

Multicell Coordinated Beamforming With Rate Outage Constraint—Part I: Complexity Analysis

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Abstract—This paper studies the coordinated beamforming (CoBF) design in the multiple-input single-output interference channel, assuming only channel distribution information given a priori at the transmitters. The CoBF design is formulated as an optimization problem that maximizes a predefined system utility, e.g., the weighted sum rate or the weighted max-min-fairness (MMF) rate, subject to constraints on the individual probability of rate outage and power budget. While the problem is non-convex and appears difficult to handle due to the intricate outage probability constraints, so far it is still unknown if this outage constrained problem is computationally tractable. To answer this, we conduct a computational complexity analysis of the outage constrained CoBF problem. Specifically, we show that the outage constrained CoBF problem with the weighted sum rate utility is intrinsically difficult, i.e., NP-hard. Moreover, the outage constrained CoBF problem with the weighted MMF rate utility is also NP-hard except the case when all the transmitters are equipped with single antenna. The presented analysis results confirm that efficient approximation methods are indispensable to the outage constrained CoBF problem.

Index Terms—Interference channel, coordinated beamforming, outage probability, complexity analysis.

I. INTRODUCTION

COORDINATED transmission has been recognized as a promising approach to improving the system performance of wireless cellular networks [2]. According to the level of cooperation, the coordinated transmission can be roughly classified into two categories, i.e., *MIMO cooperation* and *interference coordination* [3]. In MIMO cooperation, the transmitters, e.g., base stations (BSs), cooperate in data transmission by sharing all the channel state information (CSI) and data signals. In interference coordination, the BSs only coordinate in the transmission strategies for mitigating the inter-cell in-

terference. Compared with MIMO cooperation, interference coordination requires the CSI to be shared only, and hence induces less overhead on the backhaul network [4]. A common model for studying interference coordination is the interference channel (IFC) [5], where multiple transmitters simultaneously communicate with their respective receivers over a common frequency band, and thus interfere with each other.

In this paper, we consider a multiple-input single-output (MISO) IFC, wherein the transmitters are equipped with multiple antennas while the receivers are equipped with single antenna. Moreover, we are interested in the *coordinated beamforming* (CoBF) design problem; that is, the transmitters coordinate with each other to optimize their transmit beamforming vectors. A typical formulation of the CoBF design problem is to maximize a system utility function, e.g., the sum rate, proportional fairness rate, harmonic mean rate or the max-min-fairness (MMF) rate, assuming that the transmitters have perfect CSI. It turns out that the CoBF problems are in general difficult optimization problems. Specifically, it has been shown in [6] that, except for the MMF rate [7], the CoBF problem for the sum rate, proportional fairness rate and harmonic mean rate are NP-hard in general, implying that they cannot be efficiently solved in general. Due to this fact, a significant amount of research efforts has been devoted to the development of reliable and efficient methods for handling the CoBF problems. For example, the works [8]–[11] characterize the optimal beamforming structure in order to reduce the dimension of exhaustive search. Global optimization algorithms were also developed in [12]–[15] but are only efficient when the number of users is small. Another branch of works focus on suboptimal but computationally efficient approximation algorithms; see [6], [10], [16]–[22].

The works mentioned above all have assumed that the transmitters have perfect CSI. However, in practical wireless scenarios, especially in mobile channels, it is difficult for the transmitters to acquire accurate CSI due to time-varying channels. In contrast, the statistical distribution of the channel, i.e., the channel distribution information (CDI), can remain static in a relatively long period of time and thus is easier to obtain compared to the CSI. However, with only CDI at the transmitters, the transmission would suffer from rate outage; that is, the instantaneous channel may not reliably support the data transmission. In view of this, outage-aware CoBF designs, which constrain the probability of rate outage to be low, have attracted extensive attention recently; see, e.g., [23]–[25] for the outage constrained utility maximization problem, [26], [27] for the outage constrained power minimization problem and [26], [28], [29] for the

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outage balancing problem. One essential fact that is commonly observed in these works is that the outage constrained CoBF problems are usually nonconvex and appear much more difficult to handle than their perfect CSI counterparts [6], [7], due to the complicated probabilistic outage constraints. Therefore, most of the works [24], [25], [29] have concentrated on developing efficient approximation algorithms. However, unlike the perfect CSI case where the complexity status of various CoBF problems has been identified [6], [7], it is still not clear if the outage constrained CoBF problems are computational tractable or, instead, are intrinsically difficult, i.e., NP-hard [30].

Our interest in this paper lies in characterizing the computational complexity status of the outage constrained CoBF problem. Specifically, we consider the outage constrained (weighted) sum rate maximization (SRM) CoBF problem and the outage constrained (weighted) MMF CoBF problem, which respectively maximize the (weighted) sum rate and (weighted) MMF rate under rate outage constraints and individual power constraints. We analytically show that the outage constrained SRM problem is NP-hard in general, and that the outage constrained MMF problem is polynomial-time solvable when each of the transmitters is equipped with only one antenna but is NP-hard when each of the transmitters is equipped with at least two antennas. The NP-hardness of the outage constrained SRM problem is established by a polynomial-time reduction from the NP-hard Max-Cut problem [30], i.e., each problem instance of the Max-Cut problem can be transformed to a problem instance of the outage constrained SRM problem in polynomial time; while the NP-hardness of the outage constrained MMF problem in the MISO scenario is established by a polynomial-time reduction from the 3-satisfiability (3-SAT) problem, which is known to be NP-complete [30]. The proposed analysis about the NP-hardness of MMF problem can also be analogously applied to prove that the outage balancing problem studied in [28] is NP-hard when each of the transmitters has at least two antennas. The complexity analysis results further motivate the development of efficient approximation algorithms for handling the outage constrained CoBF problem; see [31].

Synopsis: In Section II, we present the system model and problem formulations. The complexity analyses for the outage constrained SRM problem and outage constrained MMF problem are presented in Section III and Section IV, respectively. Finally, we draw the conclusions in Section V.

