Robust MISO Transmit Optimization under Outage-Based QoS Constraints in Two-Tier Heterogeneous Networks

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***Abstract*—To improve wireless heterogeneous network service via macrocell and femtocells that share certain spectral resources, this paper studies the transmit beamforming design for femtocell base station (FBS), equipped with multiple antennas, under an outage-based quality-of-service (QoS) constraint at the single-antenna femtocell user equipment characterized by its signal-to-interference-plus-noise ratio. Specifically, we focus on the practical case of imperfect downlink multiple-input single-output (MISO) channel state information (CSI) at the FBS due to limited CSI feedback or CSI estimation errors. By characterizing the CSI uncertainty probabilistically, we formulate an outage-based robust beamforming design. This nonconvex optimization problem can be relaxed into a convex semidefinite programming problem, which reduces to a power control problem when all CSI vectors are independent and identically distributed. We also investigate the performance gap between the optimal transmission strategy (that allows maximum transmission degrees of freedom (DoF) equal to the number of transmit antennas) and the proposed optimal beamforming design (with the DoF equal to one) and provide some feasibility conditions, followed by their performance evaluation and trade-off through simulation results.**

***Index Terms*—Convex optimization, femtocell, macrocell, robust beamforming, semidefinite relaxation (SDR).**

I. Introduction

FEMTOCELLS, whose services are made possible by operator-certified home base stations, are low power cellular access points and have been recognized as a cost-effective way to strengthen cellular network coverage and to provide good quality of service (QoS) to mostly indoor data users. The general concept of femtocells can be found in [1], [2] and the references therein. Femtocell deployment is used to improve spectrum efficiency by letting femtocell and macrocell share and access some common spectrum. Spectral sharing between femtocell users and macrocell users will inevitably lead to mutual interference. Such interference may significantly degrade users' QoS at the femtocell and macrocell [3]. For this reason, interference management at femtocell is one of the key issues in femtocell-augmented heterogeneous networks [1]–[5].

In [4], a downlink femtocell beamforming design is considered for minimizing the total transmit power under signal-to-interference-plus-noise ratio (SINR) and interference constraints on multiple-input single-output (MISO) channels. In general, optimizing beamforming vector subject to the SINR and interference constraints is not a convex optimization problem. However, it has been shown that the problem can be equivalently reformulated into a convex second-order cone program (SOCP); thus the optimal beamformer can be efficiently obtained [4], [6]. It has been shown in [7] that the optimal beamformer can also be iteratively solved by uplink-downlink duality. In addition, one can also solve the problem by applying semidefinite relaxation (SDR) [8]. These methods offer various advantages in terms of computational complexity, and offer different implications both in theory and in practical implementations.

Transmit beamforming has been considered as an efficient approach to interference management at femtocell downlink transmission given channel state information (CSI). Many conventional interference management methods such as those reported in [4], [5], and [7] are based on the assumption of perfect CSI at basestation transmitters. Unfortunately, perfect CSI is never available in practice. There are several reasons for the lack of perfect CSI at transmitters in frequency division duplex (FDD) systems. First, in fast fading environment, receivers may not have sufficient time or bandwidth to provide an accurate estimate of the time-varying channel. Second, even when channel fading is moderate, channel estimation errors at the receiver and limited feedback bandwidth make the transmitter impossible to acquire perfect CSI knowledge [9]. Although in time division duplex (TDD) systems, the basestation transmitter can exploit channel reciprocity to estimate its downlink CSI, the short coherence time in fast fading channels would still render the CSI outdated. Thus, base stations generally do not possess accurate instantaneous CSI. They instead may only have statistical information of the channels. If inaccurate CSI is used to design the transmit beamformers without taking their CSI uncertainty into account, the users may experience severe performance loss and QoS outage.

Recently, robust transmit beamforming designs that aim to provide some QoS for each user under CSI errors have drawn

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considerable attention. Specifically, performance requirements of each user must be satisfied even with the worst possible CSI errors, or, alternatively, with sufficiently high probability (which is the focus of this paper). For example, in [10]–[12], a worst-case QoS constrained problem is considered, for which the QoS constraints are satisfied for all the possible CSI errors in a bounded uncertainty set. Such designs often tend to be pessimistic. Instead of focusing on the worst-case performance constraint, a different and often more practical robust design centers on the QoS performance based on outage in a probabilistic setting. In this work, we focus on outage-based robust beamforming (which can be thought of as one precoding method). In outage-based robust beamforming, the design criterion is to characterize QoS performance constraints in terms of an outage probability. Based on the uncertainty of random CSI errors with known distribution, we design transmit beamformers to guarantee the outage probability below a (small) preset threshold.

The challenge in robust design of transmit beamforming under outage-based QoS constraints lies in the lack of closed-form expressions of the associated probability constraints. In addition, such problem formulations may not be convex in general and hard to solve optimally. Perhaps because of this challenge, there have been very few works on the probabilistically constrained QoS design for robust femtocell beamforming. Indeed, Oh et al. [13] determined the transmit power to guarantee the *minimum* (worst-case) target SINR of femtocell user equipments (FUEs) and the beam directions based on the minimum mean square error (MMSE) criterion.

In this paper, assuming that the low cost FUE is equipped with a single antenna, we develop a minimum power downlink beamforming design for multiple-antenna femtocell base station (FBS) given imperfect CSI. In particular, we consider cases when statistical information of the downlink channels or imperfect downlink CSI parameters is known to the FBS. By modeling CSI estimation errors as complex Gaussian random vectors, SINR performance at the FUE and the interference level constraint at the macrocell user equipment (MUE) cannot be universally guaranteed for any beamforming algorithm. Instead, we design FBS downlink beamformer such that the outage event occurs below a low probability threshold. However, probabilistic constraints often have no closed-form expressions and may not be convex in general, making the outage-based QoS constrained problem hard to solve.

The main contributions of this paper are summarized as follows:

- To overcome the difficulty posed by the non-convexity of the outage-based QoS constrained problem, we apply SDR to convert it into a convex problem, which has efficient interior-point based solvers, e.g., `SeDuMi` [14] or `CVX` [15]. In addition, we show that, when the downlink channels are spatially uncorrelated, the robust beamforming design problem can be exactly solved by using simple bisection algorithm [16]. Furthermore, we investigate conditions under which optimal solutions are feasible.
- Moving beyond the beamforming transmission strategy (with the degrees of freedom (DoF) equal to one), we consider the transmit covariance matrix design (that allows maximum DoF equal to the number of transmit antennas). When the FBS has perfect CSI, it has been shown that beamforming is an optimal transmission strategy in terms of transmit power minimization [17]. However, using beamforming strategy is not necessarily optimal with only imperfect CSI knowledge at the FBS. In light of this point, we propose a new transmission strategy and show its optimality under a given condition.

The rest of this paper is organized as follows. In Section II, we present the problem formulation of outage-based robust FBS beamforming design problem. In Section III, we solve the problem for the case that the FBS has only the statistical information of the downlink channels. We also consider the case of partial CSI feedback scenario and solve the corresponding outage-based constrained optimization problem in Section IV. We then investigate the optimal robust downlink transmission strategy for FBS in Section V. We present numerical simulation results in Section VI to demonstrate the advantages of our robust designs before concluding in Section VII.

II. System Model and Problem Statement

A. Notation

The notations $\mathbb{R}^n$, $\mathbb{C}^n$, and $\mathbb{H}^n$ stand for the sets of n-dimensional real vectors, complex vectors, and Hermitian matrices, respectively. We use boldfaced lowercase letters, e.g., $\boldsymbol{a}$, to represent vectors and uppercase letters, e.g., $\boldsymbol{A}$, to represent matrices. $\mathrm{Tr}(\boldsymbol{A})$ and $\lambda_{\max}(\boldsymbol{A})$ denote the trace and maximum eigenvalue of $\boldsymbol{A}$, respectively. Superscript 'H' represents (Hermitian) conjugate transpose. $\boldsymbol{A} \succeq \boldsymbol{0}$ means that the matrix $\boldsymbol{A}$ is positive semidefinite. $\mathrm{vec}(\boldsymbol{A})$ denotes the vector obtained by stacking the column vectors of $\boldsymbol{A}$. $[\boldsymbol{a}]_i$ (or simply a_i) stands for the ith entry of $\boldsymbol{a}$. For a complex $\boldsymbol{A}$, we denote by $\mathrm{Re}\{\boldsymbol{A}\}$ and $\mathrm{Im}\{\boldsymbol{A}\}$ its real and imaginary parts, respectively. $\boldsymbol{I}_n$ denotes the $n \times n$ identity matrix. $\|\cdot\|$ and $\|\cdot\|_F$ represent the vector Euclidean norm and matrix Frobenius norm, respectively. $\mathbb{E}[\cdot]$, $\mathrm{Prob}\{\cdot\}$, and $\exp(\cdot)$ denote the statistical expectation, probability function and exponential function, respectively. We write $\boldsymbol{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{C})$ if $\boldsymbol{x} - \boldsymbol{\mu}$ is a circular symmetric complex Gaussian random vector with zero mean and covariance matrix $\boldsymbol{C} \succeq \boldsymbol{0}$.

B. System Model

We consider a two-tier heterogeneous network that includes both macrocell coverage and localized femtocells. Consider the system model illustrated in Fig. 1. The macrocell base station (MBS), equipped with N_M antennas, communicates with a single-antenna MUE. The closed access FBS, equipped with N_F antennas, serves a single-antenna FUE which shares the MUE spectrum during the downlink transmission of MBS and FBS. The MUE belongs to tier-1 with a higher priority for QoS guarantee while the FUE is in tier-2 whose service can be characterized as "best-effort" in nature. The channel between MBS and FUE is denoted by $\boldsymbol{h}_{FM} \in \mathbb{C}^{N_M}$; that between FBS and FUE is denoted by $\boldsymbol{h}_{FF} \in \mathbb{C}^{N_F}$; and that between FBS and MUE is denoted by $\boldsymbol{h}_{MF} \in \mathbb{C}^{N_F}$.

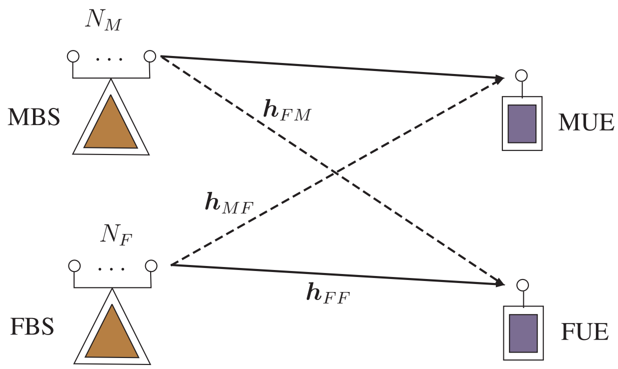


Fig. 1. System model.

The transmit signal at the FBS is given by

$$\boldsymbol{x}(t) = \boldsymbol{w}_F \cdot s_F(t), \tag{1}$$

where $s_F(t) \in \mathbb{C}$ is the information-bearing signal intended for the FUE, which is generated from a Gaussian random codebook with zero mean, and $\boldsymbol{w}_F \in \mathbb{C}^{N_F}$ is the associated beamforming vector. Let the transmit signal at the MBS be $\boldsymbol{w}_M \cdot s_M(t)$. Therefore, the received signal at the FUE can be expressed as

$$y(t) = \boldsymbol{h}_{FF}^H \boldsymbol{w}_F s_F(t) + \boldsymbol{h}_{FM}^H \boldsymbol{w}_M s_M(t) + n_F(t), \tag{2}$$

where the first term is the intended signal for FUE, the second term is the interference from macrocell, and $n_F(t) \in \mathbb{C}$ is additive noise at FUE with power $\sigma_F^2 > 0$. The QoS of the FUE is measured in terms of its SINR. Without loss of generality, suppose that $\mathbb{E}[|s_F(t)|^2] = 1$ and $\mathbb{E}[|s_M(t)|^2] = 1$. Then the SINR at FUE can be represented as

$$\mathsf{SINR}_F = \frac{|\boldsymbol{h}_{FF}^H \boldsymbol{w}_F|^2}{|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2}, \tag{3}$$

and the interference power at the MUE from the femtocell can be seen to be $|\boldsymbol{h}_{MF}^H \boldsymbol{w}_F|^2$.

