Waveguide Loss

Class: Integrated Photonic Devices  
Time: Fri. 8:00am ~ 11:00am.  
Classroom: 資電206  
Lecturer: Prof. 李明昌(Ming-Chang Lee)

Optical Loss in Waveguides

Three major losses in waveguide

- **Scattering Loss**
  - Due to surface roughness

- **Absorption Loss**
  - Due to photons annihilated in materials

- **Radiation Loss**
  - Due to waveguide bending
Scattering Loss

- Volume Scattering

- Surface Scattering (Dominant)

Surface Scattering Loss (Tien’s Model)

- Each reflection induce scattering light

\[
\sin \theta'_n = \frac{\beta_n}{k \cdot n_i}
\]

Rayleigh Criterion

\[
P_r = P_i \exp\left[-\left(\frac{4 \pi \sigma}{\lambda} \cos \theta_n\right)^2\right]
\]

\(\sigma\) : variance of surface roughness

\(m\) : mode number
Surface Scattering Loss

To quantitatively describe the optical loss, the exponential attenuation coefficient is generally used. In this case, the intensity (power per unit length) decays along the waveguide.

\[ I(z) = I_0 \exp(-\alpha z) \]

\[ I_0 \] is the initial intensity at \( z = 0 \)

\[ \alpha = A \left( \frac{1}{2} \frac{\cos^2 \theta_m}{\sin \theta_m} \right) \frac{1}{t_e + (1/\gamma_3) + (1/\gamma_2)} \]

\[ A = \frac{4\pi}{\lambda} \left( \sigma_{13}^2 + \sigma_{12}^2 \right)^{1/2} \]

Average roughness \( \sigma \)

\[ n_1 \]

\[ n_2 \]

\[ n_3 \]

Scattering Loss Analysis by Tien’s Model

Consider a planar waveguide with TE polarization

The power carried by the incident beam hit on the unit length (1 cm)

\[ \frac{c}{8\pi} n_1 E_y^2 \cos \theta_m \]

\( E_y \) is the field amplitude

According to the Rayleigh criterion, the reflected beam from the upper film surface

\[ \frac{c}{8\pi} n_1 E_y^2 \cos \theta_m \cdot \exp \left[ -\left( \frac{4\pi \sigma}{\lambda \cos \theta_m} \right)^2 \right] \]

\( \sigma \): variation of surface roughness

Rayleigh criterion
Scattering Loss Analysis by Tien’s Model

Consider the two film surface

\[
\frac{4\pi}{\lambda} \sigma \rightarrow \frac{4\pi}{\lambda} (\sigma_{11} + \sigma_{12})^{1/2}
\]

The power lost by surface scattering per unit length is

\[
\frac{c}{8\pi} n_i E_r^2 \cos \theta_u \left\{1 - \exp \left[-\left( A \cos \theta_u \right)^2\right]\right\}
\]

\[
\approx \frac{c}{8\pi} n_i E_r^2 A^2 \cos^3 \theta_u
\]

The planar waveguide mode power flow

\[
\frac{c}{4\pi} n_i E_r^2 \sin \theta_u \left( t_{\theta} + \frac{1}{\gamma_2} + \frac{1}{\gamma_3}\right)
\]

Scattering

The power attenuation per unit length

\[
\alpha = A^2 \frac{1}{2} \cos^3 \theta_u \left[ \frac{1}{t_{\theta} + (1/\gamma_2) + (1/\gamma_3)} \right]
\]
Surface Scattering Loss

- High order modes have more reflections from the surface

\[ N_m = \frac{L}{2r_c \cot \theta_m} \]

Where \( m \) is mode no.

Mode Effect

The loss for the \( m=3 \) waveguide mode is as much as 14 times that of the \( m = 0 \) waveguide mode.

