Integrated Modulator I

Class: Integrated Photonic Devices
Time: Fri. 8:00am ~ 11:00am.
Classroom: 資電206
Lecturer: Prof. 李明昌(Ming-Chang Lee)

Basic Operation Characteristic of Modulators

(1) Modulation Depth

$$\eta = \frac{(I_0 - I)}{I_0}$$ (Decrease Intensity) → Type I

I : the transmitted intensity

I_0 : the intensity without operation

$$\eta = \frac{(I - I_0)}{I_m}$$ (Increase Intensity) → Type II

I_m : the transmitted intensity with maximum signal

Therefore,

Extinction Ratio

$$\eta_{max} = \frac{(I_0 - I_m)}{I_0} \quad for \ I_m \le I_0$$ (Type I)

$$\eta_{max} = \frac{(I_m - I_0)}{I_m} \quad for \ I_m \ge I_0$$ (Type II)

Basic Operation Characteristic of Modulators

If the modulation of intensity is related to phase modulation such as MZI, the modulation depth

$$\eta = \sin^2\left(\frac{\Delta\varphi}{2}\right)$$ $\Delta\varphi$ is the phase change

If the modulation is on frequency, the modulation depth is

$$D_{\max} = \frac{|f_m - f_0|}{f_0}$$

f_0 : optical carrier frequency

f_m : shifted optical frequency

(2) Bandwidth

$$T = \frac{2\pi}{\Delta f}$$

T: minimal switching time

Δf : bandwidth (3dB)

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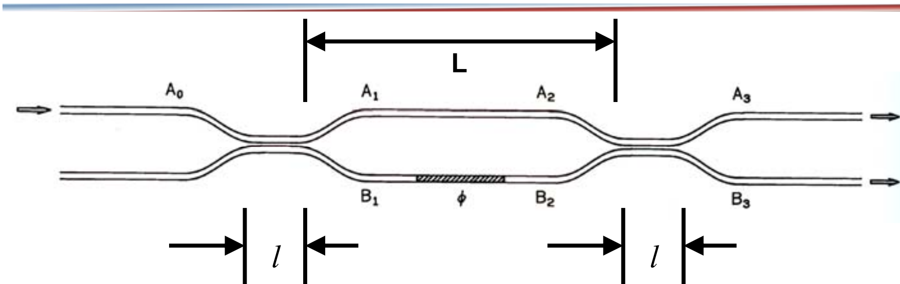
Guided-Wave Mach-Zehnder Interferometer



$$\begin{cases} |A_3|^2 = |A_0|^2 \sin^2\left(\dfrac{\phi}{2}\right) \\ |B_3|^2 = |A_0|^2 \cos^2\left(\dfrac{\phi}{2}\right) \end{cases}$$

If f is slightly modulated with δϕ

$$\begin{cases} |A_3|^2 \cong |A_0|^2 \left(\dfrac{\delta\phi}{2}\right)^2 \\ |B_3|^2 = |A_0|^2 \end{cases}$$

It is not linear modulation!

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Basic Operation Characteristic of Modulators

(3) Insertion Loss (Optical)

$$L_i = 10 \cdot \log(\frac{I_t}{I_0}) \quad \text{(Type I)}$$

$$L_i = 10 \cdot \log(\frac{I_t}{I_m}) \quad \text{(Type II)}$$

I_0 : the intensity without operation

I_t : Input light intensity

I_m : the transmitted intensity with maximum signal

(4) Power Consumption (Electrical)

$$\frac{P}{\Delta f}$$

P ← Signal Power (Electrical)

Δf ← Bandwidth

In most cases of modulators, the required drive power increases with modulation frequency.

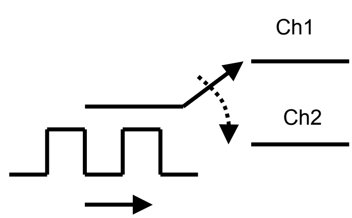
Most of power is consumed during transient time

Basic Operation Characteristic of Modulators

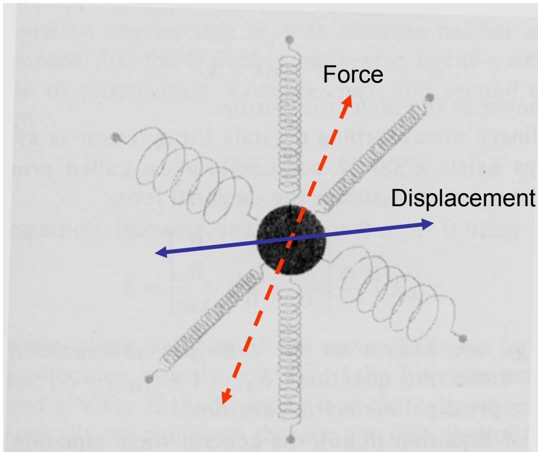
(5) Isolation (Crosstalk for switch)

$$\textbf{Isolation [dB]} = 10 \cdot \log(\frac{I_2}{I_1})$$

As channel 1 is on but channel 2 is off



Linear Anisotropic Medium



Anisotropic Materials

ε : tensor

Linear Anisotropic Medium

In a *linear* anisotropic medium, the electric displacement can be expressed in a matrix form

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \qquad (\mathbf{D} = \boldsymbol{\varepsilon} \bullet \mathbf{E})$$

If the material is nonmagnetic and lossless, the matrix ε is symmetric and can be diagonalized with real eigenvalues. And the eigenvectors are also real. Therefore,

$$\boldsymbol{\varepsilon} \rightarrow \boldsymbol{\varepsilon}'$$

and

$$x, y, z \rightarrow x', y', z'$$

$$\begin{bmatrix} D_{x'} \\ D_{y'} \\ D_{z'} \end{bmatrix} = \begin{bmatrix} \varepsilon_{x'} & 0 & 0 \\ 0 & \varepsilon_{y'} & 0 \\ 0 & 0 & \varepsilon_{z'} \end{bmatrix} \begin{bmatrix} E_{x'} \\ E_{y'} \\ E_{z'} \end{bmatrix}$$

The diagonalization is correspondent to transferring the initial coordinate system to the principle coordinate system

Linear Anisotropic Medium

The energy stored in the nonmagnetic, lossless medium is

$$U_e = \frac{1}{2}\mathbf{E}\cdot\mathbf{D} = \frac{1}{2}\sum_{i,j} E_i \varepsilon_{ij} E_j$$

