Waveguide Loss

Class: Integrated Photonic Devices
Time: Fri. 8:00am ~ 11:00am.
Classroom: 資電206
Lecturer: Prof. 李明昌(Ming-Chang Lee)

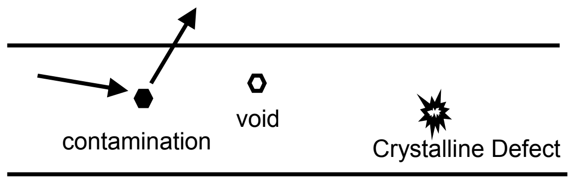
Optical Loss in Waveguides

Three major losses in waveguide

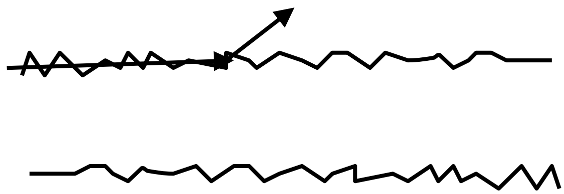
- Scattering Loss
  - Due to surface roughness
- Absorption Loss
  - Due to photons annihilated in materials
- Radiation Loss
  - Due to waveguide bending

Scattering Loss

- **Volume Scattering**

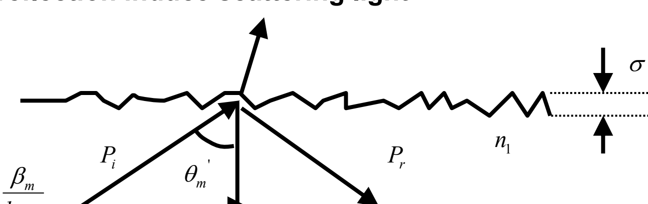


- **Surface Scattering (Dominant)**



Surface Scattering Loss (Tien's Model)

- **Each reflection induce scattering light**



$$\sin\theta_m' = \frac{\beta_m}{k_0 n_1}$$

Rayleigh Criterion $P_r = P_i \exp[-(\frac{4\pi\sigma}{\lambda}\cos\theta_m')^2]$

σ : variance of surface roughness

m: mode number

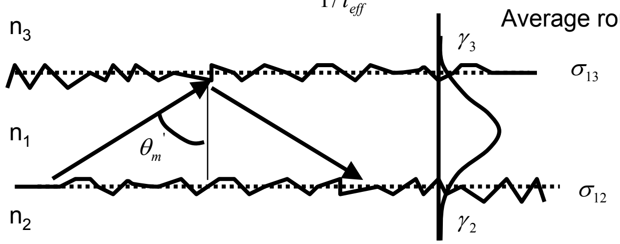
Surface Scattering Loss

To quantitatively describe the optical loss, the exponential attenuation coefficient is generally used. In this case, the intensity (power per unit length) decays along the waveguide.

$I(z) = I_0 \exp(-\alpha z)$ I_0 is the initial intensity at z = 0

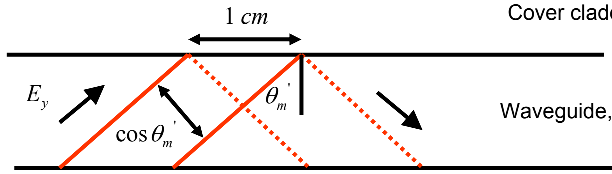
$$\alpha_s = A^2 \left(\frac{1}{2}\frac{\cos^3\theta_m'}{\sin\theta_m'}\right)\underbrace{\left(\frac{1}{t_g + (1/\gamma_2) + (1/\gamma_3)}\right)}_{1/t_{eff}} \qquad A = \frac{4\pi}{\lambda}(\sigma_{13}{}^2 + \sigma_{12}{}^2)^{1/2}$$

Average roughness



Scattering Loss Analysis by Tien's Model

Consider a planar waveguide with TE polarization



The power carried by the incident beam hit on the unit length (1 cm)

$\frac{c}{8\pi} n_1 E_y{}^2 \cos\theta_m'$ E_y is the field amplitude

According to the Rayleigh criterion, the reflected beam from the upper film surface

$$\frac{c}{8\pi} n_1 E_y{}^2 \cos\theta_m' \cdot \underbrace{\exp\left[-\left(\frac{4\pi\sigma}{\lambda}\cos\theta_m'\right)^2\right]}_{\text{Rayleigh criterion}}$$

σ : variation of surface roughness

Scattering Loss Analysis by Tien's Model

Consider the two film surface

$$\frac{4\pi}{\lambda}\sigma \longrightarrow \frac{4\pi}{\lambda}\left(\sigma_{13}+\sigma_{12}\right)^{1/2}$$

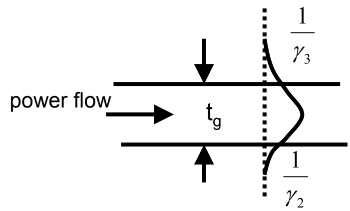
The power lost by surface scattering per unit length is

$$\frac{c}{8\pi}n_1 E_y^{\,2}\cos\theta_m^{'}\cdot\left\{1-\exp\left[-\left(A\cos\theta_m^{'}\right)^2\right]\right\}$$

$$\approx \frac{c}{8\pi}n_1 E_y^{\,2}A^2\cos^3\theta_m^{'}$$

The planar waveguide mode power flow

$$\frac{c}{4\pi}n_1 E_y^{\,2}\sin\theta_m^{'}\cdot\left(t_g+\frac{1}{\gamma_2}+\frac{1}{\gamma_3}\right)$$

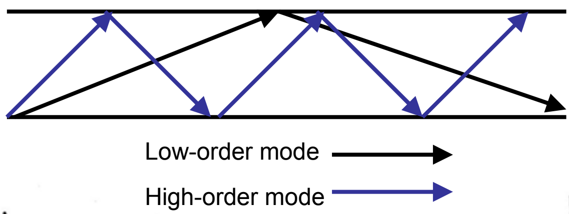


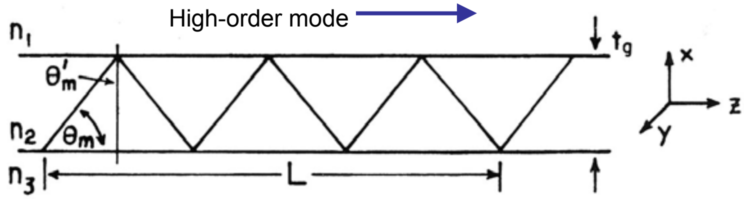
Scattering

The power attenuation per unit length

$$\alpha = A^2\,\frac{1}{2}\,\frac{\cos^3\theta_m^{'}}{\sin\theta_m^{'}}\left\{\frac{1}{t_g+(1/\gamma_2)+(1/\gamma_3)}\right\}$$