\( \frac{\Delta \alpha}{\lambda} = 4.3 \text{ cm}^{-1} \)

\( \lambda = 632.8 \text{ nm} \)

Ta\(_2\)O\(_5\)
Sidewall Scattering Loss

- Sidewall roughness is created during etching process
- The propagation loss is highly related to the roughness for a small-dimension waveguide

Optical Loss due to Surface Roughness

- Single mode waveguide
  \[ w \times t < 0.18 \mu m^2 \]
- Surface roughness required to achieve low loss
  \[ \delta_{rms} < 1 \text{ nm} \]
Absorption Loss

- **Interband absorption** → electron and hole pairs (photodetector) \( h\nu > E_g \)
- **Intraband absorption** → free carrier scattering (metal) \( h\nu < E_g \)

Free Carrier Absorption (Drude Model)

\[
\frac{d^2x}{dt^2} + mg \frac{dx}{dt} - qE_0 \exp(j\omega t) = -eE_0 \exp(j\omega t)
\]

Not harmonic oscillator!

- \( g \): damping coefficient due to scattering
- \( m \): mass of carrier

\[
x = \frac{(eE_0)}{\omega^2 - j\omega g} \exp(j\omega t)
\]

**Recall**

\[
D = \varepsilon_0 E + P_0 + P_f = \varepsilon_0 (1 + \chi_0 + \chi_f) E
\]

Dielectric polarization (Dipole Moment)  Free Carrier Effect (electron or hole)
Free Carrier Absorption

\[ P_i = -N_{ex} = \frac{-(Ne_i^2)}{\omega^2 - j\omega g} E_0 \exp(j\omega t) \]

\[ \chi_i = \frac{-(Ne_i^2)}{(m\varepsilon_e)} \frac{2}{\omega^2 - j\omega g} = \chi_i' + \chi_i'' \]

\[ \left\{ \begin{array}{l}
\chi_i' = -\frac{Ne_i^2}{\omega^2 + g^2} \\
\chi_i'' = \frac{Ne_i^2}{\omega^2 + g^2}
\end{array} \right. \]

\[ D = \varepsilon_0 E + P_0 + P_i = \varepsilon_0 [(1 + \chi_i' + \chi_i'') + j\chi_i''] E = \varepsilon_0 \left( n_i + n_i' + jn_i'' \right) \]

Where \( \chi_i' \ll 1 + \chi + \chi_i'' \)

What is \( g \)?

At steady state, electrons move as a constant speed.

That is, \( \frac{d^2x}{dt^2} = 0 \)

\[ m\frac{dx}{dt} = eE \quad (1) \]

The definition of mobility \( \mu \)

\[ \frac{dx}{dt} = \mu E \quad (2) \]

\[ g = \frac{e}{m\mu} \]
Free Carrier Absorption

\[
I \propto |E|^2 = |E_0|^2 \exp[jk_0(n' + jn^*)z] \exp[-jk_0(n' - jn^*)z] \\
= |E_0|^2 \exp[-2kn'z] = |E_0|^2 \exp[-\alpha z]
\]

\[
\alpha_p = 2kn' \approx k_0 \frac{\nu}{n} = \frac{Ne^3}{m^*n_0^*\omega^* \mu_c} \quad (\omega >> g) \text{ For optical wavelength}
\]

- Free carrier absorption is proportional to the carrier density
- The refractive index is also affected by the free carriers
Free Carrier Absorption

$p$-Si, 300k

\begin{align*}
\text{Absorption coefficient } & \text{ (cm$^{-1}$)} \\
1 & \text{1.68} \times 10^{9} \text{ cm$^{-1}$} \\
2 & \text{2.5} \times 10^{9} \text{ cm$^{-1}$} \\
3 & \text{1.4} \times 10^{9} \text{ cm$^{-1}$} \\
4 & \text{4.6} \times 10^{7} \text{ cm$^{-1}$} \\
\end{align*}

Wavelength $\lambda$ (\text{$\mu$m})

\begin{align*}
1 & \text{ (1) } 1.4 \times 10^{9} \text{ cm$^{-1}$} \\
2 & \text{ (2) } 8 \times 10^{8} \text{ cm$^{-1}$} \\
3 & \text{ (3) } 1.7 \times 10^{7} \text{ cm$^{-1}$} \\
4 & \text{ (4) } 3.2 \times 10^{7} \text{ cm$^{-1}$} \\
5 & \text{ (5) } 6.1 \times 10^{6} \text{ cm$^{-1}$} \\
6 & \text{ (6) } 1 \times 10^{6} \text{ cm$^{-1}$} \\
\end{align*}

n-Si, 300k

Resistivity vs. Impurity Concentration

\begin{align*}
\text{Impurity Concentration (cm}^{-3}) & \text{ Resistivity (Ohm-cm)} \\
0.6 & 2 \\
\end{align*}

