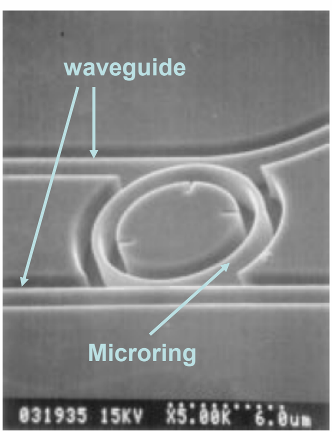
Integrated Optical Resonators II

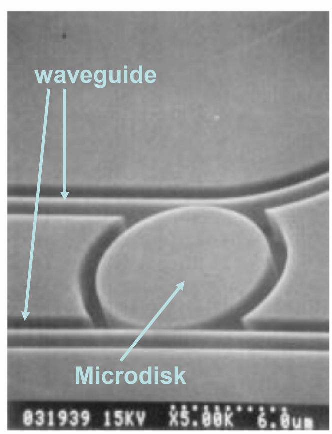
Class: Integrated Photonic Devices
Time: Fri. 8:00am ~ 11:00am.
Classroom: 資電206
Lecturer: Prof. 李明昌(Ming-Chang Lee)

SEM of Fabricated Microrings and Microdisks

Microring



Microdisk



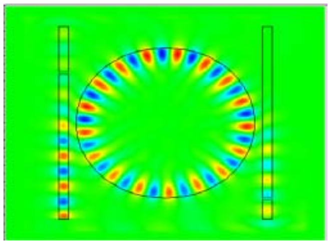
Hagness '97

Fabricated on GaAs

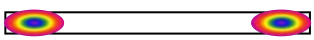
Comparison between Microrings and Microdisks

Microdisk

Top View

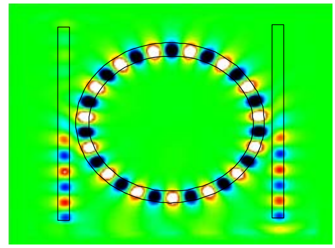


Side View

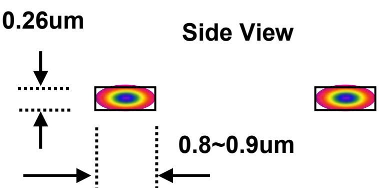


Microring

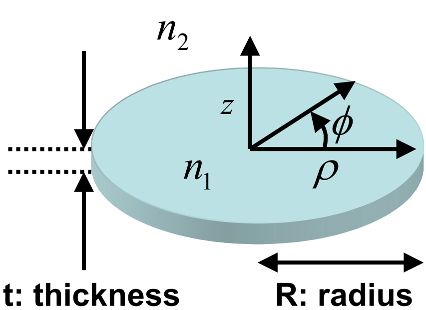
Top View



Side View



Analytic Approximation for the modes of a microdisk



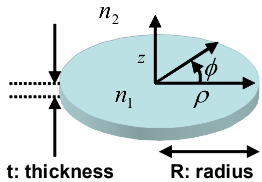
Recall Maxwell's equations for a linear, non-dispersive medium

$$\begin{cases} \nabla \times \mathbf{E}(\mathbf{r},t) = -\dfrac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \\ \nabla \times \mathbf{H}(\mathbf{r},t) = \dfrac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} \\ \nabla \cdot \mathbf{D}(\mathbf{r},t) = 0 \\ \nabla \cdot \mathbf{B}(\mathbf{r},t) = 0 \end{cases}$$

The wave equation is given by

$$\begin{cases} \nabla^2 \mathbf{E} - \dfrac{n^2(\mathbf{r})}{c^2}\dfrac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \\ \nabla^2 \mathbf{H} - \dfrac{n^2(\mathbf{r})}{c^2}\dfrac{\partial^2 \mathbf{H}}{\partial t^2} = 0 \end{cases}$$

Analytic Approximation for the modes of a microdisk



The solutions for the wave equation is given by

$$\begin{cases} \mathbf{E}(\mathbf{r},t) = \mathbf{E}(\mathbf{r})\exp(-j\omega t) \\ \mathbf{H}(\mathbf{r},t) = \mathbf{H}(\mathbf{r})\exp(-j\omega t) \end{cases}$$

