Noncoherent OSTBC-OFDM for MIMO and Cooperative Communications: Perfect Channel Identifiability and Achievable Diversity Order

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***Abstract*—This paper considers the context of orthogonal space-time block coded OFDM (OSTBC-OFDM) without channel state information at the receiver. Assuming noncoherent maximum-likelihood detection, the interest herein lies in detection within one OSTBC-OFDM block, motivated by its capability of accommodating relatively fast block fading channels. Our investigation focuses on analysis aspects, where we seek to establish practical noncoherent BPSK/QPSK OSTBC-OFDM schemes that have provably good channel identifiability and diversity properties. We consider perfect channel identifiability (PCI), a strong condition guaranteeing unique noncoherent channel identification for any (nonzero) channel. Through a judicious design involving special OSTBCs and pilot placement, we propose an OSTBC-OFDM scheme that is PCI-achieving and consumes fewer pilots compared to conventional pilot-aided channel estimation methods. We further our analysis by showing that a PCI-achieving scheme also achieves maximal noncoherent spatial diversity for the Kronecker Gaussian spatial-temporal channel fading model, which covers the popular i.i.d. Rayleigh fading channel and a variety of correlated and sparse multipath channels. All these results are developed in parallel for the centralized point-to-point MIMO scenario and a distributed relay communication scenario. For the latter scenario, our diversity analysis shows that the PCI-achieving scheme can also achieve maximal noncoherent cooperative diversity. The performance merits of the proposed PCI-achieving scheme are demonstrated by simulations.**

***Index Terms*—Distributed space-time coding, diversity, maximum-likelihood detection, noncoherent detection, OSTBC-OFDM, unique channel identifiability.**

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I. Introduction

Orthogonal space-time block coded OFDM (OSTBC-OFDM) is a popular physical-layer scheme for multi-input multi-output (MIMO) communications over frequency selective fading channels, offering diversity gains in a convenient way. Owing to its low encoding and decoding complexities, in addition, OSTBC-OFDM has been used in modern communication systems such as Wi-Fi, WiMAX and LTE [2]. To fully harvest the performance gains, the receiver requires accurate channel state information (CSI), which, however, demands sufficient resources being allocated to training or pilot data. Therefore, receiver techniques without CSI at the receiver (CSIR) are of great interest. In this scope, popular approaches include the differential methods [3], [4], and blind/semiblind signal detection methods [5]–[7]. Moreover, in the information theoretic literature [8]–[12], no-CSIR detection approaches are generally called *noncoherent detection*, to contrast with coherent detection which assumes perfect CSIR. From this perspective, both differential and blind/semiblind approaches may be regarded as being under the class of noncoherent detection. In fact, even the conventional coherent detection approach using pilot-aided channel estimation may be seen as noncoherent detection, and treated under a noncoherent analysis framework [9]–[12]. In this paper, we will alternatively call blind/semiblind detection methods noncoherent detection methods, for convenience of describing an important property—noncoherent diversity.

For noncoherent detection of OSTBC-OFDM, the differential methods [3], [4] are well known to inherently suffer from a 3 dB signal-to-noise ratio (SNR) loss. A simple alternative for noncoherent OSTBC-OFDM detection [6] is to employ noncoherent techniques for flat fading channels (see, e.g., [10]–[13]) for each subcarrier, i.e., subcarrier-by-subcarrier processing. However, this subcarrier-wise method usually assumes that the channel is static for a large number of OSTBC-OFDM blocks[1]. Another approach, which has been actively studied in signal processing, is to apply subspace-based blind and semiblind channel estimation methods [14]–[17]. Similar to the subcarrier-wise method, the subspace-based channel estimation methods [14]–[17] also require a large number of OSTBC-OFDM blocks for achieving near-coherent performance. These methods are therefore more suitable for slow fading channels.

[1]One OSTBC-OFDM block consists of T OFDM blocks, where T is the OSTBC length, as will be defined in Section II.

To deal with channels that have a shorter coherence time, the authors have proposed a 'block-wise' noncoherent maximum-likelihood (ML) detection approach [7]. This approach adopts the (semiblind) deterministic ML detection criterion [8], [18], which have been studied for flat fading noncoherent space-time coding [10]–[13]. One of the interesting results in [7] is that, by fully exploiting the orthogonal structure of OSTBC and the inter-subcarrier relationships of OFDM, the noncoherent ML detector can achieve near-coherent performance by using *one OSTBC-OFDM block only*. Therefore, the block-wise approach is particularly appealing in high user mobility environments where the channel coherence time may be as short as several OFDM blocks. For example, according to the LTE standard, a mobile channel with a moving speed 350 Km/h will experience a channel coherence time approximately equal to three OFDM blocks [19], which is equal to 1.5 OSTBC-OFDM block if the Alamouti code [20] is employed. As parallel developments, there has been interest in new subspace-based blind channel estimation methods [21]–[23] that also exploit the inter-subcarrier relationships for accommodating shorter channel coherence time. However, the present empirical results show that to achieve near-coherent performance, multiple OSTBC-OFDM blocks would be needed for the subspace-based methods in [21]–[23]. In this paper, we will focus on the noncoherent ML detection approach [7].

Our interest in noncoherent ML detection is not only for the point-to-point OSTBC-OFDM scenario but also for a relay-based distributed OSTBC-OFDM (DOSTBC-OFDM) scenario. In the distributed scenario, a set of single-antenna relays collaborates with each other to form a virtual MIMO transmission [24]–[26] in order to relay the information from a source to a destination receiver. The virtual MIMO channel between the relays and the destination receiver depends on both the physical MIMO channel and the number of cooperating relays. Hence, in addition to channel fading, the virtual MIMO channel can vary with the number of cooperating relays. We assume that the relays employ the decode-and-forward (DF) randomized relaying strategy [26], in which the relays can collaborate in an uncoordinated manner without the need of central control. Under such circumstances, a relay may or may not be able to cooperate, depending on whether the relay can correctly decode the information message from the source. Hence, the virtual MIMO channel can vary from one OSTBC-OFDM block to another. In view of this, we see a good motivation for investigating the block-wise noncoherent ML detection approach for the DOSTBC-OFDM scenario, which bypasses the need of estimating the virtual MIMO channel from time to time. To the best of our knowledge, most of the existing works on noncoherent DOSTBC detection focus only on the flat-fading scenario, e.g., see [27]–[29].

This paper focuses on BPSK/QPSK constellations. Our interest lies in seeking practical (BPSK/QPSK) OSTBC-OFDM schemes that can exhibit good noncoherent detection performance, in both the point-to-point and distributed OSTBC-OFDM scenarios. We are particularly interested in two fundamental performance aspects of the block-wise noncoherent ML detector, namely, unique channel identifiability and achievable diversity order. The former is motivated from a semiblind detection perspective, where we investigate conditions under which the unknown channel can be uniquely identified in a noise-free setting. The latter, diversity, is an important and well-known performance quantity in the space-time (or space-time-frequency) coding literature [8], [10], [11]. Different from the channel identifiability analysis in [30], which is for the subspace based channel estimation methods [21], [23], [31], our analysis is particularly for the block-wise noncoherent ML detector which possesses markedly different characteristics on the unique channel identifiability conditions. The authors have previously studied an almost-sure unique identifiability condition, wherein a block-wise noncoherent OSTBC-OFDM scheme for achieving that condition was also designed [7]. A summary of this previous result will be given in Section II. Simply speaking, that previous result is applicable to spatial-temporal i.i.d. Gaussian fading channels, but may not work for a wider class of channels, such as temporally sparse channels which are constituted only by several multipaths. Moreover, the previous work does not study the achievable diversity order of the noncoherent OSTBC-OFDM system.

Our goal in this paper is to study a strong identifiability condition, called perfect channel identifiability (PCI), and establish its connection with the achievable noncoherent diversity order[2]. A PCI-achieving scheme guarantees that any channel, as far as being nonzero, can be uniquely identified by the noncoherent ML detector. Therefore, even for the aforementioned temporally sparse channels, PCI-achieving OSTBC-OFDM schemes can guarantee unique channel identification. Moreover, as we will show, PCI-achieving schemes yield maximal achievable noncoherent diversity. We highlight our main contributions as follows.

- By exploiting a special class of OSTBCs, called non-intersecting subspace OSTBCs (NIS-OSTBCs) [13], we establish a PCI-achieving OSTBC-OFDM scheme. The proposed scheme is obtained by allocating the NIS-OSTBCs and pilot bits over the subcarriers. We show that L pilot bits, where L is the time-domain channel impulse response length, are sufficient and necessary for the proposed scheme to be PCI achieving; thus, the proposed scheme is more pilot efficient than the conventional pilot-aided channel estimation methods [33], [34].
- We present an analysis framework that connects the PCI property and the achievable noncoherent diversity order. Specifically, we show that PCI-achieving schemes can achieve the maximum noncoherent spatial diversity order of OSTBC-OFDM. Since this is also the maximum diversity order of the coherent ML detector (which has perfect CSIR), our results indicate that the PCI-achieving schemes achieve the maximum possible diversity offered by the system. The analysis is under a general channel model, namely, the Kronecker Gaussian model [35], [36], which accommodates not only the popular i.i.d. Rayleigh fading channel, but also temporally sparse channels and spatially/temporally correlated channels.
- The noncoherent diversity analysis for the relay-based DOSTBC-OFDM system is considered to be more difficult than its centralized counterpart. Nevertheless, our analysis framework is able to be extended to the DOSTBC-OFDM

[2]We should mention that diversity analysis for a noncoherent receiver is quite different from that for a coherent receiver [8], [32], owing to the different formulae of diversity in the noncoherent and coherent settings. The noncoherent diversity formula is more challenging to manage [8]; particularly, upon close inspection, one may find that space-time codes designed to yield full coherent spatial diversity does not necessarily achieve the full noncoherent spatial diversity.

scenario, showing that PCI-achieving schemes noncoherently achieve the maximum cooperative diversity of the cooperative system.

The rest of this paper is organized as follows. Section II reviews the OSTBC-OFDM system model, the block-wise noncoherent ML detector, and existing channel identifiability results in [7]. The DOSTBC-OFDM system model is also presented in that section. In Section III, the proposed PCI-achieving OSTBC-OFDM scheme is presented. Section IV presents the diversity analysis results. Simulations results are presented in Section V to examine the performance of the proposed PCI-achieving scheme. The conclusions are given in Section VI.

Notation: In this paper, we use $\mathbb{C}^n$ and $\mathbb{C}^{n\times m}$ to respectively denote the set of all n-dimensional complex vectors and the set of all n by m complex matrices. Boldfaced lowercase letters, such as $\mathbf{a}$, and boldfaced upper letters, such as $\mathbf{A}$, are used to represent vectors and matrices, respectively. $\mathbf{I}_n$ denotes the n by n identity matrix and $\mathbf{0}$ represents the zero matrix (with appropriate dimension). Superscripts "T" and "H" represent the operations of vector (matrix) transpose and (Hermitian) conjugate transpose, respectively. For vector $\mathbf{a}$, $\|\mathbf{a}\|_2$ denotes its vector Euclidean norm; while for matrix $\mathbf{A}$, $\|\mathbf{A}\|_2$ denotes its matrix 2-norm (i.e., the maximum singular value). $\|\mathbf{A}\|_F$ stands for the matrix Frobenius norm. We say $\mathbf{A}\succeq\mathbf{0}$ ($\succ\mathbf{0}$) if matrix $\mathbf{A}$ is positive semidefinite (positive definite). Matrix $\mathbf{A}^{\frac{1}{2}}$ is the square root of $\mathbf{A}$ satisfying $\mathbf{A}^{\frac{1}{2}}\mathbf{A}^{\frac{H}{2}}=\mathbf{A}$. $\mathbf{A}^{-1}$, $\mathrm{Tr}(\mathbf{A})$, $\det(\mathbf{A})$, $\mathrm{rank}(\mathbf{A})$ and $\lambda_{\max}(\mathbf{A})$ denote the inverse, trace, determinant, rank and the maximum eigenvalue of matrix $\mathbf{A}$, respectively. $\mathrm{Range}(\mathbf{A})$ represents the range space spanned by the column vectors of matrix $\mathbf{A}$. $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix with the elements of vector $\mathbf{a}$ being the diagonal entries. $\mathbb{E}(\cdot)$ denotes the statistical expectation of the argument.

II. Review of Noncoherent OSTBC-OFDM

We present a background review of the noncoherent OSTBC-OFDM system studied in [7]. The OSTBC-OFDM system model and the noncoherent ML detection approach are first reviewed in the first and second subsections. The scenario under consideration is the point-to-point MIMO. In the third subsection, we illustrate how the described technique can be extended to the relay-based distributed OSTBC-OFDM scenario. The last subsection then reviews the existing results on unique channel identifiability.

A. OSTBC-OFDM Signal Model

We consider a standard point-to-point OSTBC-OFDM system [7], [15] where the transmitter has N_t antennas and the receiver has N_r antennas. Let N_c be the number of subcarriers, and T be the space-time code length. Under the assumption that the channel is static for T OFDM symbols, or equivalently, one OSTBC-OFDM block, the received signal can be modeled as

$$\mathbf{Y}_n=\mathbf{C}_n(\mathbf{s}_n)\mathbf{H}_n+\mathbf{W}_n,\ n=1,\dots,N_c,\tag{1}$$

where

$\mathbf{Y}_n\in\mathbb{C}^{T\times N_r}$	received code matrix at subcarrier n;
$\mathbf{s}_n\in\{\pm1\}^{K_n}$	transmitted bit vector for subcarrier n where K_n is the number of bits per code;
$\mathbf{C}_n(\cdot)$: $\{\pm1\}^{K_n}\to\mathbb{C}^{T\times N_t}$	OSTBC mapping function for subcarrier n;
$\mathbf{H}_n\in\mathbb{C}^{N_t\times N_r}$	MIMO channel frequency response matrix for subcarrier n;
$\mathbf{W}_n\in\mathbb{C}^{T\times N_r}$	AWGN matrix for subcarrier n where each entry is assumed to be zero mean and have an average power σ_w^2.