Sze and Irvin
Temperature-Dependent Free Carrier Absorption

Fig. 1.4.8. Free carrier absorption versus wavelength for high purity Si at different temperatures (Rooman [166]). Temperatures are 1. 300 K, 2. 473 K, 3. 573 K, 4. 623 K, 5. 673 K (Figure reprinted with permission of McGraw-Hill Book Co.).

Free Carrier Absorption on Proton Bombardment Waveguide

The major loss comes from the evanescent wave penetrating in substrate.
Surface Plasmons

- The interaction of metals with electromagnetic radiation is largely dictated by the free electrons in the metal.
- Most metals possess a negative dielectric constant at optical frequency.
- Only the surface can support optical wave propagation (why?)

Optical Properties of Noble Metals

- **Dipole Dispersion**
- **Free-Carrier Dispersion**
Measure dielectric function of gold

Combine dipole dispersion and free-carrier dispersion

Surface Plasmons at plane interface

- EM wave and surface charge are oscillating.
- The fields in the perpendicular direction decay exponentially.
- The momentum of SP is larger than the free space photon.

W.L. Banes, review article
Mathematically, the solution has to satisfy the wave equation

\[ \nabla \times \nabla \times \mathbf{E}(r, \omega) = \frac{\omega^2}{c^2} \varepsilon(r, \omega) \mathbf{E}(r, \omega) = 0 \]

where \( \varepsilon(r, \omega) = \varepsilon_z(\omega) \quad z < 0 \quad \varepsilon(r, \omega) = \varepsilon_z(\omega) \quad z > 0 \)

---

Consider a TM wave or p-wave,

\[
E_i = \begin{cases} 
E_{i,x} & \text{exp}(jk_x x - j\omega t) \exp(jk_z z) \quad \text{where} \quad i = 1, 2 \\
0 & \end{cases}
\]

Since the x-direction wave vector is conserved or Snell's law

\[ k_x^2 + k_z^2 = \varepsilon_i k^2 \quad \text{where} \quad k = \frac{\omega}{c} \]

Since both spaces are source-free; that is, \( \nabla \cdot \mathbf{D} = 0 \)

\[ k_x E_{i,x} + k_z E_{i,z} = 0 \quad (a) \]
Surface Plasmons at plane interface

Consider the boundary condition,

\[ E_{1,z} - E_{2,z} = 0 \]
\[ \varepsilon_1 E_{1,z} - \varepsilon_2 E_{2,z} = 0 \]  \hspace{1cm} (b)

Combine (a) and (b), since the electric fields are not trivial solutions, the determinant of respective matrix has to be zero; then

\[ \varepsilon_i k_{1,z} - \varepsilon_j k_{1,z} = 0 \]

Therefore,

\[ k_i^2 - k_{k_i}^2 = \varepsilon_i k_i \]

recall \[ k_i^2 + k_{k_i}^2 = \varepsilon_i k_i \]

\[ k_i^2 = \frac{\varepsilon_i \varepsilon_2}{\varepsilon_i + \varepsilon_2} k^2 = \frac{\varepsilon_i \varepsilon_2}{\varepsilon_i + \varepsilon_2} \omega^2 \]  \hspace{1cm} and  \hspace{1cm} \[ k_{k_i} = \frac{\omega}{c} \]

Since the surface plasmonic mode are evanescent on the two sides of interface

\[ k_i \]  should be real  \hspace{1cm} and  \hspace{1cm} \[ k_{k_i} \]  should be imaginary

Therefore

\[ \varepsilon_i(\omega) \cdot \varepsilon_2(\omega) < 0 \]
\[ \varepsilon_i(\omega) + \varepsilon_2(\omega) < 0 \]

The dielectric functions must be negative with an absolute value exceeding that of the other.

