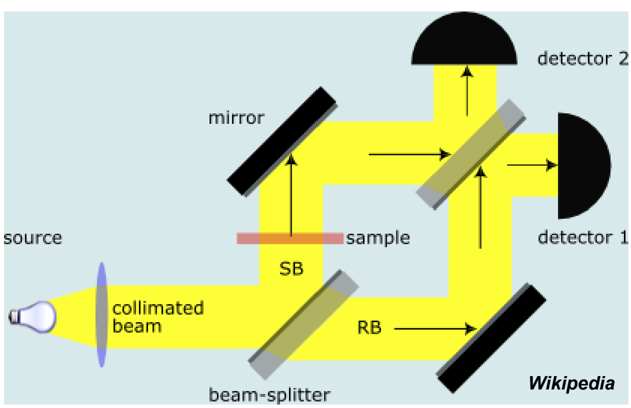
Waveguide Coupler II

Class: Integrated Photonic Devices
Time: Fri. 8:00am ~ 11:00am.
Classroom: 資電206
Lecturer: Prof. 李明昌(Ming-Chang Lee)

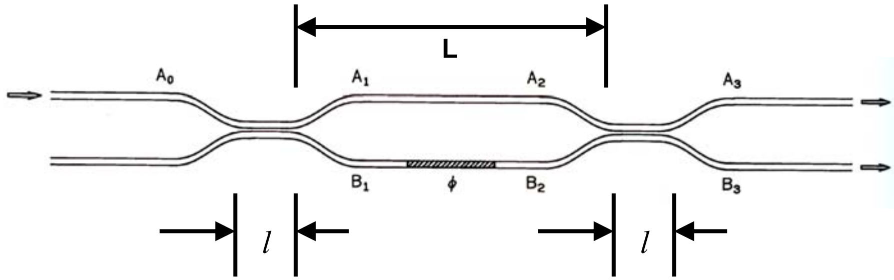
Free-Space Mach-Zehnder Interferometer



$$\Delta\phi = 2\pi\left(\frac{L_1 - L_2}{\lambda}\right) \quad \textbf{Detector 1} \quad \sim \cos^2\left(\frac{L_1 - L_2}{2\lambda}\right)$$

$$\Delta\phi = \pi + 2\pi\left(\frac{L_1 - L_2}{\lambda}\right) \quad \textbf{Detector 2} \quad \sim \sin^2\left(\frac{L_1 - L_2}{2\lambda}\right)$$

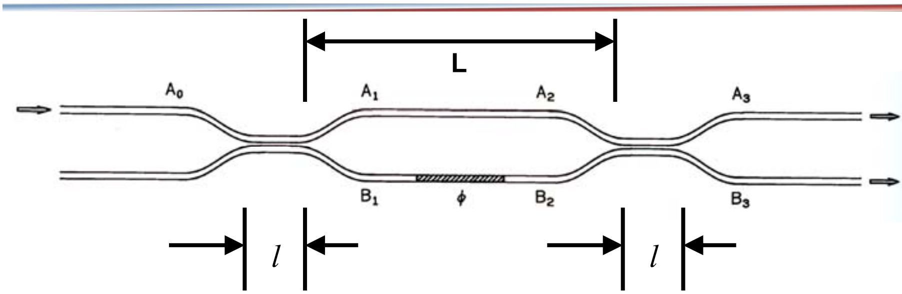
Guided-Wave Mach-Zehnder Interferometer



$$\begin{cases} A_1 = A_0 \cos(\kappa l) \\ B_1 = jA_0 \sin(kl) \end{cases}$$

if $\kappa l = \frac{\pi}{4}$ $\begin{cases} A_1 = A_0 \cos(\kappa l) = A_0 \cos(\frac{\pi}{4}) = \frac{A_0}{\sqrt{2}} \\ B_1 = jA_0 \sin(kl) = jA_0 \sin(\frac{\pi}{4}) = j\frac{A_0}{\sqrt{2}} \end{cases}$ **(3-dB Coupler)**