We will concentrate on OSTBCs with BPSK or QPSK constellations. For such cases, the OSTBC mapping functions exhibit a linear structure [37]

$$\mathbf{C}_n(\mathbf{s}_n)=\frac{1}{\sqrt{K_n}}\sum_{k=1}^{K_n}\mathbf{X}_{n,k}s_{n,k},\tag{2}$$

where $s_{n,k}\in\{\pm1\}$ is the kth entry of $\mathbf{s}_n$, and $\mathbf{X}_{n,k}\in\mathbb{C}^{T\times N_t}$ are the basis matrices of $\mathbf{C}_n(\cdot)$. Moreover, $\mathbf{C}_n(\mathbf{s}_n)$ satisfies the semi-unitary property

$$\mathbf{C}_n^H(\mathbf{s}_n)\mathbf{C}_n(\mathbf{s}_n)=\mathbf{I}_{N_t}\quad\text{for any }\mathbf{s}_n\in\{\pm1\}^{K_n}.\tag{3}$$

We should note that unlike coherent OSTBC-OFDM, where a common space-time code is often employed for all subcarriers [i.e., $\mathbf{C}_1(\cdot)=\cdots=\mathbf{C}_{N_c}(\cdot)$], assuming non-identical $\mathbf{C}_n(\cdot)$ over subcarriers is essential to the noncoherent OSTBC-OFDM development in the ensuing sections.

B. Block-Wise Noncoherent ML OSTBC-OFDM Detection

The emphasis of this paper is on noncoherent OSTBC-OFDM detection; that is, approaches that detect the information bits $\mathbf{s}_n$, $n=1,\dots,N_c$ without knowing the channels $\mathbf{H}_n$, $n=1,\dots,N_c$ *a priori*. In particular, we focus on the block-wise noncoherent ML detection approach [7], in which only one OSTBC-OFDM block is used for noncoherent detection. As mentioned previously, the block-wise approach is attractive in allowing us to deal with relatively fast fading channels where the channel coherence interval can be as short as that of one OSTBC-OFDM block. The key ingredient that leads to this block-wise noncoherent detection approach is to exploit the fact that each $\mathbf{H}_n$ is parameterized by a common time-domain MIMO channel. To illustrate this, let

$$\boldsymbol{\mathcal{H}}=[\,\mathbf{h}_{:,1},\dots,\mathbf{h}_{:,N_r}\,]\in\mathbb{C}^{LN_t\times N_r}\tag{4}$$

be the collection of all time-domain MIMO channel coefficients, where $\mathbf{h}_{:,i}=[\mathbf{h}_{1,i}^T,\dots,\mathbf{h}_{N_t,i}^T]^T$, $i=1,\dots,N_r$, in which $\mathbf{h}_{m,i}\in\mathbb{C}^L$ is the channel impulse response vector from the mth transmit antenna to the ith receive antenna, and L is the channel length in time. Moreover, let

$$\mathbf{f}_n=\frac{1}{\sqrt{N_c}}\left[1,e^{-j\frac{2\pi}{N_c}(n-1)},\dots,e^{-j\frac{2\pi}{N_c}(n-1)(L-1)}\right]^T\tag{5}$$

be the DFT vector for subcarrier n, in which $j=\sqrt{-1}$. Then, each $\mathbf{H}_n$ is physically a MIMO Fourier transform of $\boldsymbol{\mathcal{H}}$; precisely we have the following expression:

$$\mathbf{H}_n=\mathbf{A}_n\boldsymbol{\mathcal{H}}=(\mathbf{I}_{N_t}\otimes\mathbf{f}_n^T)\boldsymbol{\mathcal{H}},\ n=1,\dots,N_c\tag{6}$$

$$\mathbf{A}_n\triangleq(\mathbf{I}_{N_t}\otimes\mathbf{f}_n^T)\tag{7}$$

where $\otimes$ is the Kronecker product. Using the time-domain channel parametrization in (6), one can write the received signals in (1) in a compact form as

$$\boldsymbol{\mathcal{Y}} \triangleq \begin{bmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_{N_c} \end{bmatrix} = \begin{bmatrix} \mathbf{C}_1(\mathbf{s}_1)\mathbf{A}_1 \\ \vdots \\ \mathbf{C}_{N_c}(\mathbf{s}_{N_c})\mathbf{A}_{N_c} \end{bmatrix} \boldsymbol{\mathcal{H}} + \begin{bmatrix} \mathbf{W}_1 \\ \vdots \\ \mathbf{W}_{N_c} \end{bmatrix} = \boldsymbol{\mathcal{G}}(\boldsymbol{s})\boldsymbol{\mathcal{H}} + \boldsymbol{\mathcal{W}} \quad (8)$$

$$\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \triangleq \begin{bmatrix} \mathbf{C}_1(\mathbf{s}_1)\mathbf{A}_1 \\ \vdots \\ \mathbf{C}_{N_c}(\mathbf{s}_{N_c})\mathbf{A}_{N_c} \end{bmatrix} \quad (9)$$

where $\boldsymbol{\mathcal{G}}(\boldsymbol{s})$ is a supercode that stacks all the transmitted codewords,

$$\boldsymbol{s} = [\mathbf{s}_1^T, \ldots, \mathbf{s}_{N_c}^T]^T \in \{\pm 1\}^{\bar{K}} \quad (10)$$

contains all the transmitted bits, with $\bar{K} = \sum_{n=1}^{N_c} K_n$, and $\boldsymbol{\mathcal{W}} \triangleq [\mathbf{W}_1^T, \ldots, \mathbf{W}_{N_c}^T]^T$.

From the compact OSTBC-OFDM model in (8), the blockwise noncoherent ML detection problem is formulated as follows:

$$\min_{\boldsymbol{s} \in \{\pm 1\}^{\bar{K}}} \left\{ \min_{\boldsymbol{\mathcal{H}} \in \mathbb{C}^{LN_t \times N_r}} \|\boldsymbol{\mathcal{Y}} - \boldsymbol{\mathcal{G}}(\boldsymbol{s})\boldsymbol{\mathcal{H}}\|_F^2 \right\}. \quad (11)$$

Formulation (11) is based on the deterministic blind ML criterion[3] [38]—it seeks to achieve the least squares error by estimating the unknown data $\boldsymbol{s}$ and the unknown time-domain channel $\boldsymbol{\mathcal{H}}$ jointly. As is common in blind approaches, the noncoherent ML formulation in (11) has its solution subject to scaling ambiguity; for instance, if $(\boldsymbol{s}, \boldsymbol{\mathcal{H}})$ is an optimal solution to (11), then $(-\boldsymbol{s}, -\boldsymbol{\mathcal{H}})$ is also optimal. To resolve this ambiguity, we assume that some pilot bits, presumably small in amount, are inserted in $\boldsymbol{s}$. A general expression for the pilot placement is as follows:

$$\boldsymbol{s} \triangleq \boldsymbol{\Pi} \begin{bmatrix} \mathbf{s}_{\mathrm{p}} \\ \mathbf{s}_d \end{bmatrix} \quad (12)$$

where $\mathbf{s}_d \in \{\pm 1\}^{K_d}$ collects the K_d (unknown) information bits, $\mathbf{s}_{\mathrm{p}} \in \{\pm 1\}^{\bar{K} - K_d}$ contains the (known) pilot bits, and $\boldsymbol{\Pi}$ is a $\bar{K}$ by $\bar{K}$ permutation matrix that describes how the pilots and data are assigned. With the pilot bits $\mathbf{s}_{\mathrm{p}}$ fixed, (11) reduces to the following semiblind ML detection problem

$$\min_{\mathbf{s}_d \in \{\pm 1\}^{K_d}} \left\{ \min_{\boldsymbol{\mathcal{H}} \in \mathbb{C}^{LN_t \times N_r}} \|\boldsymbol{\mathcal{Y}} - \boldsymbol{\mathcal{G}}([\mathbf{s}_{\mathrm{p}}^T, \mathbf{s}_d^T]^T)\boldsymbol{\mathcal{H}}\|_F^2 \right\} \quad (13)$$

where only $\mathbf{s}_d$ is to be determined. It has been shown [7] that (13) can be efficiently handled. In essence, (13) can be recast as a Boolean quadratic program (BQP), and then handled by available BQP methods, such as the efficient high-performance semidefinite relaxation (SDR) method. SDR can be implemented in a low-complexity manner, e.g., using the row-by-row optimization approach in [39]. A practical divide-and-conquer method for coping with large-DFT-size OSTBC-OFDM has also been developed in [7].

[3]Formulation (11) is also known as the generalized likelihood ratio test (GLRT) in the literature of noncoherent space-time (frequency) coding [8], [10], [11], [36].

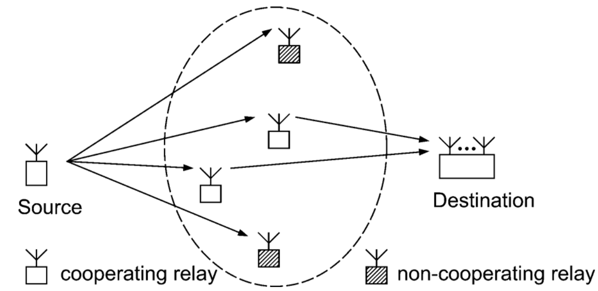

Fig. 1. The scenario setup for the considered relay-based distributed system.

C. Noncoherent ML Detection of Distributed OSTBC-OFDM

In this subsection, we shift our focus from point-to-point OSTBC-OFDM to a relay-based distributed OSTBC-OFDM (DOSTBC-OFDM) system. The scenario setup is depicted in Fig. 1. A number of relays are employed to forward information bits from the source to the destination. The relays are all equipped with one antenna, and the decode-and-forward (DF) protocol is assumed. These relays attempt to form a virtual MIMO, more precisely, a virtual OSTBC-OFDM transmission, providing diversity gain for the relays-to-destination link. For simplicity, we assume that there is no source-to-destination link. The relays are uncoordinated in the sense that they may choose to cooperate or not to cooperate at any time—the reasons for not cooperating would be that the relay fails to correctly decode the information bits sent from the source, or that the relay runs out of battery. In this problem setting, there is no central control coordinating the relays. To enable virtual OSTBC-OFDM formation in an uncoordinated fashion, we adopt the randomized distributed space-time coding approach [26].

The problem formulation is as follows. At each OSTBC-OFDM block, each relay will decide whether or not it will join the cooperative transmission. Let N_s denote the number of cooperating relays in that block. Let $\mathbf{s}_n$, $n = 1, \ldots, N_c$, be the information bits to be transmitted, defined in the same way as the previous subsections. The relays, if cooperating, are assumed to have correctly decoded $\mathbf{s}_n$, $n = 1, \ldots, N_c$. The forwarding of the information bits is done by the randomized distributed approach [26]: Each cooperating relay, say, the mth relay, transmits a time-block-coded OFDM block

$$\mathbf{C}_n(\mathbf{s}_n)\mathbf{q}_m \in \mathbb{C}^T, \quad n = 1, \ldots, N_c, \quad (14)$$

where $\mathbf{C}_n(\cdot) : \{\pm 1\}^{K_n} \rightarrow \mathbb{C}^{T \times N_t}$ is the OSTBC mapping function as defined in (2), and $\mathbf{q}_m \in \mathbb{C}^{N_t}$ is a random vector generated locally by relay m, for $m = 1, \ldots, N_s$; for example, the i.i.d. complex Gaussian randomization $\mathbf{q}_m \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_t})$ as suggested in [26]. Similar to (1) and (6), the subsequent received signal model of the relays-to-destination link is shown to be

$$\mathbf{Y}_n = \mathbf{C}_n(\mathbf{s}_n)\mathbf{Q}(\mathbf{I}_{N_s} \otimes \mathbf{f}_n^T)\boldsymbol{\mathcal{H}} + \mathbf{W}_n, \; n = 1, \ldots, N_c \quad (15)$$

where

$$\mathbf{Q} = [\mathbf{q}_1, \ldots, \mathbf{q}_{N_s}] \in \mathbb{C}^{N_t \times N_s} \quad (16)$$

collects all the randomization vectors, and the physical MIMO channel $\boldsymbol{\mathcal{H}}$ is defined as in (4) but here each $\mathbf{h}_{m,i} \in \mathbb{C}^L$ represents the channel impulse response vector between the mth cooperating relay and the ith receive antenna. By using the Kronecker product property $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC}\otimes\mathbf{BD})$ [40], we can reformulate (15) as

$$\mathbf{Y}_n = \mathbf{C}_n(\mathbf{s}_n)(\mathbf{I}_{N_t} \otimes \mathbf{f}_n^T)\boldsymbol{\mathcal{H}}_{\rm v} + \mathbf{W}_n,\ n = 1, \ldots, N_c \quad (17)$$

$$\boldsymbol{\mathcal{H}}_{\rm v} \triangleq (\mathbf{Q} \otimes \mathbf{I}_L)\boldsymbol{\mathcal{H}} \in \mathbb{C}^{LN_t \times N_r}. \quad (18)$$

It is important to note that in terms of formulations, (17) is identical to the point-to-point OSTBC-OFDM model in (8), with the physical time-domain channel $\boldsymbol{\mathcal{H}}$ replaced by the virtual time-domain channel $\boldsymbol{\mathcal{H}}_{\rm v}$. In a coherent scenario, it has been shown that the randomized distributed space-time coding approach can achieve the same spatial diversity as that of its point-to-point counterpart, for certain appropriate randomizations of $\mathbf{Q}$ [26].

