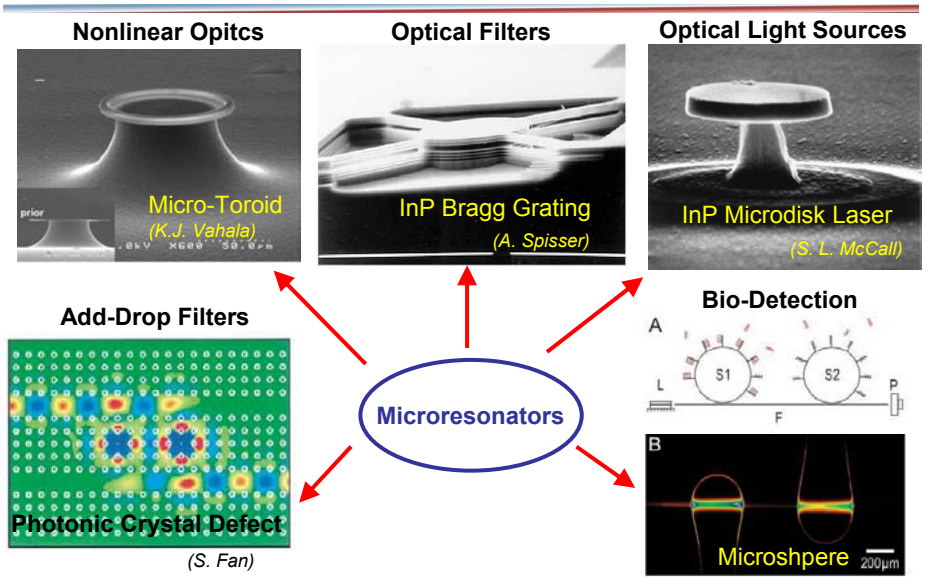
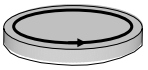
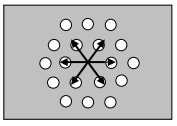
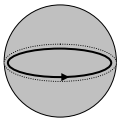
Integrated Optical Resonators I

Class: Integrated Photonic Devices
Time: Fri. 8:00am ~ 11:00am.
Classroom: 資電206
Lecturer: Prof. 李明昌(Ming-Chang Lee)

Applications of Optical Microresonators

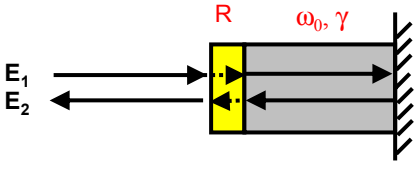


Types of Optical Resonators

Standing-Wave	Travelling-Wave
Fabry-Perot	Microring
1D PC Defect	 Microdisk
2D or 3D PC Defect 	Microsphere 

Analogy of Standing-Wave and Travelling-Wave Optical Resonators

Standing-Wave Resonators (Gires-Tournois Interferometers)



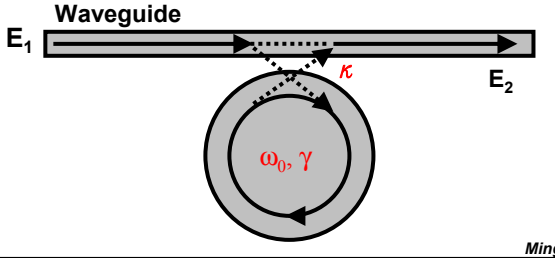
100% reflection

ω_0: Resonant Freq.

γ: Resonator Loss

R: Partial Reflectivity

Travelling-Wave Resonators (Microdisks)



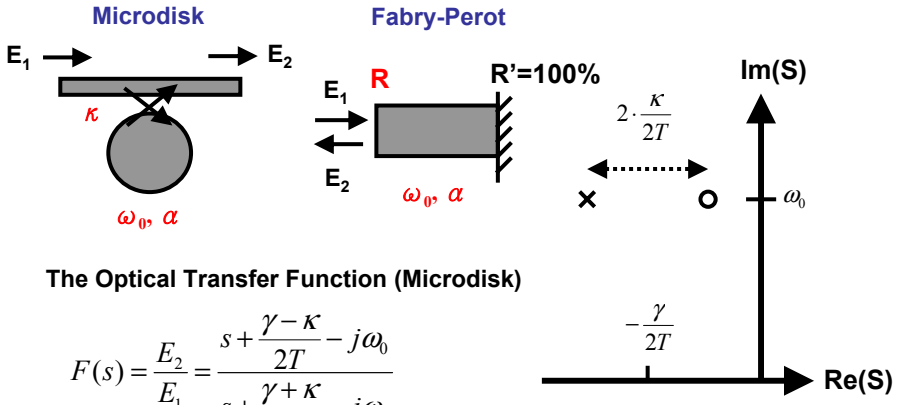
R ⟷ 1/ κ

ω_0: Resonant Freq.

γ: Resonator Loss

κ: Power coupling ratio

Transfer Function of Resonator-Coupled Waveguides



The Optical Transfer Function (Microdisk)

