Waveguide Coupler I

Class: Integrated Photonic Devices
Time: Fri. 8:00am ~ 11:00am.
Classroom: 資電206
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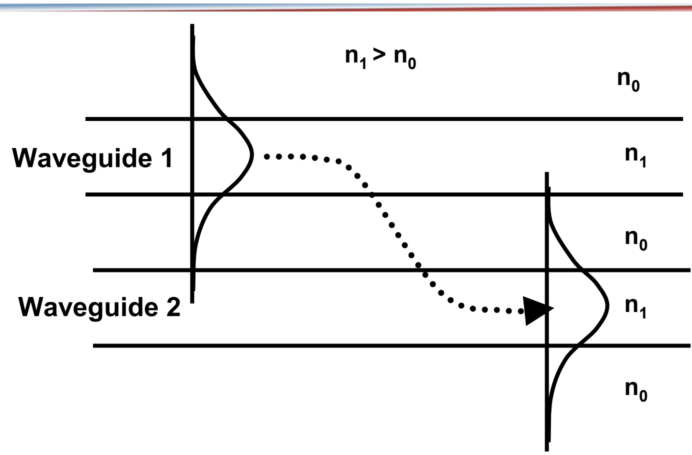
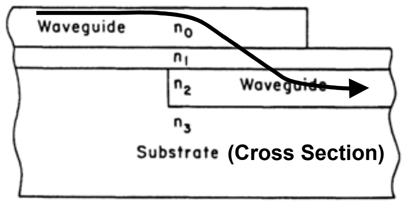
Waveguide Coupler



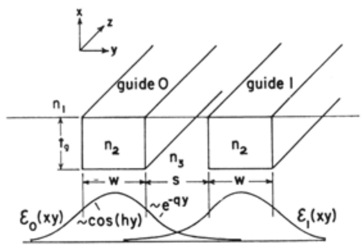
- **How to switch the power from one waveguide to other waveguides**

Two Types of Directional Couplers

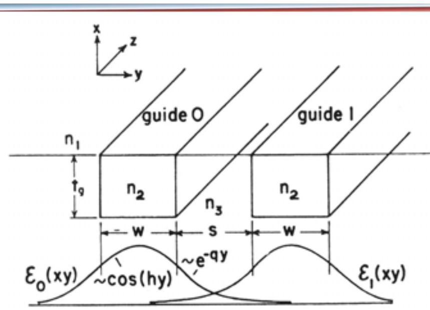
I. Planar Waveguide Coupler (Out-of-Plane)



II. Dual-Channel Waveguide Coupler (In-Plane)



Coupled Mode Theory for Directional Coupler



monochromatic waveguide modes

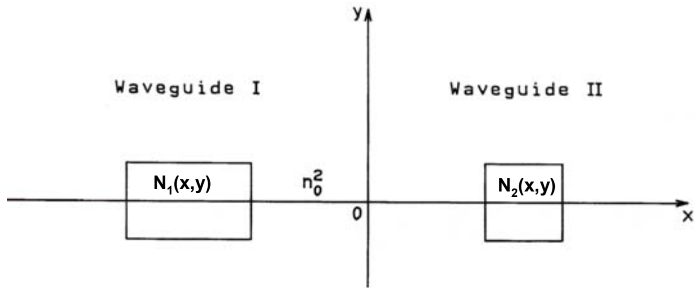
$$\vec{E}(x,y,z,t) = A\hat{L}(z)\vec{\psi}(x,y)\exp(-j\omega t)$$

Axial (Longitudinal) Part — **Transverse Part** — **Time harmonic**

$\hat{L}(z)$ **is a complex amplitude including phase term** $\exp(j\beta z)$

$\vec{\psi}(x,y)$ **represents the field distribution of a single waveguide**

Coupled Mode Theory for Directional Coupler



We define the eigen modes in each optical waveguide satisfying Maxwell's equations

$$\begin{cases} \nabla\times\widetilde{\mathbf{E}}_p = j\omega\mu_0\widetilde{\mathbf{H}}_p \\ \nabla\times\widetilde{\mathbf{H}}_p = -j\omega\varepsilon_0 N_p{}^2\widetilde{\mathbf{E}}_p \end{cases}$$

(p = 1,2) **N_p(x,y): refractive index**

The electromagnetic fields of the coupled waveguide is summation of two eigen modes

$$\begin{cases} \widetilde{\mathbf{E}} = A_1(z)\widetilde{\mathbf{E}}_1 + A_2(z)\widetilde{\mathbf{E}}_2 \\ \widetilde{\mathbf{H}} = A_1(z)\widetilde{\mathbf{H}}_1 + A_2(z)\widetilde{\mathbf{H}}_2 \end{cases}$$

A_1 and A_2 is the amplitude of optical fields

Coupled Mode Theory for Directional Coupler

The summed fields should also satisfy Maxwell's equation

$$\begin{cases} \nabla\times\widetilde{\mathbf{E}} = j\omega\mu_0\widetilde{\mathbf{H}} \\ \nabla\times\widetilde{\mathbf{H}} = -j\omega\varepsilon_0 N^2\widetilde{\mathbf{E}} \end{cases} \qquad (1)$$

N(x,y) denote the entire refractive index of coupled waveguide

The vector formula

$$\nabla\times(A\mathbf{E}) = A\nabla\times\mathbf{E} + \nabla A\times\mathbf{E} = A\nabla\times\mathbf{E} + \frac{dA}{dz}\mathbf{u}_z\times\mathbf{E} \qquad (2)$$