Notation: The sets of n -dimensional real vectors, complex vectors and complex Hermitian matrices are denoted by \mathbb{R}^n , \mathbb{C}^n and \mathbb{H}^n , respectively. The set of non-negative real vectors and the set of positive-semidefinite Hermitian matrices are denoted by \mathbb{R}_+^n and \mathbb{H}_+^n , respectively. The superscripts ‘ T ’ and ‘ H ’ represent the matrix transpose and conjugate transpose, respectively. We denote $\|\cdot\|$ and $\lceil \cdot \rceil$ as the vector Euclidean norm and ceiling function, respectively. $\mathbf{A} \succeq \mathbf{0}$ means that matrix \mathbf{A} is positive semidefinite. We use the expression $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{Q})$ if \mathbf{x} is circularly symmetric complex Gaussian distributed with mean $\boldsymbol{\mu}$ and covariance matrix \mathbf{Q} . We denote $\exp(\cdot)$ (or simply $e^{(\cdot)}$) as the exponential function, while $\ln(\cdot)$ and $\Pr\{\cdot\}$ represent the natural logarithmic function and the probability function, respectively. For a variable a_{ik} , where the domain of sub-

script pair i, k are clear from context, $\{a_{ik}\}$ denotes the set of all a_{ik} with the subscript pair i, k covering all the possible values over its domain, and $\{a_{ik}\}_k$ denotes the set of all a_{ik} with the first subscript equal to i . The sets $\{a_{ik}\}_{k \neq j}$ and $\{a_{ik}\}_{(i,k) \neq (j,\ell)}$ are defined by the set $\{a_{ik}\}_k$ excluding a_{ij} and the set $\{a_{ik}\}$ excluding $a_{j\ell}$, respectively.

II. SYSTEM MODEL AND PROBLEM STATEMENT

We consider a MISO IFC with K pairs of transmitters and receivers (i.e., K users). Each transmitter is equipped with N_t antennas and each receiver is equipped with single antenna. Transmit beamforming is assumed for the communication between each transmitter and its intended receiver. Let $s_i \sim \mathcal{CN}(0, 1)$ denote the message for the i th user, and $\mathbf{w}_i \in \mathbb{C}^{N_t}$ denote the corresponding beamforming vector. We assume a frequency flat channel model, and the channel vector between transmitter i and receiver k is modeled as $\mathbf{h}_{ik} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Q}_{ik})$, where $\mathbf{Q}_{ik} \succeq \mathbf{0}$ is the channel covariance matrix. The received signal for receiver i is then given by

$$x_i = \mathbf{h}_{ii}^H \mathbf{w}_i s_i + \sum_{k=1, k \neq i}^K \mathbf{h}_{ki}^H \mathbf{w}_k s_k + n_i, \quad (1)$$

where $n_i \sim \mathcal{CN}(0, \sigma_i^2)$ is the additive noise at receiver i with variance $\sigma_i^2 > 0$. Under the assumption that single-user detection is used by each receiver, the instantaneous achievable rate (in bits/sec/Hz) of the i th user can be expressed as

$$r_i(\{\mathbf{h}_{ki}\}_k, \{\mathbf{w}_k\}) = \log_2 \left(1 + \frac{|\mathbf{h}_{ii}^H \mathbf{w}_i|^2}{\sum_{k \neq i} |\mathbf{h}_{ki}^H \mathbf{w}_k|^2 + \sigma_i^2} \right). \quad (2)$$

To enhance the overall system performance, a typical formulation of the CoBF design is to optimize the beamforming vectors $\{\mathbf{w}_k\}_{k=1}^K$ of all the K users so as to maximize a performance measuring system utility function, e.g., the information sum rate, under power constraints [6], which can be mathematically expressed as

$$\max_{\substack{\mathbf{w}_i \in \mathbb{C}^{N_t}, R_i \geq 0, \\ i=1, \dots, K}} U(R_1, \dots, R_K) \quad (3a)$$

$$\text{subject to (s.t.) } R_i \leq r_i(\{\mathbf{h}_{ki}\}_k, \{\mathbf{w}_k\}), \quad i = 1, \dots, K, \quad (3b)$$

$$\|\mathbf{w}_i\|^2 \leq P_i, \quad i = 1, \dots, K. \quad (3c)$$

Here, R_1, \dots, R_K are the respective transmission rates of the K users, and P_1, \dots, P_K are the associated power budgets. The function $U(R_1, \dots, R_K)$ denotes the system utility. In this paper, we are interested in two system utilities in particular, i.e., the weighted sum rate, where $U(R_1, \dots, R_K) = \sum_{i=1}^K \alpha_i R_i$ and the weighted minimum rate, where $U(R_1, \dots, R_K) = \min_{i=1, \dots, K} R_i / \alpha_i$. The corresponding utility optimization problems are known as the SRM problem and the MMF problem, respectively. These two problem formulations represent two extremes of the tradeoff between system throughput and the user fairness. For the SRM problem, one aims to maximize the system throughput,

TABLE I
 SUMMARY OF COMPLEXITY ANALYSIS RESULTS

| | SRM with CSI [32, Theorem 1] | SRM with CDI Theorem 1 | MMF with CSI [6, Theorem 3.3] | MMF with CDI Theorems 2 & 3 |
|-----------|-------------------------------------|------------------------------|----------------------------------|--------------------------------|
| $N_t = 1$ | NP-hard (Max Indep. Set Problem) | NP-hard (Max-Cut Problem) | Polynomial-time solvable | Polynomial-time solvable |
| $N_t > 1$ | NP-hard | NP-hard | Polynomial-time solvable | NP-hard (3-SAT Problem) |

but the transmission may be dominated by a few of users. For the MMF problem, one places the highest emphasis on user fairness, but the achieved system throughput may not be as high. Interestingly, the complexity status of solving these two problems are also very different. Specifically, solving the SRM problem is NP-hard in general [32] which means that the problem is unlikely to be solved in a polynomial-time complexity; by contrast, the MMF problem is polynomial-time solvable [6]. Efficient approximation algorithms for handling the SRM problem have been extensively studied (see [6], [15] and references therein).

However, to solve the CoBF design problem (3), it is required that perfect instantaneous CSI is available at the transmitters, leading to enormous communication overhead. It is hence more appropriate to assume that only statistical channel information, i.e., the set of channel covariance matrices $\{\mathbf{Q}_{ik}\}$, is available at the transmitters. Under such circumstances, reliable transmission cannot be guaranteed, and the users may suffer from rate outage. Specifically, given any transmission rate $R_i > 0$, the outage event $r_i(\{\mathbf{h}_{ki}\}_k, \{\mathbf{w}_k\}) < R_i$ occurs with a non-zero probability. It is therefore desirable to constrain the probability of rate outage below a preassigned threshold. Let $\epsilon_i \in (0, 1)$ be the maximum tolerable rate-outage probability for user i . To the end, we consider the following outage constrained CoBF design problem [25]

$$\max_{\mathbf{w}_i \in \mathbb{C}^{N_t}, R_i \geq 0, i=1, \dots, K} U(R_1, \dots, R_K) \quad (4a)$$

$$\text{s.t. } \Pr\{r_i(\{\mathbf{h}_{ki}\}_k, \{\mathbf{w}_k\}) < R_i\} \leq \epsilon_i, \quad (4b)$$

$$\|\mathbf{w}_i\|^2 \leq P_i, \quad i = 1, \dots, K. \quad (4c)$$

According to [25], [33], the outage constraint (4b) can be explicitly expressed as

$$\rho_i \exp\left(\frac{(2^{R_i} - 1)\sigma_i^2}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i}\right) \prod_{k \neq i} \left(1 + \frac{(2^{R_i} - 1)\mathbf{w}_k^H \mathbf{Q}_{ki} \mathbf{w}_k}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i}\right) \leq 1, \quad (5)$$

where $\rho_i \triangleq 1 - \epsilon_i$ for $i = 1, \dots, K$ are the satisfaction probabilities required in the downlink transmission.