Surface Scattering Loss



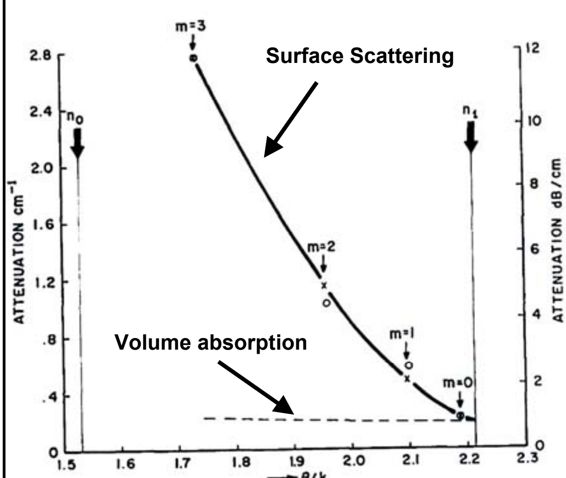


$$N_R = \frac{L}{2t_g \cot\theta_m}$$

Where m is mode no.

- **High order modes have more reflections from the surface**

Mode Effect



Tien, 1971

The loss for the m=3 waveguide mode is as much as 14 times that of the m = 0 waveguide mode.

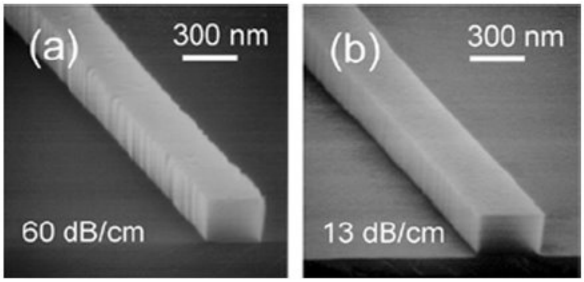
Ta_2O_5

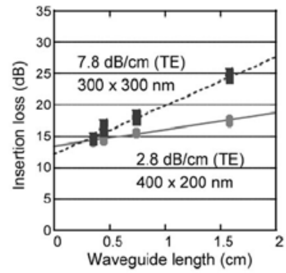
λ = 632.8 nm

$$\alpha : (dB/cm) = 4.3\ cm^{-1}$$

Sidewall Scattering Loss

Rough Sidewall | Smooth Sidewall | Measured Result

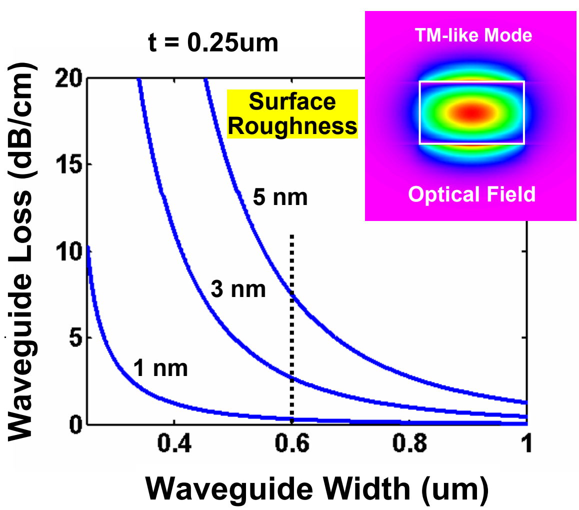




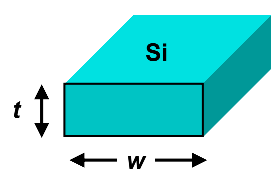
Tsuchizawa T. JSTQE 2005

- Sidewall roughness is created during etching process
- The propagation loss is highly related to the roughness for a small-dimension waveguide

Optical Loss due to Surface Roughness



Wavelength : 1550nm



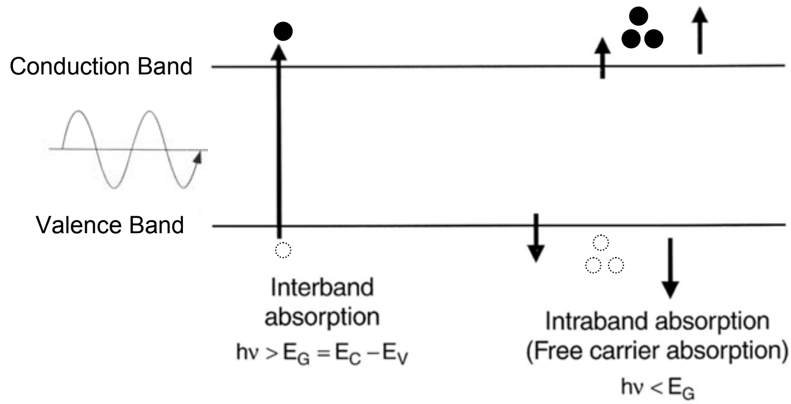
- Single mode waveguide

  $w \times t < 0.18\ \mu m^2$

- Surface roughness required to achieve low loss

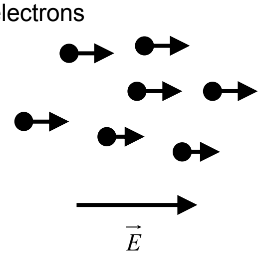
  $\delta_{rms} < 1$ nm

Absorption Loss



- **Interband absorption → electron and hole pairs (photodetector)** $h\nu > E_g$
- **Intraband absorption → free carrier scattering (metal)** $h\nu < E_g$

Free Carrier Absorption (Drude Model)



$$m\frac{d^2x}{dt^2} + mg\frac{dx}{dt} = qE_0\exp(j\omega t) = -eE_0\exp(j\omega t)$$

Not harmonic oscillator!

