







Faraday's Effect

E is harmonic time dependence $exp(j\omega t)$ and r also has the same harmonic dependence

 $-m\omega^2 \mathbf{r} + k\mathbf{r} = -e\mathbf{E} + j\omega e\mathbf{r} \times \mathbf{B}$

The polarization P of the medium is just a constant times r, namely, -Ner, hence the above equation becomes

$$(-m\omega^2 + k)\mathbf{P} = Ne^2\mathbf{E} + j\omega e\mathbf{P} \times \mathbf{B}$$

The polarization can be solved and written as a function of E

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$$

Ming-Chang Lee, Integrated Photonic Devices



Faraday's Effect

Suppose a plane wave propagating along the z direction; that is,

$$\vec{E} = \vec{E_0} \exp(jkz)$$

(The propagation direction is parallel to the magnetic field)

E has to be subjected to Helmholtz Eq. $\nabla^2 \vec{E} + (I + \chi) \frac{\omega^2}{c^2} \vec{E} = 0$ $\begin{cases}
-k^2 E_x + \frac{\omega^2}{c^2} E_x = -\frac{\omega^2}{c^2} (\chi_{11} E_x + j \chi_{12} E_y) \\
-k^2 E_y + \frac{\omega^2}{c^2} E_y = -\frac{\omega^2}{c^2} (-j \chi_{12} E_x + \chi_{11} E_y) \\
\frac{\omega^2}{c^2} E_z = -\frac{\omega^2}{c^2} \chi_{33} E_z \quad (E_z = 0 \text{ since E propagate along z})
\end{cases}$ *Ming-Chang Lee, Integrated Photonic Devices*





























