\lambda}{2\Lambda n} \quad n: \text{refractive index}$$

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Bragg-Type Modulators



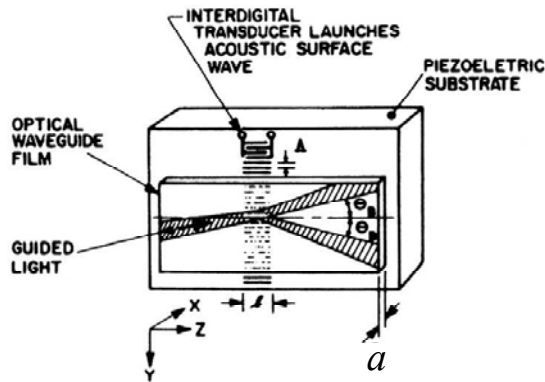
Generally, the output of modulator is taken to be the zeroth-order beam. Therefore, the modulation index is given

$$\eta_B = \frac{I_0 - I}{I_0} = \sin^2 \left(\frac{\Delta\phi'}{2} \right) \quad \text{Recall} \quad \Delta\phi' = \max \{ \Delta\phi \} = \frac{\Delta n 2\pi l}{\lambda_0} = \frac{2\pi}{\lambda_0} \sqrt{\frac{M_2 10^7 P_a l}{2a}}$$

$$\longrightarrow \eta_B = \frac{I_0 - I}{I_0} = \sin^2 \left[\frac{\pi}{\lambda_0} \sqrt{\frac{10^7 M_2 P_a l}{2a}} \right]$$

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Waveguide AO Bragg Modulators



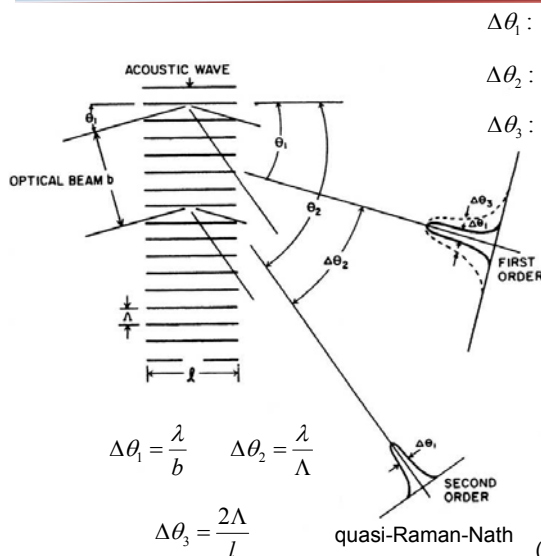
- The drive power can be reduced in waveguide-based AO modulators

$$\eta_B = \sin^2 \left[\frac{\pi}{\lambda_0} \sqrt{\frac{10^7 M_2 P_a l}{2a}} \right]$$

$$a \downarrow \text{ then } P_a \downarrow$$

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Bragg-Type Beam Deflectors and Switches



$\Delta\theta_1$: half intensity width

$\Delta\theta_2$: angular separation of the peaks

$\Delta\theta_3$: half intensity width of scanned angle

In fact, the AO frequency is equivalent to a frequency shift of optical frequency (Doppler Effect)

The variation of acoustic frequency will reduce the intensity (spoil Bragg condition)

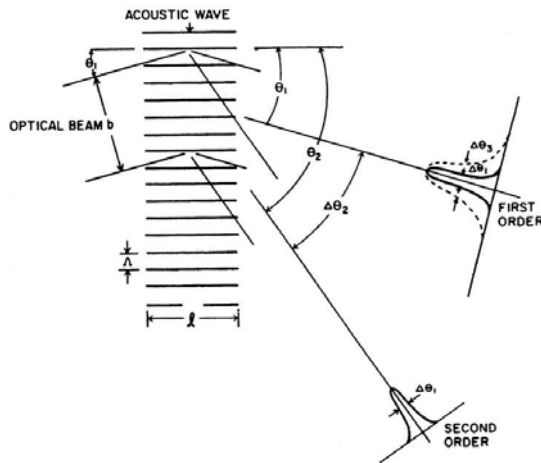
Acoustic wave velocity

$$\Delta f_0 \cong \frac{2v_a \Lambda}{\lambda l}$$

(bandwidth of half intensity due to variation)

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Bragg-Type Beam Deflectors and Switches