If E and D are presented according to the principle coordinate system

$$2U_e = \frac{D_x^{\,2}}{\varepsilon_x} + \frac{D_y^{\,2}}{\varepsilon_y} + \frac{D_z^{\,2}}{\varepsilon_z} \qquad \text{Replace} \quad \frac{\varepsilon_0 \mathbf{D}}{\sqrt{2U_e}} = (x, y, z)$$

$$\longrightarrow \quad \frac{x^2}{n_x^{\,2}} + \frac{y^2}{n_y^{\,2}} + \frac{z^2}{n_z^{\,2}} = 1 \quad \text{(Index Ellipsoid)} \quad \text{where} \quad n_i = \sqrt{\frac{\varepsilon_i}{\varepsilon_0}}$$

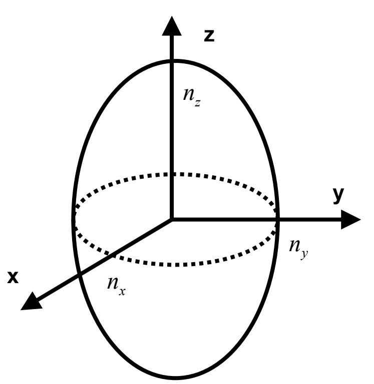
A general expression of index ellipsoid (not in the principle coordinate system but in crystal symmetry coordinate) is

$$\eta_1 (x')^2 + \eta_2 (y')^2 + \eta_3 (z')^2 + 2\eta_4 y'z' + 2\eta_5 x'z' + 2\eta_6 x'y' = 1$$

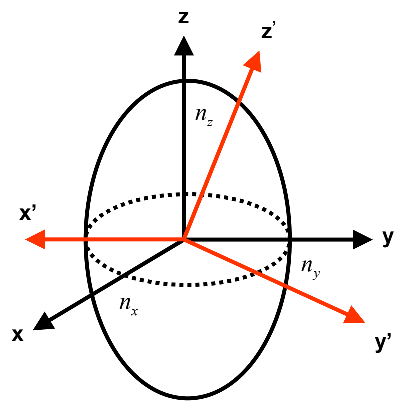
In fact $\boldsymbol{\eta} = \left(\frac{\boldsymbol{\varepsilon}}{\varepsilon_0}\right)^{-1} \equiv \frac{1}{\mathbf{n}^2}$

Linear Anisotropic Medium

Index Ellipsoid (Principle Coordinate)



Index Ellipsoid (Other Coordinate)



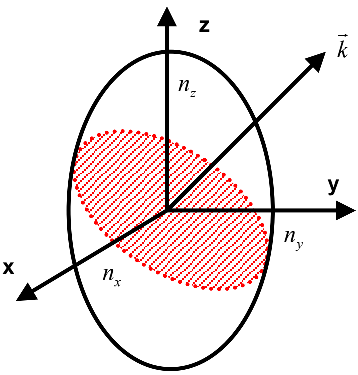
If $n_x \neq n_y \neq n_z$ $\longrightarrow$ Biaxial crystals

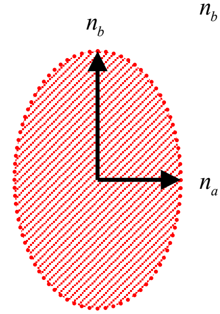
If $n_x = n_y \neq n_z$ $\longrightarrow$ Uniaxial crystals

Linear Anisotropic Medium

n_a : short axis of ellipse

n_b : long axis of ellipse





$$\vec{D} = \vec{D_a} + \vec{D_b}$$

$$k_a = \frac{2\pi}{\lambda_0} n_a$$

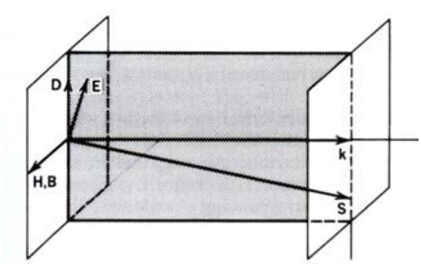
$$k_b = \frac{2\pi}{\lambda_0} n_b$$

- The wave numbers of two orthogonal displacement vectors are not the same
- Optical axis: How to choose the direction of k such that $n_a = n_b$ (circular cross section) (no bifringe)

Uniaxial: single optical axis ⟶ principle axis

biaxial: two optical axes

Wavefront Propagation Direction vs. Beam Direction (not along optical axis)



$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \longrightarrow \vec{k} \times \vec{H} = -\omega \vec{D} \qquad \vec{D} = \varepsilon \cdot \vec{E}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \nearrow \quad \vec{k} \times \vec{E} = \omega \mu_0 \vec{H}$$

$$\vec{E} \times \vec{H} = \vec{S}$$

$\vec{k}$: wavefront propagation direction

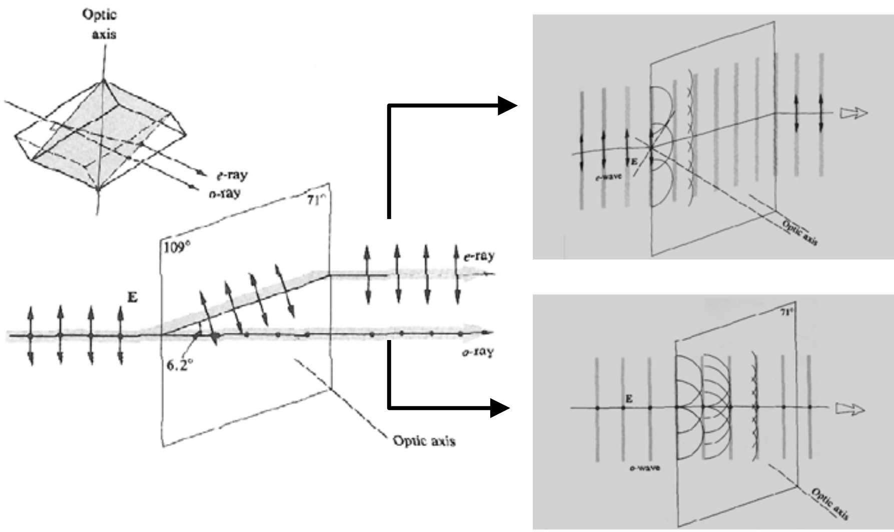
$\vec{S}$: beam propagation direction (Poynting Vector)

$\vec{k}, \vec{H}, \vec{D}$ are mutual orthogonal

$\vec{S}, \vec{H}, \vec{E}$ are mutual orthogonal

But $\vec{E}, \vec{k}$ are not perpendicular and $\vec{S}, \vec{k}$ are not parallel

Wavefront Propagation Direction vs. Beam Direction (not along optical axis)