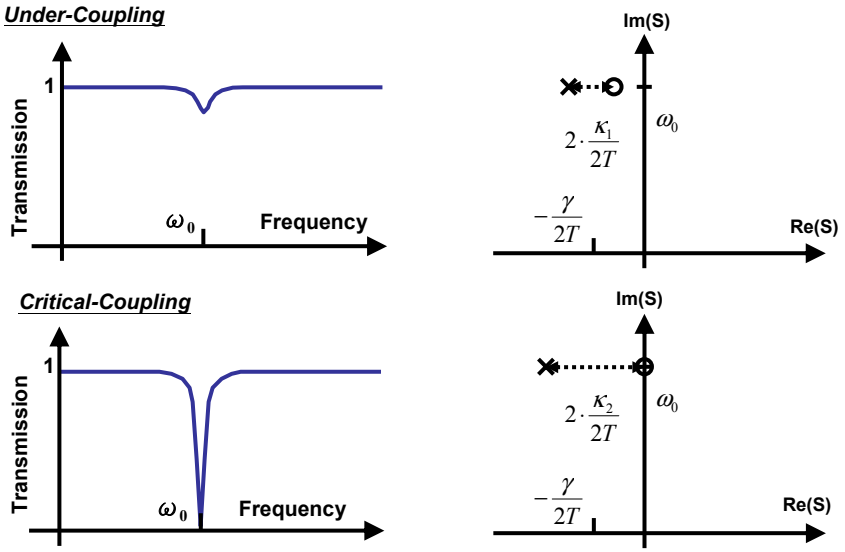
$$F(s)=\frac{E_2}{E_1}=\frac{s+\frac{\gamma-\kappa}{2T}-j\omega_0}{s+\frac{\gamma+\kappa}{2T}-j\omega_0}$$

T : **Round-trip Propagation Time**

$\gamma \approx L_{cavity}\alpha$: **Round-trip Loss**

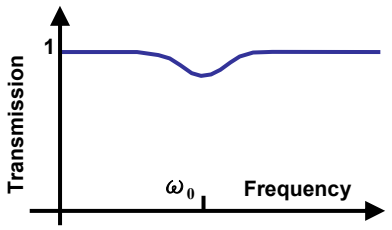
Quality Factor: $Q=\omega_0\cdot\frac{T}{\kappa+\gamma}$

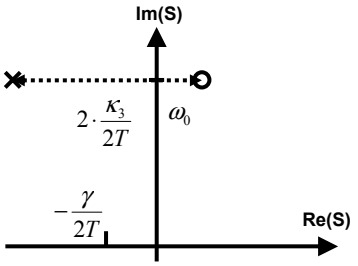
Transmittance of Resonator-Coupled Waveguide



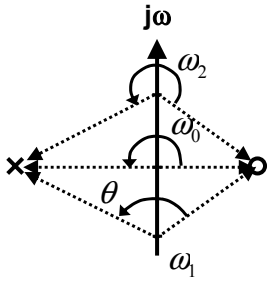
Transmittance of Resonator-Coupled Waveguide

Over-Coupling

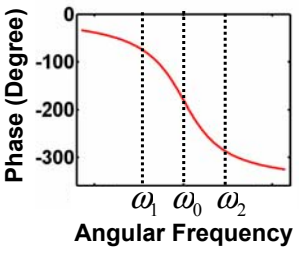




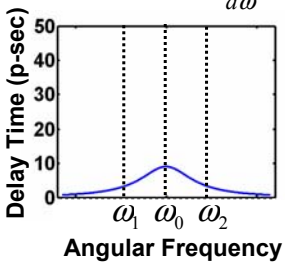
Phase Variation of Resonator-Coupled Waveguide



Phase: $\measuredangle F(j\omega)=\theta$



Group Delay : $-\frac{d\theta}{d\omega}$



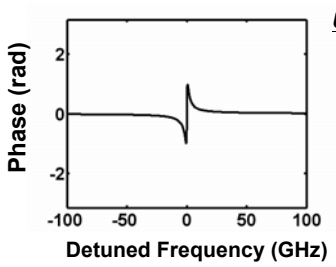
The phase response of transfer function F(s) is given by

$$\measuredangle F(j\omega)=\measuredangle\frac{E_2}{E_1}=\measuredangle\frac{j\omega+\frac{\gamma-\kappa}{2T}-j\omega_0}{j\omega+\frac{\gamma+\kappa}{2T}-j\omega_0}=\underbrace{\measuredangle\left\{j(\omega-\omega_0)+\frac{\gamma-\kappa}{2T}\right\}}_{\text{zero}}-\underbrace{\measuredangle\left\{j(\omega-\omega_0)+\frac{\gamma+\kappa}{2T}\right\}}_{\text{pole}}$$

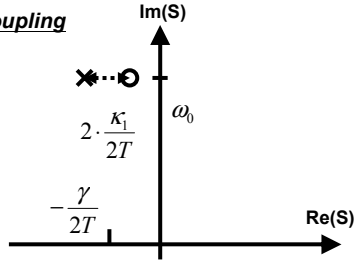
The group delay is

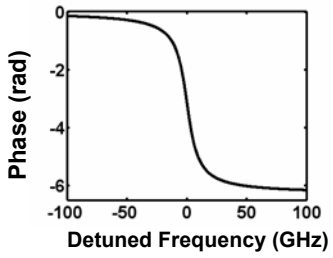
$$T(j\omega)=-\frac{\measuredangle F(j\omega)}{d\omega}$$