(the amplitude only varies at z-direction) **Perturbation!**

Combine (1) and (2)

$$\begin{cases} (\mathbf{u}_z\times\widetilde{\mathbf{E}}_1)\dfrac{dA_1}{dz} + (\mathbf{u}_z\times\widetilde{\mathbf{E}}_2)\dfrac{dA_2}{dz} = 0 \qquad (3) \\ (\mathbf{u}_z\times\widetilde{\mathbf{H}}_1)\dfrac{dA_1}{dz} + j\omega\varepsilon_0(N^2 - N_1{}^2)A_1\widetilde{\mathbf{E}}_1 + (\mathbf{u}_z\times\widetilde{\mathbf{H}}_2)\dfrac{dA_2}{dz} + j\omega\varepsilon_0(N^2 - N_2{}^2)A_2\widetilde{\mathbf{E}}_2 = 0 \qquad (4) \end{cases}$$

Coupled Mode Theory for Directional Coupler

$$\begin{cases} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\widetilde{\mathbf{E}}_1^{\,*}\cdot(4)-\widetilde{\mathbf{H}}_1^{\,*}\cdot(3)\right]dxdy=0 & (5) \\ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left[\widetilde{\mathbf{E}}_2^{\,*}\cdot(4)-\widetilde{\mathbf{H}}_2^{\,*}\cdot(3)\right]dxdy=0 & (6) \end{cases}$$

From Eq. (5)

$$\frac{dA_1}{dz}+\frac{dA_2}{dz}\cdot\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_1^{\,*}\times\widetilde{\mathbf{H}}_2+\widetilde{\mathbf{E}}_2\times\widetilde{\mathbf{H}}_1^{\,*})dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_1^{\,*}\times\widetilde{\mathbf{H}}_1+\widetilde{\mathbf{E}}_1\times\widetilde{\mathbf{H}}_1^{\,*})dxdy}$$
$$+\,jA_1\frac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N^2-N_1^{\,2})\widetilde{\mathbf{E}}_1^{\,*}\cdot\widetilde{\mathbf{E}}_1dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_1^{\,*}\times\widetilde{\mathbf{H}}_1+\widetilde{\mathbf{E}}_1\times\widetilde{\mathbf{H}}_1^{\,*})dxdy}$$
$$+\,jA_2\frac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N^2-N_2^{\,2})\widetilde{\mathbf{E}}_1^{\,*}\cdot\widetilde{\mathbf{E}}_2dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_1^{\,*}\times\widetilde{\mathbf{H}}_1+\widetilde{\mathbf{E}}_1\times\widetilde{\mathbf{H}}_1^{\,*})dxdy}=0 \qquad (7)$$

Coupled Mode Theory for Directional Coupler

From Eq. (6)

$$\frac{dA_2}{dz}+\frac{dA_1}{dz}\cdot\frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_2^{\,*}\times\widetilde{\mathbf{H}}_1+\widetilde{\mathbf{E}}_1\times\widetilde{\mathbf{H}}_2^{\,*})dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_2^{\,*}\times\widetilde{\mathbf{H}}_2+\widetilde{\mathbf{E}}_2\times\widetilde{\mathbf{H}}_2^{\,*})dxdy}$$
$$+\,jA_2\frac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N^2-N_2^{\,2})\widetilde{\mathbf{E}}_2^{\,*}\cdot\widetilde{\mathbf{E}}_2dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_2^{\,*}\times\widetilde{\mathbf{H}}_2+\widetilde{\mathbf{E}}_2\times\widetilde{\mathbf{H}}_2^{\,*})dxdy}$$
$$+\,jA_1\frac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N^2-N_1^{\,2})\widetilde{\mathbf{E}}_2^{\,*}\cdot\widetilde{\mathbf{E}}_1dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\widetilde{\mathbf{E}}_2^{\,*}\times\widetilde{\mathbf{H}}_2+\widetilde{\mathbf{E}}_2\times\widetilde{\mathbf{H}}_2^{\,*})dxdy}=0 \qquad (8)$$

Here we separate the transverse and axial dependencies of electromagnetic fields

$$\begin{cases}\widetilde{\mathbf{E}}_p=\mathbf{E}_p\exp(j\beta_pz)\\ \widetilde{\mathbf{H}}_p=\mathbf{H}_p\exp(j\beta_pz)\end{cases}\qquad \textbf{(p = 1,2)}$$

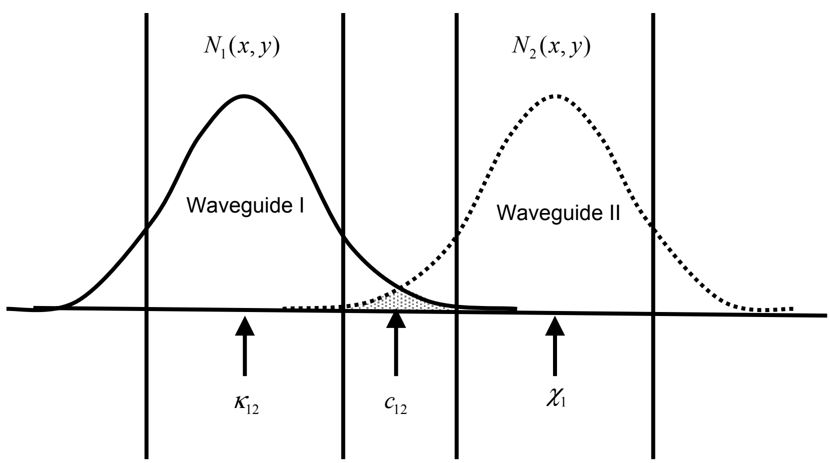
Coupled Mode Theory for Directional Coupler