This paper focuses on the noncoherent version of the DOSTBC-OFDM scheme. This scenario is motivated not only by fast fading environments as mentioned previously, but also by an issue arising from the decentralized nature of the scheme—the relays themselves may choose to join or not to join at any time, and this can cause significant variations of the virtual channel $\boldsymbol{\mathcal{H}}_{\rm v}$ from one OSTBC-OFDM block to another [28]. In this work, we propose to perform noncoherent DOSTBC-OFDM detection by treating the virtual channel $\boldsymbol{\mathcal{H}}_{\rm v}$ as if it were physical. Specifically, by directly applying the noncoherent ML formulation (13) to the DOSTBC-OFDM model in (17), we obtain the following block-wise noncoherent ML detector for the destination receiver:

$$\min_{\mathbf{s}_d\in\{\pm 1\}^{K_d}} \left\{ \min_{\boldsymbol{\mathcal{H}}_{\rm v}\in\mathbb{C}^{LN_t\times N_r}} \|\boldsymbol{\mathcal{Y}} - \boldsymbol{\mathcal{G}}(\boldsymbol{s})\boldsymbol{\mathcal{H}}_{\rm v}\|_F^2 \right\} \quad (19)$$

where we directly estimate the virtual channel $\boldsymbol{\mathcal{H}}_{\rm v}$ and the information bits without the need of knowing the number of cooperating relays and their randomized transmit vectors $\mathbf{q}_m$. Note that $\boldsymbol{\mathcal{Y}}$ and $\boldsymbol{\mathcal{G}}(\boldsymbol{s})$ are defined in the way as in (8). Since the noncoherent DOSTBC-OFDM ML detector formulation in (19) is the same as that of its point-to-point counterpart in (13), the implementation of the former can be handled in exactly the same way as the latter. In addition, the channel identifiability conditions for the former also directly apply to the latter.

D. Unique Channel Identifiability

A fundamental performance aspect of the noncoherent ML detector (13) (and (19)) is unique channel identifiability, i.e., the conditions under which the noncoherent ML detector (13) can uniquely identify the unknown channel $\boldsymbol{\mathcal{H}}$ in the noise-free situation. The mathematical definition of unique channel identifiability[4] is given below.

Definition 1 (Unique Channel Identifiability) [41]: *We say that the channel $\boldsymbol{\mathcal{H}}$ ($\neq \mathbf{0}$) is uniquely identifiable if the following ambiguity condition*

$$\mathbf{C}_n(\mathbf{s}_n)\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{C}_n(\mathbf{s}'_n)\mathbf{A}_n\boldsymbol{\mathcal{H}}',\ \forall\, n = 1, \ldots, N_c \quad (20)$$

holds only when $\boldsymbol{\mathcal{H}}' = \boldsymbol{\mathcal{H}}$, where $\boldsymbol{s}, \boldsymbol{s}' \in \{\pm 1\}^{\bar{K}}$ [see (10)] in which $\mathbf{s}_{\rm p} = \mathbf{s}'_{\rm p}$ (i.e., the pilot bit vectors of $\boldsymbol{s}$ and $\boldsymbol{s}'$ are the same).

One should notice that, for an OSTBC-OFDM system, unique identification of $\boldsymbol{\mathcal{H}}$ also implies unique identification of the unknown data $\mathbf{s}_d$ provided that $\mathbf{A}_n\boldsymbol{\mathcal{H}} \neq \mathbf{0}$, $n = 1, \ldots, N_c$; i.e., there is no channel null in the frequency domain.[5]

The channel identifiability problem has been investigated in our previous work [7], in which a mild assumption is made on $\boldsymbol{\mathcal{H}}$. The result is as follows:

Theorem 1 (One-Pilot-Code Scheme [7]): *Assume $N_c > L$, and that*

A1) $\boldsymbol{\mathcal{H}}$ is Gaussian distributed and at least one column of $\boldsymbol{\mathcal{H}}$ has a positive definite covariance matrix (e.g., spatially and temporally i.i.d. Gaussian channel).

For the noncoherent ML detector (13), the channel $\boldsymbol{\mathcal{H}}$ is uniquely identifiable with probability one if one of the subcarriers is dedicated to transmitting pilots only; e.g., $\mathbf{s}_{\rm p} = \mathbf{s}_1$.

Theorem 1 shows that the one-pilot-code scheme, which simply assigns one pilot space-time code at one subcarrier, is powerful—it uses fairly few pilots to achieve unique identification of $\boldsymbol{\mathcal{H}}$, in an almost sure sense. However, the result is under the premise of A1) which implies that the coefficients of $\boldsymbol{\mathcal{H}}$ have to be rich enough in randomness. While A1) is a common assumption in the literature of space-time-frequency coding [42], there are cases where A1) is not satisfied. One example is temporally sparse channels, which arise from environments where the channel is constituted by a few or several multipaths. In that case we are expected to see many zero coefficients in $\mathbf{h}_{m,i}$, and none of the columns of $\boldsymbol{\mathcal{H}}$ may satisfy A1). Another example where A1) may not apply is the DOSTBC-OFDM scenario. In this scenario, unique identifiability of the virtual channel $\boldsymbol{\mathcal{H}}_{\rm v} = (\mathbf{Q} \otimes \mathbf{I}_L)\boldsymbol{\mathcal{H}}$ is our concern. It can be shown that for the case of $N_t > N_s$, where $\mathbf{Q}$ is a tall matrix, $\boldsymbol{\mathcal{H}}_{\rm v}$ does not satisfy A1) inherently.

III. Perfect Channel Identifiability

The above issues motivate us to consider a stronger channel identifiability condition, called PCI. The definition of PCI will be given in the first subsection. To provide some insights, a simple way of achieving PCI will also be introduced. In the second subsection, we will establish a new PCI-achieving scheme that consumes fewer pilots than the simple scheme.

A. Definition of PCI

PCI is defined as follows:

Definition 2 (PCI): *We say that an OSTBC-OFDM scheme, characterized by the pilot placement $(\mathbf{s}_{\rm p}, \boldsymbol{\Pi})$ and the code functions $\mathbf{C}_n(\cdot)$, $n = 1, \ldots, N_c$, achieves PCI if $\boldsymbol{\mathcal{H}}$ is uniquely identifiable for any $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{LN_t\times N_r}$, $\boldsymbol{\mathcal{H}} \neq \mathbf{0}$.*

PCI is stronger than probability-one channel identifiability we employed in Theorem 1—the former guarantees unique channel identification for any nonzero $\boldsymbol{\mathcal{H}}$, without requiring any statistical assumption. Therefore, for temporally sparse channels, or perhaps even for non-Gaussian channels, a

[4]We should mention here that the channel identifiability definition given in Definition 1 is different from that defined in [21], [30], [31] for subspace based blind channel estimation methods. The two definitions can yield quite different conditions for uniquely identifying the unknown channel, as shown in [41] and [30] for the flat-fading OSTBC case. In particular, as reported in [30], increasing the number of receive antennas can improve the channel identifiability for the subspace based methods; however, the number of receive antennas has no impact on the channel identifiability defined in Definition 1.

[5]The reason is that, for an OSTBC $\mathbf{C}(\cdot)$, the code difference $\mathbf{C}_n(\mathbf{s}_n) - \mathbf{C}_n(\mathbf{s}'_n)$ has full column rank for any $\mathbf{s}_n \neq \mathbf{s}'_n$. Hence, when the channel is uniquely identified (i.e., $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}'$) and $\mathbf{A}_n\boldsymbol{\mathcal{H}} \neq \mathbf{0}$, (20) holds only if $\mathbf{s}_n = \mathbf{s}'_n$.

PCI-achieving scheme can enable us to uniquely identify the channel. The same advantage applies to the DOSTBC-OFDM scenario too, where the virtual channel $\boldsymbol{\mathcal{H}}_{\rm v}(\neq \mathbf{0})$ can always be identified unambiguously.

There is a simple way to achieve PCI, if the amount of pilots were not a concern. Let us consider an OSTBC-OFDM scheme in which, without loss of generality, the first L subcarriers are loaded only with pilots; i.e., $\mathbf{s}_{\rm p} = [\mathbf{s}_1^T, \ \ldots, \ \mathbf{s}_L^T]^T$. Then, when the ambiguity condition (20) holds, it must hold that

$$\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{A}_n\boldsymbol{\mathcal{H}}' \ \forall \ n = 1, \ldots, L. \tag{21}$$

We note that

$$\begin{bmatrix} \mathbf{A}_1 \\ \vdots \\ \mathbf{A}_L \end{bmatrix} = \bar{\mathbf{\Pi}} \left(\mathbf{I}_{N_t} \otimes \begin{bmatrix} \mathbf{f}_1^T \\ \vdots \\ \mathbf{f}_L^T \end{bmatrix} \right) \in \mathbb{C}^{LN_t \times LN_t} \tag{22}$$

where $\bar{\mathbf{\Pi}} \in \mathbb{R}^{LN_t \times LN_t}$ is a permutation matrix, has full rank due to the Vandermonde structure of $[\, \mathbf{f}_1, \ldots, \mathbf{f}_L \,]^T$. Hence, we can only have $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}'$, and PCI is achieved. In the sequel, this simple noncoherent scheme will be named the *L-pilot-code scheme*.

There is a low-complexity alternative for the receiver to exploit the inserted L pilot codes. Specifically, the receiver can first estimate $\boldsymbol{\mathcal{H}}$ by the pilot-aided LS channel estimator [33], [34]

$$\hat{\boldsymbol{\mathcal{H}}} = \left\{ \boldsymbol{\mathcal{G}}_{\rm p}^H(\mathbf{s}_{\rm p})\boldsymbol{\mathcal{G}}_{\rm p}(\mathbf{s}_{\rm p}) \right\}^{-1} \boldsymbol{\mathcal{G}}_{\rm p}^H(\mathbf{s}_{\rm p})\boldsymbol{\mathcal{Y}}_{\rm p} \tag{23}$$

where $\boldsymbol{\mathcal{Y}}_{\rm p} = [\mathbf{Y}_1^T, \ldots, \mathbf{Y}_L^T]^T$ and $\boldsymbol{\mathcal{G}}_{\rm p}(\mathbf{s}_{\rm p}) = [(\mathbf{C}_1(\mathbf{s}_1)\mathbf{A}_1)^T, (\mathbf{C}_2(\mathbf{s}_2)\mathbf{A}_2)^T, \ldots, (\mathbf{C}_L(\mathbf{s}_L)\mathbf{A}_L)^T]^T$, followed by coherently detecting the information symbol $\mathbf{s}_d$ using the LS channel estimate. Albeit simple, this estimation-detection method is suboptimal and does not perform as well when compared to the noncoherent (semiblind) detector (13). In terms of spectral efficiency, both the L-pilot-code scheme and the LS channel estimator require L pilot codes. Next, we will present a PCI-achieving scheme which can yield comparable performance as the L-pilot-code scheme but consumes L *pilot bits* only.

B. Proposed PCI-Achieving Scheme

Our OSTBC-OFDM scheme design philosophy is to use as few pilot bits as possible, while achieving PCI at the same time. To this end, we first consider the following question: What would be the minimal number of pilot bits required to attain PCI? The following lemma provides some guideline:

Lemma 1: *If an OSTBC-OFDM scheme achieves PCI, then the pilot placement must be such that at least L of the subcarriers have pilot bits.*

Proof: The proof is by contradiction. Suppose that only $L-1$ of the subcarriers are assigned pilots. Let us assume that the first $L-1$ subcarriers are the pilot embedded subcarriers, without loss of generality. Also, suppose that $\boldsymbol{\mathcal{H}}$ takes the form

$$\boldsymbol{\mathcal{H}} = [\, \mathbf{g}_1 \otimes \mathbf{h}, \ldots, \mathbf{g}_{N_r} \otimes \mathbf{h} \,]$$

where $\mathbf{h} \in \mathbb{C}^L$ is a nonzero vector lying in the nullspace of $[\, \mathbf{f}_1, \ldots, \mathbf{f}_{L-1} \,]^T \in \mathbb{C}^{(L-1)\times L}$, and $\mathbf{g}_1, \ldots, \mathbf{g}_{N_r} \in \mathbb{C}^{N_t}$ are arbitrary. Then, we have

$$\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{0} \ \text{for} \ n = 1, \ldots, L-1,$$

i.e., there are channel nulls over the first $L-1$ subcarriers. Under such circumstances, one can easily check that the ambiguity condition (20) holds for $\boldsymbol{\mathcal{H}}' = -\boldsymbol{\mathcal{H}}$, $\mathbf{s}_n' = \mathbf{s}_n$ for $n = 1, \ldots, L-1$ and $\mathbf{s}_n' = -\mathbf{s}_n$ for $n = L, \ldots, N_c$. ■

The intuition behind the proof is that, if there are less than L pilot subcarriers, then channel realizations that have frequency nulls exactly on the pilot subcarriers are not uniquely identifiable. Therefore, we need to use at least L pilot bits, each assigned over a distinct subcarrier, if PCI is desired. We should note that such a pilot requirement is necessary, but not sufficient in achieving PCI. Another key ingredient, which will lead to the proposed PCI-achieving scheme, is to employ a special class of OSTBCs, called non-intersecting subspace (NIS) OSTBCs. In the context of noncoherent ML detection of OSTBCs in flat fading channels, NIS-OSTBCs have been found indispensable in attaining PCI [41], [43]. The definition of NIS-OSTBC is given as follows:

Definition 3 [41]: *Assume BPSK/QPSK constellation. An OSTBC $\mathbf{C}(\cdot)$ is said to be an NIS-OSTBC if*

$$\text{Range}\{\mathbf{C}(\mathbf{s})\} \cap \text{Range}\{\mathbf{C}(\mathbf{s}')\} = \{\mathbf{0}\} \tag{24}$$

for any $\mathbf{s}, \mathbf{s}' \in \{\pm 1\}^K$, $\mathbf{s}' \neq \pm\mathbf{s}$.