C. Optimal Beamforming with Perfect CSI

The FBS aims to design its beamforming vector $\boldsymbol{w}_F$ such that the transmit power is minimized, subject to an SINR constraint on the FUE and an interference power constraint on the MUE, i.e.,

$$\frac{|\boldsymbol{h}_{FF}^H \boldsymbol{w}_F|^2}{|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2} \geq \gamma_F \quad \text{and} \quad |\boldsymbol{h}_{MF}^H \boldsymbol{w}_F|^2 \leq \epsilon_M, \tag{4}$$

where $\gamma_F \geq 0$ and $\epsilon_M \geq 0$ are the preset target values. Mathematically, the problem can be formulated as

$$\min_{\boldsymbol{w}_F \in \mathbb{C}^{N_F}} \ \|\boldsymbol{w}_F\|^2 \tag{5a}$$

$$\text{s.t.} \ \frac{|\boldsymbol{h}_{FF}^H \boldsymbol{w}_F|^2}{|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2} \geq \gamma_F, \tag{5b}$$

$$|\boldsymbol{h}_{MF}^H \boldsymbol{w}_F|^2 \leq \epsilon_M. \tag{5c}$$

Although problem (5) is not convex, it can be equivalently reformulated into a convex SOCP [6], [8], i.e.,

$$\min_{\boldsymbol{w}_F \in \mathbb{C}^{N_F}} \ \|\boldsymbol{w}_F\|^2 \tag{6a}$$

$$\text{s.t.} \ \boldsymbol{h}_{FF}^H \boldsymbol{w}_F \geq \sqrt{\gamma_F \left(|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 + \sigma_F^2\right)}, \tag{6b}$$

$$|\boldsymbol{h}_{MF}^H \boldsymbol{w}_F|^2 \leq \epsilon_M, \tag{6c}$$

$$\operatorname{Im}\left\{\boldsymbol{h}_{FF}^H \boldsymbol{w}_F\right\} = 0, \tag{6d}$$

and thus can be efficiently solved by using interior-point based solvers, e.g., `SeDuMi` [14] or `CVX` [15].

D. Optimal Beamforming with Imperfect CSI

Typically, femtocells are connected to macrocell network via a wired broadband backhaul link such as digital subscriber line (DSL) [18], and thus we assume that the beamforming vector at the MBS, $\boldsymbol{w}_M$, is perfectly known to the FBS. In contrast to the assumption of the perfect CSI $\boldsymbol{h}_{FF}$, $\boldsymbol{h}_{FM}$, and $\boldsymbol{h}_{MF}$ made in the conventional formulation in (5), in our work, we specifically consider two cases based on the available channel information at the FBS:

1) *No CSI feedback:* The FBS has no instantaneous channel estimate sent back from the FUE, and the FBS knows only the statistical information of the channels:

$$\boldsymbol{h}_{FF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_{h,FF}), \ \boldsymbol{h}_{FM} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_{h,FM}),$$
$$\boldsymbol{h}_{MF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_{h,MF}), \tag{7}$$

where channel covariance matrices $\boldsymbol{C}_{h,FF}$, $\boldsymbol{C}_{h,FM}$, and $\boldsymbol{C}_{h,MF}$ are positive definite. This model is more advisable for a fast fading system [19], where the feedback channel is unable to provide a reliable estimate of the current CSI, and thus no instantaneous CSI estimate is fed back to FBS.

2) *Partial CSI feedback:* The FBS receives estimated CSI $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{FM}$ from FUE, but knows only the statistical information of $\boldsymbol{h}_{MF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_{h,MF})$ (due to the lack of feedback link from MUE). Specifically, the true CSIs $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{FM}$ are modeled as

$$\boldsymbol{h}_{FF} = \hat{\boldsymbol{h}}_{FF} + \boldsymbol{e}_{FF} \quad \text{and} \quad \boldsymbol{h}_{FM} = \hat{\boldsymbol{h}}_{FM} + \boldsymbol{e}_{FM}, \tag{8}$$

where $\hat{\boldsymbol{h}}_{FF} \in \mathbb{C}^{N_F}$ and $\hat{\boldsymbol{h}}_{FM} \in \mathbb{C}^{N_M}$ are the channel estimates of $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{FM}$, respectively, and $\boldsymbol{e}_{FF} \in \mathbb{C}^{N_F}$ and $\boldsymbol{e}_{FM} \in \mathbb{C}^{N_M}$ denote the corresponding estimation error vectors. Assume that

$$\boldsymbol{e}_{FF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_{e,FF}) \quad \text{and} \quad \boldsymbol{e}_{FM} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{C}_{e,FM}). \tag{9}$$

The model given by (8) is suitable for slow fading channels [19].

In the presence of CSI uncertainty, the beamforming solution to problem (5) will no longer guarantee the QoS requirement in (4) universally. To mitigate this QoS outage, our goal is to design the beamforming vector at the FBS, $\boldsymbol{w}_F$, such that the outage occurs below a small probability threshold. Mathematically, we come up with the following robust beamforming design problem:

$$\min_{\boldsymbol{w}_F \in \mathbb{C}^{N_F}} \ \|\boldsymbol{w}_F\|^2 \tag{10a}$$

$$\text{s.t. Prob}\left\{ \frac{\left|\boldsymbol{h}_{FF}^H \boldsymbol{w}_F\right|^2}{\left|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M\right|^2 + \sigma_F^2} \geq \gamma_F \right\} \geq 1 - \rho_F, \tag{10b}$$

$$\text{Prob}\left\{ |\boldsymbol{h}_{MF}^H \boldsymbol{w}_F|^2 \leq \epsilon_M \right\} \geq 1 - \rho_M, \tag{10c}$$

where ρ_F and ρ_M denote the preset maximum tolerable outage probabilities for SINR and interference power constraints, respectively.

We note that problem (10) is almost intractable since constraints (10b) and (10c) may not be convex and have no closed-form expressions in general. In Section III and Section IV, we will solve problem (10) for the *No CSI feedback* and *Partial CSI feedback* cases, respectively.

III. No CSI Feedback Scenario

In this section, we will show that problem (10) under the channel model in (7) can be formulated as a convex optimization problem by using SDR. To this end, we first find closed-form expressions for the probability functions in (10b) and (10c). For the probability function in (10b), one can observe that the random variables $|\boldsymbol{h}_{FF}^H \boldsymbol{w}_F|^2$ and $|\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2$ are independently exponential distributed with parameters $1/(\boldsymbol{w}_F^H \boldsymbol{C}_{h,FF} \boldsymbol{w}_F)$ and $1/(\boldsymbol{w}_M^H \boldsymbol{C}_{h,FM} \boldsymbol{w}_M)$, respectively, and the closed-form expression has been shown in [20, Appendix I]. Therefore, constraint (10b) can be equivalently written as

$$\exp\left(\frac{-\gamma_F \sigma_F^2}{\boldsymbol{w}_F^H \boldsymbol{C}_{h,FF} \boldsymbol{w}_F}\right) \frac{\boldsymbol{w}_F^H \boldsymbol{C}_{h,FF} \boldsymbol{w}_F}{\boldsymbol{w}_F^H \boldsymbol{C}_{h,FF} \boldsymbol{w}_F + \gamma_F \boldsymbol{w}_M^H \boldsymbol{C}_{h,FM} \boldsymbol{w}_M} \geq 1 - \rho_F. \tag{11}$$

The probability function in (10c) is in fact a cumulative distribution function (CDF) of an exponential random variable with parameter $1/(\boldsymbol{w}_F^H \boldsymbol{C}_{h,MF} \boldsymbol{w}_F)$. Therefore, constraint (10c) can be represented as

$$\boldsymbol{w}_F^H \boldsymbol{C}_{h,MF} \boldsymbol{w}_F \leq \epsilon_M / \ln(1/\rho_M). \tag{12}$$

As a result, problem (10) can be equivalently expressed as

$$\min_{\boldsymbol{w}_F \in \mathbb{C}^{N_F}} \ \|\boldsymbol{w}_F\|^2 \tag{13a}$$

$$\text{s.t. } \exp\left(\frac{-\gamma_F \sigma_F^2}{\boldsymbol{w}_F^H \boldsymbol{C}_{h,FF} \boldsymbol{w}_F}\right) \times \frac{\boldsymbol{w}_F^H \boldsymbol{C}_{h,FF} \boldsymbol{w}_F}{\boldsymbol{w}_F^H \boldsymbol{C}_{h,FF} \boldsymbol{w}_F + \gamma_F \boldsymbol{w}_M^H \boldsymbol{C}_{h,FM} \boldsymbol{w}_M} \geq 1 - \rho_F, \tag{13b}$$

$$\boldsymbol{w}_F^H \boldsymbol{C}_{h,MF} \boldsymbol{w}_F \leq \epsilon_M / \ln(1/\rho_M). \tag{13c}$$

Although problem (13) is still not convex due to nonconvex constraint (13b), the problem has a more tractable form than problem (10).

We now handle problem (13) by SDR [16], [21], [22]. Specifically, we define a positive semidefinite matrix $\boldsymbol{W}_F = \boldsymbol{w}_F \boldsymbol{w}_F^H$ and then relax the nonconvex rank-one constraint on $\boldsymbol{W}_F$. This yields the relaxed problem (14), shown at the top of the next page. The relaxed optimization problem in (14) is now convex (by the second-order condition of convexity), and can be efficiently solved using off-the-shelf optimization software.

If the obtained solution of problem (14), denoted by $\boldsymbol{W}_F^\star$, is of rank one, then we can simply perform the rank-one decomposition $\boldsymbol{W}_F^\star = \boldsymbol{w}_F^\star (\boldsymbol{w}_F^\star)^H$, and output $\boldsymbol{w}_F^\star$ as the optimal beamforming solution to problem (10), or equivalently problem (13). If $\boldsymbol{W}_F^\star$ has rank higher than one, one can apply some rank-one approximation procedure [22] to obtain a feasible beamforming solution for problem (13).

Next let us present a special case of independent identically distributed (i.i.d.) channels for $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$, and show that the power minimization problem (13) can be exactly solved in this case. Then, we investigate the feasibility condition for problem (13).

A. Spatially i.i.d. $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$

Assume that the channels $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$ are spatially i.i.d., i.e., [see (7)]

$$\boldsymbol{C}_{h,FF} = \sigma_{h,FF}^2 \boldsymbol{I}_{N_F} \quad \text{and} \quad \boldsymbol{C}_{h,MF} = \sigma_{h,MF}^2 \boldsymbol{I}_{N_F}. \tag{15}$$

This assumption is advisable for the case that FBS is located in a rich-scattered environment and the antennas at the FBS are spatially separated enough to be uncorrelated. Then, problem (13) can be simplified to the following power minimization problem:

$$\min_{P_F \in \mathbb{R}} \ P_F \triangleq \|\boldsymbol{w}_F\|^2 \tag{16a}$$

$$\text{s.t. } g(P_F) \triangleq \exp\left(\frac{-\gamma_F \sigma_F^2}{\sigma_{h,FF}^2 P_F}\right) \times \frac{\sigma_{h,FF}^2 P_F}{\sigma_{h,FF}^2 P_F + \gamma_F \boldsymbol{w}_M^H \boldsymbol{C}_{h,FM} \boldsymbol{w}_M} \geq 1 - \rho_F, \tag{16b}$$

$$P_F \leq \frac{\epsilon_M}{\sigma_{h,MF}^2 \ln(1/\rho_M)}, \tag{16c}$$

$$P_F \geq 0. \tag{16d}$$

Since the function $g(P_F)$ in constraint (16b) is strictly increasing in P_F, the optimal transmit power, denoted by $P_F^\star$, can be found by bisection [16]. In the bisection search, one can set the upper bound on P_F, denoted by $P_{F,\max}$, to [by (16c)]

$$P_{F,\max} = \frac{\epsilon_M}{\sigma_{h,MF}^2 \ln(1/\rho_M)}, \tag{17}$$

and the lower bound $P_{F,\min} = 0$. The optimal P_F is the one such that constraint (16b) is active. The resulting bisection algorithm for finding the optimal P_F to problem (16) is summarized in Algorithm 1. The optimal transmit beamformer is determined as $\boldsymbol{w}_F^\star = \sqrt{P_F^\star}\boldsymbol{u}_F$, where $\boldsymbol{u}_F \in \mathbb{C}^{N_F}$ is an arbitrary column vector with $\|\boldsymbol{u}_F\| = 1$.