$$\begin{cases} \dfrac{dA_1}{dz} + c_{12}\dfrac{dA_2}{dz}\exp[j(\beta_2-\beta_1)z] + j\chi_1 A_1 + j\kappa_{12}A_2\exp[j(\beta_2-\beta_1)z] = 0 & \text{(9) (from (7))} \\ \dfrac{dA_2}{dz} + c_{21}\dfrac{dA_1}{dz}\exp[-j(\beta_2-\beta_1)z] + j\chi_2 A_2 + j\kappa_{21}A_1\exp[-j(\beta_2-\beta_1)z] = 0 & \text{(10) (from (8))} \end{cases}$$

where

$$\begin{cases} \kappa_{pq} = \dfrac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N^2-N_q^{\,2})\mathbf{E}_p^{\,*}\cdot\mathbf{E}_q\,dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\mathbf{E}_p^{\,*}\times\mathbf{H}_p+\mathbf{E}_p\times\mathbf{H}_p^{\,*})dxdy} & \textbf{(q perturb p)} \\ c_{pq} = \dfrac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\mathbf{E}_p^{\,*}\times\mathbf{H}_q+\mathbf{E}_q\times\mathbf{H}_p^{\,*})dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\mathbf{E}_p^{\,*}\times\mathbf{H}_p+\mathbf{E}_p\times\mathbf{H}_p^{\,*})dxdy} & \textbf{(q perturb p)} \\ \chi_p = \dfrac{\omega\varepsilon_0\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N^2-N_p^{\,2})\mathbf{E}_p^{\,*}\cdot\mathbf{E}_p\,dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\mathbf{E}_p^{\,*}\times\mathbf{H}_p+\mathbf{E}_p\times\mathbf{H}_p^{\,*})dxdy} & \end{cases}$$

Coupled Mode Theory for Directional Coupler



- **χ is much smaller than κ**

Coupled Mode Theory for Directional Coupler

(a). Consider χ_p

$$\chi_p = \chi_p^* \longrightarrow \chi_p \text{ is real number}$$

(b). Consider c_{pq}

Recall

$$\vec{E} = \frac{1}{2}\left(\vec{\mathbf{E}}\exp(-j\omega t) + \vec{\mathbf{E}}^*\exp(j\omega t)\right)$$

$$\vec{H} = \frac{1}{2}\left(\vec{\mathbf{H}}\exp(-j\omega t) + \vec{\mathbf{H}}^*\exp(j\omega t)\right)$$

$$P_p = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{2}\mathrm{Re}(\mathbf{E}_p \times \mathbf{H}_p^*)\cdot \mathbf{u}_z dxdy$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\mathbf{E}_p^* \times \mathbf{H}_p + \mathbf{E}_p \times \mathbf{H}_p^*)\cdot \mathbf{u}_z dxdy = 4P_p = 4$$

If $\mathbf{E}_p$, $\mathbf{H}_p$ is the eigen mode

$$c_{pq} = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\mathbf{E}_p^* \times \mathbf{H}_q + \mathbf{E}_q \times \mathbf{H}_p^*)dxdy}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathbf{u}_z\cdot(\mathbf{E}_p^* \times \mathbf{H}_p + \mathbf{E}_p \times \mathbf{H}_p^*)dxdy} \longrightarrow c_{21} = c_{12}^*$$

Coupled Mode Theory for Directional Coupler

(c). Consider κ_{pq}

Suppose there is no coupling loss, the total power should be constant

$$P = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{1}{2}\mathrm{Re}(\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}^*)\cdot \mathbf{u}_z dxdy \quad \text{and} \quad \frac{dP}{dz} = 0$$

$$jA_1^* A_2(\kappa_{21}^* - \kappa_{12} - 2\delta c_{12})\exp(-j2\delta z) - jA_1 A_2^*(\kappa_{21} - \kappa_{12}^* - 2\delta c_{12}^*)\exp(j2\delta z) = 0$$

for every z

where $\delta = \dfrac{(\beta_2 - \beta_1)}{2}$

$$\kappa_{21} = \kappa_{12}^* + 2\delta c_{12}^*$$

- The coupling coefficients are complicate conjugated when
  - Two modes are in the same waveguide or
  - Two waveguides are phase matching

$$\longrightarrow \delta = \frac{\beta_2 - \beta_1}{2} = 0$$

Coupled Mode Theory for Directional Coupler

General expression of coupled mode theory

$$\begin{cases} \dfrac{dA_1}{dz} = j\kappa_a A_2 \exp(j2\delta z) + j\gamma_a A_1 & [(9)-(10)\times c_{12}\exp(j2\delta z)=0] \\ \dfrac{dA_2}{dz} = j\kappa_b A_1 \exp(-j2\delta z) + j\gamma_b A_2 & [(10)-(9)\times c_{21}\exp(-j2\delta z)=0] \end{cases}$$

where $\begin{cases} \kappa_a = \dfrac{\kappa_{12} - c_{12}\chi_2}{1-|c_{12}|^2} \\ \kappa_b = \dfrac{\kappa_{21} - c_{12}^{*}\chi_1}{1-|c_{12}|^2} \end{cases}$ **and** $\begin{cases} \gamma_a = \dfrac{\kappa_{21}c_{12} - \chi_1}{1-|c_{12}|^2} \\ \gamma_b = \dfrac{\kappa_{12}c_{12}^{*} - \chi_2}{1-|c_{12}|^2} \end{cases}$

Coupled Mode Theory for Directional Coupler

In most cases, we suppose two waveguides are sufficient separated

$\chi_{p,q} \sim 0$ **and** $C_{pq} \ll 1$

Then

$\boxed{\kappa_{12} = \kappa_{21}}$ **and** $\boxed{\gamma_a = \gamma_b = 0}$

The general coupled mode theory is simplified by

$$\begin{cases} \dfrac{dA_1}{dz} = j\kappa_{12} A_2 \exp[j(\beta_2 - \beta_1)z] \\ \dfrac{dA_2}{dz} = j\kappa_{21} A_1 \exp[-j(\beta_2 - \beta_1)z] \end{cases}$$