NIS-OSTBC has the following important property:

Property 1 [41]: F*or any $\mathbf{H}, \mathbf{H}' \in \mathbb{C}^{N_t \times N_r}$, $\mathbf{H} \neq \mathbf{0}$, and $\mathbf{s}, \mathbf{s}' \in \{\pm 1\}^K$, the ambiguity equation*

$$\mathbf{C}(\mathbf{s})\mathbf{H} = \mathbf{C}(\mathbf{s}')\mathbf{H}'$$

holds only when $(\mathbf{s}', \mathbf{H}') = \pm(\mathbf{s}, \mathbf{H})$, if and only if $\mathbf{C}(\cdot)$ is an NIS-OSTBC.

By Property 1, we see that for NIS-OSTBCs, there is at most a sign ambiguity in identifying the channel $\mathbf{H}$, and that sign ambiguity can be eliminated by using one pilot bit. This property has been found important in providing PCI (up to a sign ambiguity) for the flat fading noncoherent OSTBC systems [41]. However, almost all OSTBCs, especially those developed in the context of coherent space-time coding, are not NIS. Fortunately, an NIS-OSTBC construction method has been proposed [41]. The method works by modifying an existing OSTBC:

Lemma 2 [41]: G*iven a BPSK/QPSK OSTBC $\mathbf{C}(\mathbf{s}) = \frac{1}{\sqrt{K}}\sum_{k=1}^{K}\mathbf{X}_k s_k$, where K is even, construct a new code by*

$$\mathbf{C}_{\rm NIS}(\bar{\mathbf{s}}) = \frac{1}{\sqrt{2K-1}} \left[\sum_{k=1}^{K-1} \mathbf{X}_k^T \bar{s}_k, \ \sum_{k=1}^{K} \mathbf{X}_k^T \bar{s}_{K-1+k} \right]^T \tag{25}$$

where $\bar{\mathbf{s}} \in \{\pm 1\}^{2K-1}$. Then $\mathbf{C}_{\rm NIS}(\bar{\mathbf{s}})$ is an NIS-OSTBC.

Let us take the QPSK Alamouti code ($T = 2$, $N_t = 2$, $K = 4$) [20] as an example. By Lemma 2, we can construct an NIS-OSTBC as

$$\mathbf{C}_{\rm NIS}(\mathbf{s}) = \frac{1}{\sqrt{7}} \begin{bmatrix} s_1 & s_2 - js_3 & s_4 + js_5 & s_6 - js_7 \\ s_2 + js_3 & -s_1 & s_6 + js_7 & -s_4 + js_5 \end{bmatrix}^T. \tag{26}$$

Note that in (26), the code length is doubled and is equal to $2N_t$. Moreover, note that one bit is dropped in (26), which incurs a slight bit loss. The two properties are inevitable, as it was shown

TABLE I
COMPARISON OF THE PROPOSED PCI ACHIEVING SCHEME WITH SOME EXISTING SCHEMES

	Identifiability	Data rate	Detection method
One-pilot-code scheme [7]	probability-one	$\frac{N_c K - K}{N_c T}$	Noncoherent ML (13)
Proposed L-pilot-bit scheme	PCI	$\frac{N_c K - 2L}{N_c T}$	Noncoherent ML (13)
L-pilot-code scheme	PCI	$\frac{N_c K - LK}{N_c T}$	Noncoherent ML (13)
LS channel estimation	PCI	$\frac{N_c K - LK}{N_c T}$	Coherent ML

in [41] that an NIS-OSTBC must satisfy $T \geq 2N_t$ and cannot have full rate.

We now consider constructing an OSTBC-OFDM scheme, based on NIS-OSTBCs:

Given a pilot assignment index set $\mathcal{P} \subseteq \{1, \ldots, N_c\}$, an NIS-OSTBC assignment index set $\mathcal{S} \subseteq \{1, \ldots, N_c\}$, an NIS-OSTBC function $\mathbf{C}_{\rm NIS}(\cdot)$, and an arbitrary OSTBC function $\mathbf{C}_{\rm O}(\cdot)$ whose matrix dimension is the same as that of $\mathbf{C}_{\rm NIS}(\cdot)$, do the following code assignment:

$$\mathbf{C}_n(\cdot) = \mathbf{C}_{\rm NIS}(\cdot) \; \forall \, n \in \mathcal{S} \quad (27)$$
$$\mathbf{C}_n(\cdot) = \mathbf{C}_{\rm O}(\cdot) \; \forall \, n \in \mathcal{S}^c \triangleq \{1, \ldots, N_c\} \backslash \mathcal{S}. \quad (28)$$

For the pilot placement, assign, for each $n \in \mathcal{P}$, a pilot bit to $s_{n,1}$.

The idea of the scheme above is to have part of the subcarriers employing NIS-OSTBCs, and the others ordinary OSTBCs. For the QPSK Alamouti example illustrated above, we can use (26) as the NIS code $\mathbf{C}_{\rm NIS}(\cdot)$. As for the ordinary code $\mathbf{C}_{\rm O}(\cdot)$, it is logical to choose a maximal code rate OSTBC with the same dimension as $\mathbf{C}_{\rm NIS}(\cdot)$. This can be obtained by concatenating two Alamouti codes:

$$\mathbf{C}_{\rm O}(\mathbf{s}) = \frac{1}{\sqrt{8}} \begin{bmatrix} s_1 + js_4 & s_2 - js_3 & s_5 + js_8 & s_6 - js_7 \\ s_2 + js_3 & -s_1 + js_4 & s_6 + js_7 & -s_5 + js_8 \end{bmatrix}^T. \quad (29)$$

An important question now is how many NIS-OSTBCs and pilot bits would be required to achieve PCI. Intuitively, we should use more NIS-OSTBCs and pilot bits to improve channel identifiability; in fact, a trivial PCI achieving condition is when all the transmitted codes are NIS-OSTBCs and every subcarrier has a pilot bit; i.e., $\mathcal{S} = \mathcal{P} = \{1, \ldots, N_c\}$. However, we should minimize the use of pilots and NIS-OSTBCs since they incur data rate reduction. To address this question, we make a few assumptions.

A2) $|\mathcal{P}| = L$.

Assumption A2) means that we employ just L pilot bits, which is the minimal number of pilot bits necessary for PCI, according to Lemma 1.

A3) $\mathcal{P} \subseteq \mathcal{S}$ if $|\mathcal{S}| \geq |\mathcal{P}|$, and $\mathcal{S} \subseteq \mathcal{P}$ if $|\mathcal{S}| \leq |\mathcal{P}|$.

Assumption A3) means that the pilot bits are assigned to the NIS-OSTBC subcarriers, whatever possible. We have the following theorem:

Theorem 2: *Under A2)-A3), the OSTBC-OFDM scheme in (27) and (28) is PCI-achieving if and only if* $|\mathcal{S}| \geq L$.

The proof of Theorem 2 will be detailed in the next subsection. Theorem 2 indicates that the minimal number of NIS-OSTBCs for achieving PCI is L. Concluding, we should set $\mathcal{S} = \mathcal{P}$, $|\mathcal{P}| = L$, to maximize the data rate. For convenience, we will call such a PCI-achieving scheme the *L-pilot-bit* scheme.

A comparison on the data rate/channel identifiability/detection complexity between the proposed L-pilot-bit scheme and the existing schemes is shown in Table I. One can see from the table that the proposed L-pilot-bit scheme has a higher data rate (when $K > 2$), even though it achieves the same PCI as the L-pilot-code scheme and LS channel estimation method. The detection complexity of the proposed L-pilot-bit scheme is higher than the LS channel estimation method, but, as mentioned in Section II-B, the noncoherent ML detector (13) can be efficiently implemented. As we will show in the simulation section, the proposed L-pilot-bit scheme can exhibit a better bit error performance than the LS channel estimation method.

C. Proof of Theorem 2

We prove Theorem 2 in this subsection. For ease of exposition, we will assume $\mathcal{P} = \{1, \ldots, L\}$; this is without loss of generality, as one may verify from the following proof. First, suppose that $|\mathcal{S}| \geq L$. Our aim is to show that the proposed scheme achieves PCI under such circumstances. Recall from our previous unique identifiability definitions that PCI is achieved if the ambiguity condition

$$\mathbf{C}_n(\mathbf{s}_n)\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{C}_n(\mathbf{s}'_n)\mathbf{A}_n\boldsymbol{\mathcal{H}}', \quad n = 1, \ldots, N_c \quad (30)$$

never holds for any $\boldsymbol{\mathcal{H}}' \neq \boldsymbol{\mathcal{H}}$ ($\boldsymbol{\mathcal{H}}, \boldsymbol{\mathcal{H}}' \neq \mathbf{0}$), $\boldsymbol{s}, \boldsymbol{s}' \in \{\pm 1\}^{\bar{K}}$, $\mathbf{s}_{\rm p} = \mathbf{s}'_{\rm p}$. Under A3) and $|\mathcal{S}| \geq L$, we have $\{1, \ldots, L\} \subseteq \mathcal{S}$; i.e., the pilot-bit-embedded subcarriers also bear NIS-OSTBCs. Let us consider the ambiguity condition corresponding to those subcarriers, or the first L of (30):

$$\mathbf{C}_{\rm NIS}(\mathbf{s}_n)\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{C}_{\rm NIS}(\mathbf{s}'_n)\mathbf{A}_n\boldsymbol{\mathcal{H}}', \quad n = 1, \ldots, L. \quad (31)$$

By the NIS-OSTBC property in Property 1 and by the pilot bit constraints $s_{n,1} = s'_{n,1}$, $n = 1, \ldots, L$, each equation in (31) holds only when i) $\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{A}_n\boldsymbol{\mathcal{H}}' = \mathbf{0}$, or when ii) $\mathbf{s}_n = \mathbf{s}'_n$. For case ii), it can be easily verified that $\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{A}_n\boldsymbol{\mathcal{H}}'$ must hold. Subsequently, we have the following set of equations:

$$\mathbf{A}_n\boldsymbol{\mathcal{H}} = \mathbf{A}_n\boldsymbol{\mathcal{H}}', \quad n = 1, \ldots, L. \quad (32)$$

However, we notice by using the same argument as in (22) that (32) are all satisfied only when $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}'$. In other words, the ambiguity condition (30) is violated whenever $\boldsymbol{\mathcal{H}}' \neq \boldsymbol{\mathcal{H}}$ ($\boldsymbol{\mathcal{H}}, \boldsymbol{\mathcal{H}}' \neq$

$\mathbf{0}$). Hence, the proposed scheme achieves PCI whenever $|\mathcal{S}| \geq L$.

Second, suppose that $|\mathcal{S}| < L$. We will show by construction that the proposed scheme violates PCI. Let us choose $\mathcal{S} = \{1, \ldots, L-1\}$. Moreover, set

$$\boldsymbol{\mathcal{H}} = \mathbf{g} \otimes \mathbf{h}, \qquad \boldsymbol{\mathcal{H}}' = \mathbf{g}' \otimes \mathbf{h},$$

where $\mathbf{g}, \mathbf{g}'$ are to be determined, and $\mathbf{h} \in \mathbb{C}^L$ satisfies $\mathbf{f}_n^T \mathbf{h} = 0,\ n = 1, \ldots, L-1$. As a result, we have

$$\mathbf{C}_{\rm NIS}(\mathbf{s}_n)\mathbf{A}_n \boldsymbol{\mathcal{H}} = \mathbf{C}_{\rm NIS}(\mathbf{s}'_n)\mathbf{A}_n \boldsymbol{\mathcal{H}}' = \mathbf{0},\ n = 1, \ldots, L-1. \tag{33}$$

Moreover, we claim that the following condition holds for the ordinary OSTBC $\mathbf{C}_{\rm O}(\cdot)$

$$\mathbf{C}_{\rm O}(\mathbf{u})\mathbf{g} = \mathbf{C}_{\rm O}(\mathbf{u}')\mathbf{g}', \tag{34}$$

for some $\mathbf{u}, \mathbf{u}' \in \{\pm 1\}^K$, $\mathbf{u} \neq \mathbf{u}'$, and $\mathbf{g}, \mathbf{g}' \in \mathbb{C}^{N_t}$, $\mathbf{g} \neq \pm \mathbf{g}'$; this claim is based on Property 1, which indicates that (34) can be achieved for non-NIS-OSTBCs. Moreover, we can assume that

$$u_1 = u'_1 = s_{L,1} \tag{35}$$

(i.e., the first bits of $\mathbf{u}$ and $\mathbf{u}'$ are equal to the assigned pilot bit in subcarrier L), since, if this is not true, we can flip the signs of $\mathbf{u}$ and $\mathbf{g}$, or $\mathbf{u}'$ and $\mathbf{g}'$, simultaneously such that both (34) and (35) hold. Now, by setting $\mathbf{s}_n = \mathbf{u}$ and $\mathbf{s}'_n = \mathbf{u}'$ for all $n = L, \ldots, N_c$, we show that

$$\begin{aligned} \mathbf{C}_{\rm O}(\mathbf{s}_n)\mathbf{A}_n \boldsymbol{\mathcal{H}} &= \mathbf{C}_{\rm O}(\mathbf{s}_n)\mathbf{g}(\mathbf{f}_n^T \mathbf{h}) \\ &= \mathbf{C}_{\rm O}(\mathbf{s}'_n)\mathbf{g}'(\mathbf{f}_n^T \mathbf{h}) \\ &= \mathbf{C}_{\rm O}(\mathbf{s}'_n)(\mathbf{g}' \otimes \mathbf{f}_n^T \mathbf{h}) \\ &= \mathbf{C}_{\rm O}(\mathbf{s}'_n)\mathbf{A}_n \boldsymbol{\mathcal{H}}', \end{aligned} \tag{36}$$

for all $n = L, \ldots, N_c$. We note that (33) and (36) form the ambiguity condition in (30). Hence, we have constructed a case of $|\mathcal{S}| < L$ for which the proposed scheme does not achieve PCI.