g : damping coefficient due to scattering

m : mass of carrier

$$x = \frac{(eE_0)/m}{\omega^2 - j\omega g}\exp(j\omega t)$$

Recall $\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}_0 + \mathbf{P}_1 = \varepsilon_0(1 + \chi_0 + \chi_1)\mathbf{E}$

Dielectric polarization (Dipole Moment) → χ_0

Free Carrier Effect (electron or hole) → χ_1

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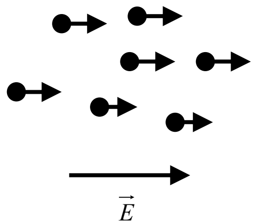
Free Carrier Absorption

electrons



$$\mathbf{P}_1 = -Nex = \frac{-(Ne^2)/m}{\omega^2 - j\omega g} E_0 \exp(j\omega t)$$

$$\chi_1 = \frac{-(Ne^2)/(m\varepsilon_0)}{\omega^2 - j\omega g} = \chi_1^{'} + j\chi_1^{"}$$

$$\begin{cases} \chi_1{'} = -\dfrac{(Ne^2)/(m\varepsilon_0)}{\omega^2 + g^2} \\ \chi_1{"} = \dfrac{(Ne^2 g)/(m\omega\varepsilon_0)}{\omega^2 + g^2} \end{cases}$$

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}_0 + \mathbf{P}_1 = \varepsilon_0[(1+\chi_0+\chi_1^{'}) + j\chi_1^{"}]\mathbf{E} = \varepsilon_0\left(n_0 + n_1^{'} + jn_1^{"}\right)^2 \mathbf{E}$$

$$n_1^{"} = \frac{\chi_1^{"}}{2(n_0 + n_1^{'})}$$ **where** $\chi_1^{"} << 1 + \chi + \chi_1^{'}$

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What is g?

At steady state, electron move as a constant speed.

That is, $d^2x/dt^2 = 0$

$$mg\frac{dx}{dt} = eE \qquad (1)$$

The definition of mobility μ

$$\frac{dx}{dt} = \mu E \qquad (2)$$

$$\boxed{g = \frac{e}{m\mu}}$$

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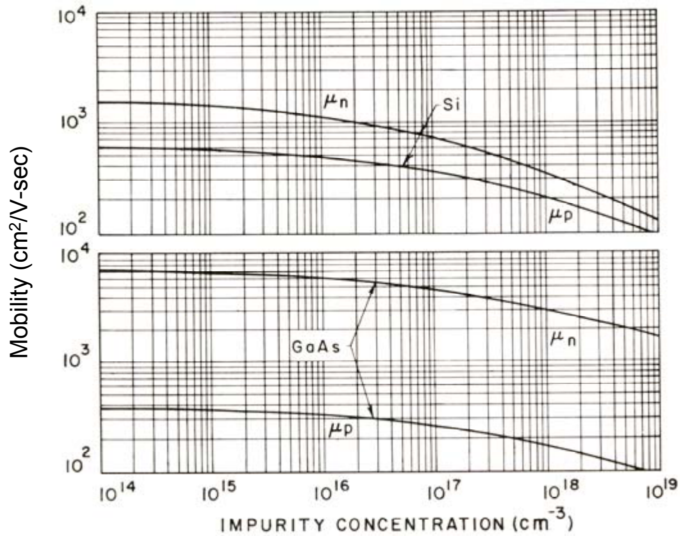
Mobility of Semiconductor



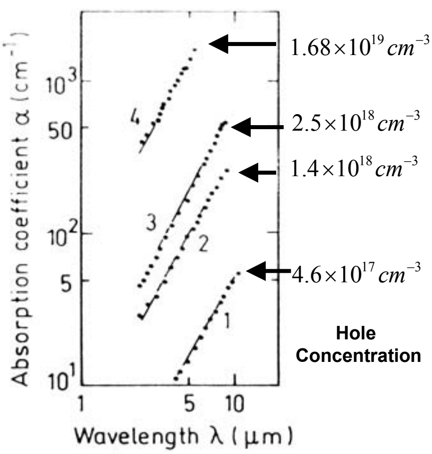
Free Carrier Absorption

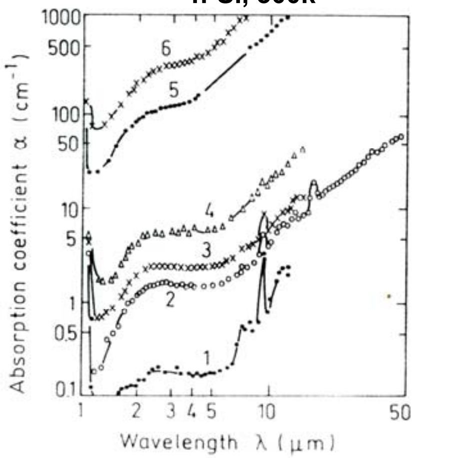
$$I \propto |E|^2 = |E_0|^2 \exp[jk_0(n' + jn'')z] \cdot \exp[-jk_0(n' - jn'')z]$$
$$= |E_0|^2 \exp[-2k_0 n'' z] = |E_0|^2 \exp[-\alpha z]$$

$$\alpha_{fc} = 2k_0 n'' \approx k_0 \frac{\chi''}{n} = \frac{Ne^3}{m^2 n \varepsilon_0 \omega^2 \mu c} \qquad (\omega >> g)$$ **For optical wavelength**

- **Free carrier absorption is proportional to the carrier density**
- **The refractive index is also affected by the free carriers**

Free Carrier Absorption





(1) $1.4\times10^{16}\,cm^{-3}$ (2) $8\times10^{16}\,cm^{-3}$ (3) $1.7\times10^{17}\,cm^{-3}$

(4) $3.2\times10^{17}\,cm^{-3}$ (5) $6.1\times10^{18}\,cm^{-3}$ (6) $1\times10^{19}\,cm^{-3}$

