Distributed-Feedback Lasers

Class: Integrated Photonic Devices
Time: Fri. 8:00am ~ 11:00am.
Classroom: 資電206
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Wavelength Dependence of Bragg Reflections

Free-Space Bragg Reflection

Waveguide Bragg Reflection

Let $\theta = 90^\circ$

- Usually only one mode will lie within the gain bandwidth of the laser

\[
2d \sin \theta = l\lambda \quad \text{and} \quad l = 1, 2, 3, \ldots
\]

\[
2\Lambda = l(\lambda_0 / n) \quad \text{and} \quad l = 1, 2, 3, \ldots
\]

Effective index
The coupling can be characterized by perturbation assumption

\[ \kappa = \frac{2\pi^2}{3\lambda_0} \left( \frac{n_1^2 - n_t^2}{n_2} \right)^{1/3} \left( \frac{a}{t_g} \right)^{1/3} \left[ 1 + \frac{3(\lambda_t/a)}{2\pi(n_2^2 - n_t^2)^{1/2}} + \frac{3(\lambda_t/a)^2}{4\pi^2(n_2^2 - n_t^2)} \right] \]

where

\[ l = \frac{\beta_m \Lambda}{\pi} \]

\( \beta_m \) is the propagation constant of the particular mode m.
For the first order (l=1) case, the reflection is essential limited to coupling between the forward and backward traveling wave

\[ A_m^-(z) = A_m^-(0) \frac{\cosh[k(z-L)]}{\cosh(kL)} \]

and

\[ A_m^+(z) = A_m^+(0) \frac{k}{k} \frac{\sinh[k(z-L)]}{\cosh(kL)} \]

L: the length of grating

The reflectance and transmittance are

\[ R_m = \left| \frac{A_m^-(0)}{A_m^+(0)} \right|^2 \quad \text{and} \quad T_m = \frac{\left| A_m^+ (L) \right|^2}{\left| A_m^+ (0) \right|^2} \]

- The incident power decreases exponentially with z, as power is reflected into the backward travelling wave.
Ray Scattering of Higher-Order Bragg Gratings

For higher-order gratings with periodicity $\Lambda$

$$\Lambda = \frac{l\lambda_0}{2n_i} \quad l > 0$$

The extra path length must be integral multiples of the wavelength to have constructive scattering ray

$$b + \Lambda = \frac{l\lambda_0}{n_i} \quad l = 0, 1, 2, 3, ... l$$

and

$$b = \Lambda \sin \theta$$

Thus,

$$\sin \theta = \frac{l\lambda_0}{n_i\Lambda} \quad l = 0, 1, 2, 3, ... l$$

For example of a second-order Bragg grating

$$\Lambda = \frac{\lambda_0}{n_i}$$

Then

$$\sin \theta = l - 1 \quad l = 0, 1, 2$$

Three diffraction modes

$$l = 0 \rightarrow$$

$$l = 1 \leftarrow$$

$$l = 2 \rightarrow$$
Coupling Efficiency of Wave Reflection

- Generally, the strong coupling in the transverse direction by second-order Bragg grating is undesirable.
- A first-order grating is required to yield optimum performance.
- However, fabrication of the first-order grating is challenging.

Lasing with Distributed Feedback

In a gain medium (gain $g$),

$$E_r(z) = E_0 \frac{\kappa e^{j \beta z} \sinh \left[ S(L - z) \right]}{(g - j \Delta \beta) \sinh (SL) - S \cosh (SL)}$$

where

$$S^2 = |k|^2 + (g - j \Delta \beta)^2$$

The parameter $E_0$ is the amplitude of a single mode incident on the grating (stimulus).
The phase mismatching term,
\[ \Delta \beta = \beta - \beta_0 \]

Where \( \beta_0 \) is the propagation constant at the Bragg wavelength.

The oscillation condition for the DFB laser corresponds to the case for

\[ \frac{E_v(0)}{E_v} \to \infty \]

That is,
\[ (g - j\Delta \beta) \sinh(SL) = S \cosh(SL) \]

In general, numerical method should be used for exactly solving \( g \) and \( \Delta \beta \) simultaneously. A special case of solution of lasing frequency

\[ \omega_m = \omega_0 - \left( n_g \frac{\pi c}{n_1 L} \right) m = 0, \pm 1, \pm 2, \ldots \]

\[ \Delta \beta = \beta - \beta_0 = \frac{(\omega - \omega_0)n_g}{c} \]

Supposing that \( g \gg |\Delta \beta| \), \( \omega_0 \) : Bragg Frequency

It is interesting to note that no oscillation can occur at exactly the Bragg frequency \( \omega_0 \). The mode spacing

\[ \Delta \omega_m = \frac{\pi c}{n_1 L} \]

However, only the lasing modes close to the Bragg frequency have the smallest threshold gain. Therefore, in a normal operating condition, the spectral feature of DFB laser often consists of the two longitudinal modes.
Separate Confinement Heterostructure Lasers

- Lattice damage is usually created during the grating fabrication. → It is better to separate the active layer out of the grating layer.

Distributed Bragg Reflection Lasers

- Two Bragg gratings are employed at both ends of the laser and outside of the electrically-pumped active region.
- To achieve a single longitudinal mode, one distributed reflector must have narrow bandwidth, high reflectivity at the lasing wavelength.
Distributed Bragg Reflection Lasers

The Transmittivity: \[ T = \frac{1}{\gamma} \exp \left[ -j \left( \beta - \Delta \beta \right) L \right] \](\alpha + j \Delta \beta) \sinh (\gamma L) + \gamma \cosh (\gamma L) \]
where \[ \gamma^2 = \kappa^2 + (\alpha + j \Delta \beta)^2 \]

The Reflectivity: \[ R = \frac{-j \kappa \sinh (\gamma L)}{(\alpha + j \Delta \beta) \sinh (\gamma L) + \gamma \cosh (\gamma L)} \]
(complex)

For the passive grating region,\[ y < x \]

Since the reflectivity is complex number, we can consider an effective cavity length \[ L_{\text{eff}} = L_1 \left( 1 + \frac{L_2}{2L_2 (\alpha L_1 + 1)} + \frac{1}{2L_2 (\alpha + \sqrt{\kappa^2 + \alpha^2})} \right) \]

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The longitudinal mode spacing between the m-th and (m±1)-th lasing mode is approximately

$$\Delta = \left| \beta_m - \beta_{m+1} \right| = \frac{\pi}{L_{\text{eff}}}$$

Wavelength Selectability

- Compared with Fabry-Perot lasers, DFB or DBR laser is easy to achieve single-longitudinal-mode operation because the spacing between the m-th and the (m±1)-th mode is generally large and the reflectivity is mode-dependent.