Index, k-vector and s-vector for a normal incident

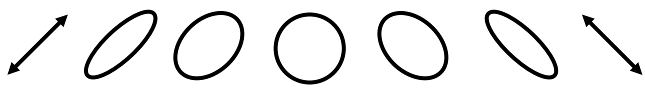
- step 1: Draw the index ellipsoid and label principle axes and indices
- step 2: Draw crystal cut
- step 3: Draw the electric displacement plane (perpendicular to the incident k vector)
- step 4: Determine two orthogonal D vectors (long axis and short axis) and the correspondent indices
- step 5: Decompose the beam into these two components
- step 6: The transmitted angles of these two s vectors are determined by the dielectric tensor

Propagation along a Principle Axis

When the wave propagates along one of the principle axes, say z. Then the field can be decomposed into two normal modes

$$E = \hat{x}E_x \exp(jk_x z - j\omega t) + \hat{y}E_y \exp(jk_y z - j\omega t)$$
$$= \left\{\hat{x}E_x + \hat{y}E_y \exp[j(k_y - k_x)z]\right\}\exp(jk_x z - j\omega t)$$

- **If it is originally linearly polarized along one of the principle axes, it remains linearly polarized in the same direction**
- **If it is originally linearly polarized at an angle** $\theta = \tan^{-1}(\frac{E_y}{E_x})$ **with respect to the x-axis, it polarization state varies periodically along z with a period** $\frac{2\pi}{|k_x - k_y|}$



Electro-Optic Effects

The electro-optic effects are traditionally defined in terms of the changes in the elements of the relative impermeability tensor η

$$\eta_\alpha(E_0) = \eta_\alpha^{(1)} + \Delta\eta_\alpha(E_0) = \eta_\alpha^{(1)} + \sum_k r_{\alpha k}E_{0k} + \sum_{k,l} s_{\alpha kl}E_{0k}E_{0l} + \cdots$$

Pockels Effect　Kerr Effect

where
$r_{\alpha k}$ **: Pockels coefficient (linear electro-optic)**
$s_{\alpha kl}$ **: Kerr coefficient (quadratic electro-optic)**

- **The Pockels effect does not exist in a material with inversion symmetry, which is called a centrosymmetric material.**
- **The Kerr effect exists in all materials**

Pockels Effects

For the Pockels effect,

$$\Delta\eta_\alpha(E_0) = \sum_k r_{\alpha k} E_{0k}$$

Which can be written explicitly in the following matrix form

$$\begin{bmatrix} \Delta\eta_1 \\ \Delta\eta_2 \\ \Delta\eta_3 \\ \Delta\eta_4 \\ \Delta\eta_5 \\ \Delta\eta_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{bmatrix}$$

- **For a noncentrosymmetric material, the number of nonvanishing independent elements in its $r_{\alpha k}$ matrix is generally reduced by its symmetry**

Matrix Form of Pockels Coefficients

Table 5.1: Matrix form of Pockels coefficients for noncentrosymmetric point groups*

Triclinine	1	$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \\ r_{41} & r_{42} & r_{43} \\ r_{51} & r_{52} & r_{53} \\ r_{61} & r_{62} & r_{63} \end{bmatrix}$
Monoclinic	2	$\begin{bmatrix} 0 & r_{21} & 0 \\ 0 & r_{22} & 0 \\ 0 & r_{23} & 0 \\ r_{41} & 0 & r_{43} \\ 0 & r_{52} & 0 \\ r_{61} & 0 & r_{63} \end{bmatrix}$
	m	$\begin{bmatrix} r_{11} & 0 & r_{13} \\ r_{21} & 0 & r_{23} \\ r_{31} & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & r_{53} \\ 0 & r_{62} & 0 \end{bmatrix}$
Orthorhombic	222	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{52} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$
	$mm2$	$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{23} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
Tetragonal	4	$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	$\bar{4}$	$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & -r_{13} \\ 0 & 0 & 0 \\ r_{41} & -r_{51} & 0 \\ r_{51} & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$
	422	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	$4mm$	$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	$\bar{4}2m$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{bmatrix}$

From I.P. Kaminow, *An Introduction to Electrooptic Devices*, (Academic Press, Orlando, 1974) pp. 110-111.

Table 5.1: *(Continued)*

Trigonal	3	$\begin{bmatrix} r_{11} & -r_{22} & r_{13} \\ -r_{11} & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ -r_{22} & -r_{11} & 0 \end{bmatrix}$
	32	$\begin{bmatrix} r_{11} & 0 & 0 \\ -r_{11} & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & -r_{11} & 0 \end{bmatrix}$
	$3m$	$\begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$
Hexagonal	6	$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ r_{41} & r_{51} & 0 \\ r_{51} & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	$\bar{6}$	$\begin{bmatrix} r_{11} & -r_{22} & 0 \\ -r_{11} & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & -r_{11} & 0 \end{bmatrix}$
	622	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & -r_{41} & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	$6mm$	$\begin{bmatrix} 0 & 0 & r_{13} \\ 0 & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	$\bar{6}m2$	$\begin{bmatrix} 0 & -r_{22} & 0 \\ 0 & r_{22} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix}$
Cubic	432	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
	23 and $\bar{4}3m$	$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$

$LiNb_3$ (beside $3m$)

III-V (beside 23 and $\bar{4}3m$)

Electro-Optic Effects

The original diagonalized dielectric tensors therefore changed due to the electro-optic effects

$$\boldsymbol{\varepsilon}^{(1)} = \begin{bmatrix} \varepsilon_x^{(1)} & 0 & 0 \\ 0 & \varepsilon_y^{(1)} & 0 \\ 0 & 0 & \varepsilon_z^{(1)} \end{bmatrix} \xrightarrow{\text{Electro-Optics}} \boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x^{(1)} + \Delta\varepsilon_{xx} & \Delta\varepsilon_{xy} & \Delta\varepsilon_{xz} \\ \Delta\varepsilon_{yx} & \varepsilon_y^{(1)} + \Delta\varepsilon_{yy} & \Delta\varepsilon_{yz} \\ \Delta\varepsilon_{zx} & \Delta\varepsilon_{zy} & \varepsilon_z^{(1)} + \Delta\varepsilon_{zz} \end{bmatrix}$$

Diagonalized

where $\frac{\Delta\varepsilon_{ij}}{\varepsilon_0} \simeq -\frac{\Delta\eta_{ij}}{\eta_i^{(1)}\eta_j^{(1)}}$ **since** $\varepsilon_i^{(1)} >> \left|\Delta\varepsilon_{i,j}\right|$

- **With electro-optic effects, not only the refractive index but also the principle axes are changed**