IV. Achievable Diversity Analysis

In the last section, we have constructed an OSTBC-OFDM scheme (the L-pilot-bit scheme) that is provably PCI-achieving. That scheme is also applicable to the relay-based DOSTBC-OFDM scenario in Section II-C. In this section, we analyze the achievable diversity of the noncoherent OSTBC-OFDM and relay-based DOSTBC-OFDM. Specifically, we show that, in both scenarios, the maximum spatial diversity can be achieved when PCI-achieving schemes are employed.

For convenience, we assume that the receiver has only one receive antenna,[6] i.e., $N_r = 1$. For $N_r = 1$, the signal model in (8) reduces to

$$\boldsymbol{y} \triangleq \boldsymbol{\mathcal{G}}(\boldsymbol{s})\boldsymbol{h} + \boldsymbol{w} \tag{37}$$

where $\boldsymbol{h} \triangleq [\mathbf{h}_1^T, \mathbf{h}_2^T, \ldots, \mathbf{h}_{N_t}^T]^T$ is the spatial-temporal channel vector in which $\mathbf{h}_m \in \mathbb{C}^L$ denotes the channel impulse response vector from the mth transmit antenna to the receiver, and $\boldsymbol{w}$ is the associated AWGN vector. Subsequently, the noncoherent ML detector in (13) is expressed as

$$\min_{\mathbf{s}_d \in \{\pm 1\}^{K_d}} \left\{ \min_{\boldsymbol{h} \in \mathbb{C}^{LN_t}} \|\boldsymbol{y} - \boldsymbol{\mathcal{G}}([\mathbf{s}_{\rm p}^T, \mathbf{s}_d^T]^T)\boldsymbol{h}\|_2^2 \right\}. \tag{38}$$

To describe the noncoherent diversity, we use the following notation

$$\text{Prob}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \rightarrow \boldsymbol{\mathcal{G}}(\boldsymbol{s}') |\ \boldsymbol{\mathcal{G}}(\boldsymbol{s}) \text{ transmitted})$$

to denote the pair-wise error probability (PEP) that the noncoherent ML detector (38) detects $\boldsymbol{s}'$, given that the transmitted bit vector is $\boldsymbol{s}$. Note that the PEP depends on the probabilistic model of $\boldsymbol{h}$, which will be delineated in the subsequent subsections. Diversity, either coherent or noncoherent, is generally defined as the high-SNR slope of the PEP in a log-log scale [26], [32]. For the considered noncoherent OSTBC-OFDM scenario, the noncoherent diversity order is defined as follows:

$$d_{\rm NC}^\star = \min_{\boldsymbol{s} \neq \boldsymbol{s}'} \left\{ \lim_{\sigma_w^2 \rightarrow 0} \frac{-\log \text{Prob}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \rightarrow \boldsymbol{\mathcal{G}}(\boldsymbol{s}') |\ \boldsymbol{\mathcal{G}}(\boldsymbol{s}))}{\log\left(\frac{1}{\sigma_w^2}\right)} \right\} \tag{39}$$

where $\boldsymbol{s}, \boldsymbol{s}' \in \{\pm 1\}^{\bar{K}}$, $\boldsymbol{s} \neq \boldsymbol{s}'$, $\mathbf{s}_{\rm p} = \mathbf{s}'_{\rm p}$. And, following the convention in the literature, such as [26], we say that an achievable noncoherent diversity order of $d_{\rm NC}$ is obtained if $d_{\rm NC} \leq d_{\rm NC}^\star$.

A. Achievable Noncoherent Diversity of Point-to-Point OSTBC-OFDM

Our noncoherent diversity analysis for the point-to-point MIMO scenario is based on a general Gaussian channel model, known as the Kronecker Gaussian model [35], [36]. In this model, the channel $\boldsymbol{h}$ follows a complex Gaussian distribution:

$$\boldsymbol{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_s \otimes \mathbf{R}_t), \tag{40}$$

where $\mathbf{R}_s \in \mathbb{C}^{N_t \times N_t}$ and $\mathbf{R}_t \in \mathbb{C}^{L \times L}$ represent the spatial and temporal covariance matrices, respectively. The Kronecker Gaussian model reduces to the popularly used i.i.d Rayleigh fading model when we choose $\mathbf{R}_s = \mathbf{I}_{N_t}$ and $\mathbf{R}_t = \mathbf{I}_L$. This model also encompasses temporally sparse channels; in such cases the sparse multipath profile is characterized by $\mathbf{R}_t$, which can be of low rank (e.g., if there are $\bar{L} < L$ paths then the rank of $\mathbf{R}_t$ is $\bar{L}$). Under the Kronecker Gaussian model, the PEP of the noncoherent detector (38) can be shown to be upper bounded by

$$\begin{aligned} &\text{Prob}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \rightarrow \boldsymbol{\mathcal{G}}(\boldsymbol{s}') |\ \boldsymbol{\mathcal{G}}(\boldsymbol{s})) \\ &\leq \det{}^{-1}\left(\mathbf{I}_{LN_t} + \left(\frac{1}{4\sigma_w^2}\right) \boldsymbol{\Psi}^{\frac{-1}{2}} \boldsymbol{\Omega}^{\frac{1}{2}} (\mathbf{R}_s \otimes \mathbf{R}_t) \boldsymbol{\Omega}^{\frac{1}{2}} \boldsymbol{\Psi}^{\frac{-1}{2}}\right) \\ &\quad \times \det{}^{-1}(\boldsymbol{\Psi}) \end{aligned} \tag{41}$$

where $\boldsymbol{\Omega} \triangleq \mathbf{I}_{LN_t} - \boldsymbol{\mathcal{G}}^H(\boldsymbol{s})\boldsymbol{\mathcal{G}}(\boldsymbol{s}')\boldsymbol{\mathcal{G}}^H(\boldsymbol{s}')\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \succeq \mathbf{0}$, and $\boldsymbol{\Psi} \triangleq \mathbf{I}_{LN_t} - \left(\frac{1}{4}\right)\boldsymbol{\Omega} \succ \mathbf{0}$. Equation (41) is obtained by using the Chernoff inequality, which has been used in the noncoherent space-time coding literature such as [36]. A concise, self-contained proof for (41) is given in Appendix A. Putting (41) into the noncoherent diversity definition (39) yields

$$d_{\rm NC}^\star \geq d_{\rm NC} \triangleq \min_{\boldsymbol{s} \neq \boldsymbol{s}'} \text{rank}\left(\boldsymbol{\Omega}^{\frac{1}{2}} (\mathbf{R}_s \otimes \mathbf{R}_t) \boldsymbol{\Omega}^{\frac{1}{2}}\right). \tag{42}$$

Our main diversity analysis result under the Kronecker Gaussian channel model is presented as follows.

[6]It is well known in the space-time coding literature [32] that the achievable diversity order of the N_r-receive-antenna case is N_r times the one-receive-antenna achievable diversity order, under a popular assumption that the spatial-temporal channel distribution of each receiver link is independent and identical.

Theorem 3: *Suppose that the channel $\boldsymbol{h}$ follows the Kronecker Gaussian model in (40), with the temporal covariance matrix satisfying*

A4) $\mathbf{f}_n^T \mathbf{R}_t \neq \mathbf{0}$ for all $n = 1, \ldots, N_c$.

Also, suppose that the OSTBC-OFDM scheme is PCI-achieving. Then

$$d_{\rm NC} = {\rm rank}(\mathbf{R}_s) \tag{43}$$

that is, the achievable diversity order of the noncoherent ML detector (38) is ${\rm rank}(\mathbf{R}_s)$.

The proof is presented in Appendix B. Theorem 3 indicates an important result—channel identifiability has a direct impact on the achievable diversity order. Specifically, by employing a PCI-achieving scheme, the full spatial diversity is attained noncoherently. Since this achievable noncoherent diversity order is the same as the diversity order of a coherent OSTBC-OFDM scheme [32], Theorem 3 implies that PCI-achieving schemes achieve the maximal possible diversity order offered by the system. Simulation results presented in Section V will also show that the one-pilot-code scheme in [7], which is not PCI-achieving, may not achieve the same diversity performance as the PCI-achieving schemes. Some additional discussions regarding Theorem 3 are in order.

1. The purpose of A4) in Theorem 3 is to ensure a nontrivial diversity result. Physically, A4) means that there is no channel null at any subcarrier. If channel nulls exist, then there is always detection error with the nulled subcarriers and it can be verified from the noncoherent diversity definition that $d_{\rm NC}$ must be zero.
2. Like coherent OSTBC-OFDM, the noncoherent PCI-achieving OSTBC-OFDM scheme achieves the full spatial diversity, but not the frequency diversity introduced by multipaths. In the coherent scenario, it is known that frequency diversity can be harvested by coding across subcarriers, e.g., by repetition coding [44] or by channel coding [45]. It is not difficult to see from the proof of Theorem 3 that the full spatial-frequency diversity can at least be obtained by repetition coding. The aspect of efficient across-subcarrier coding in the noncoherent context is beyond the scope of the present paper, but is worthwhile to investigate as a future work.

B. Achievable Noncoherent Diversity of DOSTBC-OFDM

The achievable noncoherent diversity order in the relay-based DOSTBC-OFDM scenario in Section II-C can be derived by using the analysis result established in the previous subsection. In this scenario, we assume $N_r = 1$ once again, and that the physical channel follows a complex Gaussian distribution

$$\boldsymbol{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_s \otimes \mathbf{R}_t),\ \mathbf{R}_s = \mathbf{I}_{N_s}$$

where we recall $\boldsymbol{h} \triangleq [\mathbf{h}_1^T, \mathbf{h}_2^T, \ldots, \mathbf{h}_{N_s}^T]^T$, with $\mathbf{h}_m \in \mathbb{C}^L$ denoting the time-domain channel impulse response vector between the mth relay to the receiver, and $\mathbf{R}_t \in \mathbb{C}^L$ denotes the temporal correlation matrix. Since the relays are spatially distributed, it is reasonable to assume $\mathbf{R}_s = \mathbf{I}_{N_s}$. From (18), the virtual channel is expressed as

$$\boldsymbol{h}_{\rm v} \triangleq (\mathbf{Q} \otimes \mathbf{I}_L)\boldsymbol{h} \in \mathbb{C}^{LN_s}. \tag{44}$$

Hence, given the relays-generated randomization matrix $\mathbf{Q}$, $\boldsymbol{h}_{\rm v}$ follows a distribution

$$\boldsymbol{h}_{\rm v} \sim \mathcal{CN}(\mathbf{0}, (\mathbf{Q}\mathbf{Q}^H) \otimes \mathbf{R}_t). \tag{45}$$

Applying Theorem 3 gives rise to the following result:

Theorem 4: *Consider the DOSTBC-OFDM signal model in (15) and the noncoherent ML detector in (19). Assume that $N_r = 1$, $\boldsymbol{h} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_s} \otimes \mathbf{R}_t)$ and A4) holds. Moreover, assume that the random matrix $\mathbf{Q}$ has rank equal to r with probability one. Then, for a PCI-achieving scheme, the noncoherent ML detector achieves $d_{\rm NC} = r$.*

Proof: Under the assumption that ${\rm rank}(\mathbf{Q}) = r$ with probability one, we have that

$$\begin{aligned}&{\rm Prob}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \rightarrow \boldsymbol{\mathcal{G}}(\boldsymbol{s}')|\ \boldsymbol{\mathcal{G}}(\boldsymbol{s}) \text{ transmitted})\\ &\quad = \int_{{\rm rank}(\mathbf{Q})=r} {\rm Prob}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \rightarrow \boldsymbol{\mathcal{G}}(\boldsymbol{s}')|\ \boldsymbol{\mathcal{G}}(\boldsymbol{s}), \mathbf{Q}) f(\mathbf{Q})\, d\mathbf{Q}\end{aligned}$$

where $f(\mathbf{Q})$ denotes the probability density function of $\mathbf{Q}$. By Theorem 3 and by (45), we see that given any rank-r $\mathbf{Q}$, the achievable diversity order associated with the conditioned PEP ${\rm Prob}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \rightarrow \boldsymbol{\mathcal{G}}(\boldsymbol{s}')|\ \boldsymbol{\mathcal{G}}(\boldsymbol{s}), \mathbf{Q})$ is ${\rm rank}(\mathbf{Q}\mathbf{Q}^H) = {\rm rank}(\mathbf{Q}) = r$. It follows that the diversity order associated with ${\rm Prob}(\boldsymbol{\mathcal{G}}(\boldsymbol{s}) \rightarrow \boldsymbol{\mathcal{G}}(\boldsymbol{s}')|\ \boldsymbol{\mathcal{G}}(\boldsymbol{s})$ transmitted) is r. ■

Suppose that $\mathbf{Q}$ has full rank with probability one, i.e., ${\rm rank}(\mathbf{Q}) = \min\{N_s, N_t\}$ with probability one; note that several existing randomization strategies, e.g., the Gaussian randomization [26], satisfies this condition. Then, according to Theorem 4, the PCI-achieving scheme can achieve a diversity order $\min\{N_s, N_t\}$, which is also the maximum cooperative diversity order achievable for its coherent counterpart [24], [26].