Due to the complicated constraint (5), solving the outage constrained problem (4) seems more difficult than solving its perfect CSI counterpart, i.e., problem (3). However, this intuitive observation is not mathematically precise. It is hence of interest to investigate the complexity status of problem (4). In the ensuing sections, we study the complexity of solving problem (4) with weighted sum rate and minimum rate utilities, which correspond to the SRM and MMF formulations, respectively. Our

complexity analysis will demonstrate that problem (4) is indeed more challenging. Specifically, problem (4) is NP-hard not only for the SRM formulation but also for the MMF formulation, while problem (3) is at least polynomial-time solvable for the MMF formulation [6]. The obtained results about the complexity of problem (4), together with the corresponding results in the literature about problem (3), are summarized in Table I on the top of this page.

III. COMPLEXITY ANALYSIS FOR OUTAGE CONSTRAINED SRM PROBLEM

In this section, we analyze the complexity status of the outage constrained SRM problem, which can be written as

$$\max_{\mathbf{w}_i \in \mathbb{C}^{N_t}, R_i \geq 0, i=1, \dots, K} \sum_{i=1}^K \alpha_i R_i \quad (6a)$$

$$\text{s.t. } \rho_i \exp\left(\frac{(2^{R_i} - 1)\sigma_i^2}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i}\right) \times \prod_{k \neq i} \left(1 + \frac{(2^{R_i} - 1)\mathbf{w}_k^H \mathbf{Q}_{ki} \mathbf{w}_k}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i}\right) \leq 1, \quad (6b)$$

$$\|\mathbf{w}_i\|^2 \leq P_i, \quad i = 1, \dots, K. \quad (6c)$$

Specifically, we demonstrate that problem (6) is NP-hard in general. The following theorem makes our statement precise.

Theorem 1: *The outage constrained SRM problem (6) is NP-hard in the number of users K , for all $N_t \geq 1$.*

Theorem 1 indicates that problem (6) is computationally intractable, like its perfect CSI counterpart (3) with the weighted sum rate utility [32]. While both of these two problems are NP-hard, one should note that the techniques used for proving Theorem 1 are quite different from those used in [32]. In [32], it was shown that problem (3) with the weighted sum rate utility is at least as difficult as the maximum independent set problem (which is known NP-complete) [30]. However, the same idea is not applicable to the complexity analysis for problem (6), due to the much more involved constraints (6b). Instead, we show in the next subsection that problem (6) is at least as difficult as the Max-Cut problem.

A. Proof of Theorem 1

Here, we show that problem (6) is NP-hard even when $N_t = 1$, which implies that problem (6) is NP-hard for the general case of $N_t \geq 1$. For $N_t = 1$, the CoBF design problem (6) reduces to a coordinated power control problem. Specifically, the MISO

channel $\mathbf{h}_{ki} \in \mathbb{C}^{N_t}$ reduces to the single-input single-output (SISO) channel $h_{ki} \in \mathbb{C}$, the channel covariance matrix $\mathbf{Q}_{ki} \in \mathbb{H}_+^{N_t}$ reduces to $Q_{ki} \in \mathbb{R}_+$, and the beamformer \mathbf{w}_i reduces to the square root of the transmit power p_i ; thus, problem (6) reduces to

$$\max_{\substack{p_i \in \mathbb{R}, R_i \geq 0, \\ i=1, \dots, K}} \sum_{i=1}^K \alpha_i R_i \quad (7a)$$

$$\text{s.t.} \quad \rho_i \exp\left(\frac{(2^{R_i} - 1)\sigma_i^2}{Q_{ii}p_i}\right) \times \prod_{k \neq i} \left(1 + \frac{(2^{R_i} - 1)Q_{ki}p_k}{Q_{ii}p_i}\right) \leq 1, \quad (7b)$$

$$0 \leq p_i \leq P_i, i = 1, \dots, K. \quad (7c)$$

We prove that problem (7) is NP-hard by deriving a polynomial-time transformation [6], [32] from the weighted Max-Cut problem, which is known to be NP-hard [30], to problem (7). Specifically, given an arbitrary instance of the Max-Cut problem, we can construct through the transformation a particular instance of (7) such that these two instances are equivalent. This polynomial-time transformation implies that any algorithm that can solve problem (7) in polynomial time also can solve the Max-Cut problem in polynomial time. Hence, problem (7) is at least as difficult as the Max-Cut problem, i.e., NP-hard. For ease of the ensuing presentation, the definition of the weighted Max-Cut problem is repeated as follows.

Definition 1: Consider an undirected and connected graph $G = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, |\mathcal{V}|\}$ denotes the set of vertices in G , and $\mathcal{E} = \{(i, j) | i \in \mathcal{V}, j \in \mathcal{V}, i < j, \text{ and } i, j \text{ are connected in } G\}$ denotes the set of edges in G . Each edge $(i, j) \in \mathcal{E}$ is assigned with a weight $w_{ij} > 0$. A cut, $\mathcal{E}(\mathcal{S}) \subseteq \mathcal{E}$, consists of the set of edges crossing a subset $\mathcal{S} \subseteq \mathcal{V}$ and its complement $\bar{\mathcal{S}} = \mathcal{V} \setminus \mathcal{S}$. The weighted Max-Cut problem is formulated as

$$\max_{\mathcal{S} \subseteq \mathcal{V}} \sum_{(i,j) \in \mathcal{E}(\mathcal{S})} w_{ij}. \quad (8)$$

To build the connection between the Max-Cut problem and problem (7), we consider an alternative formulation of (7):

Lemma 1: Problem (7) can be equivalently expressed as

$$\max_{p_i \in \mathbb{R}, i=1, \dots, K} \sum_{i=1}^K \alpha_i \log_2(1 + \zeta_i(\{p_k\}_{k \neq i})Q_{ii}p_i) \quad (9a)$$

$$\text{s.t.} \quad 0 \leq p_i \leq P_i, i = 1, \dots, K, \quad (9b)$$

where $\zeta_i(\{p_k\}_{k \neq i})$, which is continuously differentiable in $\{p_k\}_{k \neq i}$, is the unique solution that satisfies

$$\Psi_i(x, \{p_k\}_{k \neq i}) \triangleq \rho_i \exp(\sigma_i^2 x) \prod_{k \neq i} (1 + Q_{ki}p_k \cdot x) = 1, \quad (10)$$

for all $i, k = 1, \dots, K$.