$LiNbO_3$ (3m) Electro-Optics

$$\begin{bmatrix} \Delta\eta_1 \\ \Delta\eta_2 \\ \Delta\eta_3 \\ \Delta\eta_4 \\ \Delta\eta_5 \\ \Delta\eta_6 \end{bmatrix} = \begin{bmatrix} 0 & -r_{22} & r_{13} \\ 0 & r_{22} & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{51} & 0 \\ r_{51} & 0 & 0 \\ -r_{22} & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{0x} \\ E_{0y} \\ E_{0z} \end{bmatrix}$$

(1). $E_{0x} = E_{0y} = 0, \quad E_{0z} \neq 0$

$$\Delta\eta_1 = r_{13}E_{0z} \qquad \Delta\eta_2 = r_{13}E_{0z} \qquad \Delta\eta_3 = r_{33}E_{0z}$$

The index ellipsoid becomes

$$(\frac{1}{n_0^2} + r_{13}E_{0z})x^2 + (\frac{1}{n_0^2} + r_{13}E_{0z})y^2 + (\frac{1}{n_e^2} + r_{33}E_{0z})z^2 = 1$$

$$\boldsymbol{\varepsilon} = \varepsilon_0 \begin{bmatrix} n_0^2 - n_0^4 r_{13}E_{0z} & 0 & 0 \\ 0 & n_0^2 - n_0^4 r_{13}E_{0z} & 0 \\ 0 & 0 & n_e^2 - n_e^4 r_{33}E_{0z} \end{bmatrix}$$

- **The principle axes are not rotated. The crystal remains uniaxial with the same optical axis**
- **The indices of refraction are changed**

KDP (KH_2PO_4) $\left(\bar{4}2m\right)$

The crystal symmetry of KDP is $\bar{4}2m$

$$\begin{bmatrix}\Delta\eta_1\\ \Delta\eta_2\\ \Delta\eta_3\\ \Delta\eta_4\\ \Delta\eta_5\\ \Delta\eta_6\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\r_{41}&0&0\\0&r_{41}&0\\0&0&r_{63}\end{bmatrix}\begin{bmatrix}E_{0x}\\E_{0y}\\E_{0z}\end{bmatrix}$$

Now we consider the case when the field is applied along the z-axis:

$$E_{0x}=E_{0y}=0 \quad \textbf{and} \quad E_{0z}\neq 0$$

$$\frac{x^2+y^2}{n_0^{\ 2}}+\frac{z^2}{n_e^{\ 2}}=1 \longrightarrow \frac{x^2+y^2}{n_0^{\ 2}}+\frac{z^2}{n_e^{\ 2}}+2r_{61}E_{0z}xy=1$$

KDP (KH_2PO_4)

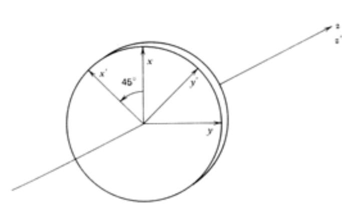
In dielectric matrix representation,

$$\boldsymbol{\varepsilon}'=\varepsilon_0\begin{bmatrix}n_o^{\ 2}&-n_o^{\ 4}r_{63}E_{0z}&0\\-n_o^{\ 4}r_{63}E_{0z}&n_o^{\ 2}&0\\0&0&n_e^{\ 2}\end{bmatrix}$$

$(x,y,z)\rightarrow(x',y',z')$ **and** $(n_x,n_y,n_z)\rightarrow(n_x',n_y',n_z')$

where

$$\begin{pmatrix}x'\\y'\\z'\end{pmatrix}=\begin{pmatrix}\frac{1}{\sqrt{2}}(x+y)\\ \frac{1}{\sqrt{2}}(-x+y)\\ z\end{pmatrix} \quad \textbf{and} \quad \begin{pmatrix}n_x'\\n_y'\\n_z'\end{pmatrix}=\begin{pmatrix}n_o-\frac{n_o^{\ 3}}{2}r_{63}E_{0z}^{\ 2}\\ n_o+\frac{n_o^{\ 3}}{2}r_{63}E_{0z}^{\ 2}\\ n_e\end{pmatrix}$$



Uniaxial ⟶ Biaxial

GaAs $(\bar{4}3m)$

For the III-V semiconductor with $(\bar{4}3m)$ symmetry, since

$$\begin{bmatrix}\Delta\eta_1\\\Delta\eta_2\\\Delta\eta_3\\\Delta\eta_4\\\Delta\eta_5\\\Delta\eta_6\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\\r_{41}&0&0\\0&r_{41}&0\\0&0&r_{41}\end{bmatrix}\begin{bmatrix}E_{0x}\\E_{0y}\\E_{0z}\end{bmatrix}$$

$$n_x = n_y = n_z = n_0$$

The only nonvanishing Pockels coefficients are

$$\gamma_{41} = \gamma_{52} = \gamma_{63}$$

We therefore consider only the case when the field is applied along the z-axis

$E_{0x} = E_{0y} = 0$ and $E_{0z} \neq 0$

Then we only have $\Delta\eta_6 = \gamma_{41}E_{0z}$

The index ellipsoid becomes

$$\frac{1}{n_0^2}x^2 + \frac{1}{n_0^2}y^2 + \frac{1}{n_0^2}z^2 + 2\gamma_{41}E_{0z}xy = 1$$

GaAs $(\bar{4}3m)$

The dielectric permittivity tensor becomes

$$\varepsilon = \varepsilon_0\begin{bmatrix} n_0^2 & -n_0^4\gamma_{41}E_{0z} & 0 \\ -n_0^4\gamma_{41}E_{0z} & n_0^2 & 0 \\ 0 & 0 & n_0^2 \end{bmatrix}$$

This results in the following new principle axes

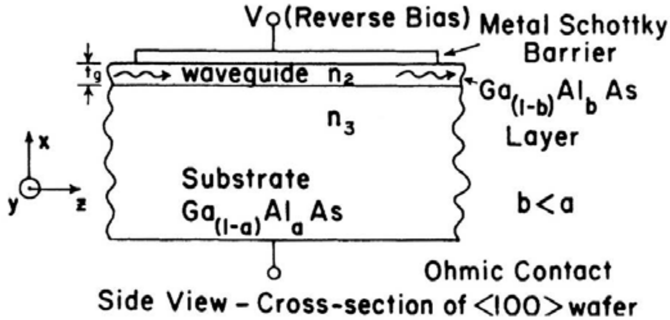
$\hat{x}' = \frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$ and $\hat{y}' = \frac{1}{\sqrt{2}}(-\hat{x}+\hat{y})$ and $\hat{z}' = \hat{z}$

with

$n_{x'} \approx n_0 - \frac{n_0^3}{2}\gamma_{41}E_{0z}$ and $n_{y'} \approx n_0 + \frac{n_0^3}{2}\gamma_{41}E_{0z}$ and $n_{z'} = n_0$

