Integrated Modulator II --- Acousto-Optics

Class: Integrated Photonic Devices  
Time: Fri. 8:00am ~ 11:00am.  
Classroom: 資電206  
Lecturer: Prof. 李明昌(Ming-Chang Lee)

Fundamental Principles of AO effect

Mechanical strain in a solid causes a change in the index of refraction

\[ \Delta n = \sqrt{n^6 p^2 10^7 P_a (2 \rho \nu_v^3 A)} \]

• In crystalline solids, the AO effect (photoelastic effect) depends strongly on the orientation (p).
• Overall, the AO effect is small (10^{-4}) even for an acoustic power density 100W/cm²
• Both bulk acoustic waves and surface acoustic waves (planar waveguide) are used

\[ \Delta n = \sqrt{M_s 10^7 P_a / (2A)} \]

where \( M_s = \frac{n^6 p^2}{\rho \nu_v^3} \) (AO figure of merit)
The acousto-optic effects are traditionally defined in terms of the changes in the elements of the relative impermeability tensor

\[ \Delta \eta_i = p_i^j S_j \quad i = 1, \ldots, 6 \]

\[ j = 1, \ldots, 6 \]

Strain-optic Tensor  
Strain (strain tensor is symmetric)

\[ \text{Acoustic Wave (strain)} \]

\[ u(z, t) = \bar{S} \sin(\Omega t - \bar{K} \cdot \bar{r}) \]

\[ |\bar{K}| = \frac{2\pi}{\lambda} \]

The equation of index ellipsoid in the presence of strain field can be written by

\[ x^2 \left( \frac{1}{n_x^2} + p_{11} S_1 + p_{12} S_2 + p_{13} S_3 + p_{14} S_4 + p_{15} S_5 + p_{16} S_6 \right) \]
\[ + y^2 \left( \frac{1}{n_y^2} + p_{21} S_1 + p_{22} S_2 + p_{23} S_3 + p_{24} S_4 + p_{25} S_5 + p_{26} S_6 \right) \]
\[ + z^2 \left( \frac{1}{n_z^2} + p_{31} S_1 + p_{32} S_2 + p_{33} S_3 + p_{34} S_4 + p_{35} S_5 + p_{36} S_6 \right) \]
\[ + 2xz (p_{14} S_1 + p_{15} S_2 + p_{16} S_3 + p_{12} S_4 + p_{13} S_5 + p_{16} S_6) \]
\[ + 2yz (p_{24} S_1 + p_{25} S_2 + p_{26} S_3 + p_{23} S_4 + p_{25} S_5 + p_{26} S_6) \]
\[ + 2zx (p_{34} S_1 + p_{35} S_2 + p_{36} S_3 + p_{33} S_4 + p_{35} S_5 + p_{36} S_6) \]
\[ + 2xy (p_{41} S_1 + p_{42} S_2 + p_{43} S_3 + p_{44} S_4 + p_{45} S_5 + p_{46} S_6) = 1 \]

where \( n_x, n_y, n_z \) are the principle indices of refraction
Examples of Strain-Optic Tensor

A. Yariv, “Optical Waves in Crystals”

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Table 9.1 (Continued).

<table>
<thead>
<tr>
<th>Isotropic (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{11}$</td>
</tr>
<tr>
<td>$P_{12}$</td>
</tr>
<tr>
<td>$P_{11}$</td>
</tr>
<tr>
<td>$0$</td>
</tr>
<tr>
<td>$0$</td>
</tr>
<tr>
<td>$0$</td>
</tr>
</tbody>
</table>

*The number inside the parentheses indicates the number of independent coefficients.

A. Yariv, “Optical Waves in Crystals”

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Examples of Strain-Optic Tensor

<table>
<thead>
<tr>
<th>Substrate</th>
<th>Wavelength λ (nm)</th>
<th>$P_{11}$</th>
<th>$P_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fused silica (SiO$_2$)</td>
<td>0.63</td>
<td>0.121</td>
<td>0.270</td>
</tr>
<tr>
<td>As$_2$S$_3$ glass</td>
<td>1.15</td>
<td>0.308</td>
<td>0.299</td>
</tr>
<tr>
<td>Water</td>
<td>0.63</td>
<td>0.311</td>
<td>0.311</td>
</tr>
<tr>
<td>Ga$_2$S$_3$/Ga$_2$S$_2$ (glass)</td>
<td>0.66</td>
<td>0.311</td>
<td>0.311</td>
</tr>
<tr>
<td>Lasing</td>
<td>0.63</td>
<td>0.311</td>
<td>0.311</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>0.63</td>
<td>0.311</td>
<td>0.311</td>
</tr>
</tbody>
</table>