Coupled Mode Theory for Directional Coupler

The coupling between mode is given by the coupled mode equations for the amplitudes of the two modes (guide 1 and guide 2)

$$\begin{cases} \dfrac{d\hat{A}_1(z)}{dz} = j\beta_1 \hat{A}_1(z) + j\kappa_{12}\hat{A}_2(z) \\ \dfrac{d\hat{A}_2(z)}{dz} = j\beta_2 \hat{A}_2(z) + j\kappa_{21}\hat{A}_1(z) \end{cases}$$

$$\hat{A}_1 = A_1 \exp(j\beta_1 z)$$
$$\hat{A}_2 = A_2 \exp(j\beta_2 z)$$

β_1 : the propagation constant of guide 1
β_2 : the propagation constant of guide 2

κ_{12} : the coupling coefficient from 2 to 1
κ_{21} : the coupling coefficient from 1 to 2

In general, κ_{12} and κ_{21} are not equal. We suppose the coupling coefficients are approximated and real

$$\kappa_{21} = \kappa_{12} = \kappa$$

Solutions of Directional Coupler

$$\begin{cases} \hat{A}_1(z) = \left\{ \left[\cos(gz) + j\dfrac{\delta}{g}\sin(gz) \right] \hat{A}_1(0) - j\dfrac{\kappa}{g}\sin(gz)\hat{A}_2(0) \right\} \exp(j(\beta_1+\delta)z) \\ \hat{A}_2(z) = \left\{ -j\dfrac{\kappa}{g}\sin(gz)\hat{A}_1(0) + \left[\cos(gz) + j\dfrac{\delta}{g}\sin(gz) \right] \hat{A}_2(0) \right\} \exp(j(\beta_2-\delta)z) \end{cases}$$

Where $g^2 \equiv \kappa^2 + \delta^2$ and $\delta \equiv \dfrac{\beta_2 - \beta_1}{2}$

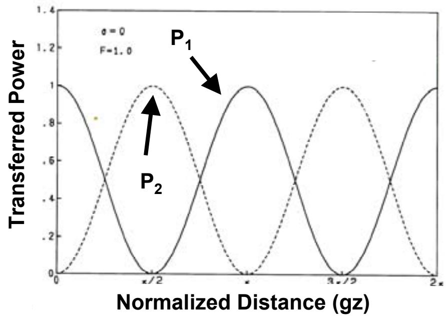
If the initial condition $A_1(0) = 1$ and $A_2(0) = 0$, the solution is

$$\begin{cases} \hat{A}_1(z) = \left[\cos(gz) - j\dfrac{\delta}{g}\sin(gz) \right] \exp(j(\beta_1+\delta)z) \\ \hat{A}_2(z) = j\dfrac{\kappa}{g}\sin(gz)\exp(j(\beta_2-\delta)z) \end{cases}$$

- The propagation constant of each eigen mode becomes the average of two individual propagation constants in the coupling region

Transferred Power

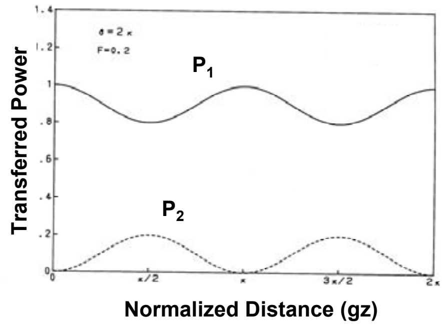
Phase Matching



$\delta = 0$

$$\begin{cases} P_1(z) = A_1(z)\cdot A_1(z)^* = \cos^2(gz) + \delta^2 \dfrac{\sin^2(gz)}{g^2} \\ P_2(z) = A_2(z)\cdot A_2(z)^* = \dfrac{\kappa^2}{g^2}\sin^2(gz) \end{cases}$$

Phase Mismatching



$\delta = 2\kappa$

Where $g^2 \equiv \kappa^2 + \delta^2$ **and** $\delta \equiv \dfrac{\beta_2 - \beta_1}{2}$

- **With a phase difference δ, the power transfer is incomplete**

Directional Coupler with Phase Matching

Suppose the two waveguides are identical

$$\beta_1 = \beta_2 = \beta$$

and

$$\kappa_{21} = \kappa_{12} = \kappa \quad \leftarrow \text{Real}$$

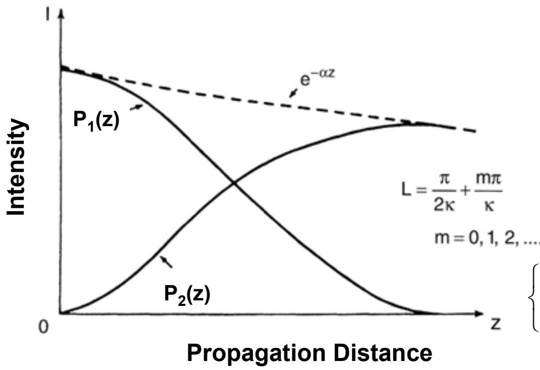
The coupled mode equation becomes

$$\begin{cases} \dfrac{d\hat{A}_1(z)}{dz} = j\beta\hat{A}_1(z) + j\kappa\hat{A}_2(z) \\ \dfrac{d\hat{A}_2(z)}{dz} = j\beta\hat{A}_2(z) + j\kappa\hat{A}_1(z) \end{cases}$$

If the initial condition $A_1(0) = 1$ and $A_2(0) = 0$, the solution is

$$\begin{cases} \hat{A}_1(z) = \cos(\kappa z)\exp(j\beta z) \\ \hat{A}_2(z) = j\sin(\kappa z)\exp(j\beta z) \end{cases}$$