Phase Variation of Resonator-Coupled Waveguide

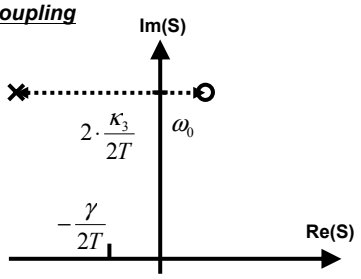


Under-Coupling





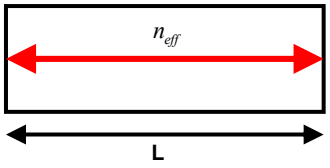
Over-Coupling



Resonant Wavelengths of Microresonators

- **Periodical Resonant Frequency (Wavelength)**

Standing Wave Resonators



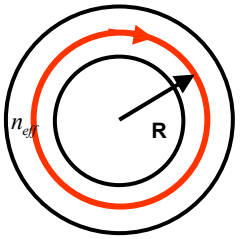
Resonant Wavelength

$$\frac{2\pi}{\lambda_m} n_{eff}(2L) = 2m\pi \quad m = 1, 2, 3, \ldots$$

Free Spectrum Range

$$\Delta f = \left(\frac{2L}{c} n_g\right)^{-1}$$

Travelling Wave Resonators



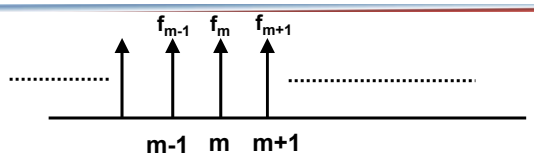
Resonant Wavelength

$$\frac{2\pi}{\lambda_m} n_{eff}(2\pi R) = 2m\pi \quad m = 1, 2, 3, \ldots$$

Free Spectrum Range

$$\Delta f = \left(\frac{2\pi R}{c} n_g\right)^{-1}$$

Free Spectrum Range



D = 2L (Cavity cycle)
c : speed of light
n(m) : effective index at mode m

$$f_m = \frac{mc}{n(m)D} \quad (1)$$

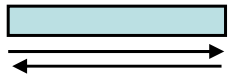
$$f_{m+1} = \frac{(m+1)c}{n(m+1)D} = \frac{(m+1)c}{(n+\Delta n)D} \quad (2)$$

Notation : n(m+1) = n(m) + Δn = n + Δn

(2)- (1)

$$\Delta f = f_{m+1} - f_m = \frac{(m+1)c}{(n+\Delta n)D} - \frac{mc}{nD} \quad (3)$$

Fabry-Prot Resonator



L

$$\Delta f(n+\Delta n) = \frac{c}{D} - f\Delta n \quad (4)$$

$$\Delta f n + \Delta f \Delta n + f\Delta n = \frac{c}{D} \quad (5)$$

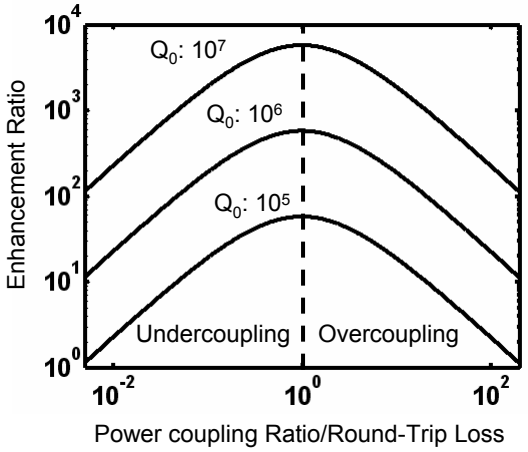
$$\Delta f\left(n + \frac{f}{\Delta f}\Delta n\right) = \frac{c}{D} \quad (6)$$

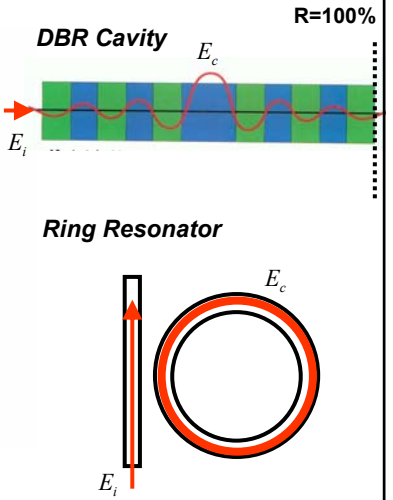
$$\Delta f \cdot n_g = \frac{c}{D} \quad (7)$$

$$\Delta f = \frac{c}{D \cdot n_g} \quad (8)$$

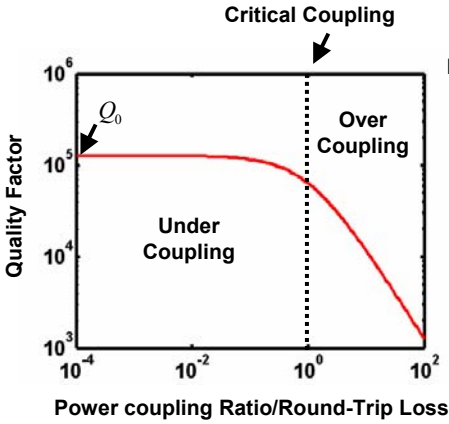
Resonant Magnification of Microresonators

- **Resonance-Enhanced Optical Fields inside Resonators**





Quality Factor vs. Power Coupling Ratio



Recall $Q = \omega_0 \cdot \frac{T}{\kappa + \gamma}$ and $0 \le \kappa, \gamma \le 1$

$$\omega_0 \cdot \frac{T}{2} < \omega_0 \cdot \frac{T}{\kappa + \gamma} \le \omega_0 \cdot \frac{T}{\gamma}$$

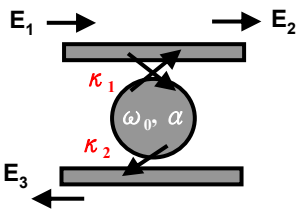
$$\rightarrow \quad \omega_0 \cdot \frac{T}{2} < Q \le Q_0$$

$Q_0 = \omega_0 \cdot \frac{T}{\gamma}$ (Intrinsic Q)

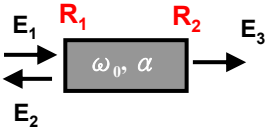
- Quality factor decreases with power coupling ratio.
- The maximal quality factor is limited by the intrinsic Q

Transfer Function of Two Waveguide-Coupled Optical resonator

Microdisk



Fabry-Perot



The Optical Transfer Function (Microdisk)

$$F_{it}(s) = \frac{E_2}{E_1} = \frac{s + \frac{(\gamma + \kappa_2 - \kappa_1)}{2T} - j\omega_0}{s + \frac{(\gamma + \kappa_2 + \kappa_1)}{2T} - j\omega_0}$$