Proof: See Appendix A for details. ■

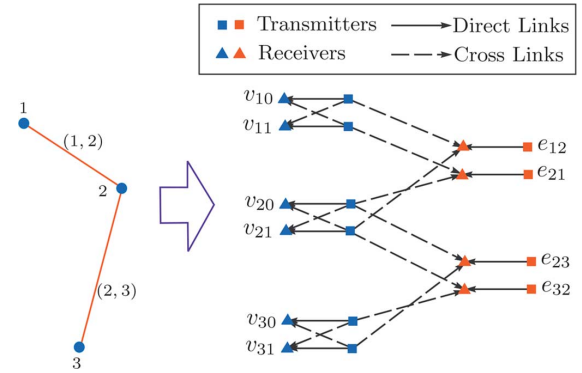


Fig. 1. An example illustrating the association between a graph $G(\mathcal{V} = \{1, 2, 3\}, \mathcal{E} = \{(1, 2), (2, 3)\})$ and an IFC with 10 users.

By reformulating problem (7) as problem (9), one can compactly characterize the relation between the achievable rates and the transmit powers, e.g., the monotonicity and convexity, using the implicit functions $\zeta_i(\cdot)$, $i = 1, \dots, K$. As a result, it is much easier to analyze the optimal power allocation pattern based on the alternative formulation (9) than based on the original formulation (7). Thus, given any instance of the Max-Cut problem, i.e., the undirected and connected graph $G = (\mathcal{V}, \mathcal{E})$ and the weights $w_{ij} > 0$ for all $(i, j) \in \mathcal{E}$, we focus on constructing a particular instance of (9) that is equivalent to the weighted Max-Cut problem (8) associated with the graph G and the weights $\{w_{ij}\}$. The construction is detailed as follows.

We associate each node $i \in \mathcal{V}$ with two distinct transmitter-receiver pairs (users) in the coordinated power control problem (9), denoted by v_{i0}, v_{i1} . Moreover, each edge $(i, j) \in \mathcal{E}$ is associated with two other users, denoted by e_{ij}, e_{ji} . The resulting set of users is the union of the user set \mathcal{U}_v associated with nodes and the user set \mathcal{U}_e associated with edges, i.e.,

$$\begin{aligned} \mathcal{U} &= \mathcal{U}_v \cup \mathcal{U}_e \\ &\triangleq \{v_{10}, v_{11}, \dots, v_{|\mathcal{V}|0}, v_{|\mathcal{V}|1}\} \cup \{e_{ij}, e_{ji} | (i, j) \in \mathcal{E}\}, \end{aligned} \quad (11)$$

which contains $K = |\mathcal{U}| = 2(|\mathcal{V}| + |\mathcal{E}|)$ users in total. For these K users, we consider a particular instance of problem (9) with the following specified system parameters:

$$\sigma_u^2 = 0.1, \rho_u = 0.95, P_u = \begin{cases} 1, & \text{if } u \in \mathcal{U}_v, \forall u \in \mathcal{U}, \\ 0.7, & \text{if } u \in \mathcal{U}_e, \end{cases} \quad (12a)$$

$$Q_{v_{ia}v_{jb}} = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{otherwise,} \end{cases} \quad Q_{e_{ij}u} = \begin{cases} 1, & \text{if } u = e_{ij}, \\ 0, & \text{otherwise,} \end{cases} \quad (12b)$$

$$Q_{v_{ia}e_{k\ell}} = \begin{cases} 1, & \text{if } (i, a) = (k, 0) \text{ or } (i, a) = (\ell, 1), \\ 0, & \text{otherwise,} \end{cases} \quad (12c)$$

$$\begin{cases} \alpha_{v_{ia}} = 1, & \forall v_{ia} \in \mathcal{U}_v, \\ \alpha_{e_{ij}} = \alpha_{e_{ji}} = \frac{w_{ij}}{2 \sum_{(k,\ell) \in \mathcal{E}} w_{k,\ell}}, & \forall e_{ij} \in \mathcal{U}_e, i < j. \end{cases} \quad (12d)$$

To clarify the association described by (11) and (12), let us consider a simple example illustrated by Fig. 1. Here, we demonstrate how a simple graph G with three vertices $\mathcal{V} = \{1, 2, 3\}$ and two edges $\mathcal{E} = \{(1, 2), (2, 3)\}$ can be

mapped to an IFC through (11) and (12). As illustrated in Fig. 1, each vertex i corresponds to two users, v_{i0} and v_{i1} , for all $i \in \{1, 2, 3\}$; and each edge (i, j) also corresponds to two users, e_{ij} and e_{ji} , for all $(i, j) \in \{(1, 2), (2, 3)\}$. For the IFC on the right-hand side in Fig. 1, user u and user u' would interfere with each other if $Q_{uu'} = 1$ and do not interfere if $Q_{uu'} = 0$. Thus, according to the parameter set in (12b), any two users associated with a common vertex in G will interfere with each other, while users associated with different vertices would not interfere with each other. Besides, the users associated with the edges only communicate with their intended receivers and do not interfere with the other users. By (12c), user v_{i0} and v_{j1} would interfere with user e_{ij} , while users v_{j0} and v_{i1} would interfere with user e_{ji} , for all edges (i, j) . All the resulting interference patterns are shown in Fig. 1.