Directional Coupler with Phase Matching and Loss



If we consider the loss α

$$\begin{cases} \hat{A}_1(z) = \cos(\kappa z)\exp(j\beta z)\exp(-\frac{\alpha}{2}z) \\ \hat{A}_2(z) = j\sin(\kappa z)\exp(j\beta z)\exp(-\frac{\alpha}{2}z) \end{cases}$$

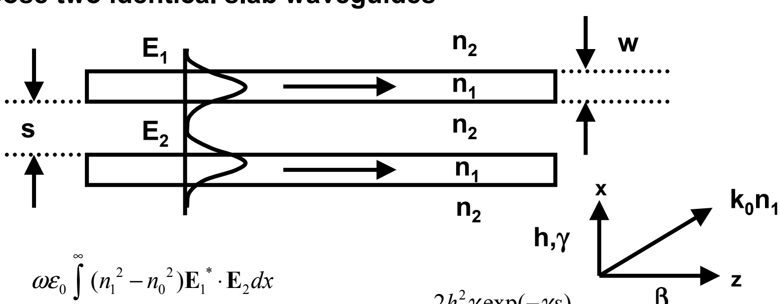
$$\begin{cases} P_1(z) = A_1(z)A_1^*(z) = \cos^2(\kappa z)\exp(-\alpha z) \\ P_2(z) = A_2(z)A_2^*(z) = \sin^2(\kappa z)\exp(-\alpha z) \end{cases}$$

The optical power is completely transferred as propagation length (L)

$$L = \frac{(2m+1)\pi}{2\kappa} \quad ,m = 0,1,2\ldots$$

What is the coupling coefficient?

Suppose two identical slab waveguides



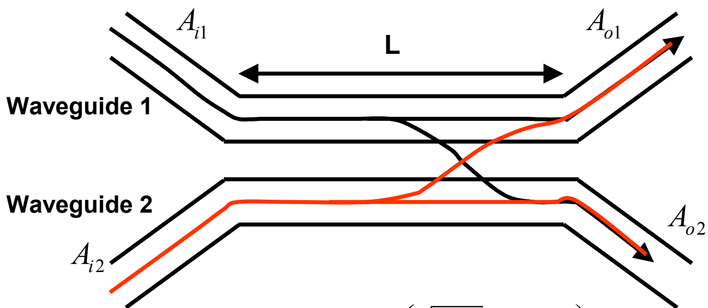
$$\kappa = \frac{\omega\varepsilon_0 \int_{-\infty}^{\infty} (n_1^2 - n_0^2)\mathbf{E}_1^* \cdot \mathbf{E}_2 dx}{\int_{-\infty}^{\infty} \hat{\mathbf{u}}_z \cdot (\mathbf{E}_1^* \times \mathbf{H}_1 + \mathbf{E}_1 \times \mathbf{H}_1^*) dx} \longrightarrow \kappa = \frac{2h^2\gamma\exp(-\gamma s)}{\beta d_{TE}(h^2+\gamma^2)}$$

$$\kappa \approx \frac{\gamma h^2}{\beta d_{TM}(h^2+\gamma^2)} \cdot \frac{2\beta^2+\gamma^2-h^2}{k_2^2}\exp(-\gamma s)$$

- The coupling coefficient is an exponential function of gap s.

Transfer Matrix Expression

Suppose a lossless coupler



recall

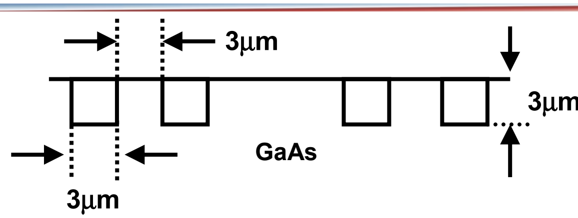
$$\begin{cases} \hat{A}_1(z) = \cos(\kappa z)\exp(j\beta z) \\ \hat{A}_2(z) = j\sin(\kappa z)\exp(j\beta z) \end{cases}$$

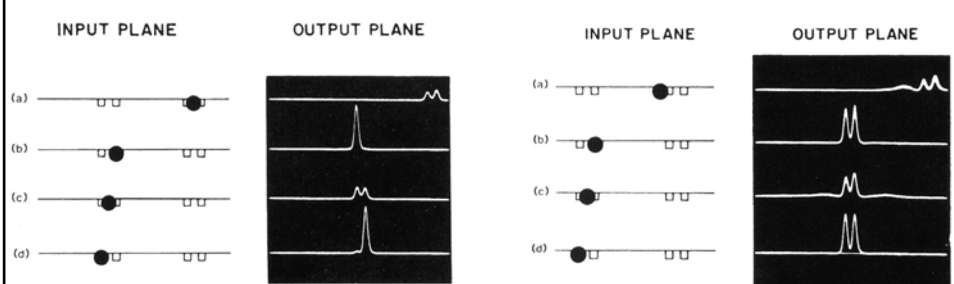
$$\begin{pmatrix} A_{o1} \\ A_{o2} \end{pmatrix} = \begin{pmatrix} \sqrt{1-c^2} & jc \\ jc & \sqrt{1-c^2} \end{pmatrix} \begin{pmatrix} A_{i1} \\ A_{i2} \end{pmatrix}$$

$A_1(0)=1$, $A_2(0)=0$ **where** $c = \sin^2(\kappa L)$ c^2 **: power coupling ratio**

Varying L affects the power coupling ratio

Experimental Measurement





L = 2.1 mm (100% coupler) **L = 1 mm (3dB coupler)**

Summary of Waveguide Coupling

When you design a waveguide coupler, you have to

- **Consider the coupling coefficient (κ)**
- **Consider the phase matching (Δβ)**
- **Consider the coupling length (L)**

$$\begin{cases} P_1(z) = \cos^2(gz)\exp(-\alpha z) + \delta^2 \dfrac{\sin^2(gz)}{g^2}\exp(-\alpha z) \\ P_2(z) = \dfrac{\kappa^2}{g^2}\sin^2(gz)\exp(-\alpha z) \end{cases}$$

$$g^2 \equiv \kappa^2 + \delta^2$$

Supermodes

Actually, the power oscillating between two waveguides can be thought as the beating between two supermodes.