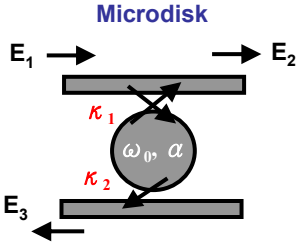
$$F_{id}(s) = \frac{E_3}{E_1} = \frac{\frac{\sqrt{\kappa_1 \kappa_2}}{T}}{s + \frac{(\gamma + \kappa_2 + \kappa_1)}{2T} - j\omega_0}$$

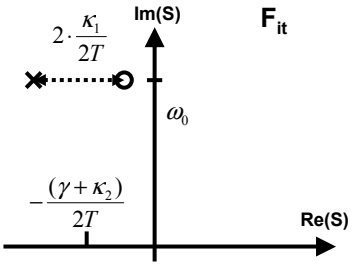
Recall

$$F(s) = \frac{E_2}{E_1} = \frac{s + \frac{\gamma - \kappa}{2T} - j\omega_0}{s + \frac{\gamma + \kappa}{2T} - j\omega_0}$$

Single Waveguide Coupling

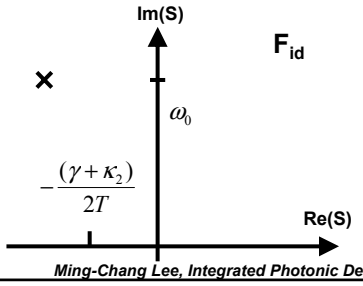
Transfer Function of Two Waveguide-Coupled Optical resonator





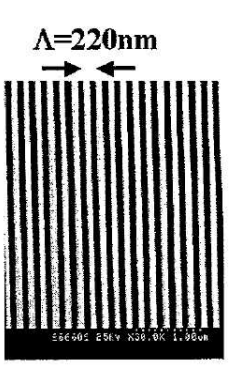
$$F_{it}(s)=\frac{E_2}{E_1}=\frac{s+\frac{(\gamma+\kappa_2-\kappa_1)}{2T}-j\omega_0}{s+\frac{(\gamma+\kappa_2+\kappa_1)}{2T}-j\omega_0}$$

$$F_{id}(s)=\frac{E_3}{E_1}=\frac{\frac{\sqrt{\kappa_1\kappa_2}}{T}}{s+\frac{(\gamma+\kappa_2+\kappa_1)}{2T}-j\omega_0}$$

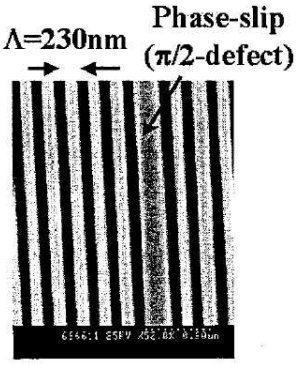


1D Photonic Crystal Resonator or Distributed Bragg Reflector Cavity

Fabricated on Silicon-on-Insulator (SOI)



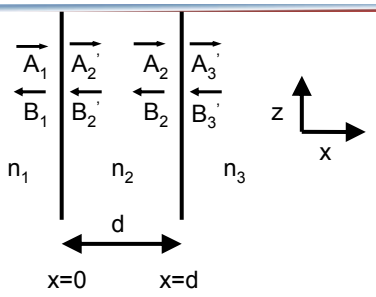
1D Photonic Crystals (Bragg Reflector)



1D Photonic Crystal Resonator (DBR Cavity)

B. Jalali

Matrix Formulation for Isotropic Layer Media



Suppose for lossless dielectric media

$$n_m(x)=\begin{cases} n_1, & \mathbf{x<0} \\ n_2, & \mathbf{0<x<d} \\ n_3, & \mathbf{d<x} \end{cases} \qquad m=1,2,3$$

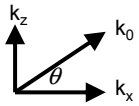
$$E_m = E_m(x)e^{i(k_{mz}z-\omega t)}$$

Right **Left**

where

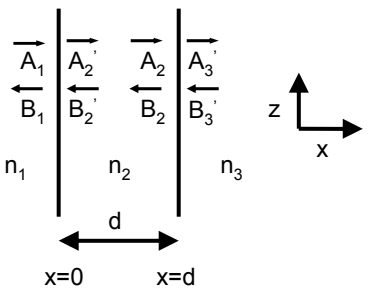
$$\begin{cases} E_m(x)=a_m(jk_{mx}x)+b_m(-jk_{mx}x) \\ k_{mz}=\dfrac{\omega}{c}n_m\cdot\sin(\theta_m) \\ k_{mx}=\dfrac{\omega}{c}n_m\cdot\cos(\theta_m) \end{cases}$$

and



$$\begin{cases} A_1=a_1(0^-), \\ B_1=b_1(0^-), \\ A_2'=a_2(0^+), \\ B_2'=b_2(0^+), \\ A_2=a_2(d^-), \\ B_2=b_2(d^-), \\ A_3'=a_3(d^+), \\ B_3'=b_3(d^+), \end{cases}$$

Matrix Formulation for Isotropic Layer Media



Consider the boundary conditions,

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = D_{1,2}\begin{pmatrix} A_2' \\ B_2' \end{pmatrix}$$

$$\begin{pmatrix} A_2 \\ B_2 \end{pmatrix} = D_{2,3}\begin{pmatrix} A_3' \\ B_3' \end{pmatrix}$$

(a)