Table 9.2: Strain-Optic Coefficients \cite{1}

A. Yariv, “Optical Waves in Crystals”

Particle Picture of Acousto-optic Interaction

Energy Conservation:

$E_{\text{photon}}' = E_{\text{photon}} + E_{\text{phonon}} \rightarrow \omega' = \omega + \Omega$

Momentum Conservation:

$p_{\text{photon}}' = p_{\text{photon}} + p_{\text{phonon}} \rightarrow \vec{k}' = \vec{k} + \vec{K}$

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Raman-Nath-Type vs. Bragg-Type Modulators

Raman-Nath-Type Modulator

\[ l < \frac{\Lambda^2}{\lambda} \]

Bragg-Type Modulator

\[ l \geq \frac{\Lambda^2}{\lambda} \]

Momentum Figures of Raman-Nath Scattering and Bragg Scattering

- In Raman-Nath cases, the direction of phonon momentum or the wavevector of acoustic wave is diverged

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Momentum Figures of Raman-Nath Scattering and Bragg Scattering

- In Bragg scattering cases, since the phonon momentum is constant, multiple scattering is difficult due to momentum non-conservative (phase mismatched).

Intensity Modulation of Raman-Nath AO Modulator

To evaluate the diffraction efficiencies associated with each diffraction order, we consider the periodically perturbed sheet acting as a phase grating.

\[
\Delta n(x, y, z, t) = \begin{cases} 
\Delta n_0 \sin(\Omega t - \vec{K} \cdot \vec{r}) & 0 < z < L \\
0 & \text{Otherwise}
\end{cases}
\]

An incident optical wave

\[
E = E_0 \exp\left[ j( \omega t - \vec{k} \cdot \vec{r} ) \right]
\]

Thus, the transmitted wave can be written

\[
E_r = E_0 \exp\left[ -j\phi + j( \omega t - \vec{k} \cdot \vec{r} ) \right]
\]

where

\[
\phi = \frac{\theta}{c} \Delta n ds
\]

s: interaction path
Intensity Modulation of Raman-Nath AO Modulator

If $L$ is small enough, the integration can simply be given by

$$\phi = \frac{\omega}{c} \frac{L}{\cos \theta} \sin(\Omega t - \vec{K} \cdot \vec{r}) = \delta \sin(\Omega t - \vec{K} \cdot \vec{r})$$

Then the transmitted wave

$$E_i = E_0 \exp \left[ -j\delta \sin(\Omega t - \vec{K} \cdot \vec{r}) + j(\omega t - \vec{k} \cdot \vec{r}) \right]$$

By using the identity for Bessel functions

$$e^{-j\delta \sin t} = \sum_{m=-\infty}^{\infty} J_m(\delta) e^{-jm\pi}$$

$$E_i = E_0 \sum_{m=-\infty}^{\infty} J_m(\delta) \exp \left[ j(\omega - m\Omega) t - j(\vec{k} - m\vec{K}) \cdot \vec{r} \right]$$

The diffraction efficiency for the $m$th-order Raman-Nath diffraction is

$$\eta_m = J_m(\delta)^2 = J_m \left( \frac{2\pi L \Delta n_l}{\lambda \cos \theta} \right)$$

The diffraction efficiency for the order of $m = 1$ or -1 is maximum when the modulation index

$$\delta = 1.85$$

The zeroth order is completely quenched as

$$\delta = 2.45$$

$$J_0(2.45) = 0$$
Raman-Nath-Type Modulators

Light passing through in the z-direction undergoes a phase shift
\[ \Delta \varphi = \Delta n 2 \pi l \frac{\sin \left( \frac{2 \pi y}{\Lambda} \right)}{\lambda_0} \]
\[ \Delta n : \text{index change (AO)} \]
\[ l : \text{interaction length} \]
\[ \Lambda : \text{acoustic wavelength} \]

\[ A = l \cdot a \]
\[ \Delta \varphi = \frac{2 \pi}{\lambda_0} \sqrt{\frac{M \cdot 10^7 P_l}{2a}} \sin \left( \frac{2 \pi y}{\Lambda} \right) \]

- The optical beam is incident transversely to the acoustic beam.
- The optical waves undergo a simple phase grating diffraction (short acoustic beam width)

Raman-Nath-Type Modulators

For Raman-Nath type diffraction, the interaction length should be small
\[ l \ll \frac{\Lambda^2}{\lambda} \]

The diffraction angle (mode)
\[ \sin \theta = \frac{m \lambda_0}{\Lambda}, \quad m = 0, \pm 1, \pm 2, \ldots \]