For example, suppose the two waveguides are identical,

$$\begin{cases} \hat{A}_1(z) = A_0\cos(\kappa z)\exp(j\beta z) \\ \hat{A}_2(z) = A_0 j\sin(\kappa z)\exp(j\beta z) \end{cases} \quad \textbf{if} \quad \begin{cases} \hat{A}_1(0) = A_0 \\ \hat{A}_2(0) = 0 \end{cases}$$

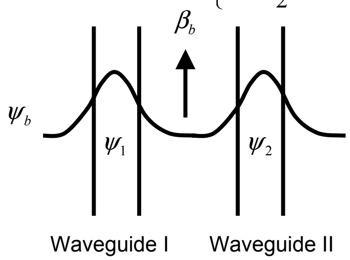
$$\begin{aligned} E(z) &= \hat{A}_1(z)\psi_1(x,y) + \hat{A}_2(z)\psi_2(x,y) \\ &= A_0\left[\psi_1(x,y)\cos(\kappa z)\exp(j\beta z) + j\psi_2(x,y)\sin(\kappa z)\exp(j\beta z)\right] \\ &= A_0\left\{\frac{1}{2}\psi_1\left[\exp(j\kappa z)+\exp(-j\kappa z)\right]\exp(j\beta z) + \frac{1}{2}\psi_2\left[\exp(j\kappa z)-\exp(-j\kappa z)\right]\exp(j\beta z)\right\} \\ &= A_0\left\{\underbrace{\frac{1}{2}(\psi_1-\psi_2)\exp\left[j(\beta-\kappa)z\right]}_{E_a(z)} + \underbrace{\frac{1}{2}(\psi_1+\psi_2)\exp\left[j(\beta+\kappa)z\right]}_{E_b(z)}\right\} \end{aligned}$$

Supermodes

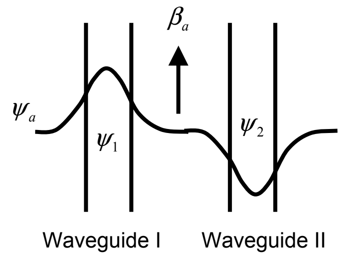
$$E(z) = A_0 \left\{ \frac{1}{2}(\psi_1 - \psi_2)\exp\left[j(\beta-\kappa)z\right] + \frac{1}{2}(\psi_1 + \psi_2)\exp\left[j(\beta+\kappa)z\right] \right\}$$

$$= A_0\psi_a \exp(j\beta_a z) + A_0\psi_b \exp(j\beta_b z) = A_0 E_a(z) + A_0 E_b(z)$$

Where $\begin{cases} \psi_a = \frac{1}{2}(\psi_1 - \psi_2) \\ \psi_b = \frac{1}{2}(\psi_1 + \psi_2) \end{cases}$ **and** $\begin{cases} \beta_a = \beta - \kappa \\ \beta_b = \beta + \kappa \end{cases}$

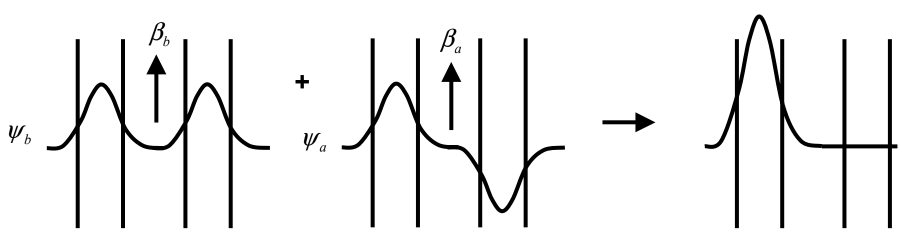


(Symmetric)

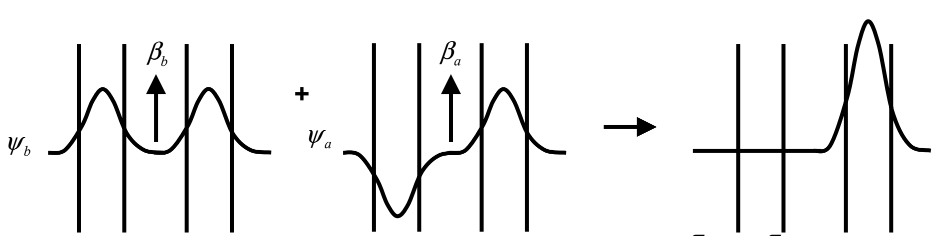


(Antisymmetric)