Nobel metals such as gold and silver, have a large negative real part of the dielectric constant along with small imaginary part
Properties of surface plasmonic waves

Consider the metal dielectric

\[ \epsilon_i = \epsilon_i' + j\epsilon_i'' \]

Suppose the imaginary part is much smaller than the real part and \( \epsilon_2 \) is positive real, the wave number of SP mode

\[ k_z = k_z' + jk_z'' \]

where

\[
\begin{align*}
  k_z' &= \sqrt{\frac{\epsilon_1\epsilon_2 - \omega^2}{\epsilon_1 + \epsilon_2}} \frac{\omega}{c} \\
  k_z'' &= \sqrt{\frac{\epsilon_1\epsilon_2 - \epsilon_1'\epsilon_2'}{2\epsilon_1'\epsilon_2'} \frac{\omega}{\epsilon_1 + \epsilon_2}} \frac{\omega}{c}
\end{align*}
\]

and

\[
\begin{align*}
  k_{z1} &= \frac{\omega}{c} \sqrt{\frac{\epsilon_1^2}{\epsilon_1 + \epsilon_2} \left[ 1 + j\frac{\epsilon_1'}{2\epsilon_1} \right]} \\
  k_{z2} &= \frac{\omega}{c} \sqrt{\frac{\epsilon_2^2}{\epsilon_1 + \epsilon_2} \left[ 1 - j\frac{\epsilon_1'}{2(\epsilon_1 + \epsilon_2)} \right]}
\end{align*}
\]

Excitation of Surface Plasmonic Wave

In this case, SPP can not be directly coupled from air.
Radiation Loss

\[ (R+X_r) \frac{d\theta}{dt} = \frac{\omega}{\beta_0} \]

\[ R \frac{d\theta}{dt} = \frac{\omega}{\beta_c} \]

The angular phase velocity should be the same.

\[ X_r = \frac{\beta_c - \beta_0}{\beta_0} R \]

What is Attenuation Coefficient (\(\alpha\))? 

\[ \alpha = -\frac{1}{P(z)} \frac{dP(z)}{dz} \quad (\text{Because } P(z) = P_0 \exp(-\alpha z)) \]

\[ \approx -\frac{1}{P(z_0)} \frac{\Delta P(z)}{\Delta z} \quad \text{Dissipated Power} \]

\[ \Delta Z \quad \text{Propagation length} \]
**What is \( \alpha \) due to Radiation Loss?**

Suppose the field:

\[
E(x) = \sqrt{C_0} \cos(hx)
\]

\[-\frac{a}{2} \leq x \leq \frac{a}{2}\]

\[
E(x) = \sqrt{C_0} \cos\left(\frac{ha}{2}\right) \exp\left\{-\frac{|x| - (a/2)}{\gamma}\right\}
\]

\[|x| \geq \frac{a}{2}\]

**Total Power** \( P(z) \)

\[
P_{\text{tot}} = \int_{-\infty}^{\infty} E^2(x) dx = C_0 \left[ \frac{a}{2} + \frac{1}{2h} \sin(ha) + \gamma \cos^2\left(\frac{ha}{2}\right) \right]
\]

**Radiated Power** \( \Delta P(z) \)

\[
P_{\text{rad}} = \int_{-\infty}^{\infty} E^2(x) dx = C_0 \left[ \frac{a}{2} \cos\left(\frac{ha}{2}\right) \exp\left(\frac{-2}{\gamma}(X_r - \frac{a}{2})\right) \right]
\]

**What is Attenuation Coefficient (\( \alpha \))?**

The propagation length of unguided mode (analogy to a truncated waveguide)

\[
Z_c = \frac{a}{\phi} = \frac{a^2}{2\lambda} \quad (\text{Because } \sin\left(\frac{\phi}{2}\right) = \frac{\lambda}{a})
\]