V. Simulation Results

In this section, we present some simulation results to examine the performance of the proposed L-pilot-bit scheme. The performance comparison results of point-to-point OSTBC-OFDM are presented in the first subsection, and that of DOSTBC-OFDM are presented in the second subsection.

A. Performance Comparison Results of Centralized OSTBC-OFDM

In the simulations, we consider a QPSK OSTBC-OFDM system as described in Section II with the DFT size equal to 256 ($N_c = 256$) The channel vectors $\mathbf{h}_{m,i}$, $m = 1, \ldots, N_t$, $i = 1, \ldots, N_r$, in (4) are assumed to be identically distributed and independent of each other. Two kinds of temporal profiles of $\mathbf{h}_{m,i}$ are considered:

- *Channel model I* (*temporally i.i.d. channel model*): Elements of each $\mathbf{h}_{m,i}$ are i.i.d. complex Gaussian distributed with zero mean and unit variance; i.e., $\mathbf{h}_{m,i} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_L)$ for all m, i.
- *Channel model II* (*temporally sparse channel model*): $\mathbf{h}_{m,i}$ contains some deterministic zero coefficients. The remaining non-zero coefficients are i.i.d. complex Gaussian distributed with zero mean and unit variance. For example, suppose that the channel order of $\mathbf{h}_{m,i}$ is equal

to 8 ($L=8$), and that the first, third and fifth coefficients of $\mathbf{h}_{m,i}$ are deterministically zero. Then, we write

$$\mathbf{h}_{m,i} \sim \mathcal{CN}(\mathbf{0}, \mathrm{diag}\{[0,1,0,1,0,1,1,1]\}) \text{ for all } m,i. \quad (46)$$

We will denote $\bar{L}$ as the number of non-zero elements in $\mathbf{h}_{m,i}$; e.g., $\bar{L}=5$ for (46).

The receive signal-to-noise ratio (SNR) was defined as

$$\mathrm{SNR} = \frac{\mathbb{E}\{\|\boldsymbol{\mathcal{G}}(\boldsymbol{s})\boldsymbol{\mathcal{H}}\|_F^2\}}{\mathbb{E}\{\|\boldsymbol{\mathcal{W}}\|_F^2\}} = \frac{\mathbb{E}\{\|\boldsymbol{\mathcal{H}}\|_F^2\}}{\sigma_w^2 T N_c N_r} = \frac{N_t \mathbb{E}\{\|\mathbf{h}_{m,i}\|_2^2\}}{\sigma_w^2 T N_c}. \quad (47)$$

All the simulation results to be presented are obtained by averaging over 15 000 channel realizations.

Throughout the simulations, we either set $L=8$ or $L=12$. For the proposed L-pilot-bit scheme in (27) and (28), (with $\mathcal{S}=\mathcal{P}, |\mathcal{P}|=L$), we set the NIS-OSTBC assignment index set as $\mathcal{S}=\{1+32q | q=0,\ldots,7\}$ for the case of $L=8$, and $\mathcal{S}=\{1+16q | q=0,\ldots,11\}$ for the case of $L=12$. For all $n \in \mathcal{S}$, the NIS OSTBC $\mathbf{C}_{\mathrm{NIS}}(\cdot)$ in (26) is employed; while for all $n \in \mathcal{S}^c$ the code in (29) is used (so $N_t=2$ and $T=4$). For the considered case of $N_c=256$, directly solving the ML detection problem (13) would be computationally too expensive; instead, we consider an implementation involving combined use of subcarrier grouping and cyclic ML (CML) [7]. Specifically, we first apply noncoherent ML detection to a subset of subcarriers, for which the problem size is reduced and the processing complexity is manageable. Let $\mathcal{S}_{\mathrm{SDR}}=\{1+8q | q=0,\ldots,31\}$ be that subcarrier subset. We handle

$$\min_{\substack{\mathbf{s}_n \in \{\pm 1\}^{K_n}, \\ n \in \mathcal{S}_{\mathrm{SDR}}}} \left\{ \min_{\boldsymbol{\mathcal{H}} \in \mathbb{C}^{LN_t \times N_r}} \sum_{n \in \mathcal{S}_{\mathrm{SDR}}} \|\mathbf{Y}_n - \mathbf{C}_n(\mathbf{s}_n)\mathbf{A}_n \boldsymbol{\mathcal{H}}\|_F^2 \right\} \quad (48)$$

using the SDR method [7], with the pilots bits in $\mathbf{s}_n, n \in \mathcal{S}$ being fixed. Note that the NIS code assignment index set $\mathcal{S}$ defined above is subsumed by $\mathcal{S}_{\mathrm{SDR}}$ for both cases of $L=8$ and $L=12$; therefore, PCI and full spatial diversity are guaranteed for (48) according to Theorem 2 and Theorem 3.

Let $\hat{\boldsymbol{\mathcal{H}}}$ be the optimal channel estimate from (48). Then the channel estimate $\hat{\boldsymbol{\mathcal{H}}}$ is used to initialize a CML procedure [18] for (13) to estimate the unknown data in all subcarriers and then refine the channel estimate in a cyclic, low-complexity fashion. We will show next that this alternative approach works very well even with one cycle of CML procedure. Readers may refer to [7] and [18] for the details about CML.

We will compare the proposed L-pilot-bit scheme with the LS channel estimator [33] [see (23)], the differential OSTBC-OFDM scheme [3], the one-pilot-code scheme [7] and the L-pilot-code scheme in Section II-C. For the LS channel estimator, all $\mathbf{s}_n, n \in \mathcal{S}$ are set as pilot data. The receiver first estimates the channel $\boldsymbol{\mathcal{H}}$ in accordance with (23) followed by detecting the unknown data based on this channel estimate. In the one-pilot-code scheme [7], the first subcarrier is dedicated for pilot signal; i.e., $\mathbf{s}_{\mathrm{p}}=\mathbf{s}_1$. The pilot placement of the L-pilot-code scheme is the same as that of the LS channel estimator. The previously mentioned CML based implementation method is used for both the one-pilot-code scheme and the L-pilot-code scheme. The performance of the coherent ML detector (which has perfect CSIR) is presented as a performance lower bound.

In Fig. 2, we present the performance comparison results. The number of receive antennas is set to 3 ($N_r=3$). Fig. 2(a) was obtained under Channel model I with $L=8$. One can observe from this figure that the proposed L-pilot-bit scheme exhibits almost the same bit error rate (BER) performance as the coherent ML detector and the L-pilot-code scheme for SNR ≥ 6 dB. Moreover, at $\mathrm{BER}=10^{-4}$, the proposed scheme has around 2 dB and 3 dB SNR advantages over the LS channel estimator and the differential scheme, respectively. We next examine the performances of the five schemes under temporally sparse channels. Fig. 2(b) shows the simulation results under Channel model II with $L=8$ and $\bar{L}=5$. The first, third and fifth coefficients of each $\mathbf{h}_{m,i}$ are set to zero; i.e., each $\mathbf{h}_{m,i}$ follows the distribution in (46). We can observe from this figure that, again the proposed L-pilot-bit scheme exhibits a near-coherent performance for SNR $\geq$6 dB. Interestingly, it is observed that the BER performance of the one-pilot-code scheme degrades, and it performs worse than the L-pilot-bit scheme for $1.2 \sim 1.5$ dB on average at $\mathrm{BER}=10^{-4}$. The performance gap between the one-pilot-code scheme and the proposed L-pilot-bit scheme becomes even evident when the channel length L increases or when the channels $\mathbf{h}_{m,i}$ are even more sparse. For example, one can see from Fig. 2(c), where the channel length is increased to 12, that the one-pilot-code scheme performs worst among the five methods. Fig. 2(d) presents the simulation results obtained under Channel model II with $L=8$ and with only the 2nd, 4th, and 6th coefficients of $\mathbf{h}_{m,i}$ being nonzero ($\bar{L}=3$). As seen from this figure, the one-pilot-code scheme is not able to decode the information bits properly. By contrast, the proposed scheme can still yield consistent BER performance.

B. Performance Comparison Results of Distributed OSTBC-OFDM

In this subsection, we present the performance comparison results of the L-pilot-bit scheme with the existing methods in the randomized DOSTBC-OFDM system as described in Section II-C. The simulation setting mainly follows that in the previous subsection. We consider the channel model II with $L=8$ and $\bar{L}=5$ where the first, third and fifth coefficients of each $\mathbf{h}_{m,i}$ are set to zero. Note that here $\mathbf{h}_{m,i}$ denotes the channel impulse response from the mth relay to the ith receiver antenna. The randomization matrix $\mathbf{Q}$ were generated following the i.i.d. complex Gaussian distribution with zero mean and unit variance. The SNR for the DOSTBC-OFDM systems is defined as

$$\begin{aligned}\mathrm{SNR} &= \frac{\mathbb{E}\{\|\boldsymbol{\mathcal{G}}(\boldsymbol{s})(\mathbf{Q}\otimes\mathbf{I}_L)\boldsymbol{\mathcal{H}}\|_F^2\}}{\sum_{n=1}^{N_c}\mathbb{E}\{\|\mathbf{W}_n\|_F^2\}} \\ &= \frac{\mathbb{E}\{\|(\mathbf{Q}\otimes\mathbf{I}_L)\boldsymbol{\mathcal{H}}\|_F^2\}}{N_c\sigma_w^2 T N_r} = \frac{N_t N_s \mathbb{E}\{\|\mathbf{h}_{m,i}\|_2^2\}}{N_c \sigma_w^2 T}. \quad (49)\end{aligned}$$

The simulation results are presented in Fig. 3. The number of receive antennas is set to two and each simulation result is obtained by averaging over 15,000 channel realizations. From Fig. 3(a) to Fig. 3(c), we can see that the proposed L-pilot-bit scheme can yield a near-coherent performance for BER $\leq 10^{-4}$, and outperforms the existing schemes. Similar

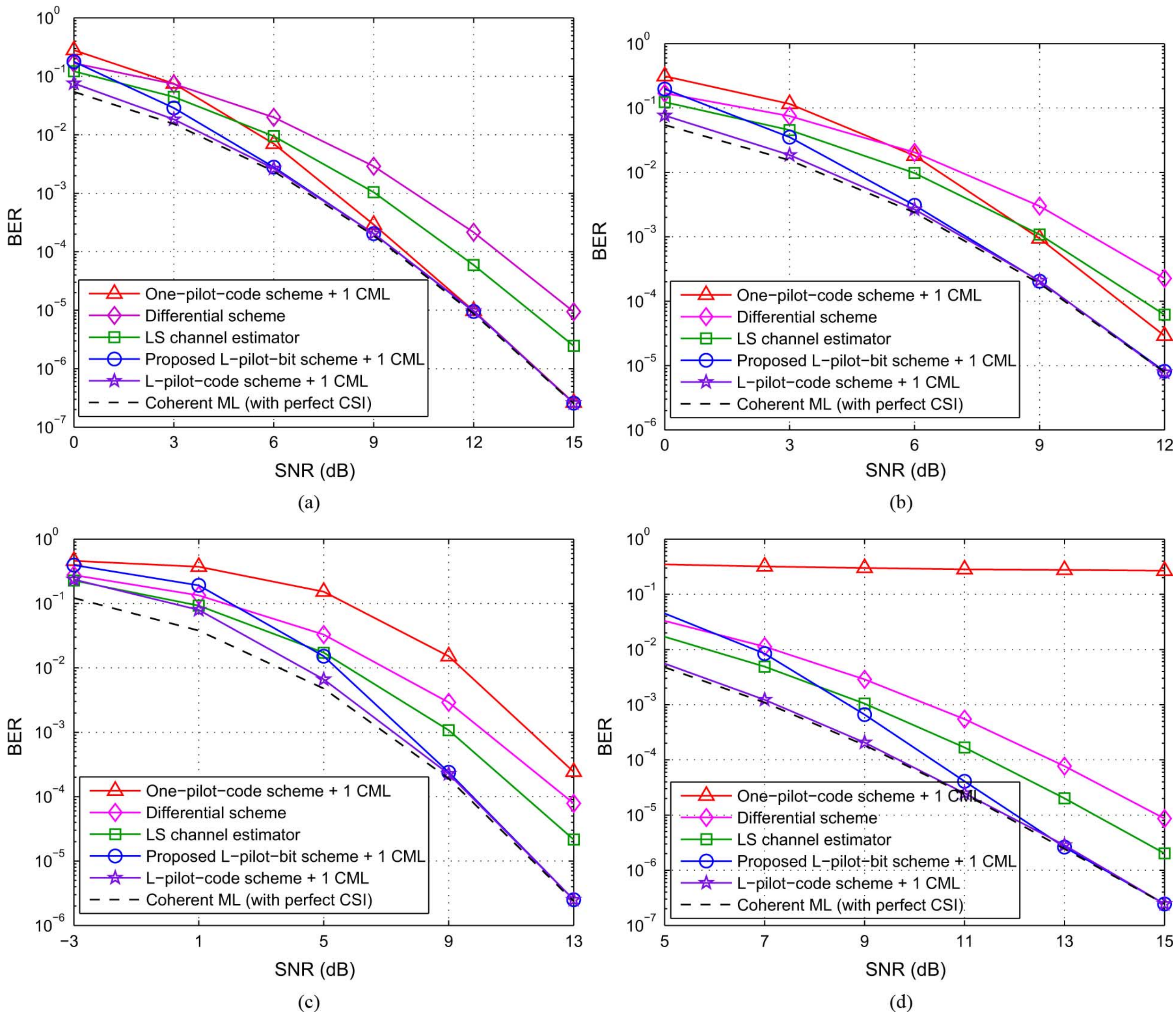

Fig. 2. Performance comparison results of the proposed L-pilot-bit scheme with the existing methods in a point-to-point OSTBC-OFDM system with $N_c = 256$, $N_t = 2, T = 4$ and $N_r = 3$. (a) Channel model I, $L = 8$; (b) Channel model II, $L = 8$; $\bar{L} = 5$; (c) Channel model II, $L = 12$, $\bar{L} = 9$; (d) Channel model II, $L = 8$, $\bar{L} = 3$.

conclusions can be obtained from the comparison results displayed in Fig. 3(d), where the number of relays N_s increases from one to eleven under $\mathrm{SNR} = 13$ dB.