Supermodes



$$(\beta_b - \beta_a)L = 2m\pi,\ m = 0,1,2,\ldots$$

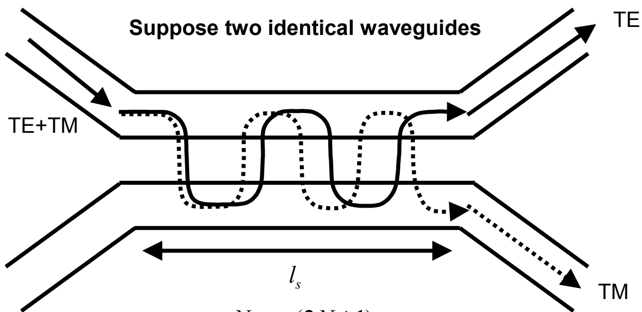


$$L_\pi = \frac{\pi}{\beta_b - \beta_a} = \frac{\pi}{2\kappa}$$

$$(\beta_b - \beta_a)L = 2(m+1)\pi,\ m = 0,1,2,\ldots$$

Polarization Dependent

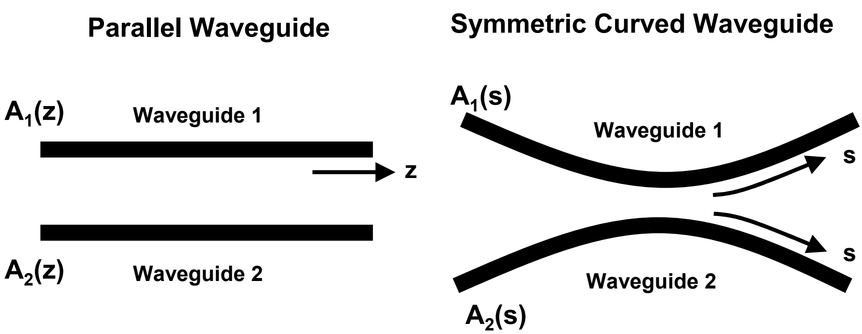
Because the coupling coefficients of TE modes and TM modes are different, we can design a polarization splitter.



$$l_s = \frac{N\pi}{\kappa_{TE}} = \frac{(2N \pm 1)\pi}{2\kappa_{TM}}$$

where N **:integer and** κ_{TE} κ_{TM} **are coupling coefficient for TE and TM**

Symmetric Curved Waveguide Coupling

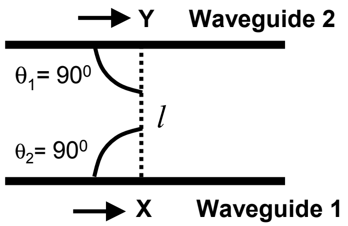


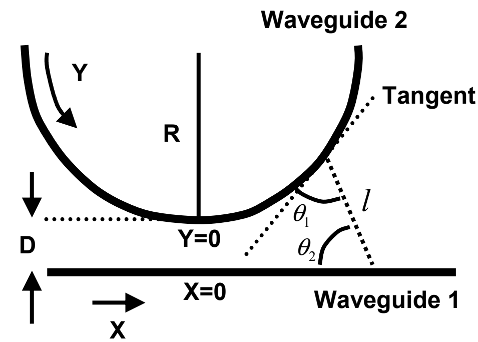
$$\begin{cases} d\hat{A}_1(z) = j\beta_1 \hat{A}_1(z)dz + j\kappa_{12}\hat{A}_2(z)dz \\ d\hat{A}_2(z) = j\beta_2 \hat{A}_2(z)dz + j\kappa_{21}\hat{A}_1(z)dz \end{cases}$$

$$\begin{cases} d\hat{A}_1(s) = j\beta_1 \hat{A}_1(s)ds + j\kappa_{12}(s)\hat{A}_2(s)ds \\ d\hat{A}_2(s) = j\beta_2 \hat{A}_2(s)ds + j\kappa_{21}(s)\hat{A}_1(s)ds \end{cases}$$

- **Line coordinate vs. curve coordinate**
- **The amplitude increment can be modelled in point-to-point correspondence**

Asymmetric Curved Waveguide Coupling





X: Axial coordinate for waveguide 2

Y: Axial coordinate for waveguide 1

$X = Y$

If $\frac{X}{2R+D} = \tan(\frac{Y}{2R})$ then $\theta_1 = \theta_2$

- **If we can define a function to link X,Y curve coordinates such that $\theta_1 = \theta_2$, the asymmetric curved waveguide coupling can be treated as a symmetric curved waveguide coupling.**

Asymmetric Curved Waveguide Coupling

The asymmetric curved waveguide coupling can be expressed by coupled mode theory.

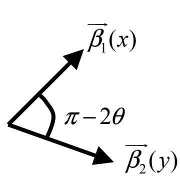
$$\begin{cases} d\hat{A}_1(x) = j\beta_1(x)\hat{A}_1(x)dx + j\tilde{\kappa}_2(y)\hat{A}_2(y)dy \\ d\hat{A}_2(y) = j\beta_2(y)\hat{A}_2(y)dy + j\tilde{\kappa}_1(x)\hat{A}_1(x)dx \end{cases}$$

where $\frac{X}{2R+D} = \tan(\frac{Y}{2R})$ **and**

$$\begin{cases} \tilde{\kappa}_1 = \kappa_0 \cdot \cos(\pi - 2\theta) \cdot \left(\frac{dy}{dx}\right)^{1/2} \\ \tilde{\kappa}_2 = \kappa_0 \cdot \cos(\pi - 2\theta) \cdot \left(\frac{dx}{dy}\right)^{1/2} \end{cases}$$

Coupling coefficient due to gap (κ_0)

The angle of $\vec{\beta_1}(x), \vec{\beta_2}(y)$ ($\pi - 2\theta$)



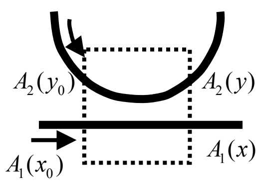
Asymmetric Curved Waveguide Coupling

Consider the coupler is lossless; that is, the coupling power is conservative

$$\tilde{\kappa}_1 dx = \tilde{\kappa}_2 dy$$

The coupled mode theory becomes

$$\begin{cases} d\hat{A}_1(x) = j\beta_1(x)\hat{A}_1(x)dx + j\tilde{\kappa}_1(x)\hat{A}_2(y)dx \\ d\hat{A}_2(y) = j\beta_2(y)\hat{A}_2(y)dy + j\tilde{\kappa}_2(y)\hat{A}_1(x)dy \end{cases} \rightarrow \begin{cases} \dfrac{d\hat{A}_1(x)}{dx} = j\beta_1(x)\hat{A}_1(x) + j\tilde{\kappa}_1(x)\hat{A}_2(y) \\ \dfrac{d\hat{A}_2(y)}{dy} = j\beta_2(y)\hat{A}_2(y) + j\tilde{\kappa}_2(y)\hat{A}_1(x) \end{cases}$$