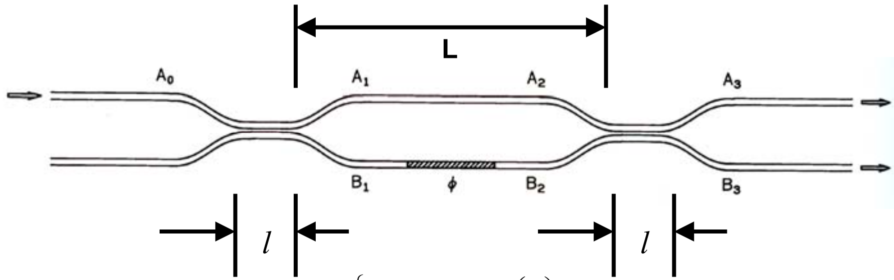
Guided-Wave Mach-Zehnder Interferometer



$$\begin{cases} A_2 = A_1 \exp(j\beta L) = \frac{A_0}{\sqrt{2}} \exp(j\beta L) \\ B_2 = B_1 \exp(j\beta L + j\phi) = j\frac{A_0}{\sqrt{2}} \exp(j\beta L + j\phi) \end{cases}$$

Then $\begin{cases} A_3 = \frac{A_2}{\sqrt{2}} + j\frac{B_2}{\sqrt{2}} = -jA_0 \sin(\frac{\phi}{2}) \exp(j\beta L + j\frac{\phi}{2}) \\ B_3 = \frac{B_2}{\sqrt{2}} + j\frac{A_2}{\sqrt{2}} = -jA_0 \cos(\frac{\phi}{2}) \exp(j\beta L + j\frac{\phi}{2}) \end{cases}$

Guided-Wave Mach-Zehnder Interferometer



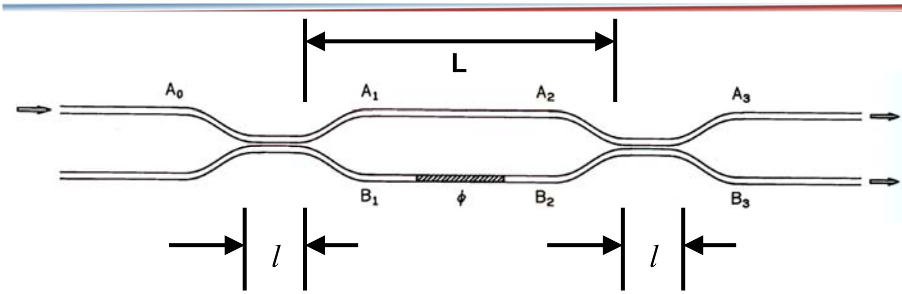
$$\begin{cases} |A_3|^2 = |A_0|^2 \sin^2\left(\dfrac{\phi}{2}\right) \\ |B_3|^2 = |A_0|^2 \cos^2\left(\dfrac{\phi}{2}\right) \end{cases}$$

If f is slightly modulated with δϕ

$$\begin{cases} |A_3|^2 \cong |A_0|^2 \left(\dfrac{\delta\phi}{2}\right)^2 \\ |B_3|^2 = |A_0|^2 \end{cases}$$

It is not linear modulation!

Guided-Wave Mach-Zehnder Interferometer

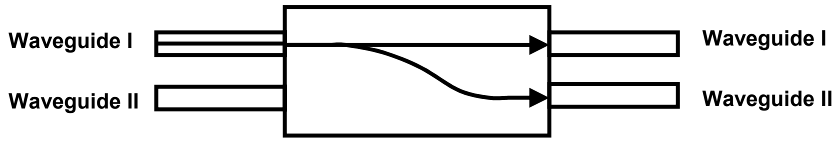


To make the modulation is more linear,

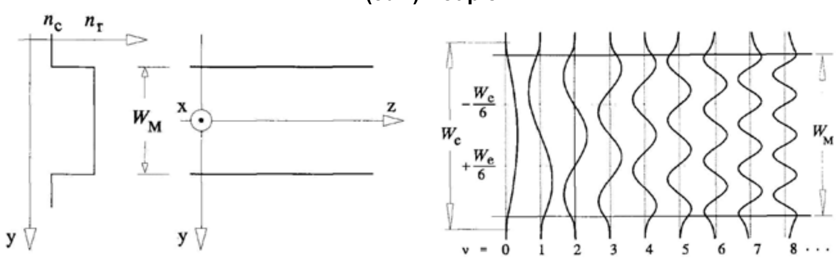
$$\phi = \frac{\pi}{2} + \delta\phi$$

$$|A_3|^2 = |A_0|^2 \sin^2\left(\frac{\pi}{4} + \frac{\delta\phi}{2}\right) = \frac{1}{2}|A_0|^2 [1 + \sin(\delta\phi)] \simeq \frac{1}{2}|A_0|^2 [1 + \delta\phi]$$