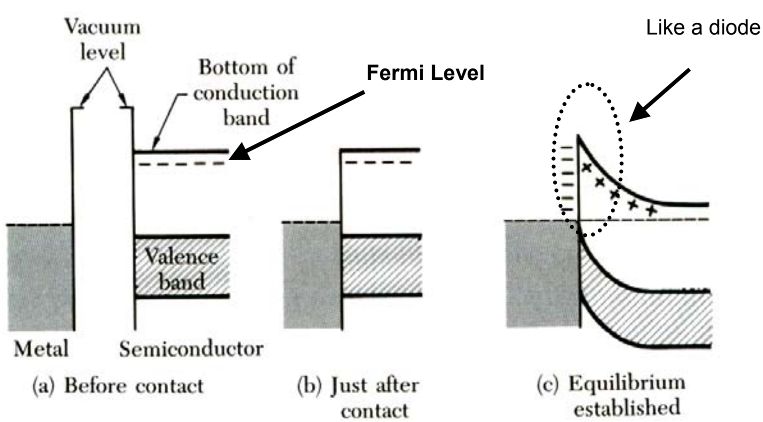
Isotropic ⟶ Biaxial (Anisotropic)

Single-Waveguide EO Modulators

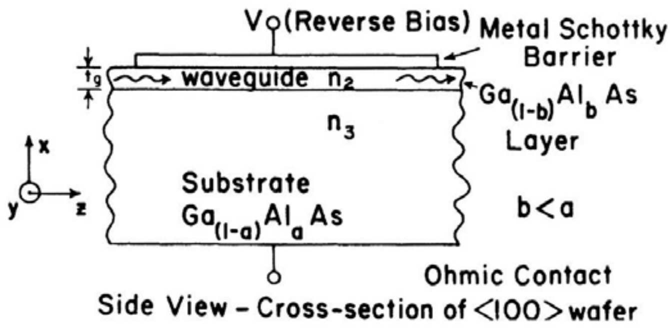


- It can operates as either phase modulator, amplitude modulator, polarization modulator or optical switch
- Various materials such as $LiNbO_3$, GaP, $LiTaO_3$ can be used.
- The metal contact forms a Schottky diode in the interface (for applying electric field

Schottky Barrier



Single-Waveguide EO Modulators



(1) Phase Modulation

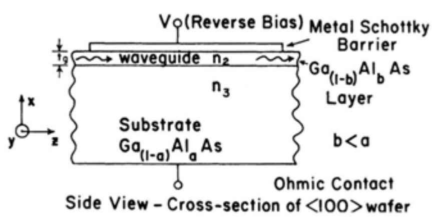
Material Index Contrast

Electro-Optic Effect

$$\Delta n_{23} = n_2 - n_3 = \Delta n_{chemical} + \Delta n_{CCR} + \Delta n_{EO}$$

Concentration Reduction (if any)

Single-Waveguide EO Modulators



Without Bias

The cut-off condition for single mode waveguide:

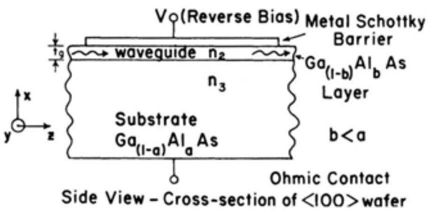
$$\frac{1}{32n_2}\left(\frac{\lambda_0}{t_g}\right)^2 < \Delta n_{chemical} + \Delta n_{CCR} < \frac{9}{32n_2}\left(\frac{\lambda_0}{t_g}\right)^2$$

When a voltage V is applied with reversed bias on the Schottky diode, the electric fields are built within the waveguide.

$$\Delta n_{EO} = n_2^3 r_{41} \frac{V}{2t_g}$$ **(TE-polarized in y-direction)**

(There is no field-induced index change in TM waves)

Single-Waveguide EO Modulators



Therefore, the index change due to electric fields becomes

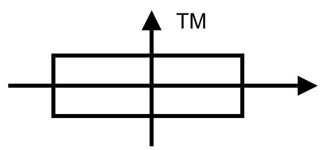
$$\Delta n = \frac{\Delta\beta}{k} = \frac{\Delta\beta\lambda_0}{2\pi}$$

(TE-polarized in y-direction)

The phase change produced by the electric field is given by

$$\Delta\varphi_{EO} = \Delta\beta L = \frac{\pi}{\lambda_0} n_2^{\,3} r_{41} \frac{VL}{t_g}$$

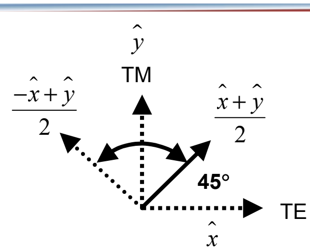
(2) Polarization Modulation



$$\Delta n_{TE} = n_2^{\,3} r_{41} \frac{V}{2t_g}$$

$$\Delta n_{TM} = 0$$

Single-Waveguide EO Modulators



$$\begin{aligned} E &= \hat{x}E_0 \exp(j\beta_{TE}z) + \hat{y}E_0 \exp(j\beta_{TM}z) \\ &= E_0 \exp(j\beta_{TM}z)\left\{\hat{x}\exp(j\Delta\beta z) + \hat{y}\right\} \\ &= E_0 \exp(j\beta_{TM}z)\left\{\hat{x}\exp(j\frac{\pi}{\lambda_0} n_2^{\,3} r_{41}\frac{Vz}{t_g}) + \hat{y}\right\} \end{aligned}$$

(3) Intensity Modulation

- **The index contrast is designed just around the cut-off condition of fundamental mode without bias.**

$$\Delta n_{23} = \Delta n_{chemical} + \Delta n_{CCR} = \frac{1}{32 n_2}\left(\frac{\lambda_0}{t_g}\right)^2$$