*For example (s-wave), E_y and H_z should continue

where

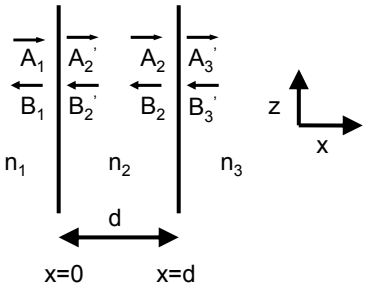
$$D_{m,m+1}=\begin{pmatrix} \frac{1}{2}\left(1+\frac{k_{(m+1)x}}{k_{mx}}\right) & \frac{1}{2}\left(1-\frac{k_{(m+1)x}}{k_{mx}}\right) \\ \frac{1}{2}\left(1-\frac{k_{(m+1)x}}{k_{mx}}\right) & \frac{1}{2}\left(1+\frac{k_{(m+1)x}}{k_{mx}}\right) \end{pmatrix}$$

s-wave (TE)

$$D_{m,m+1}=\begin{pmatrix} \frac{1}{2}\left(1+\frac{n_{m+1}^{\ 2}k_{mx}}{n_m^{\ 2}k_{(m+1)x}}\right) & \frac{1}{2}\left(1-\frac{n_{m+1}^{\ 2}k_{mx}}{n_m^{\ 2}k_{(m+1)x}}\right) \\ \frac{1}{2}\left(1-\frac{n_{m+1}^{\ 2}k_{mx}}{n_m^{\ 2}k_{(m+1)x}}\right) & \frac{1}{2}\left(1+\frac{n_{m+1}^{\ 2}k_{mx}}{n_m^{\ 2}k_{(m+1)x}}\right) \end{pmatrix}$$

p-wave (TM)

Matrix Formulation for Isotropic Layer Media



Consider the phase transformation,

$$\begin{pmatrix} A_2' \\ B_2' \end{pmatrix} = P_2 \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} \qquad \textbf{(b)}$$

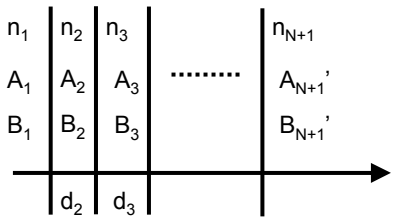
where $P_m = \begin{pmatrix} e^{-j\phi_m} & 0 \\ 0 & e^{j\phi_m} \end{pmatrix}$

and $\phi_m = k_{mx} d_m$

Therefore, combine (a) and (b),

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = D_{1,2} P_2 D_{2,3} \begin{pmatrix} A_3' \\ B_3' \end{pmatrix}$$

Matrix Formulation for Isotropic Layer Media



Therefore, for N+1 layer media

$$\begin{pmatrix} A_1 \\ B_1 \end{pmatrix} = D_{1,2} P_2 D_{2,3} P_3 \cdots D_{N,N+1} \begin{pmatrix} A_{N+1}' \\ B_{N+1}' \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} A_{N+1}' \\ B_{N+1}' \end{pmatrix}$$

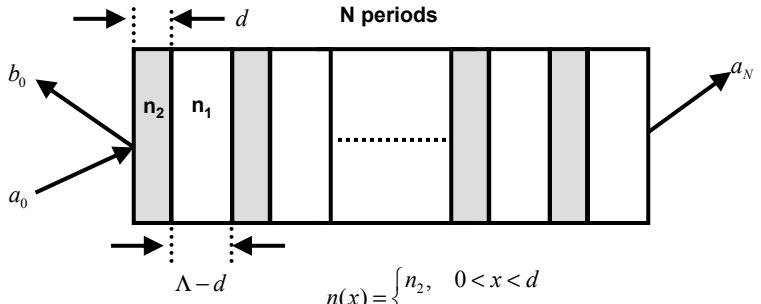
The reflection and transmission coefficients are defined as

$$r = \left(\frac{B_1}{A_1} \right)_{B_{N+1}'=0} = \frac{M_{21}}{M_{11}} \qquad \textbf{and} \qquad t = \left(\frac{A_{N+1}'}{A_1} \right)_{B_{N+1}'=0} = \frac{1}{M_{11}} \qquad \textbf{\textit{(Field)}}$$

The reflectance and transmittance are defined as

$$R = |r|^2 = \left| \frac{M_{21}}{M_{11}} \right|^2 \qquad \textbf{and} \qquad T = \frac{n_{N+1} \cos\theta_{N+1}}{n_1 \cos\theta_0} |t|^2 = \frac{n_{N+1} \cos\theta_{N+1}}{n_1 \cos\theta_0} \left| \frac{1}{M_{11}} \right|^2 \qquad \textbf{\textit{(Power)}}$$

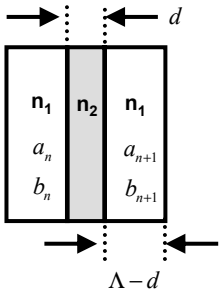
Bragg Reflectors (1D Photonic Crystals)



$$n(x) = \begin{cases} n_2, & 0 < x < d \\ n_1, & d < x < \Lambda \end{cases}$$

- **The media are arranged periodically**
- **Under a specific condition, the light is expected to be highly reflected. (reflector)**

Bragg Reflectors



For one period,

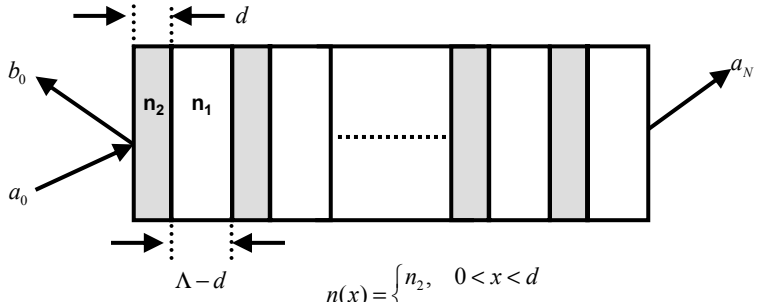
$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix}$$

where

(TE, for example)

$$\begin{cases} A = e^{jk_{1x}(\Lambda-d)} \left[\cos k_{2x} d + \frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x} d \right] \\ B = e^{-jk_{1x}(\Lambda-d)} \left[\frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x} d \right] \\ C = e^{jk_{1x}(\Lambda-d)} \left[-\frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x} d \right] \\ D = e^{-jk_{1x}(\Lambda-d)} \left[\cos k_{2x} d - \frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} + \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x} d \right] \end{cases}$$