B. *Feasibility Condition for Problem* (13)

Although problem (13) can be efficiently solved by using SDR [see (14)], it may not be feasible if the parameters, e.g., ρ_F, γ_F, ρ_M, and ϵ_M, are poorly chosen such that the feasible set under constraints (13b) and (13c) becomes empty. To this end, a sufficient and necessary condition under which problem (13) is feasible is given in the following proposition.

$$\min_{\boldsymbol{W}_F\in\mathbb{H}^{N_F}} \ \mathrm{Tr}(\boldsymbol{W}_F) \tag{14a}$$

$$\text{s.t.}\ \mathrm{Tr}(\boldsymbol{C}_{h,FF}\boldsymbol{W}_F)\geq(1-\rho_F)\left(\mathrm{Tr}(\boldsymbol{C}_{h,FF}\boldsymbol{W}_F)\exp\left(\frac{\gamma_F\sigma_F^2}{\mathrm{Tr}(\boldsymbol{C}_{h,FF}\boldsymbol{W}_F)}\right)+\gamma_F\boldsymbol{w}_M^H\boldsymbol{C}_{h,FM}\boldsymbol{w}_M\exp\left(\frac{\gamma_F\sigma_F^2}{\mathrm{Tr}(\boldsymbol{C}_{h,FF}\boldsymbol{W}_F)}\right)\right), \tag{14b}$$

$$\mathrm{Tr}(\boldsymbol{C}_{h,MF}\boldsymbol{W}_F)\leq\epsilon_M/\ln(1/\rho_M), \tag{14c}$$

$$\boldsymbol{W}_F\succeq\boldsymbol{0}. \tag{14d}$$

Algorithm 1 Bisection search for finding $P_F^\star$ to problems (16) and (29).

1: Set $P_{F,\min}=0$ and $P_{F,\max}=\epsilon_M/(\sigma_{h,MF}^2\ln(1/\rho_M))$; set a solution accuracy $\epsilon>0$;
2: **repeat**
3: find the value of $g(P_F)$ in (16b) [or $f(P_F)$ in (29b)] for $P_F=(P_{F,\min}+P_{F,\max})/2$;
4: if $g(P_F)\geq 1-\rho_F$ [or $f(P_F)\geq 1-\rho_F$], then update $P_{F,\max}=P_F$; otherwise, update $P_{F,\min}=P_F$;
5: **until** $P_{F,\max}-P_{F,\min}\leq\epsilon$; then output P_F as the desired transmit power.

Proposition 1 *Problem* (10) *under no CSI feedback scenario, or equivalently problem* (13)*, is feasible if and only if*

$$\exp\left(-\frac{\gamma_F\sigma_F^2\ln(1/\rho_M)}{\lambda_{\max}\epsilon_M}\right)\times\frac{\lambda_{\max}\epsilon_M}{\lambda_{\max}\epsilon_M+\gamma_F\boldsymbol{w}_M^H\boldsymbol{C}_{h,FM}\boldsymbol{w}_M\ln(1/\rho_M)}\geq 1-\rho_F, \tag{18}$$

where $\lambda_{\max}>0$ *is the maximum eigenvalue of* $\boldsymbol{C}_{h,MF}^{-1/2}\boldsymbol{C}_{h,FF}\boldsymbol{C}_{h,MF}^{-1/2}$.

Proof: See Appendix A. ■

Remark 1 *Following similar derivations in Proposition 1 with* $\boldsymbol{C}_{h,FF}$ *and* $\boldsymbol{C}_{h,MF}$ *replaced by* $\boldsymbol{h}_{FF}\boldsymbol{h}_{FF}^H$ *and* $\boldsymbol{h}_{MF}\boldsymbol{h}_{MF}^H$*, respectively, we can show that problem* (5) *is feasible if and only if*

$$\lambda_{\max}\epsilon_M\geq\gamma_F\left(|\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2+\sigma_F^2\right), \tag{19}$$

where $\lambda_{\max}>0$ *is the principal generalized eigenvalue for the matrix pair* $(\boldsymbol{h}_{FF}\boldsymbol{h}_{FF}^H,\boldsymbol{h}_{MF}\boldsymbol{h}_{MF}^H)$.

Remark 2 *Consider the case that only the statistical information of* $\boldsymbol{w}_M$ *(rather than the exact* $\boldsymbol{w}_M$*) is known to FBS, e.g.,* $\boldsymbol{w}_M$ *is selected from a codebook* $\mathcal{W}$*. Let* p_i *denote the probability that the MUE picks the* i*th beamforming vector* $\boldsymbol{w}_{M,i}\in\mathcal{W}$ *and feeds back its index using* B *bits to the MBS. Then constraint* (10b) *averaged over the codebook* $\mathcal{W}$ *under the channel model* (7) *can be expressed as [see* (11)*]*

$$\exp\left(\frac{-\gamma_F\sigma_F^2}{\boldsymbol{w}_F^H\boldsymbol{C}_{h,FF}\boldsymbol{w}_F}\right)\times\sum_{i=1}^{2^B}p_i\frac{\boldsymbol{w}_F^H\boldsymbol{C}_{h,FF}\boldsymbol{w}_F}{\boldsymbol{w}_F^H\boldsymbol{C}_{h,FF}\boldsymbol{w}_F+\gamma_F\boldsymbol{w}_{M,i}^H\boldsymbol{C}_{h,FM}\boldsymbol{w}_{M,i}}\geq 1-\rho_F. \tag{20}$$

Applying SDR to (20) *gives rise to*

$$\sum_{i=1}^{2^B}p_i\frac{\mathrm{Tr}(\boldsymbol{C}_{h,FF}\boldsymbol{W}_F)}{\mathrm{Tr}(\boldsymbol{C}_{h,FF}\boldsymbol{W}_F)+\gamma_F\boldsymbol{w}_{M,i}^H\boldsymbol{C}_{h,FM}\boldsymbol{w}_{M,i}}\geq(1-\rho_F)\exp\left(\frac{\gamma_F\sigma_F^2}{\mathrm{Tr}(\boldsymbol{C}_{h,FF}\boldsymbol{W}_F)}\right), \tag{21}$$

which is still a convex constraint (by the second-order condition of convexity). Thus, problem (14) *with constraint* (14b) *replaced by* (21) *remains convex for this case.*

IV. Partial CSI Feedback Scenario

In the previous section, we have presented the transmit power minimization problem when the FBS has no instantaneous CSI feedback from the FUE, but knows the statistical information of the channels. Obviously, when the FBS can obtain its downlink CSI from the FUE, its transmit power performance will be improved expectantly. In this section, we will discuss the transmit power minimization problem for the partial CSI feedback case that the FBS acquires imperfect channel estimates of $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{FM}$ modeled by (8), but knows only the statistical information of $\boldsymbol{h}_{MF}\sim\mathcal{CN}(\boldsymbol{0},\boldsymbol{C}_{h,MF})$ (due to the lack of feedback link from MUE). For this case, problem (10) can be written as

$$\min_{\boldsymbol{w}_F\in\mathbb{C}^{N_F}}\ \|\boldsymbol{w}_F\|^2 \tag{22a}$$

$$\text{s.t.}\ \mathrm{Prob}\left\{\frac{|(\hat{\boldsymbol{h}}_{FF}+\boldsymbol{e}_{FF})^H\boldsymbol{w}_F|^2}{|(\hat{\boldsymbol{h}}_{FM}+\boldsymbol{e}_{FM})^H\boldsymbol{w}_M|^2+\sigma_F^2}\geq\gamma_F\right\}\geq 1-\rho_F, \tag{22b}$$

$$\mathrm{Prob}\left\{|\boldsymbol{h}_{MF}^H\boldsymbol{w}_F|^2\leq\epsilon_M\right\}\geq 1-\rho_M. \tag{22c}$$

Again, problem (22) is difficult to solve since the probability function in (22b) has no closed-form expression and may not be convex in general.

To proceed, we first formulate constraint (22b) into a tractable form. One can show that under (9) the random variables

$$X_{FF}=\frac{2|(\hat{\boldsymbol{h}}_{FF}+\boldsymbol{e}_{FF})^H\boldsymbol{w}_F|^2}{\boldsymbol{w}_F^H\boldsymbol{C}_{e,FF}\boldsymbol{w}_F},\ X_{FM}=\frac{2|(\hat{\boldsymbol{h}}_{FM}+\boldsymbol{e}_{FM})^H\boldsymbol{w}_M|^2}{\boldsymbol{w}_M^H\boldsymbol{C}_{e,FM}\boldsymbol{w}_M} \tag{23}$$

are noncentral chi-squared distributed [23] with two DoF and noncentrality parameters ζ_{FF} and ζ_{FM}, respectively, where

$$\zeta_{FF}=\frac{2\left|\hat{\boldsymbol{h}}_{FF}^H\boldsymbol{w}_F\right|^2}{\boldsymbol{w}_F^H\boldsymbol{C}_{e,FF}\boldsymbol{w}_F}\ \text{and}\ \zeta_{FM}=\frac{2\left|\hat{\boldsymbol{h}}_{FM}^H\boldsymbol{w}_M\right|^2}{\boldsymbol{w}_M^H\boldsymbol{C}_{e,FM}\boldsymbol{w}_M}. \tag{24}$$

$$\mathbb{E}\left[\text{Prob}\left\{X_{FF} \geq \frac{2\gamma_F}{\boldsymbol{w}_F^H \boldsymbol{C}_{e,FF}\boldsymbol{w}_F}\left(\frac{\boldsymbol{w}_M^H \boldsymbol{C}_{e,FM}\boldsymbol{w}_M}{2}x_{FM} + \sigma_F^2\right) \;\middle|\; X_{FM} = x_{FM}\right\}\right] \geq 1-\rho_F$$

$$\Rightarrow \mathbb{E}\left[Q_1\left(\frac{\sqrt{2}|\hat{\boldsymbol{h}}_{FF}^H \boldsymbol{w}_F|}{\sqrt{\boldsymbol{w}_F^H \boldsymbol{C}_{e,FF}\boldsymbol{w}_F}}, \sqrt{\frac{\gamma_F\left(\boldsymbol{w}_M^H \boldsymbol{C}_{e,FM}\boldsymbol{w}_M X_{FM} + 2\sigma_F^2\right)}{\boldsymbol{w}_F^H \boldsymbol{C}_{e,FF}\boldsymbol{w}_F}}\right)\right] \geq 1-\rho_F, \tag{25}$$

Therefore, according to (23) and (24) and using the total probability theory with respect to X_{FM}, constraint (22b) can be equivalently expressed as in (25), shown at the top of the next page, where $Q_1(\alpha,\beta)$ denotes the first-order Marcum's Q-function [24], defined as

$$Q_1(\alpha,\beta) = \int_\beta^\infty x\exp\left(-\frac{x^2+\alpha^2}{2}\right) I_0(\alpha x)dx \tag{26}$$

in which $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind.

In what follows, we will solve problem (22) with constraint (22b) replaced by (25). We first consider the special case of $\boldsymbol{C}_{h,MF} = \sigma_{h,MF}^2 \boldsymbol{I}_{N_F}$ and $\boldsymbol{C}_{e,FF} = \sigma_{e,FF}^2 \boldsymbol{I}_{N_F}$, and then present a bisection-based method for obtaining the exact solution. Then, we discuss how to handle problem (22) for the case of general channel and error covariance matrices.

A. Spatially i.i.d. $\boldsymbol{h}_{MF}$ and $\boldsymbol{e}_{FF}$

When $\boldsymbol{C}_{h,MF} = \sigma_{h,MF}^2 \boldsymbol{I}_{N_F}$ and $\boldsymbol{C}_{e,FF} = \sigma_{e,FF}^2 \boldsymbol{I}_{N_F}$, constraint (22b) can be equivalently expressed as [see (25)]

$$\mathbb{E}\left[Q_1\left(\frac{\sqrt{2}|\hat{\boldsymbol{h}}_{FF}^H \boldsymbol{u}_F|}{\sigma_{e,FF}}, \sqrt{\frac{\gamma_F(\boldsymbol{w}_M^H \boldsymbol{C}_{e,FM}\boldsymbol{w}_M X_{FM} + 2\sigma_F^2)}{\sigma_{e,FF}^2\|\boldsymbol{w}_F\|^2}}\right)\right] \geq 1-\rho_F, \tag{27}$$

where $\boldsymbol{u}_F$ denotes the normalized beamforming vector $\boldsymbol{w}_F$, i.e., $\boldsymbol{u}_F \triangleq \boldsymbol{w}_F/\|\boldsymbol{w}_F\|$. Furthermore, it has been shown in [25, Theorem 1] that $Q_1(\alpha,\beta)$ is strictly increasing in $\alpha \geq 0$ for $\beta > 0$, and is strictly decreasing in $\beta \geq 0$ for $\alpha \geq 0$. According to this fact, one can see that constraint (27) must be active when the optimal solution to problem (22) is achieved; otherwise, one can obtain a lower transmit power $\|\boldsymbol{w}_F\|^2$. Also, the optimal $\boldsymbol{u}_F^\star$ can be easily seen to be $\boldsymbol{u}_F^\star = \hat{\boldsymbol{h}}_{FF}/\|\hat{\boldsymbol{h}}_{FF}\|$, for which the function on the left-hand side of constraint (27) can be represented by

$$f(P_F \triangleq \|\boldsymbol{w}_F\|^2) = \frac{1}{2}\int_0^\infty Q_1\left(\frac{\sqrt{2}\|\hat{\boldsymbol{h}}_{FF}\|}{\sigma_{e,FF}}, \sqrt{\frac{\gamma_F(\boldsymbol{w}_M^H \boldsymbol{C}_{e,FM}\boldsymbol{w}_M x_{FM}+2\sigma_F^2)}{\sigma_{e,FF}^2 P_F}}\right) \times \exp\left(-\frac{x_{FM}+\zeta_{FM}}{2}\right) I_0(\sqrt{\zeta_{FM}x_{FM}})dx_{FM}, \tag{28}$$

where ζ_{FM} is defined in (24). As a result, problem (22) can be equivalently expressed as

$$\min_{P_F\in\mathbb{R}} \; P_F \tag{29a}$$
$$\text{s.t.} \; f(P_F) \geq 1-\rho_F, \tag{29b}$$
$$P_F \leq \frac{\epsilon_M}{\sigma_{h,MF}^2 \ln(1/\rho_M)}, \tag{29c}$$
$$P_F \geq 0. \tag{29d}$$