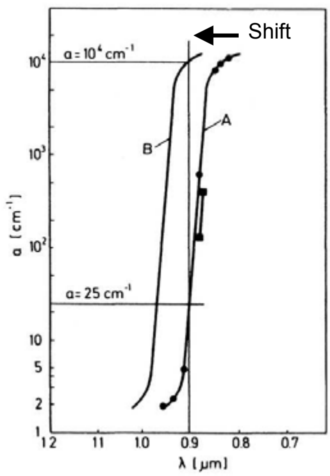
$\Delta n_{EO} > 0$ Transmitted

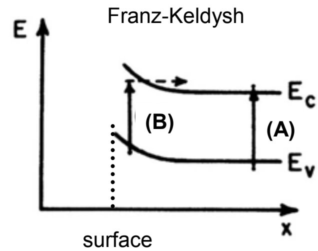
$\Delta n_{EO} < 0$ Radiated

Single-Waveguide EA Modulators

(4) Electro-Absorption Modulation

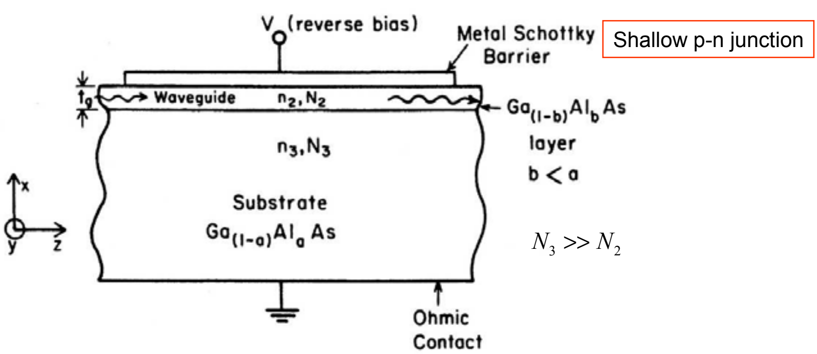
- **Franz-Keldysh effect or Quantum Confirmed Stark effect (QCSE)**





- **The band edge shifts to long wavelength with electric bias**
- **Energy band bends significantly near surface**

Single-Waveguide EA Modulators



$N_3 >> N_2$

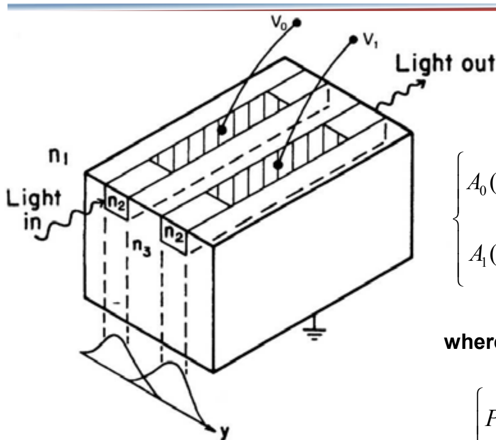
$N_3 >> N_2$ → Reduce the resistance in the substrate and increase the depletion area in the waveguide

The effective change in bandgap energy ΔE is given by

$$\Delta E = \frac{3}{2}(m^*)^{-1/3}(q\hbar\psi)^{2/3}$$

m^* : effective mass ψ : applied field

Dual-Channel Waveguide EO Modulators



Recall

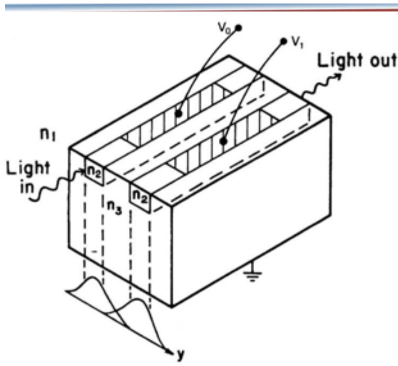
$A_0(0)=1$ **and** $A_1(0)=0$

$$\begin{cases} A_0(z)=(\cos gz - j\frac{\Delta\beta}{2g}\sin gz)\exp\left[-j(\beta_0-\frac{\Delta\beta}{2})z\right] \\ A_1(z)=-(\frac{-j\kappa}{g}\sin gz)\exp\left[-j(\beta_1+\frac{\Delta\beta}{2})z\right] \end{cases}$$

where $\Delta\beta=\beta_0-\beta_1$ **and** $g^2\equiv\kappa^2+\left(\frac{\Delta\beta}{2}\right)^2$

$$\begin{cases} P_0(z)=\cos^2(gz)-(\frac{\Delta\beta}{2})^2\frac{\sin^2(gz)}{g^2} \\ P_1(z)=\frac{\kappa^2}{g^2}\sin^2(gz) \end{cases}$$

Dual-Channel Waveguide EO Modulators



when $\Delta\beta=0$ **, 100% power is transferred if**

$$\kappa L=\frac{\pi}{2}+m\pi \qquad m=0,1,2,\ldots \qquad \textbf{(a)}$$

When a modulating voltage is applied to produce a Δβ, the coupling could be completely cancelled if

$$gL=\pi+m\pi \qquad m=0,1,2,\ldots \qquad \textbf{(b)}$$

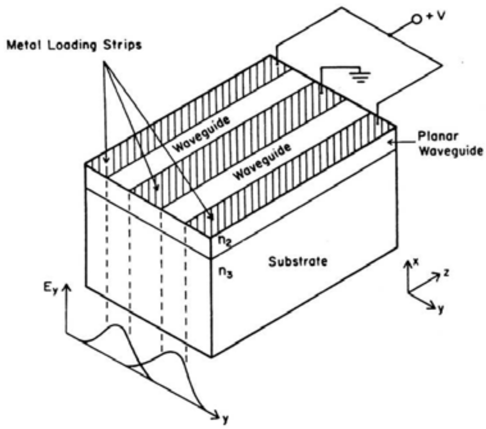
Combine (a) and (b), it can be shown that the value of Δβ required for 100% modulation is given by

$$(\Delta\beta)L=\sqrt{3}\pi$$

The change in effective index needed for 100% modulation

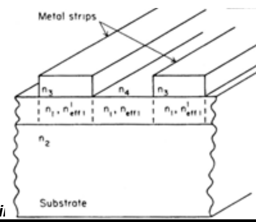
$$\Delta n_g=\frac{\sqrt{3}\pi}{k_0L}$$

Dual-Channel Waveguide EO Modulators

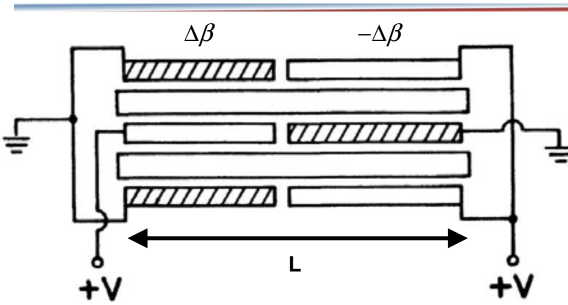


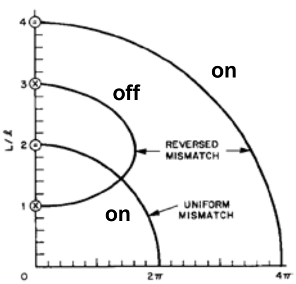
Three-electrode Dual-Channel Waveguide Modulators