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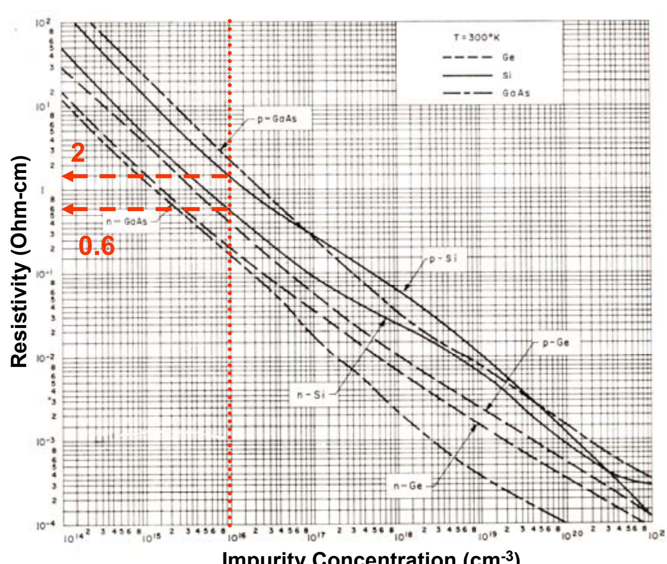
Resistivity vs. Impurity Concentration



Sze and Irvin

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Temperature-Dependent Free Carrier Absorption

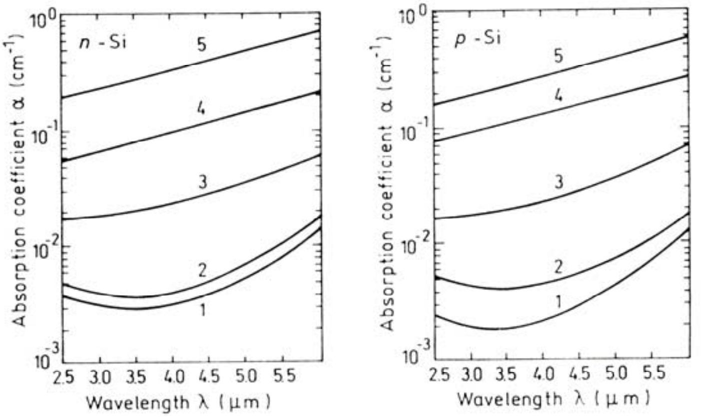
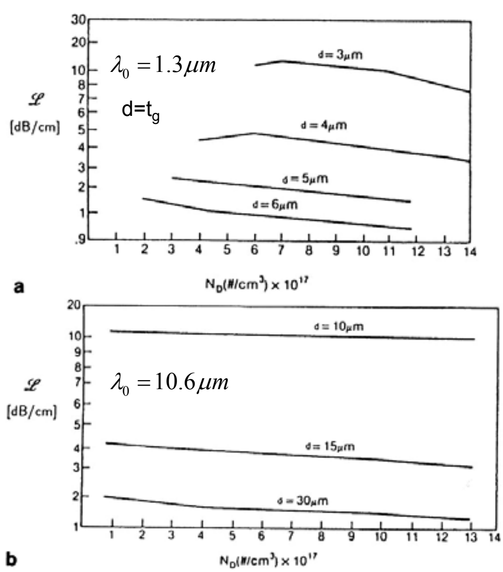
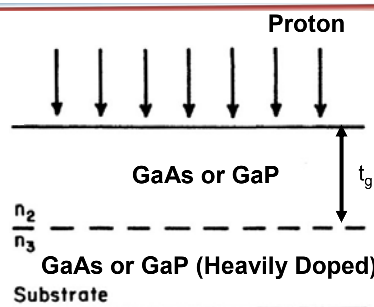


Fig. 1.4.8. Free carrier absorption versus wavelength for high purity Si at different temperatures (Runyan [1966]).
Temperatures are 1. 300 K, 2. 473 K, 3. 573 K, 4. 623 K, 5. 673 K (Figure reprinted with permission of McGraw-Hill Book Co.).

Free Carrier Absorption on Proton Bombardment Waveguide

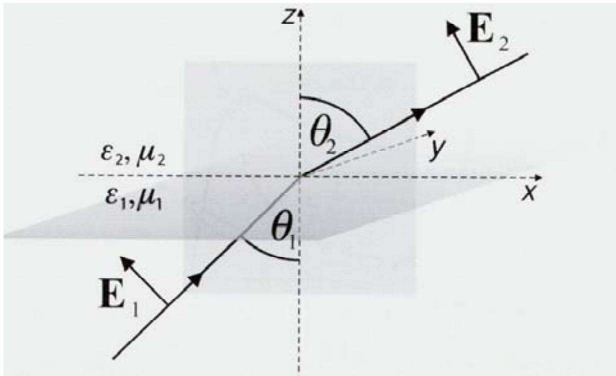




$$(N_2 - N_1) \geq \frac{\varepsilon_0 m^* \pi^2 c^2}{4 t_g{}^2 e^2}$$

The major loss comes from the evanescent wave penetrating in substrate

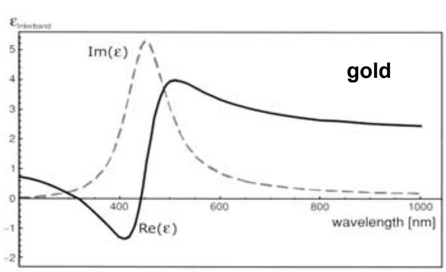
Surface Plasmons



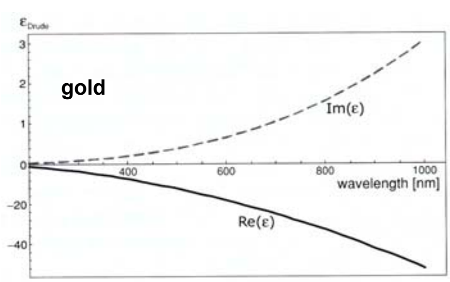
- **The interaction of metals with electromagnetic radiation is largely dictated by the free electrons in the metal.**
- **Most metals possess a negative dielectric constant at optical frequency**
- **Only the surface can support optical wave propagation (why?)**