Since $f(P_F)$ is an increasing function of P_F, again one can use the bisection method to find the optimal $P_F^\star$ for which $f(P_F^\star) = 1-\rho_F$. By setting the upper bound on P_F to $P_{F,\max} = \epsilon_M/(\sigma_{h,MF}^2\ln(1/\rho_M))$ [see (29c), which is the same as (16c)] and setting $P_{F,\min} = 0$, one can obtain the optimal P_F to problem (29) using Algorithm 1. Once the transmit power $P_F^\star$ is obtained, the optimal transmit beamformer $\boldsymbol{w}_F^\star$ is determined as $\boldsymbol{w}_F^\star = \sqrt{P_F^\star}\hat{\boldsymbol{h}}_{FF}/\|\hat{\boldsymbol{h}}_{FF}\|$.

Remark 3 *For $\boldsymbol{C}_{h,MF}=\sigma_{h,MF}^2\boldsymbol{I}_{N_F}$ and $\boldsymbol{C}_{e,FF}=\sigma_{e,FF}^2\boldsymbol{I}_{N_F}$, the optimal transmit beamformer $\boldsymbol{w}_F^\star=\sqrt{P_F^\star}\hat{\boldsymbol{h}}_{FF}/\|\hat{\boldsymbol{h}}_{FF}\|$ to problem* (22) *is feasible if and only if*

$$f(\epsilon_M/(\sigma_{h,MF}^2\ln(1/\rho_M))) \geq 1-\rho_F, \tag{30}$$

where the function $f(\cdot)$ is defined in (28)*. This result can be straightforwardly shown by utilizing the monotonicity of $f(\cdot)$.*

B. Spatially Correlated $\boldsymbol{h}_{MF}$, $\boldsymbol{e}_{FF}$, and $\boldsymbol{e}_{FM}$

We now consider problem (22) when channel vector $\boldsymbol{h}_{MF}$ and error vectors $\boldsymbol{e}_{FF}$ and $\boldsymbol{e}_{FM}$ are correlated complex Gaussian distributed. Although constraint (22b) can be represented by (25), problem (22) is still difficult to solve. To proceed, we will follow the relax-and-restrict (RAR) approach in [26] to tackle problem (22). More specifically, we first apply SDR to problem (22), and then conservatively approximate the SDR problem by tractable (convex) constraints based on a Bernstein-type inequality [27].

To illustrate the method, let us express

$$\boldsymbol{e}_{FF} = \boldsymbol{C}_{e,FF}^{1/2}\boldsymbol{v}_{FF} \;\text{ and }\; \boldsymbol{e}_{FM} = \boldsymbol{C}_{e,FM}^{1/2}\boldsymbol{v}_{FM}, \tag{31}$$

where $\boldsymbol{C}_{e,FF}^{1/2} \succeq \boldsymbol{0}$ and $\boldsymbol{C}_{e,FM}^{1/2} \succeq \boldsymbol{0}$ are the positive semidefinite square roots of $\boldsymbol{C}_{e,FF}$ and $\boldsymbol{C}_{e,FM}$, respectively, and $\boldsymbol{v}_{FF} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{I}_{N_F})$ and $\boldsymbol{v}_{FM} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{I}_{N_M})$. Let

$$\hat{\boldsymbol{h}} = \begin{bmatrix}\hat{\boldsymbol{h}}_{FF}\\ \hat{\boldsymbol{h}}_{FM}\end{bmatrix},\; \boldsymbol{C}^{1/2} = \begin{bmatrix}\boldsymbol{C}_{e,FF}^{1/2} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{C}_{e,FM}^{1/2}\end{bmatrix} \text{ and } \boldsymbol{v} = \begin{bmatrix}\boldsymbol{v}_{FF}\\ \boldsymbol{v}_{FM}\end{bmatrix}, \tag{32}$$

where $\boldsymbol{v} \sim \mathcal{CN}(\boldsymbol{0},\boldsymbol{I}_{N_F+N_M})$ since $\boldsymbol{v}_{FF}$ and $\boldsymbol{v}_{FM}$ are statistically independent. Applying SDR to problem (22), we have

$$\min_{\boldsymbol{W}_F\in\mathbb{H}^{N_F}} \; \text{Tr}(\boldsymbol{W}_F) \tag{33a}$$
$$\text{s.t.} \; \text{Prob}\left\{\boldsymbol{v}^H\boldsymbol{\Phi}(\boldsymbol{W}_F)\boldsymbol{v}+2\text{Re}\left\{\boldsymbol{v}^H\boldsymbol{\eta}(\boldsymbol{W}_F)\right\}\geq s(\boldsymbol{W}_F)\right\} \geq 1-\rho_F, \tag{33b}$$
$$\text{Tr}(\boldsymbol{C}_{h,MF}\boldsymbol{W}_F) \leq \epsilon_M/\ln(1/\rho_M), \tag{33c}$$
$$\boldsymbol{W}_F \succeq \boldsymbol{0}, \tag{33d}$$

where

$$\boldsymbol{\Phi}(\boldsymbol{W}_F) \triangleq \boldsymbol{C}^{1/2}\boldsymbol{W}\boldsymbol{C}^{1/2},\ \boldsymbol{\eta}(\boldsymbol{W}_F) \triangleq \boldsymbol{C}^{1/2}\boldsymbol{W}\hat{\boldsymbol{h}}, \tag{34a}$$

$$s(\boldsymbol{W}_F) \triangleq \sigma_F^2 - \hat{\boldsymbol{h}}^H\boldsymbol{W}\hat{\boldsymbol{h}}, \tag{34b}$$

in which

$$\boldsymbol{W} \triangleq \begin{bmatrix} \frac{1}{\gamma_F}\boldsymbol{W}_F & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{w}_M\boldsymbol{w}_M^H \end{bmatrix}. \tag{35}$$

However, problem (33) is still intractable since the probability function in (33b) does not have closed-form expression in general due to indefinite $\boldsymbol{W}$. It has been shown in [26] that the constraint in the form (33b) can be conservatively approximated by tractable convex constraints based on a Bernstein-type inequality [27] as follows:

$$\mathrm{Tr}\left(\boldsymbol{\Phi}(\boldsymbol{W}_F)\right) - \sqrt{2\delta}x - \delta y \geq s(\boldsymbol{W}_F), \tag{36a}$$

$$\sqrt{\|\boldsymbol{\Phi}(\boldsymbol{W}_F)\|_F^2 + 2\|\boldsymbol{\eta}(\boldsymbol{W}_F)\|^2} \leq x, \tag{36b}$$

$$y\boldsymbol{I}_{N_F+N_M} + \boldsymbol{\Phi}(\boldsymbol{W}_F) \succeq \boldsymbol{0}, \tag{36c}$$

$$y \geq 0, \tag{36d}$$

where $\delta \triangleq -\ln(\rho_F)$. Specifically, the constraints in (36) can be represented as in (37), shown at the top of the next page, which is a convex constraint set with respect to $(\boldsymbol{W}_F, x, y)$, and the constant ξ_{FM} in (37b) is defined as

$$\xi_{FM} \triangleq \gamma_F\sqrt{\|\boldsymbol{C}_{e,FM}^{1/2}\boldsymbol{w}_M\boldsymbol{w}_M^H\boldsymbol{C}_{e,FM}^{1/2}\|_F^2 + 2\|\boldsymbol{C}_{e,FM}^{1/2}\boldsymbol{w}_M\boldsymbol{w}_M^H\hat{\boldsymbol{h}}_{FM}\|^2}. \tag{38}$$

To minimize the transmit power $\mathrm{Tr}(\boldsymbol{W}_F)$, one can infer from (37) that y must be the principal eigenvalue of $\boldsymbol{C}_{e,FM}^{1/2}\boldsymbol{w}_M\boldsymbol{w}_M^H\boldsymbol{C}_{e,FM}^{1/2}$, i.e., $y = \|\boldsymbol{C}_{e,FM}^{1/2}\boldsymbol{w}_M\|^2$, when the optimal $\boldsymbol{W}_F$ is achieved. As a result, by (33) and (37) with y replaced by $\|\boldsymbol{C}_{e,FM}^{1/2}\boldsymbol{w}_M\|^2$, a tractable approximation to problem (22) is given by

$$\min_{\boldsymbol{W}_F\in\mathbb{H}^{N_F},\ x\in\mathbb{R}} \ \mathrm{Tr}(\boldsymbol{W}_F) \tag{39a}$$

$$\text{s.t. } \frac{1}{\gamma_F}\mathrm{Tr}\big((\boldsymbol{C}_{e,FF} + \hat{\boldsymbol{h}}_{FF}\hat{\boldsymbol{h}}_{FF}^H)\boldsymbol{W}_F\big) - \sqrt{2\delta}x \geq \sigma_F^2 + \boldsymbol{w}_M^H\big((1+\delta)\boldsymbol{C}_{e,FM} + \hat{\boldsymbol{h}}_{FM}\hat{\boldsymbol{h}}_{FM}^H\big)\boldsymbol{w}_M, \tag{39b}$$

$$\frac{1}{\gamma_F}\left\|\begin{bmatrix} \mathrm{vec}\big(\boldsymbol{C}_{e,FF}^{1/2}\boldsymbol{W}_F\boldsymbol{C}_{e,FF}^{1/2}\big) \\ \sqrt{2}\mathrm{vec}\big(\boldsymbol{C}_{e,FF}^{1/2}\boldsymbol{W}_F\hat{\boldsymbol{h}}_{FF}\big) \\ \xi_{FM} \end{bmatrix}\right\| \leq x, \tag{39c}$$

$$\mathrm{Tr}(\boldsymbol{C}_{h,MF}\boldsymbol{W}_F) \leq \epsilon_M/\ln(1/\rho_M), \tag{39d}$$

$$\boldsymbol{W}_F \succeq \boldsymbol{0}, \tag{39e}$$

where $\delta \triangleq -\ln(\rho_F)$ and ξ_{FM} is defined in (38). Problem (39) is convex, and can be efficiently solved to yield a global optimal $\boldsymbol{W}_F$.

If the obtained solution $\boldsymbol{W}_F^\star$ is not of rank one, then the rank-one approximation procedure [22] can be applied to obtain a feasible (conservative) beamforming solution to problem (22).

V. Optimal Transmission Strategy

A. Problem Generalization

In the original beamforming problem (10), it was implicitly assumed the transmitted signal $\boldsymbol{x}(t)$ at FBS [see (1)] has zero mean and rank-one covariance matrix $\boldsymbol{w}_F\boldsymbol{w}_F^H$. In spite of low implementation complexity of the designed beamformer at the FBS, the system performance may not be optimal in the sense of minimum transmit power. In this section, we consider a more general linear precoding matrix $\boldsymbol{P}_F$ such that the covariance matrix of the transmit signal $\boldsymbol{x}(t)$ is given by $\boldsymbol{Q}_F = \mathbb{E}[\boldsymbol{x}(t)\boldsymbol{x}(t)^H] = \boldsymbol{P}_F\boldsymbol{P}_F^H$[1]. We can directly optimize the covariance matrix $\boldsymbol{Q}_F \succeq \boldsymbol{0}$ such that transmit power $\mathrm{Tr}(\boldsymbol{Q}_F)$ is minimized while satisfying the QoS and interference constraints. Mathematically, the problem can be reformulated as

$$\min_{\boldsymbol{Q}_F\in\mathbb{H}^{N_F}} \ \mathrm{Tr}(\boldsymbol{Q}_F) \tag{40a}$$

$$\text{s.t. } \mathrm{Prob}\left\{\frac{\boldsymbol{h}_{FF}^H\boldsymbol{Q}_F\boldsymbol{h}_{FF}}{\left|\boldsymbol{h}_{FM}^H\boldsymbol{w}_M\right|^2 + \sigma_F^2} \geq \gamma_F\right\} \geq 1-\rho_F, \tag{40b}$$

$$\mathrm{Prob}\left\{\boldsymbol{h}_{MF}^H\boldsymbol{Q}_F\boldsymbol{h}_{MF} \leq \epsilon_M\right\} \geq 1-\rho_M, \tag{40c}$$

$$\boldsymbol{Q}_F \succeq \boldsymbol{0}. \tag{40d}$$

Obviously, the optimal transmission power obtained by solving problem (40) will be lower than that obtained by solving problem (10), due to the transmit signal has higher DoF for the former. Similar to problem (10), problem (40) is not tractable either because of the intricate probability functions in (40b) and (40c).