Multi-Mode Interference (MMI) Coupler

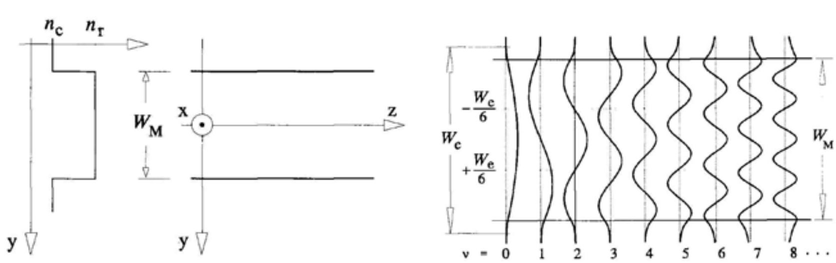


MMI (3dB) Coupler



- The MMI region can be seen as a multi-mode waveguide.
- The fundamental of the power transfer comes from multiple modes' beating.

Multi-Mode Interference (MMI) Coupler



$$h_{y\nu}^{2} + \beta_{\nu}^{2} = k_0^{2} n_r^{2}$$ **v = 0,1,2,…**

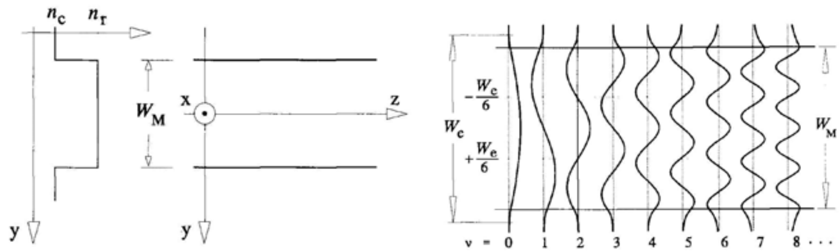
where $k_0 = \dfrac{2\pi}{\lambda_0}$ **and** $h_{y\nu} = \dfrac{(\nu+1)\pi}{W_{e\nu}}$ ← Effective width

Suppose the MMI is well-confined

$$W_{e\nu} \simeq W_{e0} = W_M + \left(\frac{\lambda_0}{\pi}\right)\left(\frac{n_c}{n_r}\right)^{2\sigma}(n_r^{2} - n_c^{2})^{-\frac{1}{2}} \qquad \sigma = 1 \text{ for TM}, 0 \text{ for TE}$$

$\beta_0 \simeq k_0 n_r$

Multi-Mode Interference (MMI) Coupler



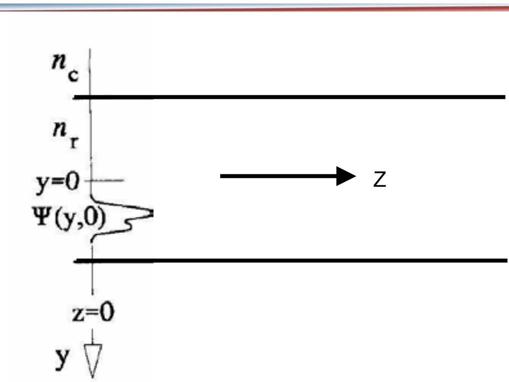
$$\beta_\nu \simeq k_0 n_r - \frac{(\nu+1)^2 \pi \lambda_0}{4 n_r W_e^2} \quad \textbf{and} \quad (\beta_0 - \beta_\nu) \simeq \frac{\nu(\nu+2)\pi\lambda_0}{4 n_r W_e^2}$$