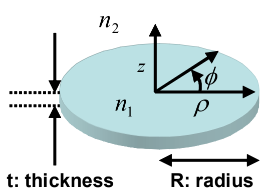
which satisfy the wave equation in the cylinder coordinate

$$\begin{cases} \left\{\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2} + \left(\frac{\omega}{c}\right)^2 n^2(\mathbf{r})\right\}\mathbf{E}(\mathbf{r}) = 0 \\ \left\{\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2} + \frac{\partial^2}{\partial z^2} + \left(\frac{\omega}{c}\right)^2 n^2(\mathbf{r})\right\}\mathbf{H}(\mathbf{r}) = 0 \end{cases}$$

The polarization of the optical mode is either perpendicular to the disk (TM) or parallel to the disk (TE). In each case, we choose the z-component

$$\begin{cases} E_z = W(\rho,\phi)Z(z) & \text{For TM polarization} \\ H_z = W(\rho,\phi)Z(z) & \text{For TE polarization} \end{cases}$$

Analytic Approximation for the modes of a microdisk



The wave equation becomes

$$\frac{1}{W}\left(\frac{\partial^2 W}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial W}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2 W}{\partial\phi^2}\right) + \frac{1}{Z}\frac{d^2 Z}{dz^2} + k_0^2 n^2(\mathbf{r}) = 0$$

where $k_0 = \frac{\omega}{c}$

By effective index method, the wave equation can be decomposed into

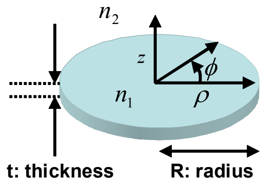
$$\left(\frac{\partial^2 W}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial W}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2 W}{\partial\phi^2}\right) + k_0^2 \bar{n}^2(\rho)W = 0 \qquad \text{(a)}$$

$$\frac{d^2 Z}{dz^2} + k_0^2(n^2(z) - \bar{n}^2)Z = 0 \qquad \text{(b)}$$

The equation (b) is similar to solve a slab waveguide. Therefore we can obtain an effective index

$$\bar{n}$$

Analytic Approximation for the modes of a microdisk



The equation (a) can be further decomposed into

$$\frac{\partial^2\Psi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\Psi}{\partial\rho}+\left(k_0{}^2\bar{n}^{2}(\rho)-\frac{m^2}{\rho^2}\right)\Psi=0 \quad \text{(c)}$$

$$\frac{d^2\Omega}{dz^2}+m^2\Omega=0 \quad \text{(d)}$$

where $W(\rho,\phi)=\Psi(\rho)\Omega(\phi)$

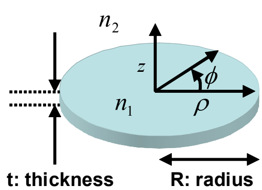
The solution of Equation (d) should be

$\Omega(\phi)\sim\exp(jm\phi)$ **m :integer (resonant mode)**

The solution of Equation (c) should be

$$\Psi(\rho)\sim\begin{cases}J_m(k_0\bar{n}\rho) & ,\rho\le R\\ J_m(k_0\bar{n}R)\exp[-\alpha(\rho-R)] & ,\rho>R\end{cases}$$

Analytic Approximation for the modes of a microdisk



Put the Ψ into Equation (c) at ρ = R to satisfy boundary condition.

$$k_0\bar{n}J_{m+1}(k_0\bar{n}R)=\left(\frac{m}{R}+\eta\alpha\right)J_m(k_0\bar{n}R)$$

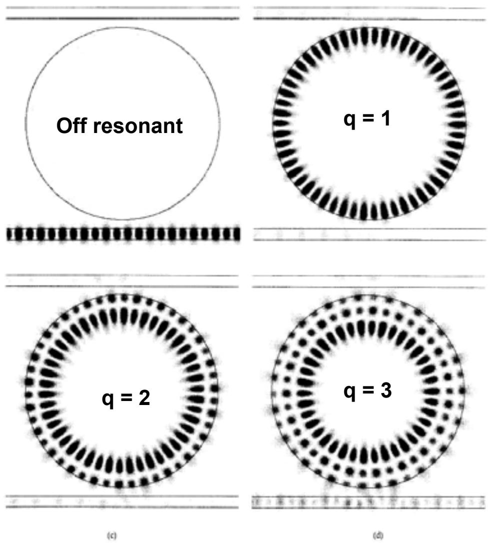
where $\eta=\begin{cases}\dfrac{\bar{n}^{2}}{n_0{}^2} & ,TE\\ 1 & ,TM\end{cases}$