B. Spatially i.i.d. $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$ (No CSI Feedback)

Let us consider the case that the FBS knows only the statistical information of the downlink channels and both the channels $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$ are spatially i.i.d. [see (15)], as considered in Section III-A for beamforming design.

To solve problem (40), let us express

$$\boldsymbol{Q}_F = \mathrm{Tr}(\boldsymbol{Q}_F)\tilde{\boldsymbol{Q}}_F,\ \boldsymbol{h}_{FF} = \sigma_{h,FF}\cdot\boldsymbol{v}_{FF},\ \boldsymbol{h}_{MF} = \sigma_{h,MF}\cdot\boldsymbol{v}_{MF} \tag{41}$$

where $\tilde{\boldsymbol{Q}}_F \succeq \boldsymbol{0}$, $\mathrm{Tr}(\tilde{\boldsymbol{Q}}_F) = 1$; $\boldsymbol{v}_{FF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{N_F})$; and $\boldsymbol{v}_{MF} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I}_{N_F})$ are power normalized counterparts of $\boldsymbol{Q}_F$, $\boldsymbol{h}_{FF}$, and $\boldsymbol{h}_{MF}$, respectively. With the expressions in

[1]The detailed discussion on the signal processing for generating a signal with covariance matrix $\boldsymbol{Q}_F$ at the FBS can be found in [28]. This technique can be found in several published works, e.g., [28]–[30] to improve system performance under the single-user MISO scenario.

$$\frac{1}{\gamma_F}\mathrm{Tr}\left(\left(\boldsymbol{C}_{e,FF}+\hat{\boldsymbol{h}}_{FF}\hat{\boldsymbol{h}}_{FF}^H\right)\boldsymbol{W}_F\right)-\sqrt{2\delta}x-\delta y\geq\sigma_F^2+\boldsymbol{w}_M^H\left(\boldsymbol{C}_{e,FM}+\hat{\boldsymbol{h}}_{FM}\hat{\boldsymbol{h}}_{FM}^H\right)\boldsymbol{w}_M, \tag{37a}$$

$$\frac{1}{\gamma_F}\left\|\begin{bmatrix}\mathrm{vec}\left(\boldsymbol{C}_{e,FF}^{1/2}\boldsymbol{W}_F\boldsymbol{C}_{e,FF}^{1/2}\right)\\ \sqrt{2}\mathrm{vec}\left(\boldsymbol{C}_{e,FF}^{1/2}\boldsymbol{W}_F\hat{\boldsymbol{h}}_{FF}\right)\\ \xi_{FM}\end{bmatrix}\right\|\leq x, \tag{37b}$$

$$y\boldsymbol{I}_{N_F+N_M}+\begin{bmatrix}\frac{1}{\gamma_F}\boldsymbol{C}_{e,FF}^{1/2}\boldsymbol{W}_F\boldsymbol{C}_{e,FF}^{1/2} & \boldsymbol{0}\\ \boldsymbol{0} & -\boldsymbol{C}_{e,FM}^{1/2}\boldsymbol{w}_M\boldsymbol{w}_M^H\boldsymbol{C}_{e,FM}^{1/2}\end{bmatrix}\succeq\boldsymbol{0}, \tag{37c}$$

$$y\geq 0, \tag{37d}$$

(41), problem (40) can be equivalently written as

$$\min_{\substack{\tilde{\boldsymbol{Q}}_F\in\mathbb{H}^{N_F},\\ P_F\in\mathbb{R}}}\ P_F\triangleq\mathrm{Tr}(\boldsymbol{Q}_F) \tag{42a}$$

$$\text{s.t. }\mathrm{Prob}\left\{\boldsymbol{v}_{FF}^H\tilde{\boldsymbol{Q}}_F\boldsymbol{v}_{FF}\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}\times\left(|\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2+\sigma_F^2\right)\right\}\leq\rho_F, \tag{42b}$$

$$\mathrm{Prob}\left\{\boldsymbol{v}_{MF}^H\tilde{\boldsymbol{Q}}_F\boldsymbol{v}_{MF}\leq\frac{\epsilon_M}{\sigma_{h,MF}^2P_F}\right\}\geq 1-\rho_M, \tag{42c}$$

$$\tilde{\boldsymbol{Q}}_F\succeq\boldsymbol{0},\ \mathrm{Tr}(\tilde{\boldsymbol{Q}}_F)=1,\ P_F\geq 0. \tag{42d}$$

We should point out that constraint (42c) actually provides an upper bound on P_F for which problem (42) is feasible. Therefore, to minimize the transmit power P_F, we only need to consider constraints (42b) and (42d).

Let us first consider problem (42) with constraint (42c) relaxed, i.e.,

$$\min_{\substack{\tilde{\boldsymbol{Q}}_F\in\mathbb{H}^{N_F},\\ P_F\in\mathbb{R}}}\ P_F \tag{43a}$$

$$\text{s.t. }\mathrm{Prob}\left\{\boldsymbol{v}_{FF}^H\tilde{\boldsymbol{Q}}_F\boldsymbol{v}_{FF}\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}\times\left(|\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2+\sigma_F^2\right)\right\}\leq\rho_F, \tag{43b}$$

$$\tilde{\boldsymbol{Q}}_F\succeq\boldsymbol{0},\ \mathrm{Tr}(\tilde{\boldsymbol{Q}}_F)=1,\ P_F\geq 0. \tag{43c}$$

One can observe that constraint (43b) must be active when the optimal $\tilde{\boldsymbol{Q}}_F$ and P_F are achieved. To proceed, we need the following lemma, and the proof is presented in Appendix B.

Lemma 1 *Minimizing the probability function in* (43b) *with respect to $\tilde{\boldsymbol{Q}}_F$ under the constraints in* (43c) *will minimize P_F.*

According to Lemma 1, the optimal $\tilde{\boldsymbol{Q}}_F$ is obtained by solving

$$\min_{\substack{\tilde{\boldsymbol{Q}}_F\succeq\boldsymbol{0},\\ \mathrm{Tr}(\tilde{\boldsymbol{Q}}_F)=1}}\ \mathbb{E}\left[\mathrm{Prob}\left\{\boldsymbol{v}_{FF}^H\tilde{\boldsymbol{Q}}_F\boldsymbol{v}_{FF}\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}\times(z+\sigma_F^2)\ \middle|\ |\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2=z\right\}\right], \tag{44}$$

where the expectation is taken with respect to the exponential random variable $|\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2$. With the optimal $\tilde{\boldsymbol{Q}}_F$, denoted by $\tilde{\boldsymbol{Q}}_F^\star$, the optimal $P_F^\star$ to problem (43) can be obtained by solving

$$\mathbb{E}\left[\mathrm{Prob}\left\{\boldsymbol{v}_{FF}^H\tilde{\boldsymbol{Q}}_F^\star\boldsymbol{v}_{FF}\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}\times(z+\sigma_F^2)\ \middle|\ |\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2=z\right\}\right]=\rho_F. \tag{45}$$

However, the optimal transmission strategy $\boldsymbol{Q}_F^\star=P_F^\star\tilde{\boldsymbol{Q}}_F^\star$ [see (41)] for problem (43) may not be feasible to problem (42) because it may not satisfy the probability constraint (42c). Due to this concern, in the next subsection, we will propose a feasibility condition under which $\boldsymbol{Q}_F^\star$ is feasible and optimal to problem (42).

We now concentrate on solving problem (44). Consider the eigenvalue decomposition of $\tilde{\boldsymbol{Q}}_F=\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^H$, where $\boldsymbol{U}\in\mathbb{C}^{N_F\times N_F}$ is a unitary matrix and $\boldsymbol{\Lambda}\in\mathbb{R}^{N_F\times N_F}$ is a diagonal matrix with eigenvalues $\lambda_1,\ldots,\lambda_{N_F}\geq 0$ being the diagonal elements. Since any unitary transformation of a Gaussian random vector will not make any change in its probability density distribution, the expectation function in (44) can be equivalently written as

$$\begin{aligned}&\mathbb{E}\left[\mathrm{Prob}\left\{\boldsymbol{v}_{FF}^H\boldsymbol{\Lambda}\boldsymbol{v}_{FF}\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}\times(z+\sigma_F^2)\ \middle|\ |\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2=z\right\}\right]\\ &=\mathbb{E}\left[\mathrm{Prob}\left\{\sum_{i=1}^{N_F}\lambda_i|v_i|^2\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}\times(z+\sigma_F^2)\ \middle|\ |\boldsymbol{h}_{FM}^H\boldsymbol{w}_M|^2=z\right\}\right],\end{aligned} \tag{46}$$

where v_i denotes the ith entry of $\boldsymbol{v}_{FF}$. Moreover, it has been shown in [30], [31] that for a given transmission DoF equal to d, i.e., the number of positive eigenvalues in $\tilde{\boldsymbol{Q}}_F$, the probability function in (46) is minimized by uniformly allocating the total power over d DoF, namely,

$$\begin{aligned}&\mathrm{Prob}\left\{\frac{1}{d}\sum_{i=1}^{d}|v_i|^2\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}(z+\sigma_F^2)\right\}\\ &\leq\mathrm{Prob}\left\{\sum_{i=1}^{d}\lambda_i|v_i|^2\leq\frac{\gamma_F}{\sigma_{h,FF}^2P_F}(z+\sigma_F^2)\right\},\end{aligned} \tag{47}$$

for all $z \geq 0$, where $\lambda_i > 0$, $i = 1, \ldots, d$, and $\sum_{i=1}^{d} \lambda_i = 1$.

According to (47), one can easily show that the optimal $\tilde{\boldsymbol{Q}}_F$ to problem (44), or equivalently problem (43), can be expressed by

$$\tilde{\boldsymbol{Q}}_F^{\star} = \boldsymbol{U}^{\star} \boldsymbol{\Lambda}(d^{\star}) (\boldsymbol{U}^{\star})^H, \tag{48}$$

where $d^{\star}$ denotes the optimal DoF (to be presented below); $\boldsymbol{U}^{\star} \in \mathbb{C}^{N_F \times N_F}$ can be any arbitrary unitary matrix and $\boldsymbol{\Lambda}(d^{\star}) \in \mathbb{R}^{N_F \times N_F}$ is a diagonal matrix with the first $d^{\star}$ diagonal elements being nonzero and equal to $1/d^{\star}$ (due to $\mathrm{Tr}(\tilde{\boldsymbol{Q}}_F^{\star}) = 1$).