Define the L_π as the beating length of the two lowest-order modes

$$L_\pi = \frac{\pi}{\beta_0 - \beta_1} = \frac{4 n_r W_e^2}{3\lambda_0}$$

then $(\beta_0 - \beta_\nu) = \dfrac{\nu(\nu+2)\pi}{3 L_\pi}$

Multi-Mode Interference (MMI) Coupler



Suppose the input field profile $\psi(y,0)$ imposed at z = 0

$\psi(y,0) = \sum_\nu c_\nu \varphi(y)$ $\quad \varphi(y)$ is the eigenmode (include radiation mode)

where $c_\nu = \dfrac{\int \psi(y,o)\varphi_\nu(y)dy}{\sqrt{\int \varphi_\nu^2(y)dy}}$ Field-orthogonality relations

Multi-Mode Interference (MMI) Coupler

If the "spatial spectrum" of the input field Ψ is narrow enough not to excite unguided modes, it may be decomposed into the guided modes

$$\psi(y,0)=\sum_{\nu=0}^{m-1} c_\nu \varphi_\nu(y) \qquad \varphi_\nu(y):\begin{cases}\textbf{even mode } (\nu \text{ is even}) & \text{symmetric}\\ \textbf{odd mode } (\nu \text{ is odd}) & \text{antisymmetric}\end{cases}$$

The field profile at a distance z

$$\psi(y,z)=\sum_{\nu=0}^{m-1} c_\nu \varphi_\nu(y)\exp(j\beta_\nu z)$$

The relative phase correspondent to fundamental mode

$$\psi(y,z)=\sum_{\nu=0}^{m-1} c_\nu \varphi_\nu(y)\exp[j(\beta_0-\beta_\nu)z]$$

$$\downarrow$$

$$\psi(y,L)=\sum_{\nu=0}^{m-1} c_\nu \varphi_\nu(y)\exp[j\frac{\nu(\nu+2)\pi}{3L_\pi}L]$$

Multi-Mode Interference (MMI) Coupler

(A) Single Images

If $L=p(3L_\pi)$ **p = 0,2,4,…(even)**

Then $\exp[j\frac{\nu(\nu+2)\pi}{3L_\pi}L]=1$

$$\downarrow$$

$\psi(y,L)=\psi(y,0)$ **(Image is reproduced)**

If $L=p(3L_\pi)$ **p = 1,3,5,…(odd)**

Then $\exp[j\frac{\nu(\nu+2)\pi}{3L_\pi}L]=1\ (\nu \text{ is even}) \text{ or } -1\ (\nu \text{ is odd})$

$$\psi(y,L)=\sum_{\nu:even} c_\nu\varphi_\nu(y)+\sum_{\nu:odd} -c_\nu\varphi_\nu(y)$$

$$=\sum_{\nu:even} c_\nu\varphi_\nu(-y)+\sum_{\nu:odd} c_\nu\varphi_\nu(-y)$$

$=\psi(-y,0)$ **(Image is mirrored at y = 0)**

$$\varphi_\nu(-y)=\begin{cases}\varphi_\nu(y) & \textbf{for } \nu \textbf{ even (cos)}\\ -\varphi_\nu(y) & \textbf{for } \nu \textbf{ odd (sin)}\end{cases}$$

Multi-Mode Interference (MMI) Coupler

(B) Multiple Images

If $L = \frac{p}{2}(3L_\pi)$ **p = 1,3,5,…(odd)**

Then $\psi(y, \frac{p}{2}3L_\pi) = \sum_{\nu=0}^{m-1} c_\nu \varphi_\nu(y) \exp[j\nu(\nu+2)p\left(\frac{\pi}{2}\right)]$

$$\downarrow$$

$$\psi(y, \frac{p}{2}3L_\pi) = \sum_{\nu:even} c_\nu \varphi_\nu(y) + \sum_{\nu:odd} (-j)^p c_\nu \varphi_\nu(y)$$