Find the solution k_0, $k_0=\dfrac{2\pi}{\lambda_0}$ → **Find** λ_0 (Resonant Wavelength)

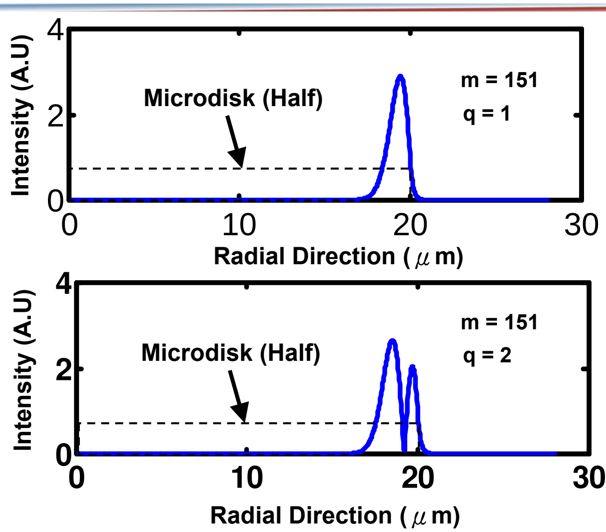
- **For each m, the solution is not unique. There are Q solutions (resonant wavelength). Sometimes these modes are also called Whispering Gallery Modes**

$$W_{(m,q)}(\rho,\phi),k_0{}^{(m,q)}$$

High Order Radial Modes



Mode Profile of Microdisk Resonator



Suppose for a silicon microdisk with radius of 20 μm

The thickness of microdisk is 0.25 μm

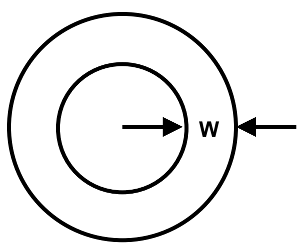
- The mode profile is not symmetric at the peak intensity

Calculated Resonant Wavelength of Microdisk

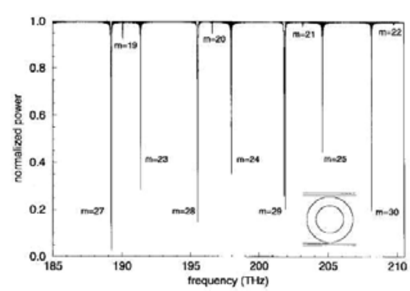
Azimuthal Mode No.	Wavelength (nm)	Propagation Constant (μm^{-1})	Mode Width (μm)	Effective Index
m = 148	1563.68	7.67	1.408	1.9088
m = 149	1559.65	7.72	1.404	1.9164
m = 150	1555.67	7.77	1.398	1.9242
m = 151	1551.72	7.82	1.394	1.9319
m = 152	1547.75	7.87	1.388	1.9394
m = 153	1543.83	7.92	1.384	1.9470
m = 154	1539.95	7.97	1.380	1.9547

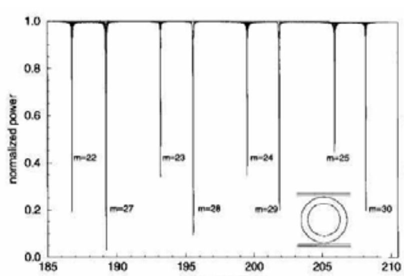
q = 1

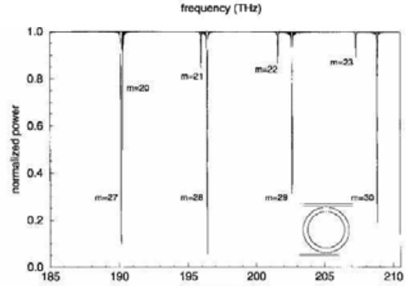
From Microdisks to Microrings



The variation of ring width

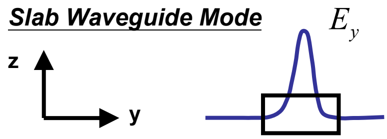






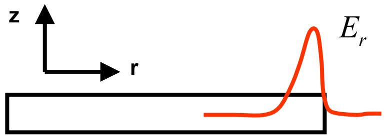
Effective Propagation Constant of Microdisk