The overall response time τ (bandwidth) is limited by the three factors

$$\tau = \frac{1}{\Delta f_0} + \frac{1}{\Delta f_a} + t \quad t = \frac{b}{v_a}$$

frequency variation range

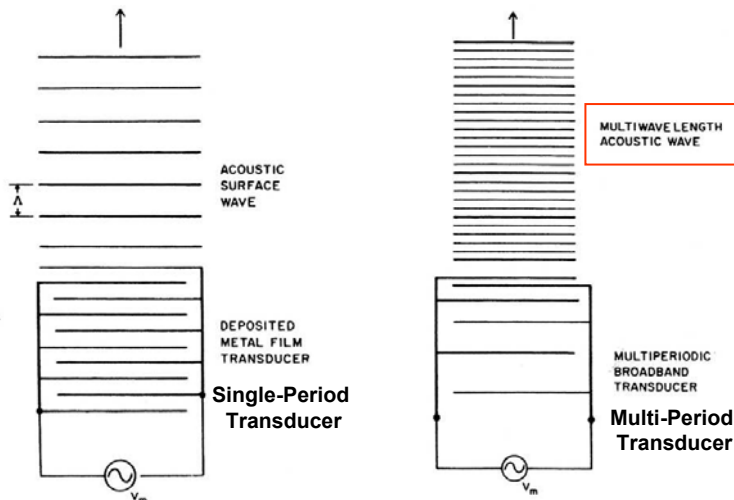
acoustic transit time

transducer bandwidth

How to increase the bandwidth Δf_0

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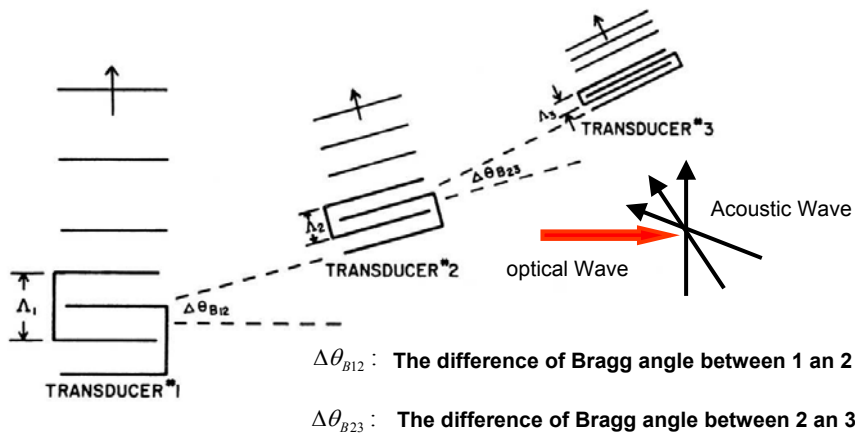
Single Chirp-Transducer



- Both wide bandwidth and high diffraction efficiency are obtained

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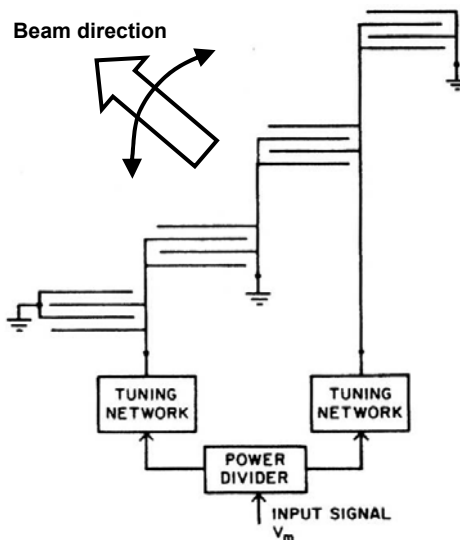
Multi-Period Transducers in a Tilted Array



- The composite of surface acoustic waves can satisfy the Bragg condition in multiple frequency ranges.

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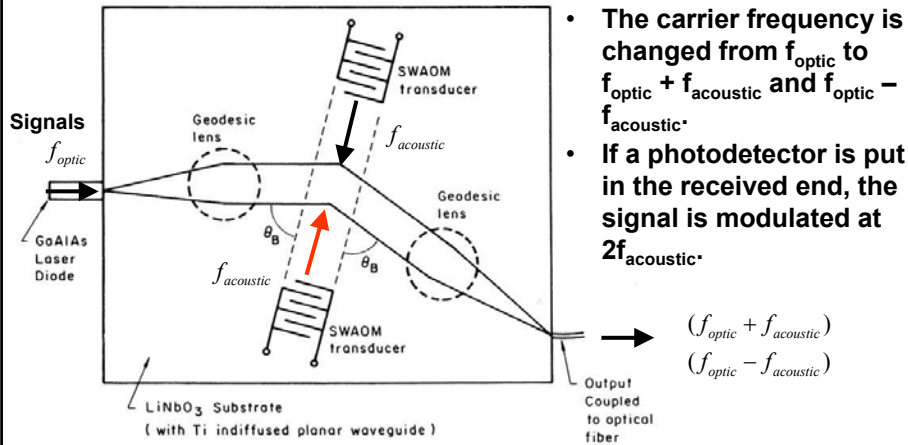
Phase Array Transducer



- The transducers are arranged in a stepped configuration, resulting in a spatial phase shift.
- The acoustic wave beam is scanned as the frequency changes (like a scanning radar)
- The scanning acoustic beam automatically tracks the Bragg-condition

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Acousto-Optic Frequency Shifter



- The carrier frequency is changed from f_{optic} to $f_{\text{optic}} + f_{\text{acoustic}}$ and $f_{\text{optic}} - f_{\text{acoustic}}$.
- If a photodetector is put in the received end, the signal is modulated at $2f_{\text{acoustic}}$.

- It can be applied in RF photonics and frequency division multiplexing.