The intensity of these order modes
\[ I = \frac{[J_m(\Delta \varphi)]^2}{2}, \quad |m| > 0 \]
\[ I_0 = [J_m(\Delta \varphi)]^2, \quad |m| = 0 \]

where \( \Delta \varphi = \max \{ \Delta \varphi \} \)
\[ \Delta \varphi = \frac{\Delta n 2 \pi l}{\lambda_0} = \frac{2 \pi}{\lambda_0} \sqrt{\frac{M \cdot 10^7 P_l}{2a}} \]

\( I_0 \) : the intensity of the transmitted optical beam in the absence of acoustic waves

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Raman-Nath-Type Modulators

- Raman-Nath modulators generally have a smaller modulation index.
- They cannot be used as switches because the light is distributed over many orders.

The output channel is usually taken to be the zeroth-order mode. The modulation index is defined by:

$$\eta_{RN} = \left[ \frac{I_0 - I(m=0)}{I_0} \right] = 1 - \left[ I_0 \left( \Delta \phi \right) \right]^2$$

Intensity Modulation of Bragg-type AO Modulator

$$\hat{E} = A_1 \hat{e}_1 \exp \left[ j(\omega t - \overrightarrow{k_1} \cdot \overrightarrow{r}) \right] + A_2 \hat{e}_2 \exp \left[ j(\omega t - \overrightarrow{k_2} \cdot \overrightarrow{r}) \right]$$

$$= A_1 \hat{e}_1 \exp \left[ j(\omega t + \alpha_1 x - \beta_1 z) \right] + A_2 \hat{e}_2 \exp \left[ j(\omega t - \alpha_2 x - \beta_2 z) \right]$$

All the wavevectors are on x-z plane.
Intensity Modulation of Bragg-type AO Modulator

The electric field \( E \) must satisfy the following wave equation:

\[
(\nabla^2 + \omega^2 \varepsilon \mu + \omega^2 \mu \Delta \varepsilon) E = 0
\]

According to the expression of electric field in the previous slide

\[
\sum_{m=1}^{2} \left[ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} - 2j\beta_m \frac{\partial}{\partial z} - 2j\alpha_m \frac{\partial}{\partial x} \right] A_m^i \hat{e}_m \exp\left[j(\omega_{m}\,t - \alpha_m x - \beta_m z)\right]
\]

\[
= -\omega^2 \mu \sum_{m=1}^{2} \Delta x A_m \hat{e} \exp\left[j(\omega_{m}\,t - \alpha_m x - \beta_m z)\right]
\]

\[
e_i \left[ \exp\left[j(\Omega \,t - Kz)\right] + \exp\left[-j(\Omega \,t - Kz)\right]\right]
\]

Suppose the second derivatives are neglected and only the first derivative remains

\[
\frac{\partial^2}{\partial x^2} = 0 \quad \frac{\partial^2}{\partial z^2} = 0
\]

Intensity Modulation of Bragg-type AO Modulator

Suppose \( A_1 \) and \( A_2 \) only vary in x direction

\[
-2j\alpha \frac{dA_i}{dx} \hat{e} \exp\left[j(\omega_{m}\,t - \alpha_m x - \beta_m z)\right] - 2j\alpha \frac{dA_i}{dx} \hat{e} \exp\left[j(\omega_{m}\,t - \alpha_m x - \beta_m z)\right]
\]

\[
= -\omega^2 \mu [\hat{e}_i \exp\left[j(\Omega \,t - Kz)\right] + \hat{e}_i \exp\left[-j(\Omega \,t - Kz)\right]]
\]

\[
x \left[ A_i \hat{e}_i \exp\left[j(\omega_{m}\,t - \alpha_m x - \beta_m z)\right] + A_i \hat{e}_i \exp\left[j(\omega_{m}\,t - \alpha_m x - \beta_m z)\right]\right]
\]

By scalar multiplication \( \hat{e}_i \exp\left[-j(\omega_{m}\,t - \alpha_m x - \beta_m z)\right] \) \( i=1,2 \)

\[
\frac{dA_i}{dx} = -j\kappa_{i2} A_i \exp(j\Delta a x)
\]

\[
\frac{dA_i}{dx} = -j\kappa_{i2}^* A_i \exp(-j\Delta a x)
\]

\[
\theta_{\pm} = \sin^{-1}\left(\frac{K}{2\kappa}\right) = \sin^{-1}\left(\frac{A}{2\Lambda}\right)
\]

supposing \( \beta_2 = \beta_1 \pm K \) \( \beta_2 = \alpha_2 \pm \Omega \) (\( \pm \) : phonon absorbed \( \pm \) : phonon generated)

\[
\kappa_{i2} = \sqrt{\frac{\omega_2 \omega_1}{4\mu}} [\hat{e}_i \cdot \hat{e}_2]
\]