Slab Waveguide Mode



$$P_{waveguide} = \frac{\beta}{2k_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\int_{-\infty}^{\infty} E_y^{\,2}(y)dy \qquad \text{(a)}$$

Disk Whispering Gallery Mode

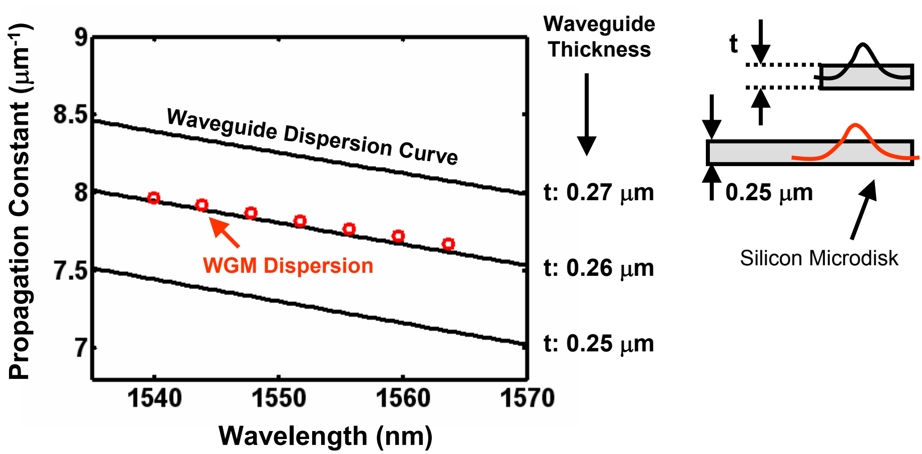


$$P_{disk} = \frac{m}{2k_0}\sqrt{\frac{\varepsilon_0}{\mu_0}}\int_{0}^{\infty} \frac{E_r^{\,2}(r)}{r}dr \qquad \text{(b)}$$

If the whispering gallery mode approximates a waveguide mode (r ~ y), then (a) should be equal to (b)

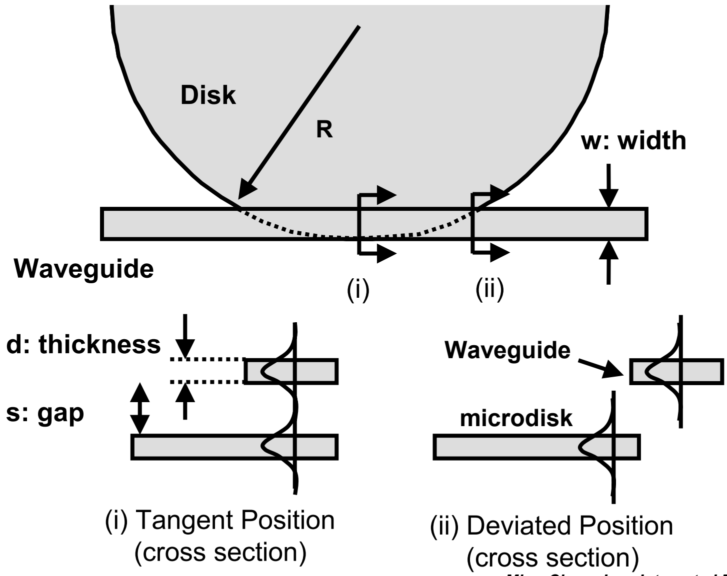
$$\beta_{disk} \simeq \frac{m\int_0^{\infty}\frac{E_r(r)^2}{r}dr}{\int_0^{\infty}E_r(r)^2 dr}$$