Bragg Reflectors



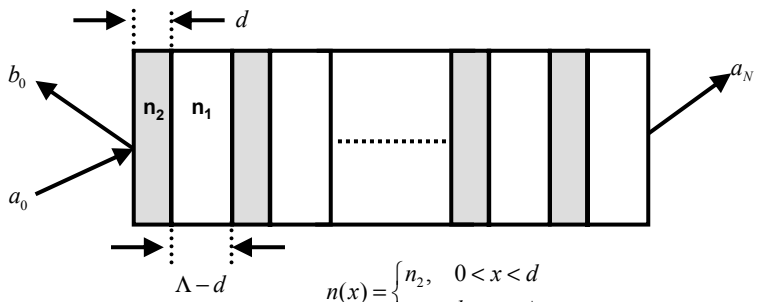
$$n(x) = \begin{cases} n_2, & 0 < x < d \\ n_1, & d < x < \Lambda \end{cases}$$

For N periods of layers,

$$\begin{pmatrix} a_0 \\ b_0 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}^N \begin{pmatrix} a_N \\ b_N \end{pmatrix} \quad \textbf{where} \quad \begin{pmatrix} A & B \\ C & D \end{pmatrix}^N = \begin{pmatrix} AU_{N-1} - U_{N-2} & BU_{N-1} \\ CU_{N-1} & DU_{N-1} - U_{N-2} \end{pmatrix}$$

$$\textbf{and} \quad U_N = \frac{\sin(N+1)K\Lambda}{\sin K\Lambda} \qquad K = \frac{1}{\Lambda}\cos^{-1}\left(\frac{A+D}{2}\right)$$

Bragg Reflectors



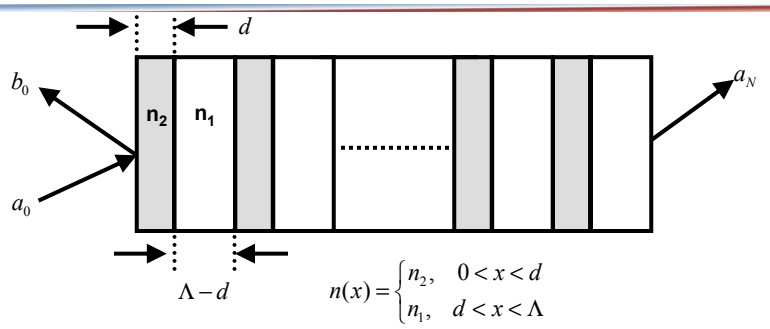
$$n(x) = \begin{cases} n_2, & 0 < x < d \\ n_1, & d < x < \Lambda \end{cases}$$

The reflection coefficient is

$$r_N = \frac{CU_{N-1}}{AU_{N-1} - U_{N-2}} \longrightarrow |r_N|^2 = \frac{|C|^2}{|C|^2 + (\sin K\Lambda / \sin NK\Lambda)^2}$$

As $K\Lambda = m\pi$ $\longrightarrow$ $|r_N|^2 = \dfrac{|C|^2}{|C|^2 + (1/N)^2} \approx 1$ (For large N)

Bragg Reflectors



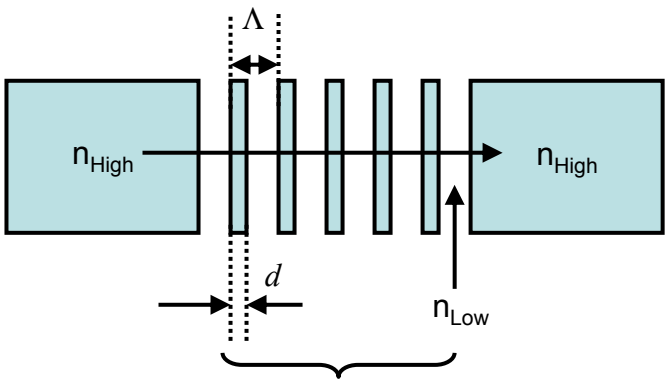
$$n(x) = \begin{cases} n_2, & 0 < x < d \\ n_1, & d < x < \Lambda \end{cases}$$

How to make a high reflection? → C is as large as possible

Recall $C = e^{jk_{1x}(\Lambda - d)} \left[-\frac{1}{2} j \left(\frac{k_{2x}}{k_{1x}} - \frac{k_{1x}}{k_{2x}} \right) \sin k_{2x} d \right]$ $\qquad |C|^2 = \frac{1}{4} \left| \frac{k_{2x}^2 - k_{1x}^2}{k_{2x} k_{1x}} \right|^2 \sin^2 k_{2x} d$

$\begin{cases} k_{2x}^2 - k_{1x}^2 \text{ large} \rightarrow \text{High index contrast} \\ k_{2x} d = \frac{(2m+1)\pi}{2} \rightarrow \text{d is quarter wavelength (m=1)} \end{cases}$