- Fabricated in GaAs
- A pair of strip-loaded waveguides
- Extinction Ratio: 13dB
- Applied Voltage: 35V
- 7-ns rise time
- The power-bandwidth ratio: 180mW/MHz



Dual-Channel Waveguide EO Modulators

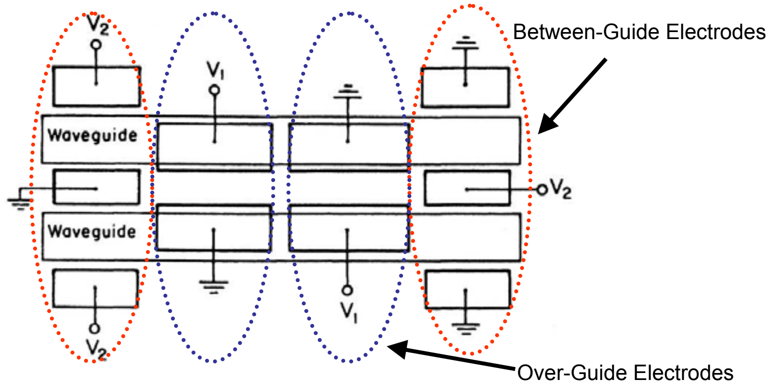




Split-Electrode Dual-Channel Waveguide Modulators

- 100% power transfer is insensitive to the waveguide length.
- The on and off states are controlled by $\Delta\beta$.
- It allows a maximum extinction ratio and a minimum crosstalk.
- The waveguide length $L > \frac{\pi}{2\kappa} = l$

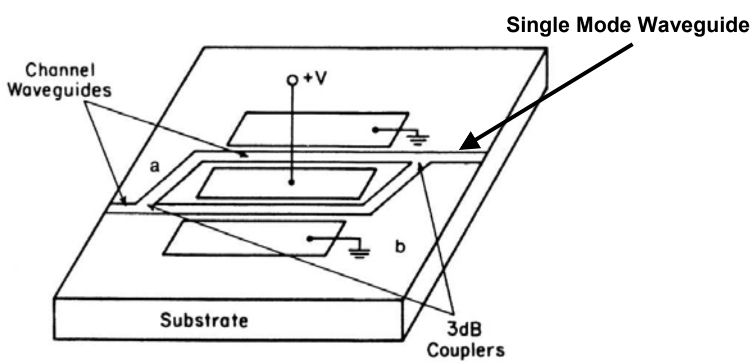
Dual-Channel Waveguide EO Modulators



Polarization Insensitive Modulators

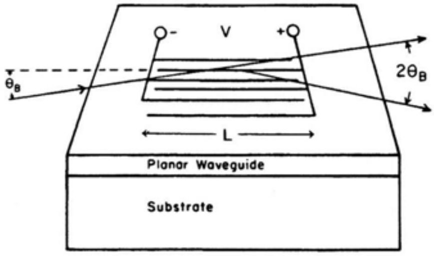
- **Stepped Δβ reversal electrodes are both for TE and TM polarization**

Mach-Zehnder Type Electro-Optic Modulators



- **Ideally, the path lengths and guide characteristics are identical.**
- **The Y-branch is a perfect 3dB splitter.**
- **The input and output waveguides are single mode.**
- **The optical wave radiates as the phase difference is equal to π.**

Bragg-Effect EO Modulators



Suppose

$$2\pi\lambda_0 L \gg \Lambda^2 \quad \text{(thick grating)}$$

The Bragg angle θ_B is given by

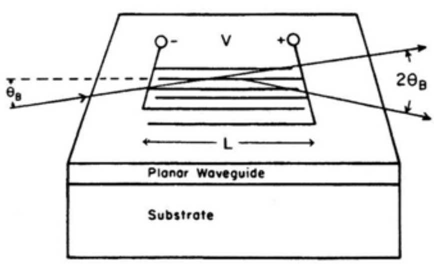
$$\sin\theta_B = \frac{\lambda_0}{2\Lambda n_g}$$

Λ : **Grating Space**

$n_g = \frac{\beta}{k_0}$ **(effective index)**

- **Index grating is patterned by a interlace, com-like, pair of electrodes**
- **The index difference is modulated by electro-optic effects**
- **The input angle of guided wave is aligned to the Bragg angle.**
- **The output wave is diffracted by $2\theta_B$ angle**

Bragg-Effect EO Modulators



If the input angle is deviated from the Bragg angle by $\Delta\theta_B$, the diffracted efficiency is reduced.

The 50%-reduction deviation angle is

$$\Delta\theta_B = \frac{2\Lambda}{L}$$

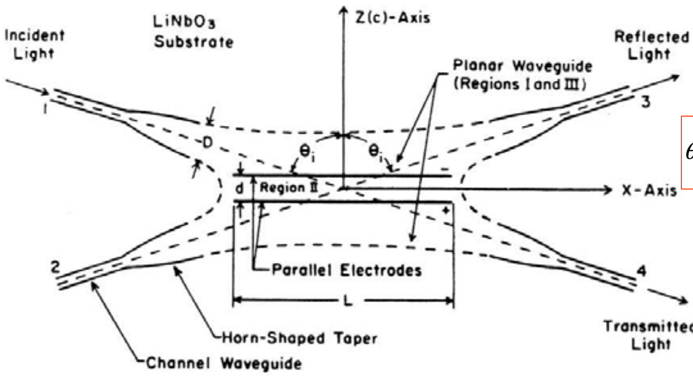
The intensity of light diffraction is dependent on the applied voltage, which is given by

$$\frac{I_{diffracted}}{I_{transmitted}} = \sin^2 VB$$

B: constant dependent on waveguide

- **Demonstrated in ZnO, $LiTaO_3$, …**
- **The extinction ratio can be 24.7dB**

EO Reflection Modulators



$$\theta_c = \sin^{-1}\left[1 - \frac{1}{2} n_1^2 r_{33} \left(\frac{V}{d}\right)\right]$$

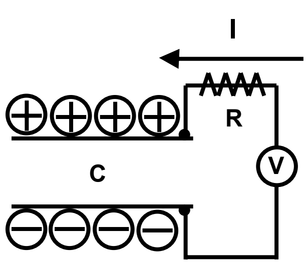
For a given input angle θ_i, the applied voltage should be

$$\left.\frac{V}{d}\right|_{TIR} = \frac{2(1-\sin\theta_i)}{n_1^2 r_{33}} \cong \frac{1}{n_1^2 r_{33}} \left(\frac{\pi}{2} - \theta_i\right)^2$$