Phase Matching Condition for a Waveguide-coupled Microdisk Resonator



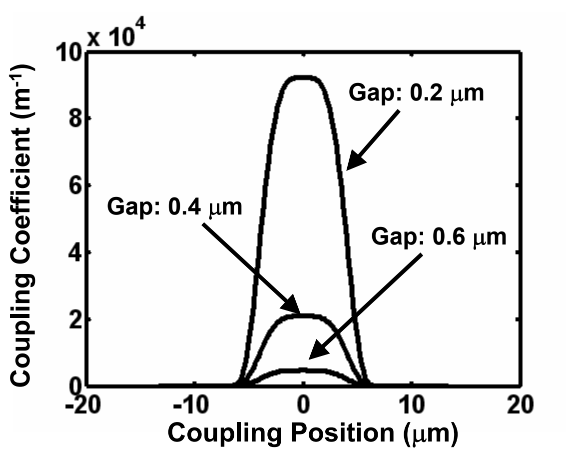
- **For phase matching, the dimensions of microdisks and waveguides may not equal**

Waveguide-Microdisk Coupling (Vertical)



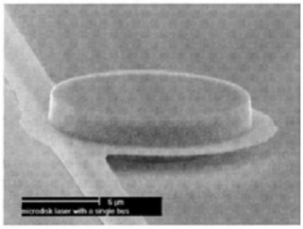
Coupling Coefficient

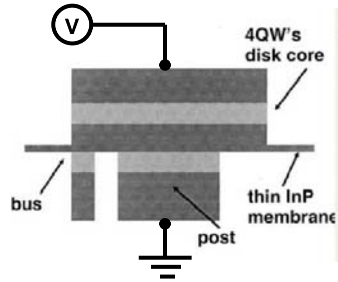
For a silicon microdisk with a 20-um radius coupled with a waveguide, the coupling coefficient is analyzed as follow:

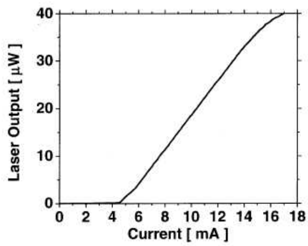


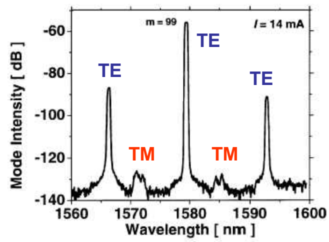
The effective coupling length is only from -5μm to 5μm