The optimal $d^{\star}$ is obtained as

$$d^{\star} = \arg\min \{P_F(d),\ d = 1, \ldots, N_F\}, \tag{49}$$

where $P_F(d)$ is the power by numerically solving (45), which can be equivalently expressed as

$$\mathbb{E}\left[\mathrm{Prob}\left\{\chi_{2d}^2 \leq \frac{2d\gamma_F}{\sigma_{h,FF}^2 P_F}(z+\sigma_F^2) \;\middle|\; |\boldsymbol{h}_{FM}^H \boldsymbol{w}_M|^2 = z\right\}\right] = \rho_F$$
$$\Rightarrow \frac{1}{(d-1)!\boldsymbol{w}_M^H \boldsymbol{Q}_{FM} \boldsymbol{w}_M} \int_0^{\infty} \Gamma\left(d, \frac{d\gamma_F}{\sigma_{h,FF}^2 P_F}(z+\sigma_F^2)\right) \times \exp\left(-\frac{z}{\boldsymbol{w}_M^H \boldsymbol{Q}_{FM} \boldsymbol{w}_M}\right) dz = \rho_F, \tag{50}$$

where χ_d^2 denotes the central chi-square random variable with d DoF, and $\Gamma(\alpha, \beta)$ is lower incomplete gamma function, defined as

$$\Gamma(\alpha, \beta) = \int_0^{\beta} t^{\alpha-1} \exp(-t)\, dt. \tag{51}$$

As a result, the optimal transmission strategy for problem (43) can be expressed as [see (41)]

$$\boldsymbol{Q}_F^{\star} = P_F(d^{\star}) \tilde{\boldsymbol{Q}}_F^{\star}. \tag{52}$$

On the other hand, constraint (42c) with $d = d^{\star}$ can be represented by

$$\mathrm{Prob}\left\{\frac{1}{2d^{\star}} \chi_{2d^{\star}}^2 \leq \frac{\epsilon_M}{\sigma_{h,MF}^2 P_F}\right\} \geq 1 - \rho_M$$
$$\Rightarrow P_F \leq \frac{\epsilon_M}{\sigma_{h,MF}^2 F_{2d^{\star}}^{-1}(1-\rho_M)} \triangleq \bar{P}_F(d^{\star}), \tag{53}$$

where $F_d^{-1}(\cdot)$ is the inverse function of $F_d(\cdot)$, and

$$F_d(x) \triangleq \mathrm{Prob}\left\{\frac{1}{d}\chi_d^2 \leq x\right\}, \tag{54}$$

which is a continuous, monotonically increasing, and invertible function. Thus, as long as the condition $P_F(d^{\star}) \leq \bar{P}_F(d^{\star})$ is satisfied, the transmission strategy in (52) is also optimal to problem (42).

Remark 4 *We should emphasize that when $P_F(d^{\star}) > \bar{P}_F(d^{\star})$, the transmission strategy in* (52) *is not feasible to problem* (42) *any more. For this case, one can determine the transmission DoF by solving* (50) *for $d = 1, \ldots, N_F$, and then choose the transmission DoF with the minimum power satisfying* (53)*. However, the obtained $\boldsymbol{Q}_F$ is a suboptimal solution to problem* (42) *since there may exist a non-uniform power allocation strategy such that constraint* (53) *is satisfied with lower power. The optimal solution for this case is still an open problem.*

C. Feasibility Condition for Transmission Strategy (52)

Next, we present a sufficient condition under which the transmission strategy in (52) is feasible, and thus is optimal to the optimization problem (42). To proceed, we need the following lemma:

Lemma 2 [31] *For positive integers $d \geq 1$ and $\ell \geq 1$, let $x(d, d+\ell)$ represent the point at which the CDFs $F_d(x)$, which is defined in* (54)*, and $F_{d+\ell}(x)$ intersect, and*

$$p(d, d+\ell) = F_d(x(d, d+\ell)) = F_{d+\ell}(x(d, d+\ell)). \tag{55}$$

Then $p(d, d+\ell)$ is unique, greater than 0.5, and decreases to 0.5 as d increases.

Values of $p(d, d+\ell)$ for $d \geq 1$ and $\ell \geq 1$ can be computed numerically, and some concrete values of $p(d, d+2)$ are listed in Table I. By Lemma 2, the following proposition can be proven.

Proposition 2 *The transmission strategy in* (52) *is a feasible solution to problem* (42) *if*

$$\exp\left(-\frac{\gamma_F \sigma_F^2 \sigma_{h,MF}^2 F_{2\theta}^{-1}(1-\rho_M)}{\sigma_{h,FF}^2 \epsilon_M}\right) \times \frac{\sigma_{h,FF}^2 \epsilon_M}{\sigma_{h,FF}^2 \epsilon_M + \gamma_F \sigma_{h,MF}^2 \boldsymbol{w}_M^H \boldsymbol{C}_{h,FM} \boldsymbol{w}_M F_{2\theta}^{-1}(1-\rho_M)} \geq 1 - \rho_F \tag{56}$$

holds true, where

$$\theta = \begin{cases} n, & \forall (1-\rho_M) \in [p(2n, 2n+2), p(2n-2, 2n)), \\ & \quad n = 1, \ldots, N_F - 1, \\ N_F, & \forall (1-\rho_M) \in [0, p(2N_F - 2, 2N_F)), \end{cases}$$

in which $p(0, 2) \triangleq 1$.

Proof: See Appendix C. ■

VI. Simulation Results

For ease of showing the simulation results, the proposed transmission strategies together with the associated CSI scenarios and optimization problems are summarized in Table II. The reader can refer to this table in all the simulation examples to be presented.

We consider FBS and MBS each equipped with four transmit antennas, i.e., $N_F = N_M = 4$. Let maximum tolerable outage probabilities $\rho_F = \rho_M = 0.1$, i.e., the SINR requirement at FUE and the interference power constraint on MUE must be satisfied with higher than 90% probability. Assume $\sigma_F^2 = 0.01$ and $\epsilon_M = -3$ dB. The beamforming vector $\boldsymbol{w}_M$ at MBS is randomly generated uniformly on the unit-norm sphere $\|\boldsymbol{w}_M\|^2 = 1$.

In the simulations, the SDR-based problems (14) and (39) are solved using CVX [15]. For each obtained $\boldsymbol{W}_F^{\star}$, we first check whether or not it is of rank one. If yes, then the beamforming solution $\boldsymbol{w}_F^{\star}$ can be simply obtained via rank-one decomposition, i.e., $\boldsymbol{W}_F^{\star} = \boldsymbol{w}_F^{\star}(\boldsymbol{w}_F^{\star})^H$. Otherwise, Gaussian randomization procedure [22] is adopted to generate

TABLE I. Some values of $p(d, d+2)$ defined in (55).

d	2	4	6	8	10	12	14	16
$p(d, d+2)$	0.7153	0.6663	0.6407	0.6243	0.6125	0.6036	0.5965	0.5907

TABLE II. Summary of the proposed transmission strategies.

CSI scenario	Transmission strategy
Perfect CSI (Naive CSI)	Solving problem (6)
No CSI	Solving problem (14); or solving problem (16) using Algorithm 1 (for i.i.d. $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$)
Partial CSI	Solving problem (29) using Algorithm 1 (for i.i.d. $\boldsymbol{h}_{MF}$ and $\boldsymbol{e}_{FF}$)
Partial CSI (RAR)	Solving problem (39)
No CSI (OTS)	Solving problem (42) (for i.i.d. $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$)

a feasible $\boldsymbol{w}_F^\star$. Numerically, the obtained $\boldsymbol{W}_F$ is considered to be of rank one if

$$\frac{\lambda_{\max}(\boldsymbol{W}_F)}{\mathrm{Tr}(\boldsymbol{W}_F)} \geq 0.9999, \tag{57}$$

i.e., the largest eigenvalue of $\boldsymbol{W}_F$ is at least 10^4 times larger than any of the other eigenvalues.

Example 1: In this example, we illustrate the impact of CSI uncertainty on the achievable SINR for FUE, i.e., the value of SINR_F in (3). We consider channel vectors $\boldsymbol{h}_{FF}$, $\boldsymbol{h}_{MF}$, and $\boldsymbol{h}_{FM}$ to be i.i.d. complex Gaussian with zero mean and covariance matrices $\boldsymbol{C}_{h,FF} = \sigma_{h,FF}^2 \boldsymbol{I}_{N_F}$, $\boldsymbol{C}_{h,MF} = \sigma_{h,MF}^2 \boldsymbol{I}_{N_F}$, and $\boldsymbol{C}_{h,FM} = \sigma_{h,FM}^2 \boldsymbol{I}_{N_M}$, respectively. Considering the indoor scenario for FBS where the channel strength of $\boldsymbol{h}_{MF}$ and $\boldsymbol{h}_{FM}$ are much lower than that of $\boldsymbol{h}_{FF}$ due to penetration losses [3], we set $\sigma_{h,FF}^2 = 1$, and $\sigma_{h,MF}^2 = \sigma_{h,FM}^2 = 0.01$. The target SINR γ_F is set to 15 dB.

Let us first consider the partial CSI feedback scenario. The CSI error vectors $\boldsymbol{e}_{FF}$ and $\boldsymbol{e}_{FM}$ are assumed to be spatially i.i.d. complex Gaussian with zero mean and covariance matrices $\boldsymbol{C}_{e,FF} = \sigma_{e,FF}^2 \boldsymbol{I}_{N_F}$ and $\boldsymbol{C}_{e,FM} = \sigma_{e,FM}^2 \boldsymbol{I}_{N_M}$, respectively, where $\sigma_{e,FF}^2 = \sigma_{e,FM}^2 = \sigma_e^2 = 0.002$. We generate a set of presumed CSI $\hat{\boldsymbol{h}}_{FF} \sim \mathcal{CN}(\boldsymbol{0}, (\sigma_{h,FF}^2 - \sigma_e^2)\boldsymbol{I}_{N_F})$, $\hat{\boldsymbol{h}}_{MF} \sim \mathcal{CN}(\boldsymbol{0}, (\sigma_{h,MF}^2 - \sigma_e^2)\boldsymbol{I}_{N_F})$, and $\hat{\boldsymbol{h}}_{FM} \sim \mathcal{CN}(\boldsymbol{0}, (\sigma_{h,FM}^2 - \sigma_e^2)\boldsymbol{I}_{N_M})$. Using the presumed CSI, the optimal beamforming solution to problem (22), or equivalently problem (29), is obtained by using Algorithm 1. To provide a comparison of the proposed RAR approach to problem (22), we also solve problem (39). The optimal beamforming solution to the conventional perfect-CSI-based problem (5) is also obtained by using the presumed CSI as if they were true channel vectors.

The achievable SINR values of FUE are obtained over 10^5 randomly generated realizations of the CSI errors $\boldsymbol{e}_{FF}$ and $\boldsymbol{e}_{FM}$. Figure 2 displays the distribution of the achievable SINR values of the FUE, where "Partial CSI" denotes the results obtained by solving problem (29) using Algorithm 1; "Partial CSI (RAR)" denotes the results obtained by solving problem (39); and "Naive CSI" denotes the results obtained by solving problem (6). From this figure, one can see that the robust beamforming designs, i.e., problem (29) ("Partial CSI") and problem (39) ("Partial CSI (RAR)"), can meet 10% outage probability, while the results using the conventional perfect-CSI-based beamforming design yields 54.5% outage probability. This reveals that the perfect-CSI-based conventional beamforming design is quite sensitive to CSI errors. Moreover, the SINR satisfaction probability using "Partial CSI (RAR)" is nearly 99%, which indicates that the proposed RAR design criterion may be too conservative in trying to meet the SINR requirement (at the cost of higher transmit power).

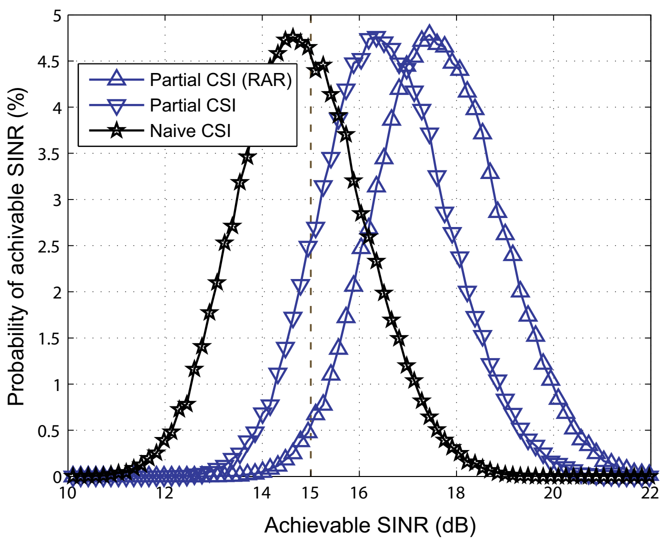


Fig. 2. Probability distributions of achievable SINR values of FUE for partial CSI feedback scenario at $\gamma_F = 15$ dB. $\sigma_{e,FF}^2 = \sigma_{e,FM}^2 = 0.002$. The SINR outage probabilities of "Partial CSI (RAR)", "Partial CSI", and "Naive CSI" are 1.3%, 10%, and 54.5%, respectively.