Optical Properties of Nobel Metals

Dipole Dispersion



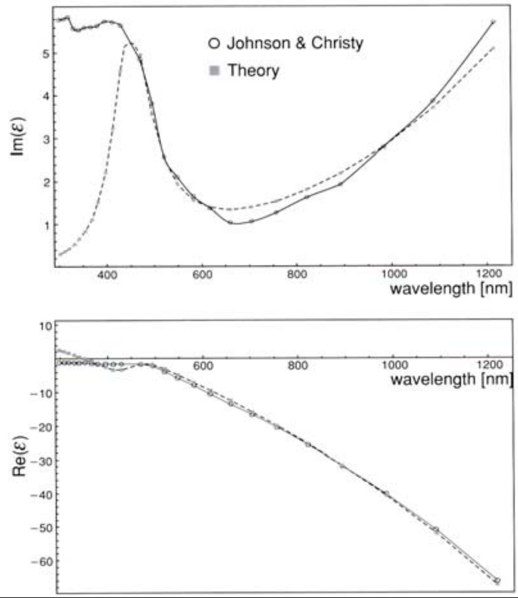
Free-Carrier Dispersion



$$\begin{cases} \chi'_{dipole}(\omega) = \dfrac{(Ne^2/m)(\omega_0^2-\omega^2)}{\varepsilon_0[(\omega_0^2-\omega^2)^2+\zeta^2\omega^2]} \\ \chi''_{dipole}(\omega) = \dfrac{(Ne^2/m)(\zeta\omega)}{\varepsilon_0[(\omega_0^2-\omega^2)^2+\zeta^2\omega^2]} \end{cases}$$

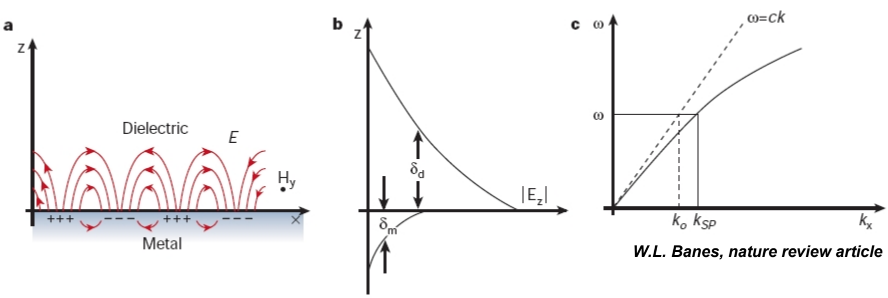
$$\begin{cases} \chi'_{free-carrier}(\omega) = -\dfrac{(Ne^2)/(m\varepsilon_0)}{\omega^2+g^2} \\ \chi''_{free-carrier}(\omega) = \dfrac{(Ne^2 g)/(m\omega\varepsilon_0)}{\omega^2+g^2} \end{cases}$$

Measure dielectric function of gold



Combine dipole dispersion and free-carrier dispersion

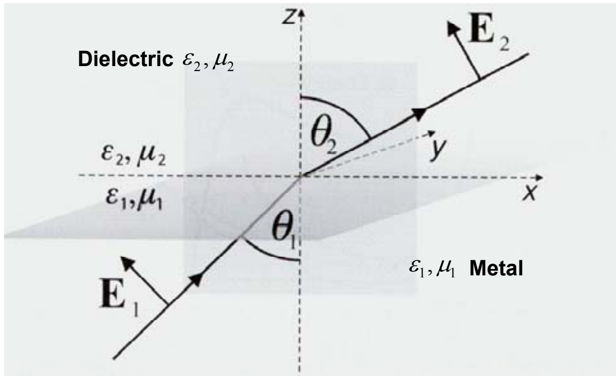
Surface Plasmons at plane interface



W.L. Banes, nature review article

- EM wave and surface charge are oscillating.
- The fields in the perpendicular direction decay exponentially.
- The momentum of SP is larger than the free space photon.

Surface Plasmons at plane interface



Mathematically, the solution has to satisfy the wave equation

$$\nabla \times \nabla \times \mathrm{E}(r,\omega) - \frac{\omega^2}{c^2}\varepsilon(r,\omega)\mathrm{E}(r,\omega) = 0$$

where $\varepsilon(r,\omega) = \varepsilon_1(\omega)$ z < 0 $\varepsilon(r,\omega) = \varepsilon_2(\omega)$ z > 0

Surface Plasmons at plane interface

TE wave or s-wave can not be a solution of surface plasmonic wave (H can not satisfy the boundary condition)

Consider a TM wave or p-wave,

$$E_i = \begin{pmatrix} E_{i,x} \\ 0 \\ E_{i,z} \end{pmatrix} \exp(jk_x x - j\omega t)\exp(jk_z z) \qquad \text{where} \qquad i = 1,2$$

Since the x-direction wave vector is conserved or Snell's law

$$k_x{}^2 + k_{i,z}{}^2 = \varepsilon_i k^2 \qquad \text{where} \qquad k = \frac{\omega}{c}$$

Since both spaces are source-free; that is, $\nabla \cdot \mathrm{D} = 0$

$$k_x E_{i,x} + k_{i,z} E_{i,z} = 0 \qquad \text{(a)}$$

Surface Plasmons at plane interface

Consider the boundary condition,

$$E_{1,x} - E_{2,x} = 0$$
$$\varepsilon_1 E_{1,z} - \varepsilon_2 E_{2,z} = 0 \qquad \text{(b)}$$

Combine (a) and (b), since the electric fields are not trivial solutions, the determinant of respective matrix has to be zero; then

$$\varepsilon_1 k_{2,z} - \varepsilon_2 k_{1,z} = 0$$

Therefore,

recall $k_x^{\ 2} + k_{i,z}^{\ 2} = \varepsilon_i k^2$

$$k_x^{\ 2} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} k^2 = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2} \frac{\omega^2}{c^2} \quad \text{and} \quad k_{i,z} = \frac{\varepsilon_i^{\ 2}}{\varepsilon_1 + \varepsilon_2} k^2 \qquad i = 1,2$$

Surface Plasmons at plane interface

Since the surface plasmonic mode are evanescent on the two sides of interface

k_x should be real and $k_{i,z}$ should be imaginary

Therefore

$$\varepsilon_1(\omega) \cdot \varepsilon_2(\omega) < 0$$
$$\varepsilon_1(\omega) + \varepsilon_2(\omega) < 0$$

The dielectric functions must be negative with an absolute value exceeding that of the other.