Microdisk Lasers Vertically Coupled to Output Waveguides



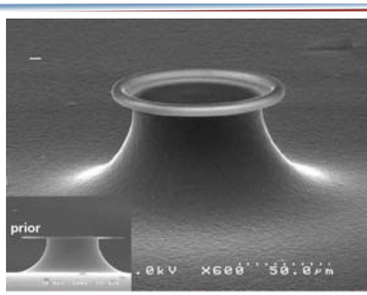


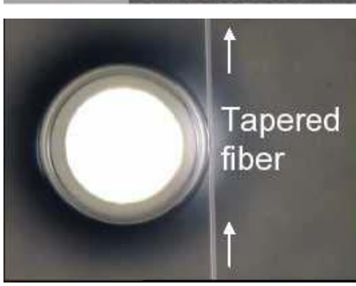


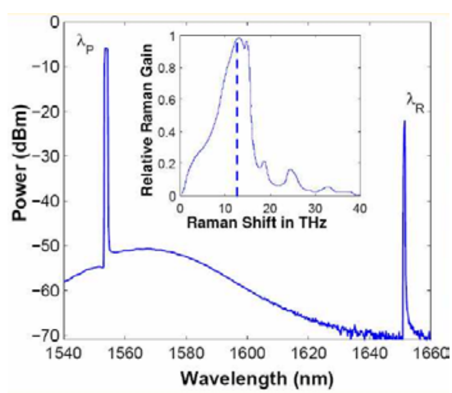


S.J. Choi, IEEE PTL, 2003

Ultra-Low Threshold Raman Laser



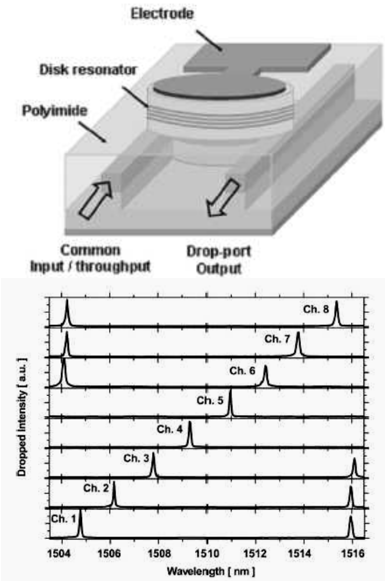


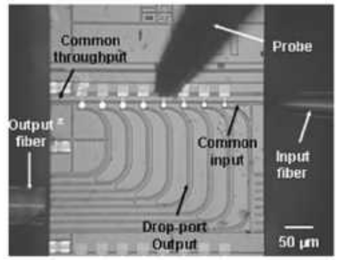


- **Threshold Power < 100mW**
- **Quantum Efficiency: 45%**

K. C. Vahala, Nature, 2004

Channel Demultiplexing in WDM

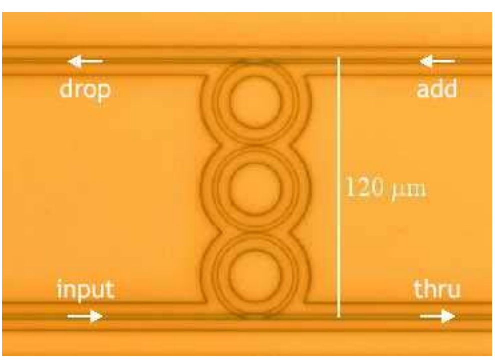




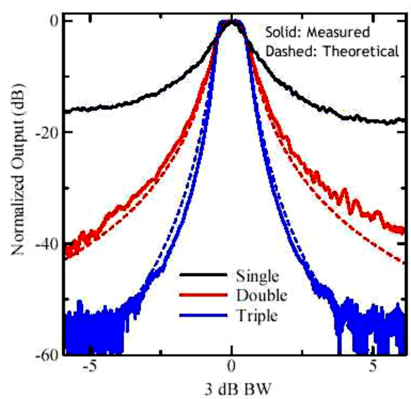
Seung June Choi, PTL, 2004

- **Microdisk resonators are made of InGaAsP (active materials)**
- **Free-carrier injection was used for shifting the resonant wavelengths**

High-Order Optical Filters

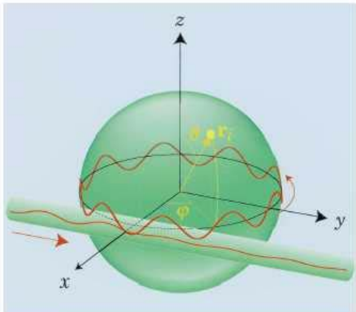


Little Optics



- **The microrings are cascaded in a lattice configuration**
- **The extinction ratio increases with the number of rings**
- **The 3-dB bandwidth also increases with the number of rings**

Bio-detection

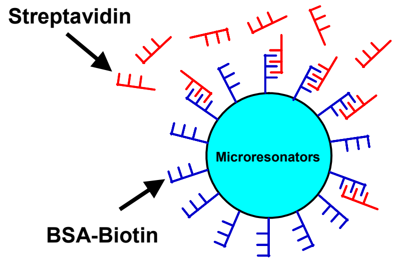


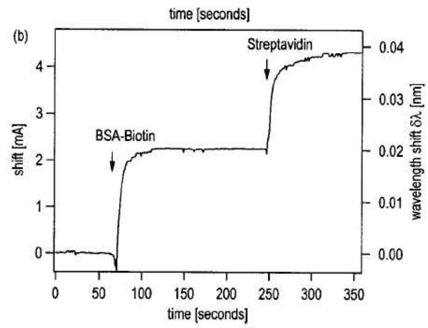
$$\frac{\delta\omega}{\omega} \simeq -\frac{\alpha_{ex}\sigma_p}{\varepsilon_0 (n_s^2 - n_m^2) R}$$

α_{ex} : excess polarizability

σ_p : surface density

Frank Vollmer, Bio-Physics,2003





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