Based on the construction described by (11) and (12), the resulting problem instance of the coordinated power control problem (9) is given by

$$\begin{aligned} \max_{\substack{p_{ia}^v \in \mathbb{R}, \forall v_{ia} \in \mathcal{U}_v, \\ p_{jk}^e \in \mathbb{R}, \forall e_{jk} \in \mathcal{U}_e}} \left\{ \sum_{v_{ia} \in \mathcal{U}_v} \log_2(1 + p_{ia}^v \zeta^v(p_{i\bar{a}}^v)) \right. \\ \left. + \sum_{e_{ij} \in \mathcal{U}_e} \alpha_{e_{ij}} \log_2(1 + p_{ij}^e \zeta^e(p_{i0}^v, p_{j1}^v)) \right\} \end{aligned} \quad (13a)$$

$$\text{s.t. } 0 \leq p_{ia}^v \leq 1, v_{ia} \in \mathcal{U}_v, \quad (13b)$$

$$0 \leq p_{jk}^e \leq 0.7, e_{ij} \in \mathcal{U}_e, \quad (13c)$$

where p_{ia}^v and p_{jk}^e denote the transmission powers of user v_{ia} and e_{jk} , respectively; $\bar{a} = 1 - a$ for $a \in \{0, 1\}$; and by (10), $\zeta^v(p)$ and $\zeta^e(p_1, p_2) = \zeta^e(p_2, p_1)$ are the unique solutions of

$$\Psi^v(x, p) \triangleq 0.95 e^{0.1x} (1 + px) = 1, \quad (14a)$$

$$\Psi^e(x, p_1, p_2) \triangleq 0.95 e^{0.1x} (1 + p_1 x)(1 + p_2 x) = 1, \quad (14b)$$

respectively. Next, we show that problem (13) is equivalent to the Max-Cut problem (8). To this end, we need to demonstrate that the optimal solution to problem (13) lies in a discrete set, as stated in the following lemma, which is proved in Appendix B.

Lemma 2: *The optimal solution of (13) must satisfy*

$$\begin{bmatrix} p_{i0}^v \\ p_{i1}^v \end{bmatrix} \in \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\} \text{ for } i = 1, \dots, |\mathcal{V}|, \quad (15a)$$

$$p_{jk}^e = 0.7, \forall e_{jk} \in \mathcal{U}_e. \quad (15b)$$

Based on Lemma 2, we can restrict the feasible set of (13) to the subset defined in (15) without loss of optimality. Let

$$\mathcal{S} = \{i \mid [p_{i0}^v \ p_{i1}^v]^T = [0 \ 1]^T\} \subseteq \mathcal{V},$$

and denote $1_{\{i \in \mathcal{S}\}}$ as the indicator function which is equal to one if $i \in \mathcal{S}$ and zero otherwise. Then, by (15), we have $p_{i0}^v = 1_{\{i \in \bar{\mathcal{S}}\}}$, $p_{i1}^v = 1_{\{i \in \mathcal{S}\}}$ and $p_{jk}^e = 0.7$. With these substitutions in problem (13), problem (13) can be equivalently reformulated

as the following problem where \mathcal{S} is now the optimization variable:

$$\begin{aligned} \max_{\mathcal{S} \subseteq \mathcal{V}} \sum_{i=1}^{|\mathcal{V}|} & [\log_2(1 + 1_{\{i \in \bar{\mathcal{S}}\}} \zeta^v(1_{\{i \in \mathcal{S}\}})) \\ & + \log_2(1 + 1_{\{i \in \mathcal{S}\}} \zeta^v(1_{\{i \in \bar{\mathcal{S}}\}}))] \\ & + \sum_{(i,j) \in \mathcal{E}} [\alpha_{e_{ij}} \log_2(1 + 0.7 \zeta^e(1_{\{i \in \bar{\mathcal{S}}\}}, 1_{\{j \in \mathcal{S}\}}))] \\ & + \alpha_{e_{ji}} \log_2(1 + 0.7 \zeta^e(1_{\{j \in \bar{\mathcal{S}}\}}, 1_{\{i \in \mathcal{S}\}}))] \end{aligned} \quad (16)$$

$$\begin{aligned} = \max_{\mathcal{S} \subseteq \mathcal{V}} \sum_{i=1}^{|\mathcal{V}|} & \log_2(1 + 1 \cdot \zeta^v(0)) + \sum_{(i,j) \in \mathcal{E} \setminus \mathcal{E}(\mathcal{S})} \alpha_{e_{ij}} (c_{01} + c_{10}) \\ & + \sum_{(i,j) \in \mathcal{E}(\mathcal{S})} \alpha_{e_{ij}} (c_{00} + c_{11}) \end{aligned} \quad (17)$$

$$\begin{aligned} = \max_{\mathcal{S} \subseteq \mathcal{V}} \sum_{i=1}^{|\mathcal{V}|} & \log_2(1 + 1 \cdot \zeta^v(0)) + \sum_{(i,j) \in \mathcal{E}} \alpha_{e_{ij}} (c_{01} + c_{10}) \\ & + \sum_{(i,j) \in \mathcal{E}(\mathcal{S})} \alpha_{e_{ij}} (c_{00} + c_{11} - c_{01} - c_{10}) \end{aligned} \quad (18)$$

$$= \max_{\mathcal{S} \subseteq \mathcal{V}} |\mathcal{V}| \cdot \log_2(1 + 1 \cdot \zeta^v(0)) + \frac{c_{01} + c_{10}}{2} \\ + \frac{c_{00} + c_{11} - c_{01} - c_{10}}{2 \sum_{(i,j) \in \mathcal{E}} w_{ij}} \cdot \sum_{(i,j) \in \mathcal{E}(\mathcal{S})} w_{ij},$$

where $c_{00} = \log_2(1 + 0.7 \zeta^e(0, 0))$, $c_{11} = \log_2(1 + 0.7 \zeta^e(1, 1))$, $c_{01} = \log_2(1 + 0.7 \zeta^e(0, 1))$, and $c_{10} = \log_2(1 + 0.7 \zeta^e(1, 0))$ are constants. Note that, when $(i, j) \in \mathcal{E}(\mathcal{S})$, vertex i and vertex j belong to different subsets \mathcal{S} and $\bar{\mathcal{S}}$, respectively, so it holds true that $1_{\{i \in \bar{\mathcal{S}}\}} = 1_{\{j \in \mathcal{S}\}}$ and $1_{\{i \in \mathcal{S}\}} = 1_{\{j \in \bar{\mathcal{S}}\}}$; on the other hand, when $(i, j) \in \mathcal{E} \setminus \mathcal{E}(\mathcal{S})$, it holds true that $1_{\{i \in \bar{\mathcal{S}}\}} \neq 1_{\{j \in \mathcal{S}\}}$ and $1_{\{i \in \mathcal{S}\}} \neq 1_{\{j \in \bar{\mathcal{S}}\}}$, by which we obtain (17). By (14b), one can show that $c_{00} + c_{11} - c_{01} - c_{10} > 0$, so solving (13) is equivalent to solving the Max-Cut problem (8). Thus, we have presented a polynomial-time transformation that equivalently converts all the problem instances of the Max-Cut problem (8) to a subset (i.e., (13) with the parameters specified by (12)) of the instances of (6). Therefore, any algorithm (if exists) that can solve the outage constrained SRM problem (6) with any number of antennas $N_t \geq 1$ in polynomial time also can solve the Max-Cut problem in polynomial time, implying that problem (6) is NP-hard for all $N_t \geq 1$. ■