VI. Conclusion

In this paper, we have analyzed the unique channel identifiability conditions and achievable diversity order of the block-wise noncoherent ML detector, for both the point-to-point OSTBC-OFDM and relay-based DOSTBC-OFDM systems. By employing the NIS-OSTBCs and by a judicious placement of pilot bits, we have constructed a pilot-efficient L-pilot-bit scheme that achieves the powerful PCI. We have also shown that PCI-achieving schemes can achieve the maximum noncoherent spatial diversity order in point-to-point OSTBC-OFDM as well as the maximum noncoherent cooperative diversity order in DOSTBC-OFDM. The presented simulation results have demonstrated that the proposed L-pilot-bit scheme, in either i.i.d., sparse, or dispersive multipath channels, outperforms the existing methods and exhibits consistent BER performance with our theoretical claims.

Our study of noncoherent OSTBC-OFDM for cooperative communications has been emphasizing the decode-and-forward strategy, wherein we focus on harvesting the noncoherent diversity offered by the relays-to-destination link. As a future direction, it would be interesting to consider the alternative relaying strategy of amplify-and-forward (AF), e.g., [46]. In the coherent scenario, it has been shown that both the source-to-relays and relays-to-destination links in AF can be exploited to enhance the diversity order. Hence, an open question is whether we may achieve the same diversity performance in the noncoherent scenario, and if yes, how we may design the OSTBC-OFDM scheme to attain that.

Appendix A
Proof of (41)

For notational simplicity, we use $\boldsymbol{\mathcal{G}}_1$ and $\boldsymbol{\mathcal{G}}_2$ to represent the two distinct codewords $\boldsymbol{\mathcal{G}}(\boldsymbol{s})$ and $\boldsymbol{\mathcal{G}}(\boldsymbol{s}')$, respectively, and use $\mathbf{R}$ to represent the channel covariance matrix $\mathbf{R}_s \otimes \mathbf{R}_t$. It has been shown [7] that (38) can be reformulated as the following BQP:

$$\max_{\mathbf{s}_d \in \{\pm 1\}^{K_d}} \boldsymbol{y}^H \boldsymbol{\mathcal{G}}(\boldsymbol{s}) \boldsymbol{\mathcal{G}}^H(\boldsymbol{s}) \boldsymbol{y}. \tag{A1}$$

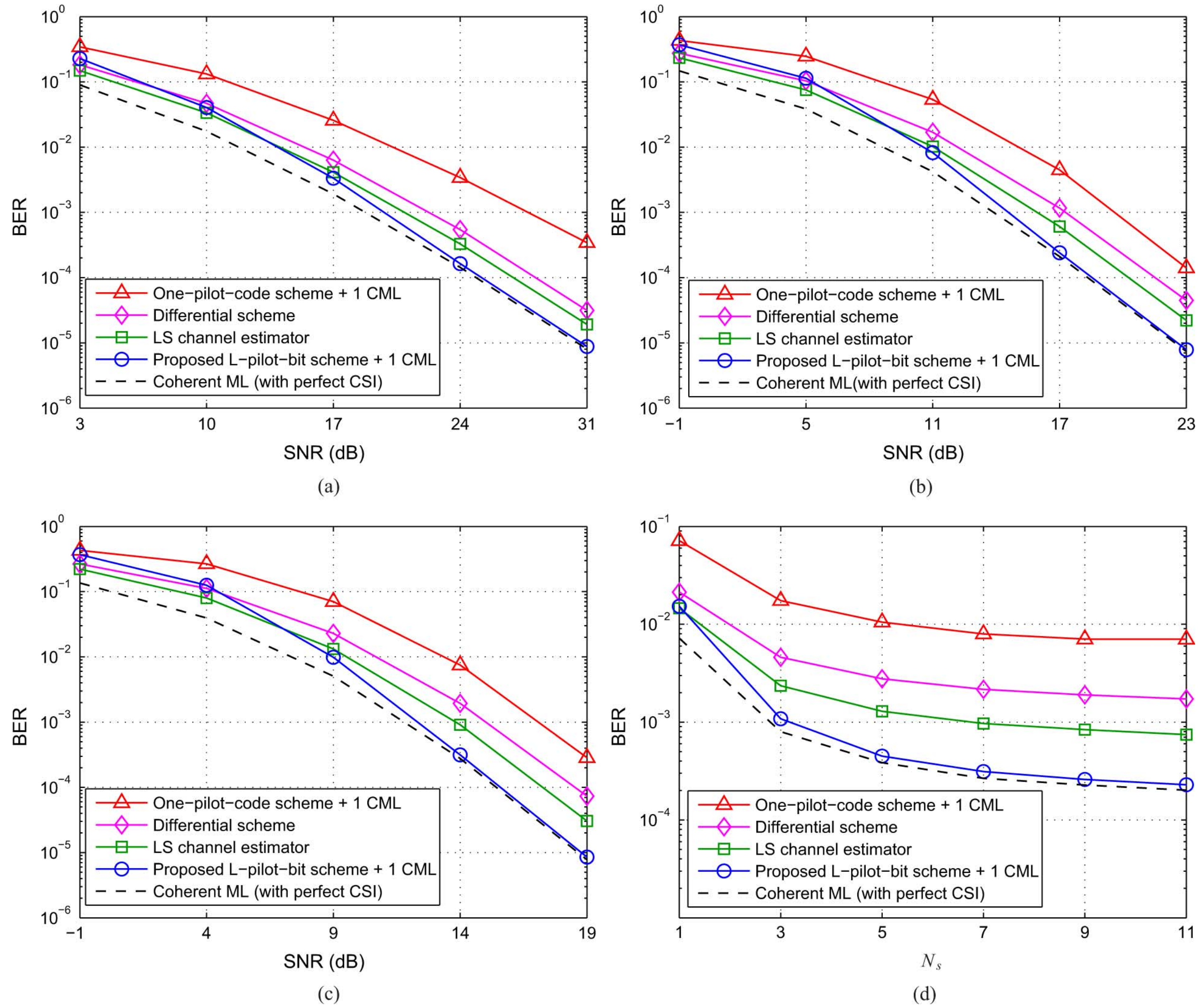

Fig. 3. Performance comparison results of the proposed L-pilot-bit scheme with the existing methods in the randomized distributed OSTBC-OFDM system with $N_c = 256$, $N_t = 2$, $T = 4$ and $N_r = 2$. Channel model II with $L = 8$ and $\bar{L} = 5$ (the first, third and fifth coefficients are set to zero) is used. (a) $N_s = 1$; (b) $N_s = 2$; (c) $N_s = 4$; and (d) $\mathrm{SNR} = 13$ dB.

By (A1), and by the Chernoff bound [47], the PEP between $\mathcal{G}_1$ and $\mathcal{G}_2$ can be upper bounded by (A2), shown at the bottom of the page, where α is chosen such that the right-hand side (RHS) of (A2) for all $0 \leq s \leq \alpha$ is finite and bounded. By the Schur complement and some tedious derivations [36], the RHS of (A2) can be shown to be

$$\min_{0\leq s\leq\alpha} \det{}^{-1}\left(\mathbf{I}_{LN_t} + \left(\frac{s}{\sigma_w^2}\right)\left((1-s)\mathbf{R} - s\sigma_w^2\mathbf{I}_{LN_t}\right)\mathbf{\Omega}\right) \tag{A3}$$

where $\mathbf{\Omega} = \mathbf{I}_{LN_t} - \mathcal{G}_1^H\mathcal{G}_2\mathcal{G}_2^H\mathcal{G}_1$. Note that $\mathbf{\Omega} \succeq \mathbf{0}$ and $\lambda_{\max}(\mathbf{\Omega}) \leq 1$ due to the fact that $\|\mathcal{G}_1^H\mathcal{G}_2\|_2 \leq \|\mathcal{G}_1\|_2\|\mathcal{G}_2\|_2 = 1$. By choosing a suboptimal value of $s = \frac{1}{2}$, we have the upper bound of PEP as

$$\begin{aligned}
&\mathrm{Prob}(\mathcal{G}_1 \to \mathcal{G}_2 \mid \mathcal{G}_1)\\
&\leq \det{}^{-1}\left(\left(\mathbf{I}_{LN_t} - \left(\frac{1}{4}\right)\mathbf{\Omega}\right) + \left(\frac{1}{4\sigma_w^2}\right)\mathbf{\Omega}^{\frac{1}{2}}\mathbf{R}\mathbf{\Omega}^{\frac{1}{2}}\right)\\
&= \det{}^{-1}\left(\mathbf{I}_{LN_t} + \left(\frac{1}{4\sigma_w^2}\right)\mathbf{\Psi}^{\frac{-1}{2}}\mathbf{\Omega}^{\frac{1}{2}}\mathbf{R}\mathbf{\Omega}^{\frac{1}{2}}\mathbf{\Psi}^{\frac{-1}{2}}\right)\\
&\quad\times \det{}^{-1}(\mathbf{\Psi})
\end{aligned} \tag{A4}$$

where $\mathbf{\Psi} = \mathbf{I}_{LN_t} - \left(\frac{1}{4}\right)\mathbf{\Omega} \succ \mathbf{0}$ due to $\lambda_{\max}(\mathbf{\Omega}) \leq 1$. The proof is complete. ■

$$\mathrm{Prob}(\mathcal{G}_1 \to \mathcal{G}_2 \mid \mathcal{G}_1) \leq \min_{0\leq s\leq\alpha} \det{}^{-1}\left(\mathbf{I}_{2LN_t} - \left(\frac{s}{\sigma_w^2}\right)\begin{bmatrix} -\mathcal{G}_1^H \\ \mathcal{G}_2^H \end{bmatrix}(\mathcal{G}_1\mathbf{R}\mathcal{G}_1 + \sigma_w^2\mathbf{I}_{N_cT})[\mathcal{G}_1\ \mathcal{G}_2]\right) \tag{A2}$$

APPENDIX B
PROOF OF THEOREM 3

Similar to the proof in Appendix A, we use $\mathcal{G}_1$ and $\mathcal{G}_2$ to represent the two distinct codewords $\mathcal{G}(\boldsymbol{s})$ and $\mathcal{G}(\boldsymbol{s}')$, respectively. Thus $\boldsymbol{\Omega}$ in (41) can be expressed as

$$\boldsymbol{\Omega} = \mathbf{I}_{LN_t} - \mathcal{G}_1^H \mathcal{G}_2 \mathcal{G}_2^H \mathcal{G}_1 \succeq \mathbf{0}. \tag{A5}$$