Next, let us show some results of the distribution of achievable SINR of FUE when FBS knows only the statistical CSI, obtained over 10^5 realizations of true i.i.d. channels $\boldsymbol{h}_{FF} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_{h,FF}^2 \boldsymbol{I}_{N_F})$, $\boldsymbol{h}_{MF} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_{h,MF}^2 \boldsymbol{I}_{N_F})$, and $\boldsymbol{h}_{FM} \sim \mathcal{CN}(\boldsymbol{0}, \sigma_{h,FM}^2 \boldsymbol{I}_{N_M})$, where $\sigma_{h,FF}^2 = 1$ and $\sigma_{h,MF}^2 = \sigma_{h,FM}^2 = 0.01$. The optimal beamforming solution to problem (10) under no CSI feedback scenario, or equivalently problem (16), is obtained by using Algorithm 1. We also consider optimal transmission strategy for this case, i.e., problem (42). One can show that the transmission strategy in (52) is optimal to problem (42) according to Proposition 2, and the optimal DoF $d^\star$, defined in (49), can be obtained numerically as $d^\star = N_F$. Figure 3 shows the distribution of the achievable SINR of FUE, where "No CSI" denotes the results obtained by solving problem (16) using Algorithm 1; and "No CSI (OTS)" denotes the results obtained by solving problem (42) using the optimal transmission strategy in (52) with $d = N_F$,

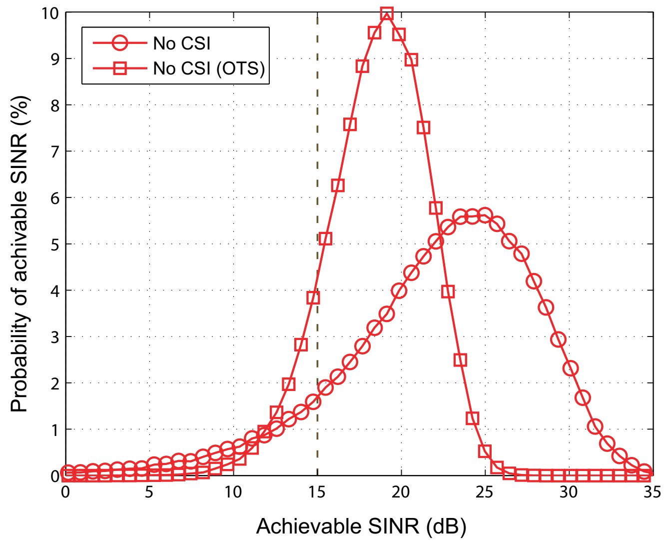


Fig. 3. Probability distributions of achievable SINR values of FUE for no CSI feedback scenario at $\gamma_F = 15$ dB. $\sigma^2_{h,FF} = 1$ and $\sigma^2_{h,MF} = \sigma^2_{h,FM} = 0.01$. Both the SINR outage probabilities of "No CSI" and "No CSI (OTS)" are 10%.

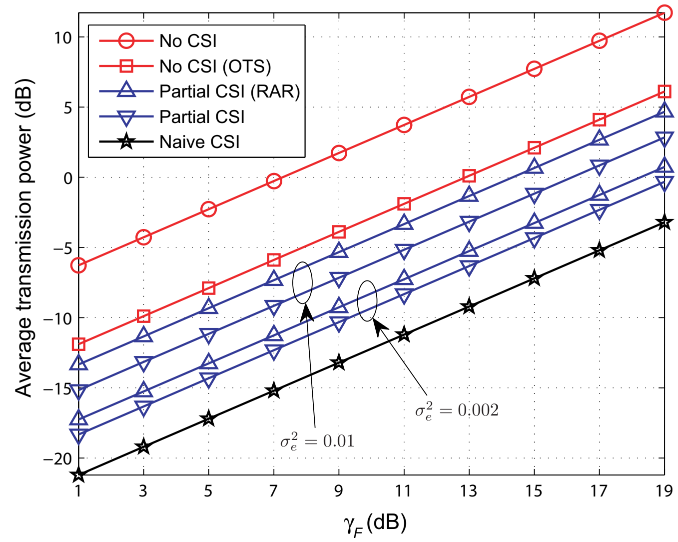


Fig. 4. Transmit power performance for spatially i.i.d. channel distributions. $\sigma^2_{h,FF} = 1$, $\sigma^2_{h,MF} = \sigma^2_{h,FM} = 0.01$, and $\sigma^2_{e,FF} = \sigma^2_{e,FM} = \sigma^2_e$.

where "OTS" stands for the optimal transmission strategy. From this figure, we can also see that both methods can achieve 90% SINR satisfaction probability. In addition, the mean SINR value of the FUE using the optimal transmission strategy is 19.85 dB, which is lower than the corresponding value 25.45 dB using the optimal beamforming strategy, and this indicates that FBS can save transmit power in achieving the same desired SINR performance when using the optimal transmission strategy.

Example 2: In this example, we compare the transmit power performance of the proposed methods. All the parameter settings in generating CSI are the same as those in Example 1. We show the results of the average transmit power obtained by the proposed methods over 500 channel realizations. The obtained results are shown in Fig. 4. Specifically, when FBS uses the proposed beamformer, the performance difference in terms of transmit power between the case that FBS knows only the statistical information of channels ("No CSI") and the case that FBS has partial CSI for $\sigma^2_e = 0.002$ ("Partial CSI") is around 11 dB, verifying that the proposed beamformer is more power efficient when more CSI information is provided. Moreover, for the scenario of no CSI feedback, the transmit power difference between the beamforming strategy ("No CSI") and the optimal transmission strategy ("No CSI (OTS)") is around 5.6 dB. Thus, the optimal transmission strategy can significantly save transmit power than the optimal beamforming strategy, illustrating a typical performance-complexity tradeoff. In addition, for the scenario of partial CSI, the difference between the transmit power obtained by using the exact beamforming solution to problem (22) ("Partial CSI") and that obtained by using RAR approach ("Partial CSI (RAR)") is within 2 dB, which indicates that the proposed RAR approach is not too conservative for this case. This is consistent with a similar observation reported in [26]. As a reference, we also plot the transmit power of the "Naive CSI" method in the figure, in order to illustrate of how much additional transmit power may be needed for the robust methods to accommodate the outage specification. In all the simulation instances, the solutions to the SDR-based problem, i.e., results labeled by "Partial CSI (RAR)", are always rank-one.

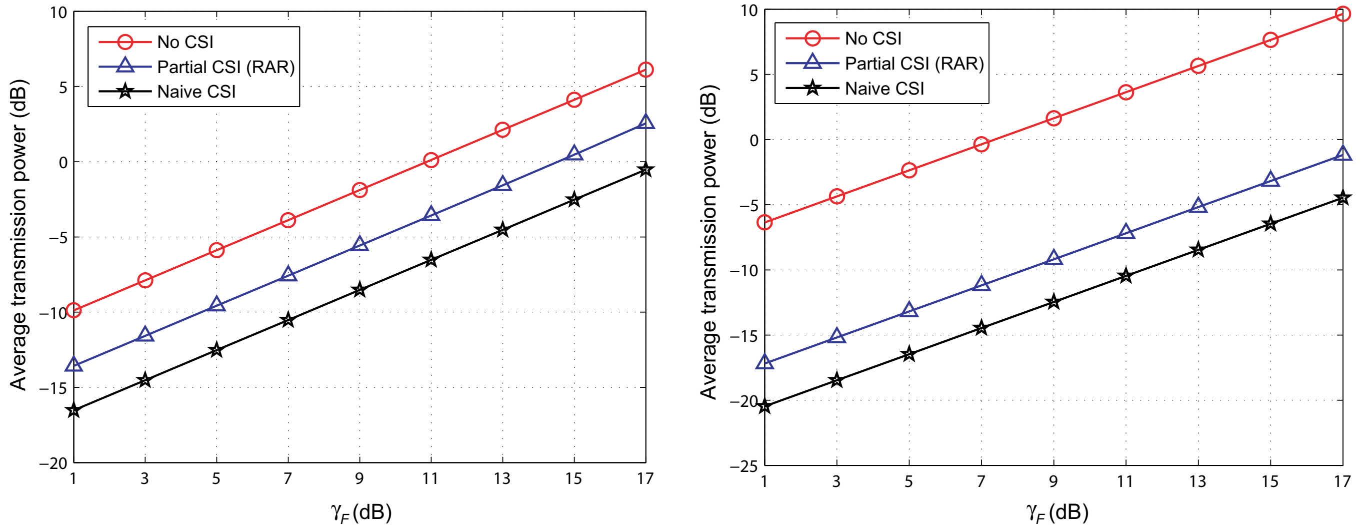
Example 3: In this example, we examine the transmit power performance for spatially correlated channels. Let $\boldsymbol{C}_{h,FF} = \sigma^2_{h,FF}\boldsymbol{C}_h$, $\boldsymbol{C}_{h,MF} = \sigma^2_{h,MF}\boldsymbol{C}_h$, and $\boldsymbol{C}_{h,FM} = \sigma^2_{h,FM}\boldsymbol{C}_h$,

$$[\boldsymbol{C}_h]_{m,n} = \varrho^{|m-n|}, \tag{58}$$

where $\varrho = 0.9$, $\sigma^2_{h,FF} = 1$ and $\sigma^2_{h,MF} = \sigma^2_{h,FM} = 0.01$. Other parameter settings are the same as those associated with Fig. 4. Figure 5(a) shows the average transmit power of the proposed methods for the spatially correlated channels. In this figure, curves "No CSI" and "Partial CSI (RAR)" denote the results obtained from solving problem (14) and problem (39), respectively. We should mention that the proposed methods of "Partial CSI" [problem (29)] and "No CSI (OTS)" [problem (42)] are not applicable for this example (see Table II). We also show the corresponding simulation results for $\varrho = 0.01$ in Fig. 5(b). From Fig. 5(a) and Fig. 5(b), we can observe similar performance trends as seen in Fig. 4, while the performance difference in terms of transmit power under no CSI feedback scenario and under partial CSI feedback scenario will be reduced as the channels become spatially more correlated (i.e., larger ϱ). In all the simulation instances, the solutions to the SDR-based problems are always rank-one.

VII. Conclusions

We have presented several robust transmit beamforming designs, i.e., transmit signal with one DoF, as summarized in Table II, for FBS equipped with N_F antennas, formulated as an outage constrained optimization problem (10), provided that either statistical CSI, i.e., no CSI feedback from FUE, or partial CSI is known to FBS. For the scenario of no CSI feedback [modeled by (7)], the optimal beamformer of problem (10) is obtained by solving (14) using off-the-shelf convex solvers, together with the feasibility condition given



(a) $\varrho = 0.9$ (b) $\varrho = 0.01$

Fig. 5. Transmit power performance for correlated channel distributions. $\sigma^2_{h,FF} = 1$, $\sigma^2_{h,MF} = \sigma^2_{h,FM} = 0.01$, and $\sigma^2_e = 0.002$.

in Proposition 1. For the case of spatially i.i.d. channels $\boldsymbol{h}_{FF}$ and $\boldsymbol{h}_{MF}$, the optimal beamformer can be exactly obtained using the computationally efficient Algorithm 1. Furthermore, the optimal transmission strategy, i.e., transmit signal with maximum DoF equal to N_F, together with its feasibility condition given in Proposition 2, has also been presented as the trade-off of complexity and performance (in terms of transmit power) for this case.

For the scenario of partial CSI feedback [modeled by (8) and (9)], the desired beamformer at FBS, a conservative beamforming solution to problem (10), is obtained by solving problem (39) using convex solvers. For the case of spatially i.i.d. channel $\boldsymbol{h}_{MF}$ and channel error $\boldsymbol{e}_{FF}$, the optimal beamformer can also be exactly obtained using Algorithm 1. Then we have shown some simulation results for evaluating the performance of the proposed methods, also verifying that the designed beamformer with more accurate CSI (e.g., partial CSI feedback) performs better than with only statistical CSI, and that the optimal transmission strategy outperforms the designed beamformer at the expense of larger DoF required, i.e., higher complexity.

Appendix A
Proof of Proposition 1

Let $\lambda \in \mathbb{R}$ and $\boldsymbol{x} \in \mathbb{C}^{N_F}$ be, respectively, the generalized eigenvalue and generalized eigenvector for the matrix pair $(\boldsymbol{C}_{h,FF}, \boldsymbol{C}_{h,MF})$, i.e.,

$$\boldsymbol{C}_{h,FF}\boldsymbol{x} = \lambda \boldsymbol{C}_{h,MF}\boldsymbol{x}. \tag{A.1}$$

Denoting the principal generalized eigenvalue and eigenvector by $\lambda_{\max}$ and $\boldsymbol{x}_{\max}$, respectively, we have

$$\boldsymbol{y}^H\boldsymbol{C}_{h,FF}\boldsymbol{y} \leq \lambda_{\max}\boldsymbol{y}^H\boldsymbol{C}_{h,MF}\boldsymbol{y}, \quad \forall \boldsymbol{y} \in \mathbb{C}^{N_F} \tag{A.2}$$

where the equality holds when $\boldsymbol{y} = \alpha\boldsymbol{x}_{\max}$, $\alpha \in \mathbb{C}$, i.e.,

$$\boldsymbol{x}^H_{\max}\boldsymbol{C}_{h,FF}\boldsymbol{x}_{\max} = \lambda_{\max}\boldsymbol{x}^H_{\max}\boldsymbol{C}_{h,MF}\boldsymbol{x}_{\max}. \tag{A.3}$$

Furthermore, one can easily show that the generalized eigenvalues in (A.1) are the eigenvalues of matrix $\boldsymbol{C}^{-1/2}_{h,MF}\boldsymbol{C}_{h,FF}\boldsymbol{C}^{-1/2}_{h,MF}$ [32]. Now, we are ready to prove the main result.