IV. COMPLEXITY ANALYSIS FOR OUTAGE CONSTRAINED MMF PROBLEM

In this section, we turn our attention to the outage constrained MMF problem, i.e.,

$$\max_{\substack{\mathbf{w}_i \in \mathbb{C}^{N_t}, R_i \geq 0, \\ i=1, \dots, K}} R \triangleq \min_{i=1, \dots, K} R_i / \alpha_i \quad (19a)$$

$$\text{s.t. } \rho_i \exp \left(\frac{(2^{R_i} - 1) \sigma_i^2}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i} \right) \\ \times \prod_{k \neq i} \left(1 + \frac{(2^{R_i} - 1) \mathbf{w}_k^H \mathbf{Q}_{ki} \mathbf{w}_k}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i} \right) \leq 1, \quad (19b)$$

$$\|\mathbf{w}_i\|^2 \leq P_i, \quad i = 1, \dots, K. \quad (19c)$$

In contrast to its perfect CSI counterpart, which can be transformed into a quasiconvex problem for multiple antennas and multiple users [6], we will show that problem (19) is polynomial-time solvable only for the single antenna case, i.e., $N_t = 1$, but NP-hard in the number of users K when $N_t \geq 2$.

To proceed with the complexity analysis, let us first introduce a feasibility problem. That is, given a target rate $\bar{R} \geq 0$,

$$\text{Find } \mathbf{w}_1, \dots, \mathbf{w}_K \quad (20a)$$

$$\text{s.t. } \rho_i \exp\left(\frac{(2^{\alpha_i \bar{R}} - 1)\sigma_i^2}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i}\right) \times \prod_{k \neq i} \left(1 + \frac{(2^{\alpha_i \bar{R}} - 1)\mathbf{w}_k^H \mathbf{Q}_{ki} \mathbf{w}_k}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i}\right) \leq 1, \quad (20b)$$

$$\|\mathbf{w}_i\|^2 \leq P_i, \quad i = 1, \dots, K. \quad (20c)$$

Note that problem (20) is closely related to problem (19). Their relation is specified in the following lemma.

Lemma 3: *Let R^* denote the optimal value of problem (19). It holds true that, for any $\bar{R} \geq 0$, problem (20) is feasible if and only if $\bar{R} \leq R^*$. Furthermore, the set of optimal beamformers to problem (19) is a subset of the feasible set of problem (20) when $\bar{R} < R^*$, and these two sets coincide when $\bar{R} = R^*$.*

Proof: Let $\{\mathbf{w}_i^*\}$ denote an optimal beamformer to problem (19). Since the left-hand side of constraint (20b) is strictly increasing in \bar{R} , we know that $\{\mathbf{w}_i^*\}$ is feasible to problem (20) if $\bar{R} \leq R^*$. Hence, problem (20) is feasible if $\bar{R} \leq R^*$, and all the optimal beamformers to problem (19) are feasible to problem (20). On the other hand, suppose that problem (20) is feasible and $\{\bar{\mathbf{w}}_i\}$ is a feasible point. Then, $\{\bar{\mathbf{w}}_i\}$ is clearly a feasible beamformer of (19) that achieves objective value \bar{R} , implying $R^* \geq \bar{R}$. Furthermore, when $\bar{R} = R^*$, all the feasible beamformers to problem (20) achieve the optimal objective value of problem (19), and hence are optimal beamformers to (19). ■

Lemma 3 infers that problem (19) is polynomial-time solvable if and only if problem (20) is polynomial-time solvable. In particular if problem (20) can be efficiently solved, then problem (19) can be efficiently solved by a bisection algorithm which involves solving a series of problem (20) ([34] §4.2.5). On the other hand, if one can solve problem (19), then one can correctly determine whether problem (20) is feasible, and, if yes, obtain a set of feasible beamformers. Therefore, these two problems belong to the same complexity class. In the next two subsections, we investigate the complexity of problem (19) for the SISO case ($N_t = 1$), and MISO case ($N_t > 1$) based on this observation.

A. Single-Antenna Case

In this subsection, we consider the SISO case, and present a polynomial-time bisection algorithm, which involves solving a

¹The initial bisection interval of \bar{R} can be found as follows. Firstly, $\bar{R} = 0$ is obviously a lower bound to the optimal value of (21). Secondly, by (21b), we have $\rho_i \exp\left(\frac{(2^{R_i} - 1)\sigma_i^2}{p_i Q_{ii}}\right) \leq 1, i = 1, \dots, K$, and thus the optimal value of problem (21) is upper bounded by $\bar{R} = \min_i \frac{1}{\alpha_i} \log_2\left(1 + \frac{P_i Q_{ii} \ln(1/\rho_i)}{\sigma_i^2}\right)$.

finite number of the feasibility problem (20), for attaining the global optimum of problem (19). When $N_t = 1$, problem (19) degenerates to the following power control problem

$$\max_{\substack{p_i \geq 0, R_i \geq 0, \\ i=1, \dots, K}} R \quad (21a)$$

$$\text{s.t. } \rho_i \exp\left(\frac{(2^{R_i} - 1)\sigma_i^2}{p_i Q_{ii}}\right) \times \prod_{k \neq i} \left(1 + \frac{(2^{R_i} - 1)p_k Q_{ki}}{p_i Q_{ii}}\right) \leq 1, \quad (21b)$$

$$p_i \leq P_i, \quad i = 1, \dots, K, \quad (21c)$$

where p_i is the transmit power of transmitter i , and Q_{ki} is the variance of the SISO channel h_{ki} , for all $i, k = 1, \dots, K$. Similarly, problem (20) reduces to

$$\text{Find } (p_1, \dots, p_K) \quad (22a)$$

$$\text{s.t. } \rho_i \exp\left(\frac{(2^{\alpha_i \bar{R}} - 1)\sigma_i^2}{p_i Q_{ii}}\right) \times \prod_{k \neq i} \left(1 + \frac{(2^{\alpha_i \bar{R}} - 1)p_k Q_{ki}}{p_i Q_{ii}}\right) \leq 1, \quad (22b)$$

$$p_i \leq P_i, \quad i = 1, \dots, K, \quad (22c)$$

which is feasible if and only if the optimal value of (21) is no less than \bar{R} . First of all, we show that problem (22) is polynomial-time solvable.