Nobel metals such as gold and silver, have a large negative real part of the dielectric constant along with small imaginary part

Properties of surface plasmonic waves

Consider the metal dielectric

$$\varepsilon_1 = \varepsilon_1^{'} + j\varepsilon_1^{"}$$

Suppose the imaginary part is much smaller than the real part and ε_2 is positive real, the wave number of SP mode

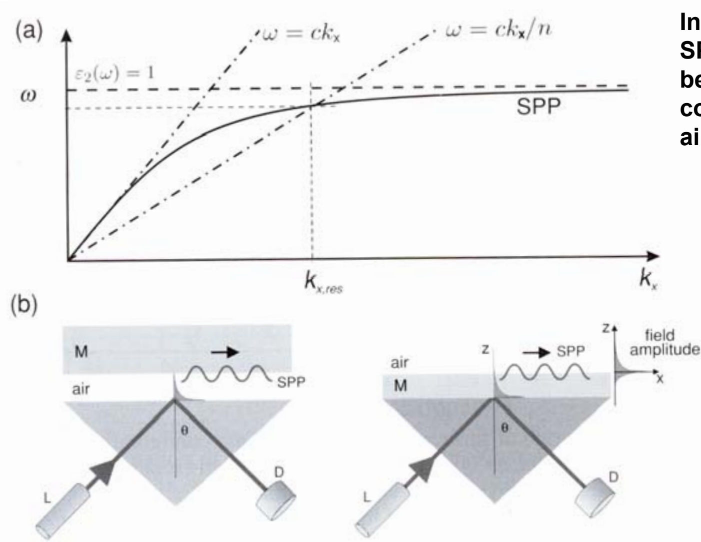
$$k_x = k_x^{'} + jk_x^{"}$$

where

$$\begin{cases} k_x^{'} \approx \sqrt{\dfrac{\varepsilon_1^{'}\varepsilon_2}{\varepsilon_1^{'}+\varepsilon_2}}\dfrac{\omega}{c} \\ k_x^{"} \approx \sqrt{\dfrac{\varepsilon_1^{'}\varepsilon_2}{\varepsilon_1^{'}+\varepsilon_2}}\dfrac{\varepsilon_1^{"}\varepsilon_2}{2\varepsilon_1^{'}(\varepsilon_1^{'}+\varepsilon_2)}\dfrac{\omega}{c} \end{cases}$$

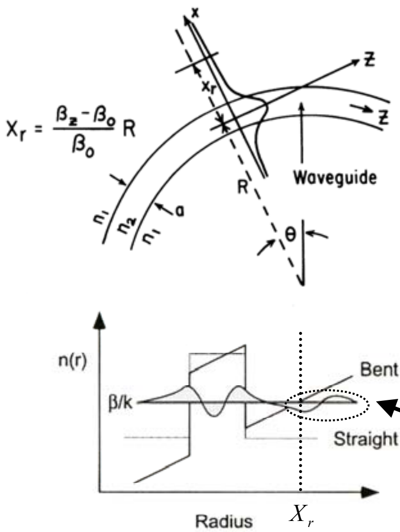
and $k_{1,z} = \dfrac{\omega}{c}\sqrt{\dfrac{\varepsilon_1^{'2}}{\varepsilon_1^{'}+\varepsilon_2}}\left[1 + j\dfrac{\varepsilon_1^{"}}{2\varepsilon_1^{'}}\right]$ $\quad k_{2,z} = \dfrac{\omega}{c}\sqrt{\dfrac{\varepsilon_2^{2}}{\varepsilon_1^{'}+\varepsilon_2}}\left[1 - j\dfrac{\varepsilon_1^{"}}{2(\varepsilon_1^{'}+\varepsilon_2)}\right]$

Excitation of Surface Plasmonic Wave



In this case, SPP can not be directly coupled from air

Radiation Loss



$$(R + X_r)\frac{d\theta}{dt} = \frac{\omega}{\beta_0}$$

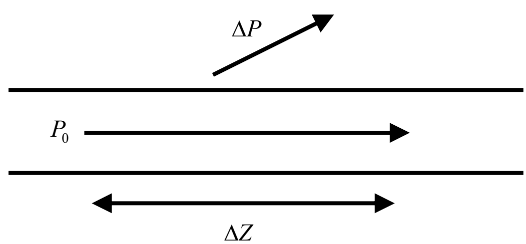
$$R\frac{d\theta}{dt} = \frac{\omega}{\beta_z}$$

The angular phase velocity should be the same.

$$X_r = \frac{\beta_z - \beta_0}{\beta_0} R$$

Radiation

What is Attenuation Coefficient (α)?



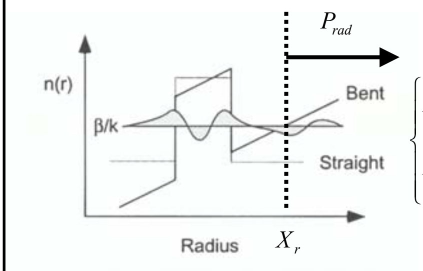
$$\alpha = -\frac{1}{P(z)}\frac{dP(z)}{dz} \quad (\textbf{Because } P(z) = P_0 \exp(-\alpha z))$$

$$\approx -\frac{1}{P(z_0)}\frac{\Delta P(z)}{\Delta z}$$

Dissipated Power

Propagation length

What is α due to Radiation Loss?



Suppose the field

$$\begin{cases} E(x) = \sqrt{C_0}\cos(hx) & -\frac{a}{2} \le x \le \frac{a}{2} \\ E(x) = \sqrt{C_0}\cos(\frac{ha}{2})\exp[-\left(\frac{|x|-(a/2)}{\gamma}\right) & |x| \ge \frac{a}{2} \end{cases}$$

Total Power ➞ $P(z_0)$

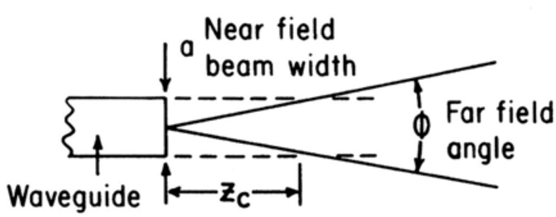
$$P_{total} = \int_{-\infty}^{\infty} E^2(x)dx = C_0[\frac{a}{2} + \frac{1}{2h}\sin(ha) + \gamma\cos^2(\frac{ha}{2})]$$

Radiated Power ➞ $\Delta P(z)$

$$P_{rad} = \int_{X_r}^{\infty} E^2(x)dx = C_0\frac{\gamma}{2}\cos(\frac{ha}{2})\exp(-\frac{2}{\gamma}(X_r - \frac{a}{2}))$$

What is Attenuation Coefficient (α)?