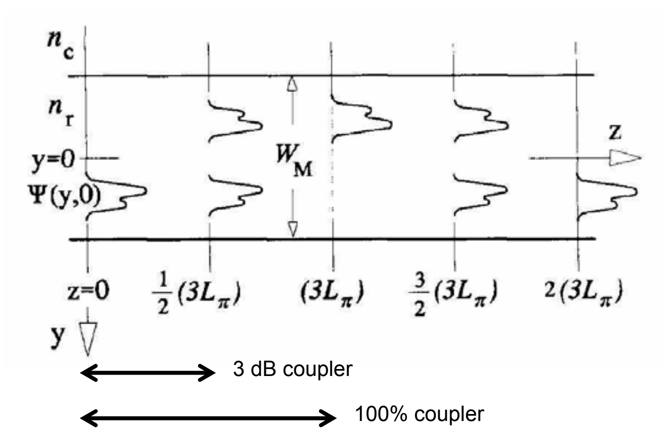
$$= \frac{1+(-j)^p}{2}\psi(y,0) + \frac{1-(-j)^p}{2}\psi(-y,0)$$

$$\sum_{\nu:even} c_\nu \varphi_\nu(y) = \frac{\psi(y,0)+\psi(-y,0)}{2}$$

$$\sum_{\nu:odd} c_\nu \varphi_\nu(y) = \frac{\psi(y,0)-\psi(-y,0)}{2}$$

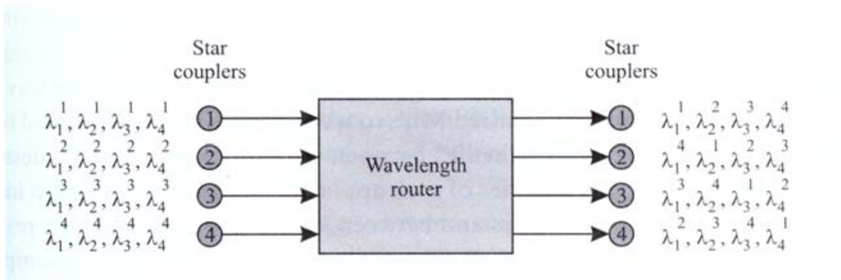
Two Images

Multi-Mode Interference (MMI) Coupler



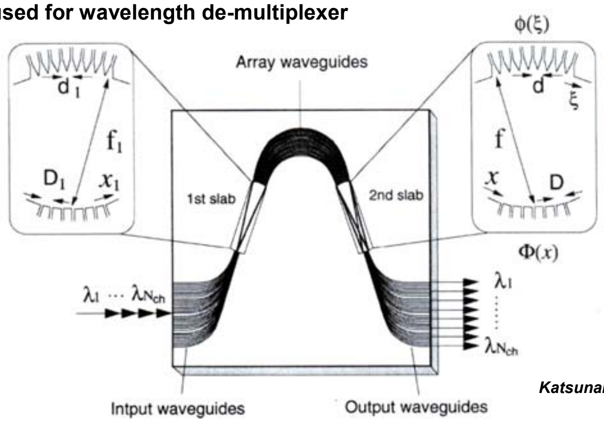
- **Advantage of MMI:**
  - **The length could be shorter**
  - **It could be extend to a N × N coupler/splitter**

Arrayed Waveguide for Star Coupler



Principle of Arrayed-Waveguide Grating (AWG)

AWG can be used for wavelength de-multiplexer



Katsunari Okamoto

- **Input/Ouput Waveguides**
- **Two focus slab regions**
- **A phase array of multiple channel waveguides**

Principle of Arrayed-Waveguide Grating (AWG)

For the phase array of waveguides, the path-length difference ΔL between neighboring waveguides results in phase difference by

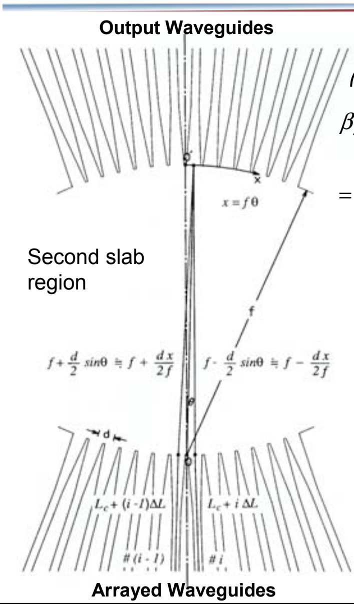
$$\Delta L / \lambda$$

In the first slab, the input waveguide separation is D_1, the array waveguide separation is d_1, and the radius of curvature is f_1. Similar definition is shown in the second slab.