\( a \): waveguide width

\( \lambda \): wavelength

The attenuated coefficient:

\[
\alpha = \frac{\gamma}{2} \cos^2\left(\frac{ha}{2}\right) \exp\left(\frac{-2}{\gamma} \frac{\beta_z - \beta_i}{\beta_z} R 2 \lambda \exp\left(\frac{a}{\gamma}\right) \right)
\]

\[
\left[ \frac{a}{2} + \frac{1}{2h} \sin(ha) + \gamma \cos^2\left(\frac{ha}{2}\right) \right] a^2
\]

*Ming-Chang Lee, Integrated Photonic Devices*
What is $\alpha$ due to Radiation Loss?

\[
\alpha = \frac{\gamma}{2} \cos^2 \left( \frac{ha}{2} \right) \exp \left( -\frac{2}{\gamma} \frac{\beta_i - \beta_n}{\beta_n} R \right) 2\lambda \exp \left( \frac{a}{\gamma} \right) C_i \exp \left( -C_i R \right)
\]

\[
C_i = \frac{2}{\gamma} \frac{\beta_i - \beta_n}{\beta_n}
\]

- The attenuation coefficient decreases with the bending radius
- The attenuation coefficient decrease with the index contrast
Other Losses --- Intersection

![Diagram of waveguide intersection]

Fig. 10.13 Crossthrough (or intersection) loss versus cross angle in the silica single-mode crossthrough waveguide.\(^1\) A. Himeno, M. Kobayashi, and H. Terui, Electron. Lett. 1985, IEE.

Waveguide Loss Measurement

- How to distinguish the loss?
  1. Waveguide loss or coupling loss?
  2. Waveguide loss of fundamental mode or high-order modes?
  3. Scattering loss, absorption loss, or radiation loss?
End-Fire Coupling Loss Measurement

\[ \alpha = \ln\left(\frac{P_i}{P_z}\right) \frac{Z_z - Z_i}{\text{for } Z_z > Z_i} \]

- **Advantage**
  - Simple and direct

- **Disadvantage**
  - Alignment sensitive
  - End face condition should be consistent
  - Can't distinguish the loss associated with different mode number

---

Prism-Coupled Loss Measurement

- **Advantage**
  - Can measure the loss from different modes
  - Alignment insensitive
  - End face quality is not required

- **Disadvantage**
  - Less accurate (It is difficult to reproduce the coupling loss)
**Fabry-Perot**

\[ U_i = U_i \cdot t \exp(-j\phi) \exp(-\frac{\alpha L}{2}) \]

\[ U_o = U_i \cdot \gamma^2 \exp(-j2\phi) \exp(-\alpha L) \]

\[ U_o = \sum U_o \]

\[ I_o = U_o \cdot U_o^* = \frac{(1-\gamma^2)^2 \exp(-\alpha L)}{(1-R)^2 + 4R\sin^2 \phi} \]

where \( R = \gamma^2 \exp(-\alpha L) \)

\[ \alpha = \frac{1}{L} \ln \left( \frac{1}{\gamma^2} \frac{I_{\text{max}}}{I_{\text{max}}} - 1 \right) \]

\[ I_{\text{max}}: \phi = n\pi \]

\[ I_{\text{max}}: \phi = \frac{1}{2}(2n+1)\pi \]

\( \gamma \): reflection

\( t \): transmission

\( t^2 = 1 - \gamma^2 \)

**Fabry-Perot Loss Measurement**

- **Advantage**
  - Alignment insensitive

- **Disadvantage**
  - End face condition should be consistent
  - Only for single mode waveguide
  - Light source should be single frequency

\( \gamma^2 \): Reflectivity

\[ \alpha = \frac{1}{L} \ln \left( \frac{1}{\gamma^2} \frac{I_{\text{max}}}{I_{\text{max}}} - 1 \right) \]
Scattering Loss Measurement

Scattering loss Measurement by Image Analysis

Fig. 3. Intensity of light scattered vertically from the reference optical circuit of Fig. 3b with a 450 nm wide step waveguide for TE polarized light. Blue curve corresponds to the wavelength of 1550 nm and black to 1310 nm. Inset: Image of the vertically scattered light acquired by the IR camera. The end of each image was averaged to produce the traces in the main figure.