The propagation length of unguided mode (analogy to a truncated waveguide)



$$Z_c = \frac{a}{\phi} = \frac{a^2}{2\lambda} \quad (\textbf{Because} \sin(\frac{\phi}{2}) = \frac{\lambda}{a})$$

a: waveguide width
λ: wavelength

The attenuated coefficient:

$$\alpha = \frac{\frac{\gamma}{2}\cos^2(\frac{ha}{2})\exp(-\frac{2}{\gamma}\frac{\beta_z - \beta_0}{\beta_0}R)2\lambda_1\exp(\frac{a}{\gamma})}{[\frac{a}{2} + \frac{1}{2h}\sin(ha) + \gamma\cos^2(\frac{ha}{2})]a^2}$$

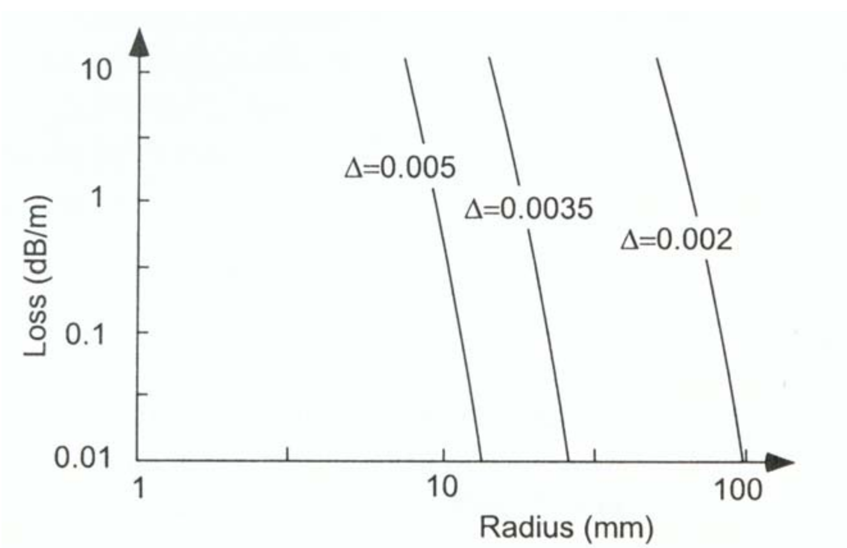
What is α due to Radiation Loss?

$$\alpha = \frac{\frac{\gamma}{2}\cos^2(\frac{ha}{2})\exp(-\frac{2}{\gamma}\frac{\beta_z-\beta_0}{\beta_0}R)2\lambda_1\exp(\frac{a}{\gamma})}{[\frac{a}{2}+\frac{1}{2h}\sin(ha)+\gamma\cos^2(\frac{ha}{2})]a^2} = C_1\exp(-C_2R)$$

$$C_2 = \frac{2}{\gamma}\frac{\beta_z-\beta_0}{\beta_0}$$

- **The attenuation coefficient decreases with the bending radius**
- **The attenuation coefficient decrease with the index contrast**

What is Attenuation Coefficient (α)?



Other Losses --- Intersection

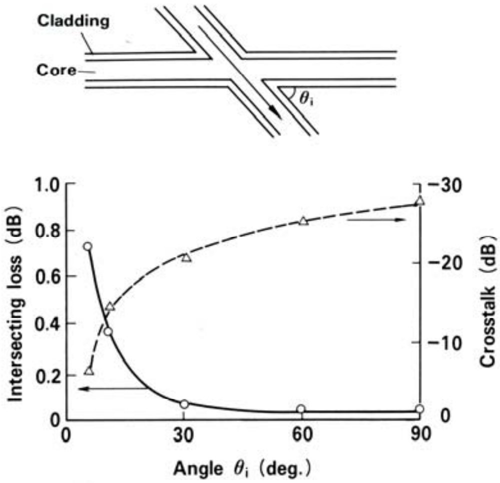
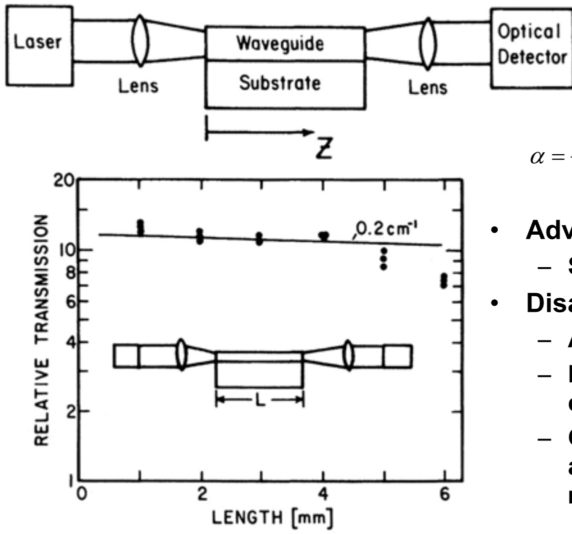


FIG. 10.13 Crossthrough (or intersection) loss versus cross angle in the silica single-mode crossthrough waveguide.[32] A. Himeno, M. Kobayashi, and H. Terui, *Electron. Lett.* 1985, IEE.

Waveguide Loss Measurement

- How to distinguish the loss?
  1. Waveguide loss or coupling loss?
  2. Waveguide loss of fundamental mode or high-order modes?
  3. Scattering loss, absorption loss, or radiation loss?