- **High speed (6GHz) due to low device capacitance**

Figure of Merit --- P/Δf

In EO modulators, the drive power is usually consumed during state transition



The average external power P_e is defined by operating the modulator at a maximum frequency equal to its bandwidth (Δf).

$$P_e = (\Delta f)\Phi$$

Where Φ is the energy supplied from an external source to switch the modulator on or off

$$\Phi = \frac{1}{2} C V^2 \rightarrow \Phi = \frac{1}{2}\int \varepsilon E_a^2 dv$$

volume

E_a **: the peak amplitude of the applied field**

Suppose all the electric fields are uniformly distributed in the modulator volume,

$$\Phi = \frac{\varepsilon W t L E_a^2}{2}$$

W: width t: thickness L: length

Figure of Merit --- P/Δf

Therefore, the external drive power is

$$P_e = \frac{(\Delta f)\varepsilon WtLE_a^{\ 2}}{2}$$

For the specific case of an EO modulator formed in GaAs, the resulting change in index of refraction are related by

$$E_a = \frac{2\Delta n}{n_2^{\ 3} r_{41}}$$

Therefore,

$$P_e = \frac{2(\Delta f)\varepsilon WtL}{n_2^{\ 6} r_{41}^{\ 2}} \Delta n^2 \qquad \text{(a)}$$

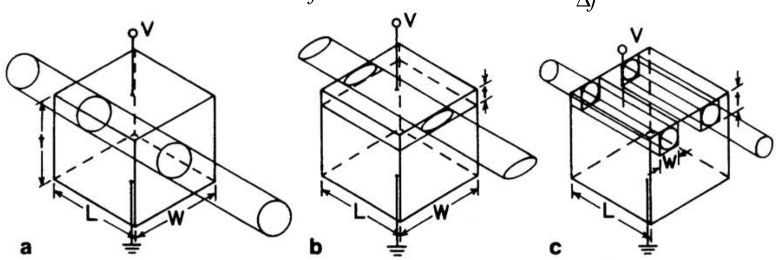
Suppose for a dual-channel modulator, it has shown that

$$\Delta n_g = \frac{\sqrt{3}\pi}{k_0 L} \qquad \text{(b)}$$

Comparison of P/Δf

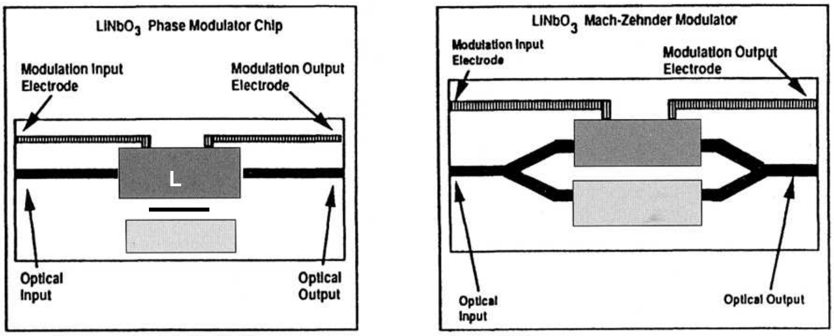
Combine (a) and (b), $\dfrac{P_e}{\Delta f} = \dfrac{3\varepsilon Wt\lambda_0^{\ 2}}{2n_2^{\ 6} r_{41}^{\ 2} L}$

$\dfrac{P_e}{\Delta f} \approx 148 \sim 1480 mW/MHz$ $\qquad \dfrac{P_e}{\Delta f} \approx 1.48 mW/MHz$ $\qquad \dfrac{P_e}{\Delta f} \approx 0.148 mW/MHz$



Bulk EO Modulator — **Planar Waveguide EO Modulator** — **Channel EO Modulator**

Travelling Wave Electrode Configuration

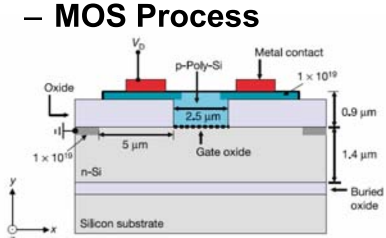


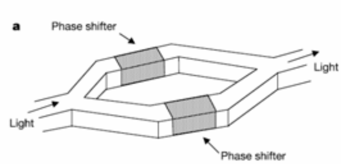
a Phase Modulator **b** Mach-Zehnder Modulator

- **It operates at high frequency (microwave).** $\omega_m \frac{L}{v_p} > \frac{\pi}{2}$
- **The signal bandwidth is limited by the difference of microwave phase velocity (vm) and optical wave phase velocity (vp).** $\left|\frac{L}{v_p} - \frac{L}{v_m}\right| \omega_m < \frac{\pi}{2}$

Silicon-based Electro-optic Device

- Free carrier plasma dispersion effect
  - **Refractive index is changed by the density of free carrier**
  - **Ridge waveguide (Ridge part : poly-silicon, slab part : single crystal)**
  - **Free carrier is accumulated around the gate oxide**
  - **MOS Process**



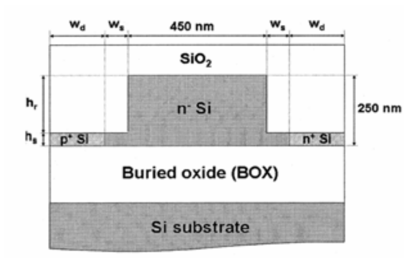


Asymmetric MZI modulator

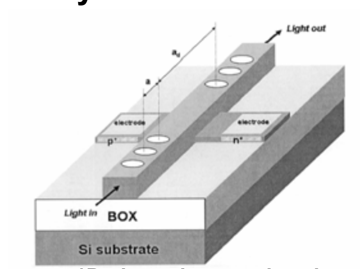
"A high-speed silicon optical modulator based on a metal-oxide-semiconductor capacitor", Nature '03, Ansheng Liu, Intel

Silicon-based Electro-optic Device

- Free Carrier Injection Effect
  - **p-i-n junction around the microcavity**
  - **Refractive index in microcavity is changed**
  - **2 distributed Bragg reflectors (DBR) are on the two sides of microcavity**



Waveguide Cross Section



1D photonic crystal cavity intensity modulator

"Electrooptic Modulation of Silicon-on-insulator Submicron-Size Waveguide device", J. of Lightwave Tech., 2003
C. Angulo Barrios (Cornell University)