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Intensity Modulation of Bragg-type AO Modulator

Suppose the x-direction momentum matched
\[ \Delta \alpha = 0 \]

The coupled equations become
\[
\frac{dA_1}{dx} = -j \kappa_{12} A_2 \\
\frac{dA_2}{dx} = -j \kappa_{21}^* A_1
\]

The solution is
\[
A_1(x) = A_1(0) \cos \kappa x - j \frac{\kappa_{12}}{\kappa} A_2(0) \sin \kappa x \\
A_2(x) = A_2(0) \cos \kappa x - j \frac{\kappa_{21}^*}{\kappa} A_1(0) \sin \kappa x
\]

where \( \kappa = |\kappa_{12}| \)

Intensity Modulation of Bragg-type AO Modulator

In the special case of a single wave incident at \( x = 0 \), the solution is
\[
A_1(x) = A_1(0) \cos \kappa x \\
A_2(x) = -j \frac{\kappa_{21}^*}{\kappa} A_1(0) \sin \kappa x
\]

The fraction of the power of the incident beam transferred in a distance \( L \) into the diffracted beam
\[
\frac{I_{\text{diffused}}}{I_{\text{incident}}} = \left| \frac{A_2(L)}{A_1(0)} \right|^2 = \sin^2 \kappa L
\]
For Bragg-type diffraction, the interaction length between the optical and acoustic beams must be relatively long. Therefore, the multiple diffraction can occur

\[ l \gg \frac{\Lambda^2}{\lambda} \]

\( l \) effective wavelength

In the case of Bragg-type modulators, the input angle of incident beam should be the Bragg angle

\[ \sin \theta_b = \frac{\lambda}{2\Lambda_t} \quad n: \text{refractive index} \]

Generally, the output of modulator is taken to be the zeroth-order beam. Therefore, the modulation index is given

\[ \eta_s = \frac{I_s - I}{I_0} = \sin^2 \left( \frac{\Delta \varphi}{2} \right) \]

Recall \( \Delta \varphi = \max \{ \Delta \varphi \} = \frac{\Delta n \pi I}{\lambda_0} = \frac{2\pi}{\lambda_0} \sqrt{\frac{M_s 10^{7} P}{2a}} \)

\[ \eta_s = \frac{I_s - I}{I_0} = \sin^2 \left( \frac{\pi}{\lambda_0} \sqrt{\frac{10^{7} M_s P}{2a}} \right) \]
Waveguide AO Bragg Modulators

- The drive power can be reduced in waveguide-based AO modulators

\[ \eta_a = \sin^2 \left( \frac{\pi}{\lambda_0} \sqrt{\frac{10^7 M P f}{2a}} \right) \]

\[ a \downarrow \text{ then } P_a \downarrow \]

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Bragg-Type Beam Deflectors and Switches

- \( \Delta \theta_i \): half intensity width
- \( \Delta \theta_p \): angular separation of the peaks
- \( \Delta \theta_s \): half intensity width of scanned angle

In fact, the AO frequency is equivalent to a frequency shift of optical frequency (Doppler Effect)

The variation of acoustic frequency will reduce the intensity (spoil Bragg condition)

Acoustic wave velocity

\[ \Delta f_o \approx \frac{2 \pi \Delta \lambda}{\lambda \Delta f} \]

(bandwidth of half intensity due to variation)

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The overall response time $\tau$ (bandwidth) is limited by the three factors:

$$\tau = \frac{1}{f_0} + \frac{1}{\Delta f} + \frac{t}{\nu_a}$$

- $f_0$: frequency variation range
- $\Delta f$: transducer bandwidth
- $\nu_a$: acoustic transit time

How to increase the bandwidth $\Delta f_0$

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**Single Chirp-Transducer**

- Both wide bandwidth and high diffraction efficiency are obtained
Multi-Period Transducers in a Tilted Array

- The composite of surface acoustic waves can satisfy the Bragg condition in multiple frequency ranges.

Phase Array Transducer

- The transducers are arranged in a stepped configuration, resulting in a spatial phase shift.
- The acoustic wave beam is scanned as the frequency changes (like a scanning radar).
- The scanning acoustic beam automatically tracks the Bragg-condition.
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**Acousto-Optic Frequency Shifter**

- The carrier frequency is changed from $f_{optic}$ to $f_{optic} + f_{acoustic}$ and $f_{optic} - f_{acoustic}$.
- If a photodetector is put in the received end, the signal is modulated at $2f_{acoustic}$.
- It can be applied in RF photonics and frequency division multiplexing.

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