An Example of Bragg Reflector

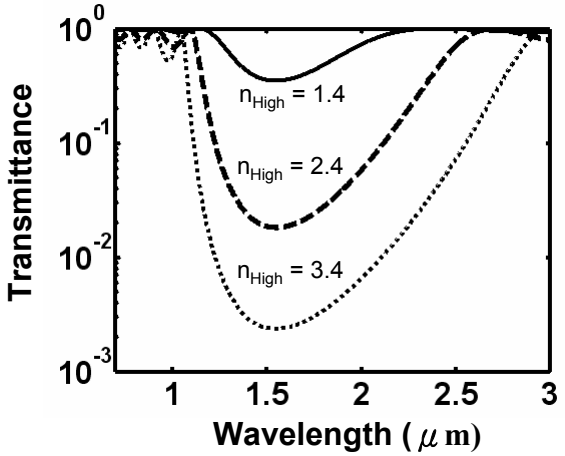


If $\lambda_0 = 1.55 \mu m$

$$d = \frac{\lambda_0}{4 n_{High}} \qquad \Lambda = \frac{\lambda_0}{4 n_{High}} + \frac{\lambda_0}{4 n_{Low}}$$

What is d?

Transmittance Varied by High Refractive Index

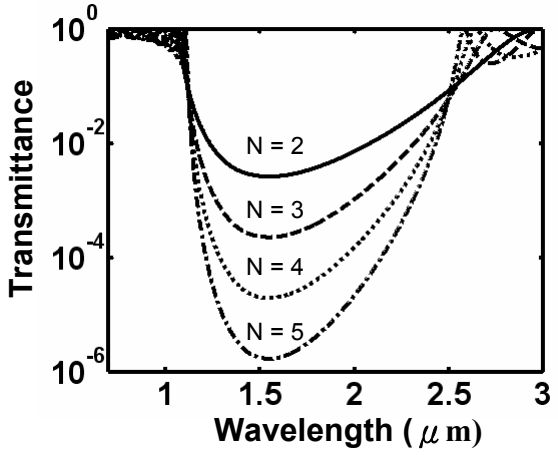


n_{Low} = 1

N = 2

- **The maximum reflection (minimum transmission) occurs at 1.55 μm**
- **High index contrast has larger reflection**

Transmittance Varied by Number of Period



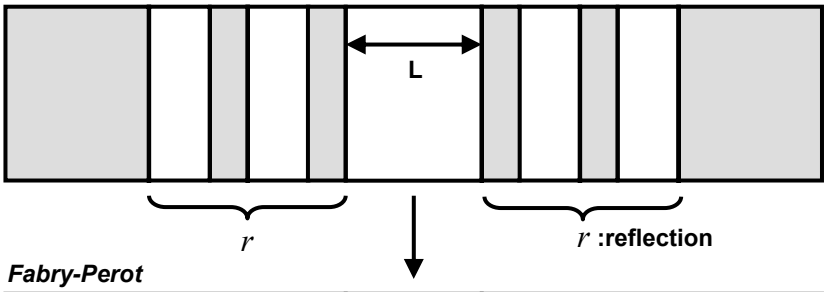
n_{Low} = 1

n_{High} = 3.4

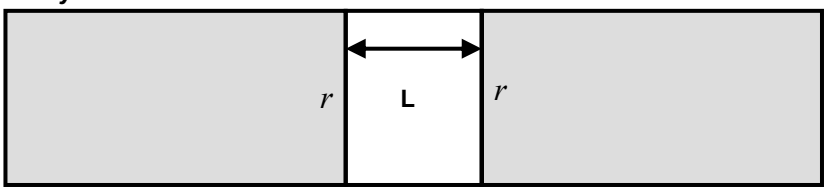
- **The maximum reflection (minimum transmission) occurs at 1.55 μm**
- **More periods have larger reflection**

Comparison of DBR Cavity and Fabry-Perot

DBR (Distributed Bragg Reflector) Cavity



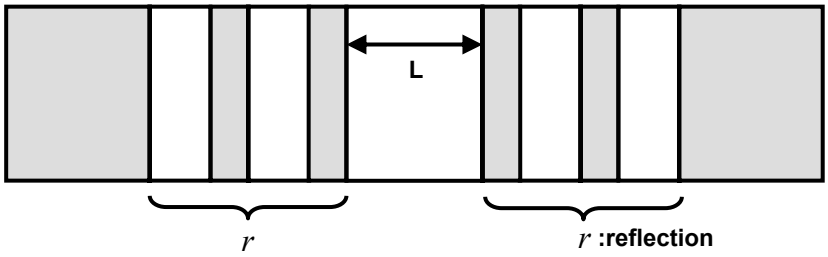
Fabry-Perot



But, what is difference?

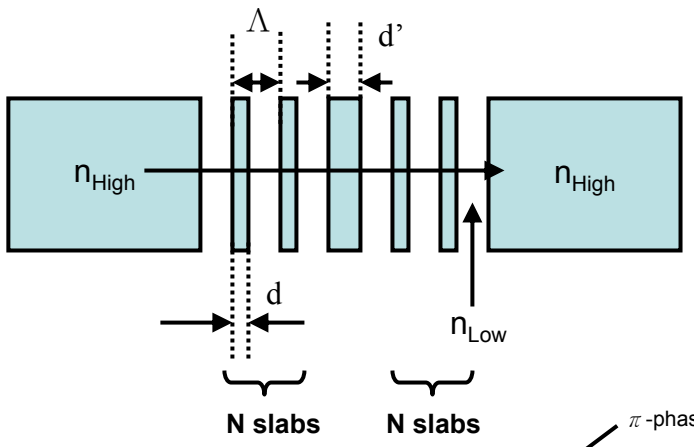
Distributed Bragg Reflector

DBR (Distributed Bragg Reflector) Cavity



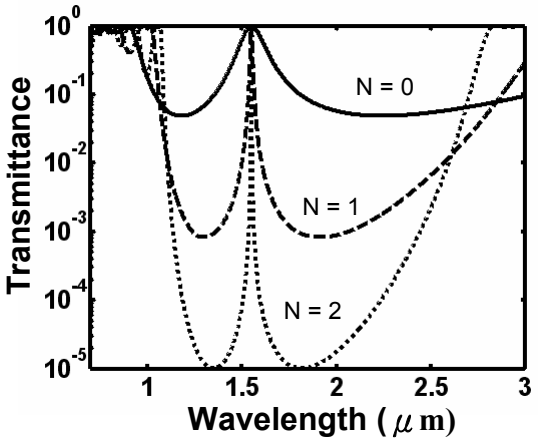
- **The reflectance is strongly depends on the wavelength**
- **The phase shift is also dependent on the wavelength**