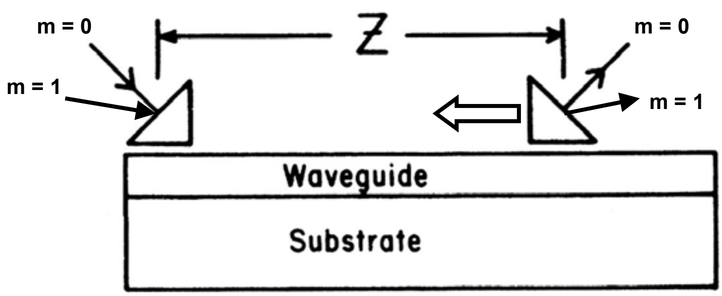
End-Fire Coupling Loss Measurement



$$\alpha = \frac{\ln(P_1 / P_2)}{Z_2 - Z_1} \quad \text{for} \quad Z_2 > Z_1$$

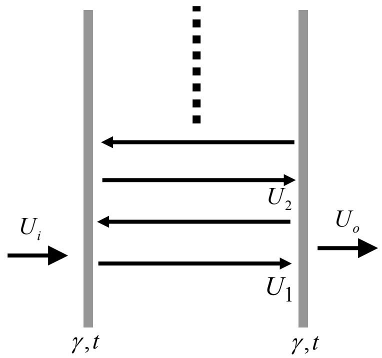
- Advantage
  - Simple and direct
- Disadvantage
  - Alignment sensitive
  - End face condition should be consistent
  - Can't distinguish the loss associated with different mode number

Prism-Coupled Loss Measurement



- Advantage
  - Can measure the loss from different modes
  - Alignment insensitive
  - End face quality is not required
- Disadvantage
  - Less accurate (It is difficult to reproduce the coupling loss)

Fabry-Perot



γ : *reflection*

t : *transmission*

$t^2 = 1 - \gamma^2$

$$U_1 = U_i \cdot t \exp(-j\varphi)\exp(-\frac{\alpha}{2}L)$$

$$U_2 = U_1 \cdot \gamma^2 \exp(-j2\varphi)\exp(-\alpha L)$$

$$\vdots$$

$$U_0 = \sum_n U_n$$

$$I_0 = U_o \cdot U_o^* = \frac{(1-\gamma^2)^2 \exp(-\alpha L)}{(1-R)^2 + 4R\sin^2\varphi}$$

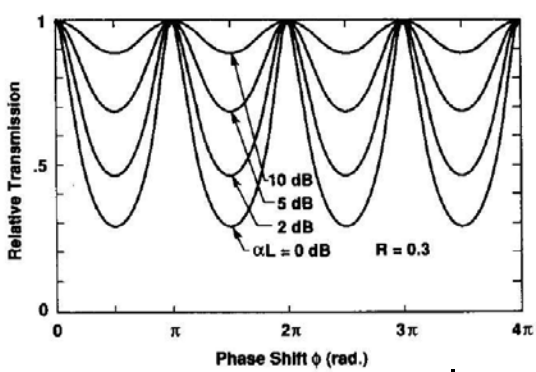
where $R = \gamma^2 \exp(-\alpha L)$

$$\alpha = -\frac{1}{L}\ln\left(\frac{1}{\gamma^2}\frac{\sqrt{I_{max}/I_{min}}-1}{\sqrt{I_{max}/I_{min}}+1}\right)$$

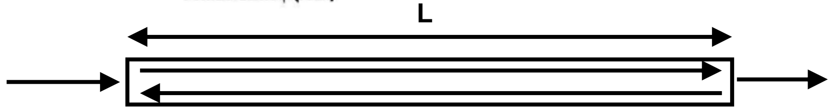
$I_{max} : \varphi = n\pi$

$I_{min} : \varphi = \frac{1}{2}(2n+1)\pi$

Fabry-Perot Loss Measurement



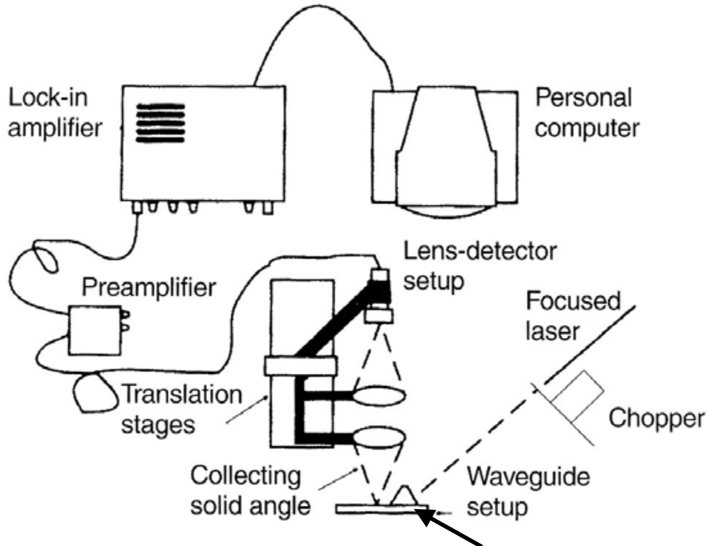
- **Advantage**
  - **Alignment insensitive**
- **Disadvantage**
  - **End face condition should be consistent**
  - **Only for single mode waveguide**
  - **Light source should be single frequency**



γ^2 : Reflectivity

$$\alpha = -\frac{1}{L}\ln\left(\frac{1}{\gamma^2}\frac{\sqrt{I_{max}/I_{min}}-1}{\sqrt{I_{max}/I_{min}}+1}\right)$$

Scattering Loss Measurement



Scattering loss Measurement by Image Analysis

McNab S.J.

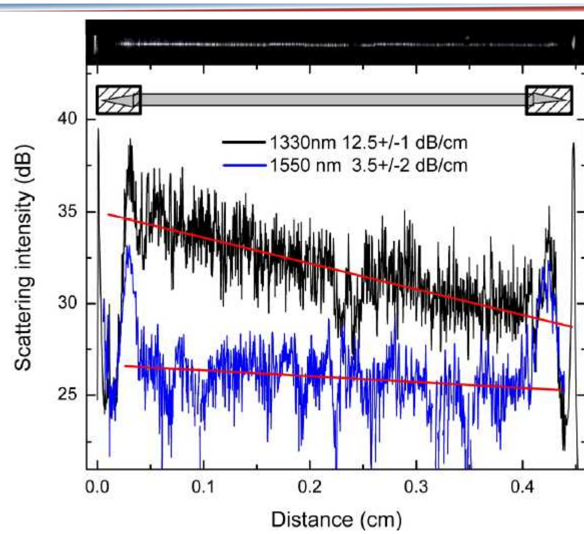


Fig. 3. Intensity of light scattered vertically from the reference optical circuit of Fig.1(b) with a 450nm wide strip waveguide for TE polarized light. Blue curve corresponds to the wavelength of 1550nm and black to 1330nm. Inset: Image of the vertically scattered light acquired by the IR camera. Several of such images were averaged to produce the traces in the main figure.