Let $\text{rank}(\mathbf{R}_s) = r_s$ and $\text{rank}(\mathbf{R}_t) = r_t$, and let

$$\mathbf{R}_s = \mathbf{V}_s \boldsymbol{\Lambda}_s \mathbf{V}_s^H,\ \mathbf{R}_t = \mathbf{V}_t \boldsymbol{\Lambda}_t \mathbf{V}_t^H \tag{A6}$$

be the eigenvalue decompositions of $\mathbf{R}_s$ and $\mathbf{R}_t$, respectively, where $\mathbf{V}_s \in \mathbb{C}^{N_t \times r_s}$ and $\mathbf{V}_t \in \mathbb{C}^{L \times r_t}$ are semi-unitary matrices, and $\boldsymbol{\Lambda}_s \in \mathbb{R}^{r_s \times r_s}$ and $\boldsymbol{\Lambda}_t \in \mathbb{R}^{r_t \times r_t}$ are (full rank) diagonal matrices with the positive eigenvalues of $\mathbf{R}_s$ and $\mathbf{R}_t$ being the diagonal elements, respectively. Let $\mathbf{U}_s \in \mathbb{C}^{r_s \times r_s}$ and $\mathbf{U}_t \in \mathbb{C}^{r_t \times r_t}$ be two arbitrary unitary matrices. Then it follows from (A6) that

$$\mathbf{R}_s^{\frac{1}{2}} \triangleq \mathbf{V}_s \boldsymbol{\Lambda}_s^{\frac{1}{2}} \mathbf{U}_s,\ \mathbf{R}_t^{\frac{1}{2}} \triangleq \mathbf{V}_t \boldsymbol{\Lambda}_t^{\frac{1}{2}} \mathbf{U}_t, \tag{A7}$$

are square roots of $\mathbf{R}_s$ and $\mathbf{R}_t$, respectively, satisfying $\mathbf{R}_s^{\frac{1}{2}} \mathbf{R}_s^{\frac{H}{2}} = \mathbf{R}_s$ and $\mathbf{R}_t^{\frac{1}{2}} \mathbf{R}_t^{\frac{H}{2}} = \mathbf{R}_t$. We thus have the chain in (A8) and (A9), shown at the bottom of the page, where the equality in (A8) is owing to the definition of $\boldsymbol{\Omega}$ in (A5), and the last equality is due to the fact that $(\mathbf{U}_s^H \boldsymbol{\Lambda}_s^{\frac{1}{2}} \otimes \mathbf{U}_t^H \boldsymbol{\Lambda}_t^{\frac{1}{2}})$ is of full rank. Moreover, since $\mathcal{G}_1$, $\mathcal{G}_2$ and $(\mathbf{V}_s \otimes \mathbf{V}_t)$ are all semi-unitary matrices, we have

$$\|\mathcal{G}_2^H \mathcal{G}_1 (\mathbf{V}_s \otimes \mathbf{V}_t)\|_2 \leq \|\mathbf{V}_s \otimes \mathbf{V}_t\|_2 \|\mathcal{G}_1\|_2 \|\mathcal{G}_2\|_2 \leq 1 \tag{A10}$$

which implies that the maximum eigenvalue value of $(\mathbf{V}_s^H \otimes \mathbf{V}_t^H)\mathcal{G}_1^H \mathcal{G}_2 \mathcal{G}_2^H \mathcal{G}_1 (\mathbf{V}_s \otimes \mathbf{V}_t)$ in (A9) is no larger than one. Therefore, determining the rank value in (A9) is equivalent to determining the number of singular values of $\mathcal{G}_2^H \mathcal{G}_1 (\mathbf{V}_s \otimes \mathbf{V}_t)$ that are strictly less than one. Let $\eta \geq 0$ be the number of singular values of $\mathcal{G}_2^H \mathcal{G}_1 (\mathbf{V}_s \otimes \mathbf{V}_t)$ that are equal to one. It follows from (A9) and (A9) that

$$\text{rank}\left(\boldsymbol{\Omega}^{\frac{1}{2}} (\mathbf{R}_s \otimes \mathbf{R}_t) \boldsymbol{\Omega}^{\frac{1}{2}}\right) = r_s r_t - \eta. \tag{A11}$$

To determine the value of η, we need the following lemma:

Lemma 3: *Let* $\mathbf{W} \in \mathbb{C}^{k \times p}$ *and* $\mathbf{Z} \in \mathbb{C}^{k \times q}$ *be two semi-unitary matrices. The matrix* $\mathbf{W}^H \mathbf{Z} \in \mathbb{C}^{p \times q}$ *has* $d\ (\leq \min\{p, q\})$ *singular values equal to one if and only if there exist two linearly independent sets* $\{\mathbf{x}_1, \ldots, \mathbf{x}_d\} \in \mathbb{C}^p$ *and* $\{\mathbf{y}_1, \ldots, \mathbf{y}_d\} \in \mathbb{C}^q$ *such that*

$$\mathbf{W}\mathbf{x}_i = \mathbf{Z}\mathbf{y}_i,\ i = 1, \ldots, d.$$

The proof of Lemma 3 is given in Appendix C. By applying Lemma 3 to $\mathcal{G}_2$ and $\mathcal{G}_1(\mathbf{V}_s \otimes \mathbf{V}_t)$, we see that there exist two linearly independent sets, say, $\{\mathbf{x}_1, \ldots, \mathbf{x}_\eta\} \in \mathbb{C}^{LN_t}$ and $\{\mathbf{y}_1, \ldots, \mathbf{y}_\eta\} \in \mathbb{C}^{r_s r_t}$, such that, for $i = 1, \ldots, \eta$,

$$\begin{bmatrix} \mathbf{C}_1(\mathbf{s}_1)(\mathbf{I}_{N_t} \otimes \mathbf{f}_1^T) \\ \vdots \\ \mathbf{C}_{N_c}(\mathbf{s}_{N_c})(\mathbf{I}_{N_t} \otimes \mathbf{f}_{N_c}^T) \end{bmatrix} \mathbf{x}_i = \begin{bmatrix} \mathbf{C}_1(\mathbf{s}_1')(\mathbf{I}_{N_t} \otimes \mathbf{f}_1^T) \\ \vdots \\ \mathbf{C}_{N_c}(\mathbf{s}_{N_c}')(\mathbf{I}_{N_t} \otimes \mathbf{f}_{N_c}^T) \end{bmatrix} \times (\mathbf{V}_s \otimes \mathbf{V}_t)\mathbf{y}_i \tag{A12}$$

where the definition of $\mathcal{G}(\cdot)$ in (9) is applied. Given that $\mathcal{G}(\cdot)$ is PCI achieving, (A12) holds only if

$$\mathbf{x}_i = (\mathbf{V}_s \otimes \mathbf{V}_t)\mathbf{y}_i,\ i = 1, \ldots, \eta. \tag{A13}$$

Now let us define a subcarrier subset $\mathcal{E} = \{n_1, \ldots, n_{|\mathcal{E}|}\} \subseteq \{1, \ldots, N_c\}$ and its complementary set $\mathcal{E}^c \subseteq \{1, \ldots, N_c\}$ where $\mathcal{E} \cup \mathcal{E}^c = \{1, \ldots, N_c\}$ and $\mathcal{E} \cap \mathcal{E}^c = \emptyset$. Suppose that $\mathbf{s}_{n_k} \neq \mathbf{s}_{n_k}'$ for all $k = 1, \ldots, |\mathcal{E}|$, and $\mathbf{s}_n = \mathbf{s}_n'$ for all $n \in \mathcal{E}^c$. Since $\mathbf{C}_1(\mathbf{s}_{n_k}) - \mathbf{C}_1(\mathbf{s}_{n_k}')$ is of full column rank for each k, we obtain from (A12) and (A13) that

$$\begin{bmatrix} \mathbf{I}_{N_t} \otimes \mathbf{f}_{n_1}^T \\ \vdots \\ \mathbf{I}_{N_t} \otimes \mathbf{f}_{n_{|\mathcal{E}|}}^T \end{bmatrix} (\mathbf{V}_s \otimes \mathbf{V}_t)\mathbf{y}_i = \tilde{\boldsymbol{\Pi}} \left(\mathbf{I}_{N_t} \otimes \mathbf{F}_{\mathcal{E}}\right) (\mathbf{V}_s \otimes \mathbf{V}_t)\mathbf{y}_i$$
$$= \tilde{\boldsymbol{\Pi}} \left(\mathbf{V}_s \otimes \mathbf{F}_{\mathcal{E}} \mathbf{V}_t\right) \mathbf{y}_i = \mathbf{0},\ i = 1, \ldots, \eta \tag{A14}$$

where $\mathbf{F}_{\mathcal{E}} = [\mathbf{f}_{n_1}, \ldots, \mathbf{f}_{n_{|\mathcal{E}|}}]^T$ and $\tilde{\boldsymbol{\Pi}}$ is a $|\mathcal{E}|N_t$ by $|\mathcal{E}|N_t$ permutation matrix. Under A4) that $\mathbf{f}_{n_k}^T \mathbf{V}_t \neq \mathbf{0}$ for all k (thus $\mathbf{F}_{\mathcal{E}} \mathbf{V}_t \neq \mathbf{0}$), and by the fact that $\text{rank}(\mathbf{A} \otimes \mathbf{B}) = \text{rank}(\mathbf{A})\text{rank}(\mathbf{B})$ for any two matrices $\mathbf{A}$ and $\mathbf{B}$ [40],

$$\text{rank}\left(\mathbf{V}_s \otimes \mathbf{F}_{\mathcal{E}} \mathbf{V}_t\right) = \text{rank}(\mathbf{V}_s)\text{rank}\left(\mathbf{F}_{\mathcal{E}} \mathbf{V}_t\right)$$
$$\geq \text{rank}(\mathbf{V}_s) = r_s,$$

and thus $\eta \leq r_s r_t - r_s$ according to the dimension theorem. By substituting it into (A11), we then obtain

$$\text{rank}\left(\boldsymbol{\Omega}^{\frac{1}{2}} (\mathbf{R}_s \otimes \mathbf{R}_t) \boldsymbol{\Omega}^{\frac{1}{2}}\right) \geq r_s.$$

By choosing the worst case of $|\mathcal{E}| = 1$, we thus obtain $d_{\text{NC}} = r_s$, as desired. ∎

APPENDIX C
PROOF OF LEMMA 3

The proof for the sufficiency of Lemma 3 is trivial and is omitted. To show the necessity, suppose that $\mathbf{W}^H \mathbf{Z}$ has d singular values equal to one. Then $\mathbf{Z}^H \mathbf{W} \mathbf{W}^H \mathbf{Z}$ has d eigenvalues

$$\text{rank}\left(\boldsymbol{\Omega}^{\frac{1}{2}} (\mathbf{R}_s \otimes \mathbf{R}_t) \boldsymbol{\Omega}^{\frac{1}{2}}\right) = \text{rank}\left((\mathbf{R}_s^{\frac{H}{2}} \otimes \mathbf{R}_t^{\frac{H}{2}}) \boldsymbol{\Omega} (\mathbf{R}_s^{\frac{1}{2}} \otimes \mathbf{R}_t^{\frac{1}{2}})\right)$$
$$= \text{rank}\left((\mathbf{R}_s^{\frac{H}{2}} \mathbf{R}_s^{\frac{1}{2}} \otimes \mathbf{R}_t^{\frac{H}{2}} \mathbf{R}_t^{\frac{1}{2}}) - (\mathbf{R}_s^{\frac{H}{2}} \otimes \mathbf{R}_t^{\frac{H}{2}}) \mathcal{G}_1^H \mathcal{G}_2 \mathcal{G}_2^H \mathcal{G}_1 (\mathbf{R}_s^{\frac{1}{2}} \otimes \mathbf{R}_t^{\frac{1}{2}})\right) \tag{A8}$$
$$= \text{rank}\left((\mathbf{U}_s^H \boldsymbol{\Lambda}_s^{\frac{1}{2}} \otimes \mathbf{U}_t^H \boldsymbol{\Lambda}_t^{\frac{1}{2}}) \left[\mathbf{I}_{r_s r_t} - (\mathbf{V}_s^H \otimes \mathbf{V}_t^H) \mathcal{G}_1^H \mathcal{G}_2 \mathcal{G}_2^H \mathcal{G}_1 (\mathbf{V}_s \otimes \mathbf{V}_t)\right] (\boldsymbol{\Lambda}_s^{\frac{1}{2}} \mathbf{U}_s \otimes \boldsymbol{\Lambda}_t^{\frac{1}{2}} \mathbf{U}_t)\right)$$
$$= \text{rank}\left(\mathbf{I}_{r_s r_t} - (\mathbf{V}_s^H \otimes \mathbf{V}_t^H) \mathcal{G}_1^H \mathcal{G}_2 \mathcal{G}_2^H \mathcal{G}_1 (\mathbf{V}_s \otimes \mathbf{V}_t)\right), \tag{A9}$$

equal to one, and thus there exist linear independent vectors $\mathbf{y}_1, \ldots, \mathbf{y}_d \in \mathbb{C}^q$ such that

$$\mathbf{y}_i^H \mathbf{Z}^H \mathbf{W}\mathbf{W}^H \mathbf{Z}\mathbf{y}_i = \mathbf{y}_i^H \mathbf{y}_i,\ i = 1, \ldots, d. \tag{A15}$$

Let $\mathbf{z}_i = \mathbf{Z}\mathbf{y}_i \in \mathbb{C}^k, i = 1, \ldots, d$, which are linearly independent. Since $\mathbf{y}_i^H \mathbf{y}_i = \mathbf{y}_i^H \mathbf{Z}^H \mathbf{Z}\mathbf{y}_i = \mathbf{z}_i^H \mathbf{z}_i$ due to semi-unitary $\mathbf{Z}$, we obtain from (A15) that

$$\mathbf{z}_i^H (\mathbf{I}_k - \mathbf{W}\mathbf{W}^H)\mathbf{z}_i = 0,\ i = 1, \ldots, d. \tag{A16}$$

Equation (A16) implies that $\mathbf{z}_1, \ldots, \mathbf{z}_d$ lie in the range space of $\mathbf{W}$. Hence there exist linear independent vectors $\mathbf{x}_1, \ldots, \mathbf{x}_d \in \mathbb{C}^p$ such that

$$\mathbf{z}_i = \mathbf{Z}\mathbf{y}_i = \mathbf{W}\mathbf{x}_i,\ i = 1, \ldots, d. \tag{A17}$$

Lemma 3 is thus proved. ∎

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