Consider the change of variables $\tilde{p}_i = \ln p_i$, for $i = 1, \dots, K$. Then, we can solve problem (22) by solving the following convex problem

$$\min_{\substack{\tilde{\alpha}, \tilde{p}_i, \\ i=1, \dots, K}} \tilde{\alpha} \quad (23a)$$

$$\text{s.t. } \ln \rho_i + \frac{(2^{\alpha_i \bar{R}} - 1)\sigma_i^2}{Q_{ii}} \cdot e^{-\tilde{p}_i} + \sum_{k \neq i} \ln \left(1 + \frac{(2^{\alpha_i \bar{R}} - 1)Q_{ki}}{Q_{ii}} \cdot e^{-\tilde{p}_i + \tilde{p}_k}\right) \leq \tilde{\alpha}, \quad (23b)$$

$$\tilde{p}_i \leq \ln P_i, \quad i = 1, \dots, K. \quad (23c)$$

Specifically, it is not difficult to verify by using an argument similar to Lemma 3 that problem (22) is feasible if and only if the optimal $\tilde{\alpha}$ of problem (23) is less than or equal to zero, and that every optimal solution of (23) directly serves as a feasible point of problem (22) provided that problem (22) is feasible. Therefore, based on Lemma 3, one can solve problem (21) in a bisection manner by solving a sequence of convex problem (23). The bisection algorithm is described in Algorithm 1.¹

Problem (23) is in fact equivalent to the outage balancing power control problem studied in [28], which can be efficiently solved by a nonlinear Perron-Frobenius theory-based algorithm with overall complexity of $\mathcal{O}(K \ln(K/\epsilon))$ ([28], Algorithm 1), where ϵ specifies the solution accuracy. Therefore, the overall complexity of Algorithm 1 is $\kappa \cdot \mathcal{O}(K \ln(K/\epsilon))$, where

$$\kappa \triangleq \lceil \log_2(\delta^{-1} \bar{R}_u) \rceil$$

is the number of bisection iterations required for Algorithm 1 to achieve a solution accuracy δ . Algorithm 1 has a polynomial-time complexity since κ is finite in practical situations. Hence, we have proven the complexity of the MMF CoBF problem for the case of $N_t = 1$ as stated in the following theorem.

Theorem 2: *When $N_t = 1$, the outage constrained MMF CoBF problem (19) (i.e., problem (21)) is polynomial-time solvable.*

Algorithm 1 Bisection algorithm for solving problem (21)

- 1: **Set** $\bar{R}_\ell := 0$, $\bar{R}_u := \min_i \frac{1}{\alpha_i} \log_2(1 + \frac{P_i Q_{ii} \ln(1/\rho_i)}{\sigma_i^2})$, and set the solution accuracy to $\delta > 0$;
 - 2: **repeat**
 - 3: Set $\bar{R} := (\bar{R}_\ell + \bar{R}_u)/2$;
 - 4: Solve problem (23), and denote the solution as $(\{\tilde{p}_i^*\}_{i=1}^K, \tilde{\alpha}^*)$;
 - 5: Set $\bar{R}_\ell := \bar{R}$ if $\tilde{\alpha}^* \leq 0$; otherwise, set $\bar{R}_u := \bar{R}$;
 - 6: **until** $\bar{R}_u - \bar{R}_\ell < \delta$;
 - 7: **Output** $p_i = e^{\tilde{p}_i^*}$, $R = \bar{R}$, $i = 1, \dots, K$, as a solution to problem (21).
-

B. Multiple-Antenna Case

In contrast to the single transmit antenna case, in this subsection, we show that the outage constrained MMF CoBF problem (19) is NP-hard when each of the transmitters is equipped with multiple antennas.

Theorem 3: *When $N_t \geq 2$, the outage constrained MMF CoBF problem (19) is NP-hard in the number of users K .*

Proof: As inferred from Lemma 3, it suffices to show that solving the feasibility problem (20) is NP-hard when $N_t \geq 2$. Analogous to the proof of Theorem 1, the main idea of this proof is to show that the 3-satisfiability (3-SAT) problem, which is known to be NP-complete [30], is reducible to problem (20) by deriving a polynomial-time transformation from the 3-SAT problem instances to a subset of the instances of (20) with $N_t \geq 2$. The 3-SAT problem is defined as follows.

Definition 2: *Given N Boolean variables and M clauses each containing exactly three literals of different Boolean variables, the 3-SAT problem is to determine whether there exists a truth assignment of the Boolean variables such that all the M clauses hold true.²*

For ease of exposition, we use “ \vee ”, “ \neg ” to denote the logical disjunction (OR), negation (NOT), and formulate a 3-SAT problem as a feasibility problem. Specifically, a 3-SAT problem instance with N Boolean variables x_1, \dots, x_N and M clauses $d_m = y_{i_m} \vee y_{j_m} \vee y_{k_m}$, $i_m, j_m, k_m \in \{1, \dots, N\}$, $m = 1, \dots, M$, where y_{i_m} is either x_{i_m} or its negation $\neg x_{i_m}$,

²Given a set of Boolean variables, a literal refers to either a Boolean variable or the negation, i.e., logical “NOT”, of a Boolean variable. A clause is a Boolean expression consisting of disjunction, i.e., logical “OR”, of literals.

and so are y_{j_m} and y_{k_m} , can be written as the following feasibility problem:

$$\text{Find } [x_1, \dots, x_N]^T \in \{0, 1\}^N \quad (24a)$$

$$\text{s.t. } d_m = y_{i_m} \vee y_{j_m} \vee y_{k_m} = 1, \quad m = 1, \dots, M. \quad (24b)$$

Given any N Boolean variables x_1, \dots, x_N and M clauses d_1, \dots, d_M , we construct a problem instance of (20) that is equivalent to the corresponding 3-SAT problem, i.e., problem (24), as follows.