The light input at the x_1 position (x_1 is measured counterclockwise from the center of the input waveguide) is radiated to the first slab and then excites the arrayed waveguides. The amplitude profile of a_i, electric field at each arrayed waveguide, is usually a Gaussian distribution.

After traveling through the arrayed waveguides, the light beams constructively interfere into one focal point x in the second slab.

Principle of Arrayed-Waveguide Grating (AWG)



Input slab — Arrayed waveguide — Output slab

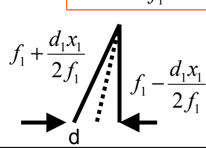
$$\beta_s(\lambda_0)\left(f_1-\frac{d_1x_1}{2f_1}\right)+\beta_c(\lambda_0)[L_c+(i-1)\Delta L]+\beta_s(\lambda_0)\left(f+\frac{dx}{2f}\right)$$

$$=\beta_s(\lambda_0)\left(f_1+\frac{d_1x_1}{2f_1}\right)+\beta_c(\lambda_0)[L_c+i\Delta L]+\beta_s(\lambda_0)\left(f-\frac{dx}{2f}\right)-2m\pi$$

where β_s and β_c denote the propagation constants in the slab region and array waveguide

λ_0 is the center wavelength of the WDM system, and L_c is the minimum array waveguide length

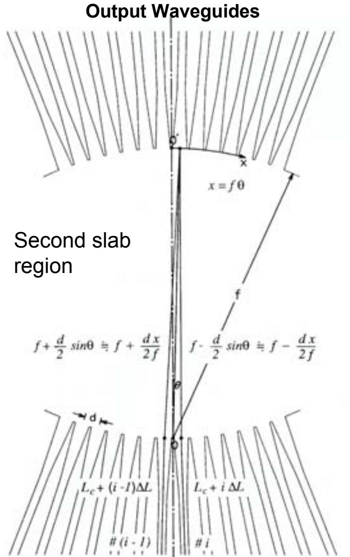
$$\beta_s(\lambda_0)\frac{d_1x_1}{2f_1}-\beta_s(\lambda_0)\frac{dx}{2f}+\beta_c(\lambda_0)\Delta L=2m\pi \quad \text{(a)}$$



m: Integer, diffractive order

Principle of Arrayed-Waveguide Grating (AWG)

Output Waveguides



Arrayed Waveguides

To satisfy the equation, if

$$\beta_c(\lambda_0)\Delta L = 2m\pi$$

and

$$\frac{d_1 x_1}{f_1} = \frac{dx}{f}$$

, the light input position x_1 will focus on the output position x

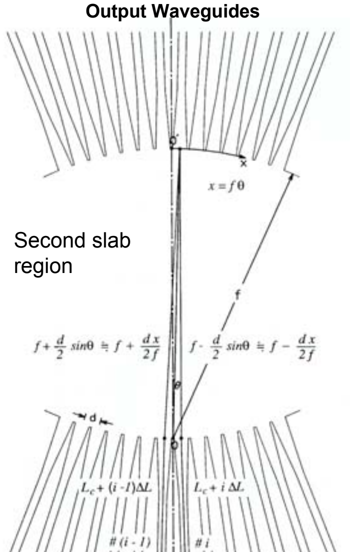
We define the effective index of the array waveguide

$$n_c = \frac{\beta_c}{k} \quad \text{and} \quad n_s = \frac{\beta_s}{k}$$

and group index $N_c = n_c - \lambda \dfrac{dn_c}{d\lambda}$

Principle of Arrayed-Waveguide Grating (AWG)

Output Waveguides



Arrayed Waveguides

The dispersion of the focal position x with respect to wavelength λ for the fixed light input position x_1 is given by

$$\frac{\Delta x}{\Delta \lambda} = -\frac{N_c f \Delta L}{n_s d \lambda_0} \quad \text{(differentiate (a))}$$