An Example of DBR Cavity



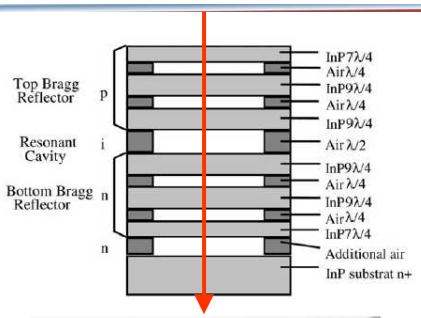
$$d = \frac{\lambda_0}{4n_{High}} \qquad \Lambda = \frac{\lambda_0}{4n_{High}} + \frac{\lambda_0}{4n_{Low}} \qquad d' = \frac{\lambda_0}{2n_{High}} \qquad \lambda_0 = 1.55\mu m$$

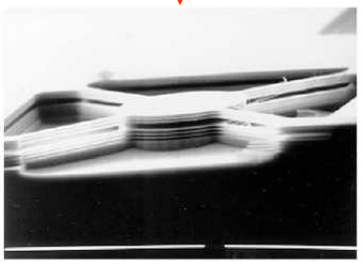
Transmittance Varied by Number of Period

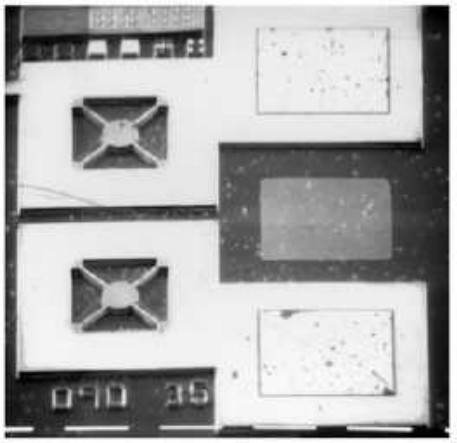


- **The extinction ratio increases with slab number.**
- **The bandwidth reduces with slab number**

InP Tunable Filters



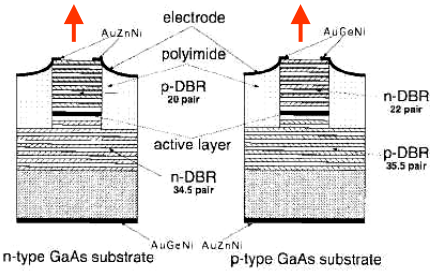




Ming-Chang Lee, Integrated Photonic Devices

Vertical-Cavity Surface-Emitting Laser

Yoshitaka Kohama, PTL, 1997



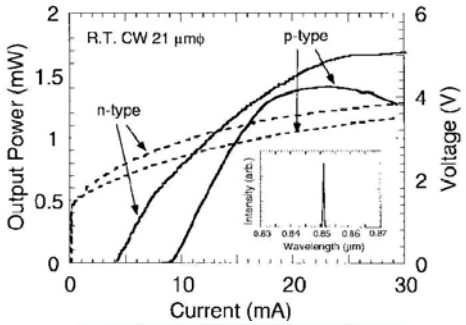
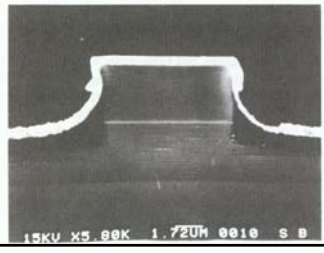


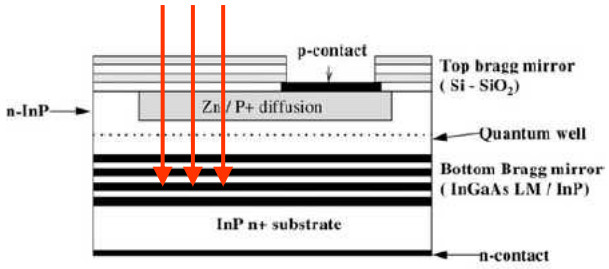
TABLE I
COMPARISON OF LASER CHARACTERISTICS BETWEEN n-SUB TYPE AND p-SUB TYPE VCSELS

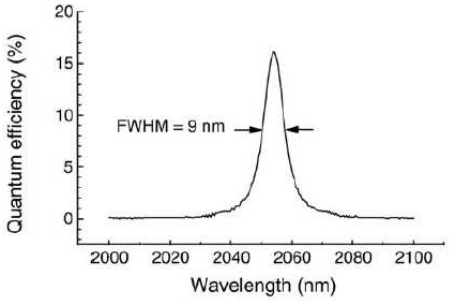
	n-sub type	p-sub type
Number of pairs of n-DBR	34.5	22
Number of pairs of p-DBR	20	35.5
Reflectivity (Calculation)	0.998	
Threshold current density (kA/cm^2)	1.2	1.7
Threshold voltage(V)	2.1	2.5



: Devices

Resonance-Enhanced Photodetectors





- **At resonant wavelength, photons are accumulated in the cavity**
- **It increases the quantum efficiency**