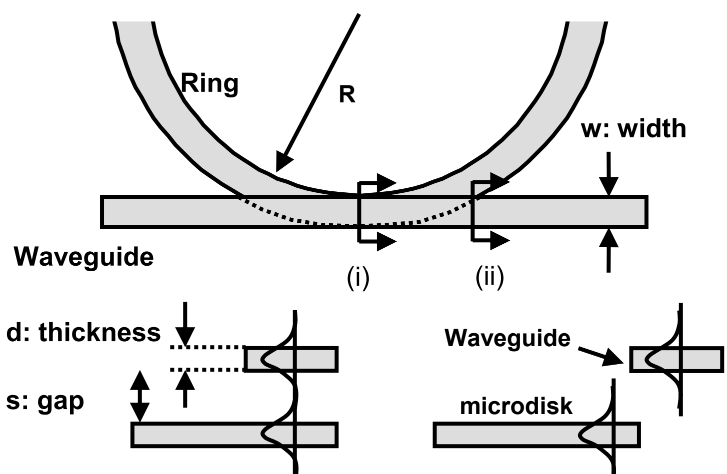
$$\begin{bmatrix} A_1(x) \\ A_2(y) \end{bmatrix} = \begin{bmatrix} C_{11}(x,y) & C_{12}(x,y) \\ C_{21}(x,y) & C_{22}(x,y) \end{bmatrix} \cdot \begin{bmatrix} A_1(x_0) \\ A_2(y_0) \end{bmatrix}$$

$$|C_{11}(x,y)|^2 + |C_{21}(x,y)|^2 = |C_{12}(x,y)|^2 + |C_{22}(x,y)|^2 = 1$$

$$C_{11}(x,y)\cdot C_{12}(x,y)^* + C_{21}(x,y)\cdot C_{22}(x,y)^* = 0$$

Due to power conservation

Example of Asymmetric Curved Waveguide Coupling (Vertical Coupling)

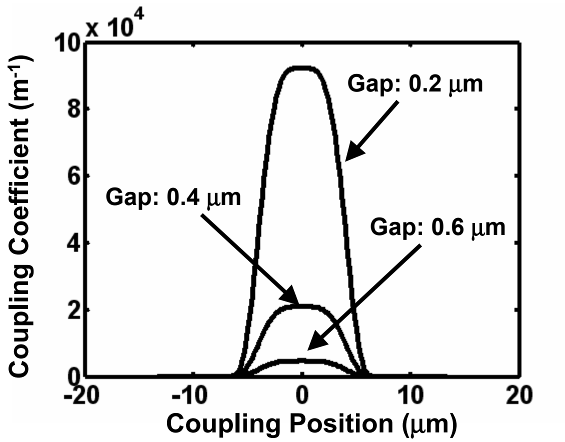


(i) Tangent Position (cross section)

(ii) Deviated Position (cross section)

Example of Asymmetric Curved Waveguide Coupling (Vertical Coupling)

For a silicon curved waveguide with 20-um radius of curvature and 0.8 um width, coupling coefficient (k_0) is analyzed as follow:



The effective coupling length is only from -5μm to 5μm

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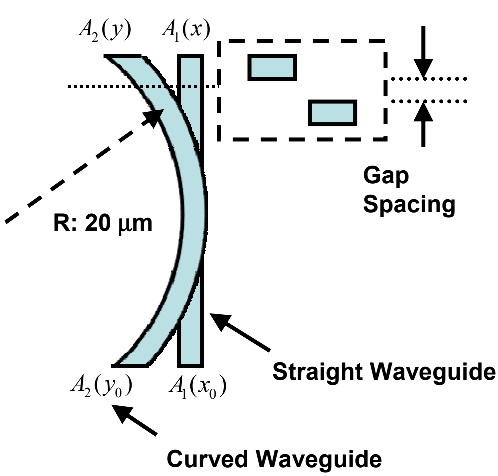
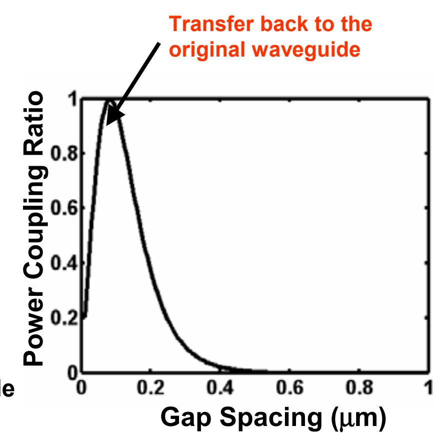
Example of Asymmetric Curved Waveguide Coupling (Vertical Coupling)

Because the coupling coefficient is dependent on the gap spacing, the power coupling ratio is also a function of gap spacing.





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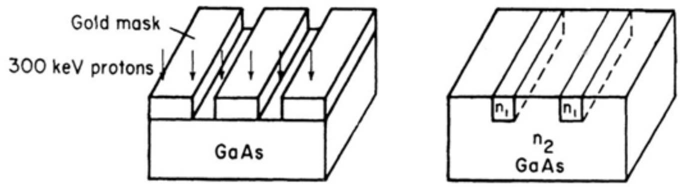
Coupler Fabrication



Gold