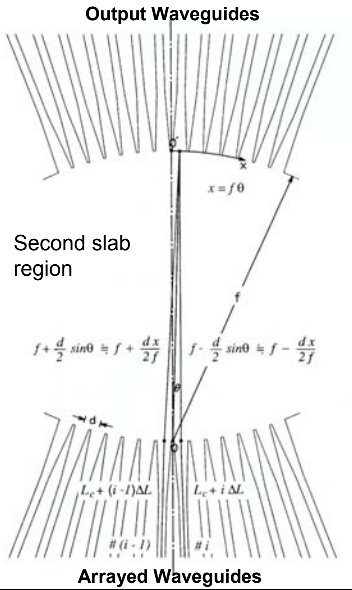
The dispersion of the input-side position x_1 with respect to wavelength λ for the fixed light output position x is given by

$$\frac{\Delta x_1}{\Delta \lambda} = \frac{N_c f_1 \Delta L}{n_s d_1 \lambda_0} \quad \text{(differentiate (a))}$$

The input and output waveguide separations are

$$|\Delta x_1| = D_1 \quad \text{and} \quad |\Delta x| = D$$

Principle of Arrayed-Waveguide Grating (AWG)



The wavelength spacing in the output side for the fixed light input position x_1

$$\Delta\lambda_{out} = \frac{n_s d D \lambda_0}{N_c f \Delta L}$$

The wavelength spacing in the input side for the fixed light output position x

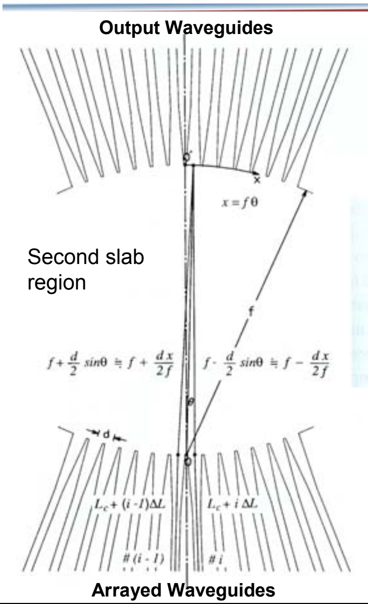
$$\Delta\lambda_{in} = \frac{n_s d_1 D_1 \lambda_0}{N_c f_1 \Delta L}$$

Generally, the waveguide parameters in the first and second slab regions are the same. Then the channel spacings are the same

$$\Delta\lambda_{in} = \Delta\lambda_{out} \equiv \Delta\lambda$$

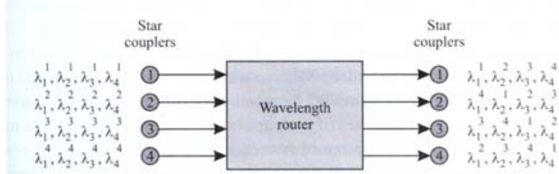
WDM channel spacing

Principle of Arrayed-Waveguide Grating (AWG)



The path-length difference ΔL is obtained as

$$\Delta L = \frac{n_s d D \lambda_0}{N_c f \Delta\lambda}$$



Free Spatial Range of AWG (m=0,1,2,…)

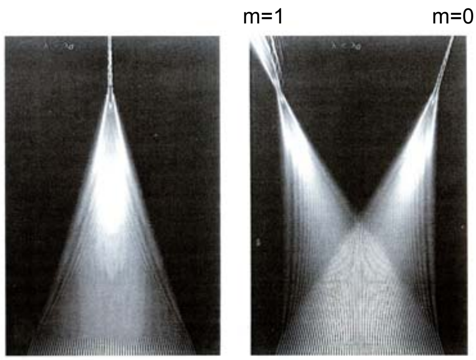


Figure 9.10: BPM simulation of the light focusing property in the second slab region for (a) the central wavelength λ_0 and (b) the shorter-wavelength component $\lambda < \lambda_0$.

Katsunari Okamoto

The spatial separation of the mth and (m+1)th focus beams for the same wavelength is given

$$X_{FSR} = x_m - x_{m+1} = \frac{\lambda_0 f}{n_s d}$$

X_{FSR} represents for free spatial range of AWG. The number of available wavelength channels N_{ch} is given by

$$N_{ch} = \frac{X_{FSR}}{D} = \frac{\lambda_0 f}{n_s d}$$