First, we prove that if (18) holds true, then problem (10), or equivalently problem (13), is feasible. Let $\boldsymbol{w}_F$ be the principal generalized eigenvector for matrix pair $(\boldsymbol{C}_{h,FF}, \boldsymbol{C}_{h,MF})$ such that constraint (13c) is active. Then, according to (A.3), we have

$$\lambda_{\max}\epsilon_M = \ln(1/\rho_M)\boldsymbol{w}^H_F\boldsymbol{C}_{h,FF}\boldsymbol{w}_F. \tag{A.4}$$

Substituting (A.4) into (18) leads to constraint (13b), and thus problem (13) is feasible.

Next, we show that if problem (13) is feasible, then (18) is true. Let $\boldsymbol{w}_F$ be a feasible point to problem (13). Since the function on the left-hand side (LHS) of (13b) is increasing in $\boldsymbol{w}^H_F\boldsymbol{C}_{h,FF}\boldsymbol{w}_F$, one can show that $\tilde{\boldsymbol{w}}_F \triangleq a\boldsymbol{w}_F$, where $a \geq 1$ is chosen such that (13c) is active, must also be feasible to problem (13), i.e.,

$$\exp\left(\frac{-\gamma_F\sigma^2_F}{\tilde{\boldsymbol{w}}^H_F\boldsymbol{C}_{h,FF}\tilde{\boldsymbol{w}}_F}\right)\frac{\tilde{\boldsymbol{w}}^H_F\boldsymbol{C}_{h,FF}\tilde{\boldsymbol{w}}_F}{\tilde{\boldsymbol{w}}^H_F\boldsymbol{C}_{h,FF}\tilde{\boldsymbol{w}}_F + \gamma_F\boldsymbol{w}^H_M\boldsymbol{C}_{h,FM}\boldsymbol{w}_M} \geq 1-\rho_F, \tag{A.5a}$$

$$\tilde{\boldsymbol{w}}^H_F\boldsymbol{C}_{h,MF}\tilde{\boldsymbol{w}}_F = \epsilon_M/\ln(1/\rho_M). \tag{A.5b}$$

According to (A.2), we have

$$\tilde{\boldsymbol{w}}^H_F\boldsymbol{C}_{h,FF}\tilde{\boldsymbol{w}}_F \leq \lambda_{\max}\tilde{\boldsymbol{w}}_F\boldsymbol{C}_{h,MF}\tilde{\boldsymbol{w}}_F = \lambda_{\max}\epsilon_M/\ln(1/\rho_M). \tag{A.6}$$

Since the function on the LHS of (A.5a) is increasing in $\tilde{\boldsymbol{w}}^H_F\boldsymbol{C}_{h,FF}\tilde{\boldsymbol{w}}_F$, by (A.6), one can conclude that (18) holds true. The proof is hence complete. ■

Appendix B
Proof of Lemma 1

The probability function in (43b), i.e.,

$$\psi(\tilde{\boldsymbol{Q}}_F, P_F) \triangleq \text{Prob}\left\{\boldsymbol{v}^H_{FF}\tilde{\boldsymbol{Q}}_F\boldsymbol{v}_{FF} \leq \frac{\gamma_F}{\sigma^2_{h,FF}P_F}\times \left(|\boldsymbol{h}^H_{FM}\boldsymbol{w}_M|^2 + \sigma^2_F\right)\right\} \tag{A.7}$$

is parameterized by $\tilde{\boldsymbol{Q}}_F$ and P_F.

Remark 5 *We will show later in* (46) *that* $\psi(\tilde{\boldsymbol{Q}}_F, P_F)$ *can be expressed as an expectation of a cumulative distribution function (CDF) of weighted sum of exponential random variables. The weighted sum of exponential random variables is known as a hypoexponential random variable [33], and the CDF is continuous and monotonically decreasing in* P_F. *Therefore, the function* $\psi(\tilde{\boldsymbol{Q}}_F, P_F)$ *is monotonically decreasing, and thus is invertible with respective to* P_F.

Since constraint (43b) must be active when the optimal $\tilde{\boldsymbol{Q}}_F$ and P_F are achieved, constraint (43b) can be expressed without loss of optimal solution of problem (43) as

$$\psi(\tilde{\boldsymbol{Q}}_F, P_F) = \rho_F. \tag{A.8}$$

Therefore, the power satisfying the equality of constraint (43b), or equivalently (A.8), can be expressed as

$$P_F(\tilde{\boldsymbol{Q}}_F, \rho_F) = \psi^{-1}(\tilde{\boldsymbol{Q}}_F, \rho_F). \tag{A.9}$$

On the other hand, let

$$\psi(\tilde{\boldsymbol{Q}}_F^\star, P_F) = \min_{\tilde{\boldsymbol{Q}}_F \succeq \boldsymbol{0}, \mathrm{Tr}(\tilde{\boldsymbol{Q}}_F)=1} \psi(\tilde{\boldsymbol{Q}}_F, P_F), \tag{A.10}$$

where $\tilde{\boldsymbol{Q}}_F^\star$ denotes an optimal solution to the minimization problem[2]. It has been shown that $\psi(\tilde{\boldsymbol{Q}}_F^\star, P_F)$ is a continuous and monotonically decreasing function of P_F [31], [34], and thus $\psi(\tilde{\boldsymbol{Q}}_F^\star, P_F)$ is invertible.

Letting P_F' be a nominal value of P_F and

$$\alpha = \psi^{-1}(\tilde{\boldsymbol{Q}}_F^\star, \rho_F) - P_F' \quad \text{and} \quad \beta = \psi^{-1}(\tilde{\boldsymbol{Q}}_F, \rho_F) - P_F', \tag{A.11}$$

we have

$$\psi(\tilde{\boldsymbol{Q}}_F^\star, P_F' + \alpha) = \psi(\tilde{\boldsymbol{Q}}_F, P_F' + \beta) = \rho_F. \tag{A.12}$$

If we can show $\alpha \leq \beta$, i.e., $\psi^{-1}(\tilde{\boldsymbol{Q}}_F^\star, \rho_F) \leq \psi^{-1}(\tilde{\boldsymbol{Q}}_F, \rho_F)$, by (A.9), we have $P_F(\tilde{\boldsymbol{Q}}_F^\star, \rho_F) \leq P_F(\tilde{\boldsymbol{Q}}_F, \rho_F)$, which is the desired result.

We now need to show that $\alpha \leq \beta$ which can be proved by contradiction. Supposing that $\alpha > \beta$ is true, due to that $\psi(\tilde{\boldsymbol{Q}}_F, P_F)$ is a decreasing function of P_F and (A.12), we have

$$\psi(\tilde{\boldsymbol{Q}}_F, P_F' + \alpha) < \psi(\tilde{\boldsymbol{Q}}_F, P_F' + \beta) = \rho_F. \tag{A.13}$$

However, according to (A.10) and (A.12), we have

$$\psi(\tilde{\boldsymbol{Q}}_F, P_F' + \alpha) \geq \psi(\tilde{\boldsymbol{Q}}_F^\star, P_F' + \alpha) = \rho_F, \tag{A.14}$$

which contradicts with (A.13), and therefore we conclude $\alpha \leq \beta$. The proof is thus complete. ■

[2]Precisely speaking, the transmit covariance matrix $\tilde{\boldsymbol{Q}}_F^\star$ depends on P_F via $d^\star$; see (48) and (49). However, without affecting the analysis, we will omit the dependence between $\tilde{\boldsymbol{Q}}_F^\star$ and P_F for notational brevity.

Appendix C
Proof of Proposition 2

We prove Proposition 2 by considering problem (42) with DoF $d = 1$, which is the same as problem (16). To present the proof, let us first consider the following problem [i.e., problem (16) with (16c) replaced by (A.15c)]:

$$\min_{P_F \in \mathbb{R}} \; P_F \tag{A.15a}$$

$$\text{s.t.} \; \exp\left(\frac{-\gamma_F \sigma_F^2}{\sigma_{h,FF}^2 P_F}\right) \frac{\sigma_{h,FF}^2 P_F}{\sigma_{h,FF}^2 P_F + \gamma_F \boldsymbol{w}_M^H \boldsymbol{C}_{h,FM} \boldsymbol{w}_M} \geq 1 - \rho_F, \tag{A.15b}$$

$$P_F \leq \frac{\epsilon_M}{\sigma_{h,MF}^2 F_{2\theta}^{-1}(1-\rho_M)} \triangleq \bar{P}_F(\theta), \tag{A.15c}$$

$$P_F \geq 0, \tag{A.15d}$$

where

$$\theta \triangleq \arg\max\{F_{2d}^{-1}(1-\rho_M), \; d = 1, \ldots, N_F\} = \begin{cases} n, & \forall (1-\rho_M) \in [p(2n, 2n+2), p(2n-2, 2n)), \\ & \qquad n = 1, \ldots, N_F - 1, \\ N_F, & \forall (1-\rho_M) \in [0, p(2N_F - 2, 2N_F)), \end{cases} \tag{A.16}$$

which has been proven in [34] using Lemma 2. With $\boldsymbol{C}_{h,FF}$, $\boldsymbol{C}_{h,MF}$, $\lambda_{\max}$, and $\ln(1/\rho_M)$ replaced by $\sigma_{h,FF}^2 \boldsymbol{I}_{N_F}$, $\sigma_{h,MF}^2 \boldsymbol{I}_{N_F}$, $\sigma_{h,FF}^2/\sigma_{h,MF}^2$, and $F_{2\theta}^{-1}(1-\rho_M)$, respectively, (18) becomes

$$\exp\left(-\frac{\gamma_F \sigma_F^2 \sigma_{h,MF}^2 F_{2\theta}^{-1}(1-\rho_M)}{\sigma_{h,FF}^2 \epsilon_M}\right) \times \frac{\sigma_{h,FF}^2 \epsilon_M}{\sigma_{h,FF}^2 \epsilon_M + \gamma_F \sigma_{h,MF}^2 \boldsymbol{w}_M \boldsymbol{C}_{h,FM} \boldsymbol{w}_M F_{2\theta}^{-1}(1-\rho_M)} \geq 1 - \rho_F, \tag{A.17}$$

which is exactly (56). Therefore, if (A.17) holds true, problem (A.15) is feasible by Proposition 1. If problem (A.15) is feasible, then problem (16) is also feasible since the feasible set of constraint (A.15c) is a subset of constraint (16c) due to $F_{2\theta}^{-1}(1-\rho_M) \geq F_2^{-1}(1-\rho_M) = \ln(1/\rho_M)$ [by the definition in (A.16) and (54)], i.e.,

$$\bar{P}_F(\theta) \leq \frac{\epsilon_M}{\sigma_{h,MF}^2 \ln(1/\rho_M)}. \tag{A.18}$$

That is to say, problem (42) with DoF $d = 1$ is feasible if problem (A.15) is feasible.

In the following, we show that the optimal transmit power to problem (42) with DoF $d = 1$, denoted as $P_F(1)$ [which is obtained by solving (50)], must be less than or equal to $\bar{P}_F(\theta)$, i.e.,

$$P_F(1) \leq \bar{P}_F(\theta). \tag{A.19}$$

Since one can always find a feasible power to problem (A.15), say $\tilde{P}_F$, which is also feasible to problem (42) with DoF $d = 1$ [due to (A.18)], if $P_F(1) > \bar{P}_F(\theta)$ holds true, then we have $\tilde{P}_F \leq \bar{P}_F(\theta) < P_F(1)$, which is a contradiction to that $P_F(1)$ is the optimal (minimum) power to problem (42) with DoF $d = 1$.

Since $P_F(1) \geq P_F(d^\star)$ [by (49)] and $\bar{P}_F(\theta) \leq \bar{P}_F(d^\star)$ [by (A.15c) and (A.16)], according to (A.19), we have $P_F(d^\star) \leq$

$\bar{P}_F(d^\star)$, which implies that the transmission strategy in (52) is feasible to problem (42) [by (53)]. ■

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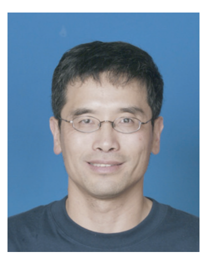

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