We associate each x_n with five users, denoted by the set $\mathcal{V}_n \triangleq \{v_{n0}, v_{n1}, v_{n2}, v_{n3}, v_{n4}\}$, for $n = 1, \dots, N$, and associate the M clauses d_1, \dots, d_M with M users $\mathcal{C} \triangleq \{c_1, \dots, c_M\}$. The entire set of users is thus

$$\mathcal{U} = \mathcal{V}_1 \cup \dots \cup \mathcal{V}_N \cup \mathcal{C},$$

which contains a total of $K = 5N + M$ users. For these K users, we consider a particular problem instance of (20) with the following specified system parameters:

$$N_t = 2, \bar{R} = 1, \alpha_u = 1, P_u = 1, \rho_u = \rho = 0.9, \forall u \in \mathcal{U}, \quad (25a)$$

$$\sigma_{v_{n\ell}}^2 = \begin{cases} \ln \rho^{-1}, & \ell = 0, \\ \ln \left(\frac{10}{11} \cdot \rho^{-1} \right), & \ell \neq 0, \end{cases} \sigma_{c_m}^2 = 0.01, \forall n, m, \quad (25b)$$

$$\mathbf{Q}_{uu} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \forall u \in \mathcal{U}, \mathbf{Q}_{v_{n0}, v_{n\ell}} = \frac{\mathbf{A}_\ell}{10}, \forall n, \forall \ell \neq 0, \quad (25c)$$

$$\mathbf{Q}_{v_{n0}, c_m} = \begin{cases} \frac{1}{25} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, & \text{if } x_n \in \{y_{i_m}, y_{j_m}, y_{k_m}\}, \\ \frac{1}{25} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, & \text{if } \neg x_n \in \{y_{i_m}, y_{j_m}, y_{k_m}\}, \end{cases} \quad \forall n, \quad (25d)$$

$$\mathbf{Q}_{u_1, u_2} = \mathbf{0}, \text{ for all } u_1, u_2 \text{ not specified above,} \quad (25e)$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \mathbf{A}_3 = \begin{bmatrix} 1 & \iota \\ -\iota & 1 \end{bmatrix}, \\ \mathbf{A}_4 = \begin{bmatrix} 1 & -\iota \\ \iota & 1 \end{bmatrix},$$

and $\iota^2 = -1$.

An illustrative example for the association between the 3-SAT problem and the outage constrained MMF CoBF problem (19) described by (25) is depicted in Fig. 2. In this example, we consider a 3-SAT problem with four variables, x_1, x_2, x_3, x_4 , and two clauses, $d_1 = y_{i_1} \vee y_{j_1} \vee y_{k_1}$, $d_2 = y_{i_2} \vee y_{j_2} \vee y_{k_2}$, where $y_{i_1} = x_1$, $y_{j_1} = x_2$, $y_{k_1} = x_3$, $y_{i_2} = x_2$, $y_{j_2} = \neg x_3$, and $y_{k_2} = \neg x_4$. In the corresponding IFC, each Boolean variable x_n is associated with five users, i.e., $\mathcal{V}_n = \{v_{n0}, v_{n1}, \dots, v_{n4}\}$, for $n = 1, \dots, 4$. For each $n = 1, \dots, 4$, only user v_{n0} interferes with users v_{n1}, \dots, v_{n4} where the cross-link channel covariance matrices are given in (25c). Furthermore, user c_m , which is due to clause d_m , is interfered by user v_{n0} if the Boolean variable x_n or its negation $\neg x_n$ appears in clause d_m . As indicated by (25d), the covariance matrix of the channel from v_{n0} to c_m depends on whether x_n or the negation $\neg x_n$ appears in d_m . In Fig. 2, the users are

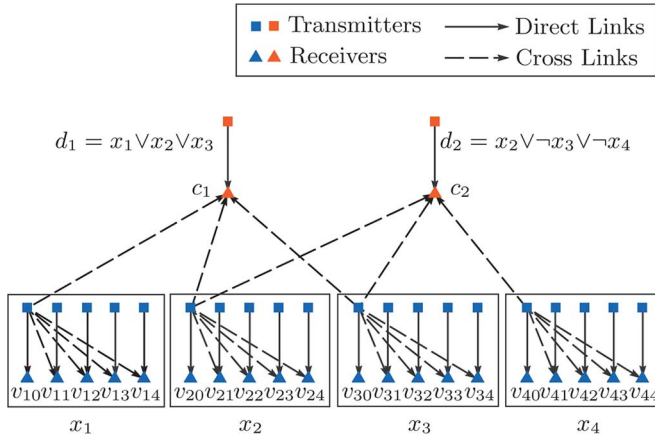


Fig. 2. An example illustrating the association between a 3-SAT problem instance $d_1 = x_1 \vee x_2 \vee x_3$, $d_2 = x_2 \vee \neg x_3 \vee \neg x_4$ and an outage constrained MMF CoBF problem (19) with $N_t = 2$ and $K = 22$ users.

not connected if the associated cross-link channel covariance matrix is zero as indicated by (25e).

With the specified system parameters given in (25), the constructed problem instance of (20) can be expressed as

$$\text{Find } \{\mathbf{w}_{1\ell}^v\}_{\ell=0}^4, \dots, \{\mathbf{w}_{N\ell}^v\}_{\ell=0}^4, \{\mathbf{w}_m^c\}_{m=1}^M \quad (26a)$$

$$\text{s.t. } \rho e^{\frac{\sigma_{v_{n0}}^2}{\|\mathbf{w}_{n0}^v\|^2}} \leq 1, \forall n, \quad (26b)$$

$$\rho e^{\frac{\sigma_{v_{n\ell}}^2}{\|\mathbf{w}_{n\ell}^v\|^2}} \left(1 + \frac{(\mathbf{w}_{n0}^v)^H \mathbf{A}_\ell \mathbf{w}_{n0}^v}{10 \|\mathbf{w}_{n\ell}^v\|^2} \right) \leq 1, \forall n, \forall \ell \neq 0, \quad (26c)$$

$$\rho e^{\frac{\sigma_{c_m}^2}{\|\mathbf{w}_m^c\|^2}} \prod_{\tau \in \{i,j,k\}} \left(1 + \frac{(\mathbf{w}_{\tau m 0}^v)^H \mathbf{Q}_{v_{\tau m 0}, c_m} \mathbf{w}_{\tau m 0}^v}{\|\mathbf{w}_m^c\|^2} \right) \leq 1, \forall m, \quad (26d)$$

$$\|\mathbf{w}_{n\ell}^v\|^2 \leq 1, \|\mathbf{w}_m^c\|^2 \leq 1, \forall n, \forall m, \forall \ell, \quad (26e)$$

where $\mathbf{w}_{n\ell}^v$ and \mathbf{w}_m^c denote the beamformers of users $v_{n\ell}$ and c_m , respectively. Next, we show that the given 3-SAT problem instance is satisfiable if and only if (26) is feasible.

To begin with, we show that any feasible point of problem (26) corresponds to a truth assignment satisfying the 3-SAT problem (24). Note that constraint (26b) can be written as

$$\|\mathbf{w}_{n0}^v\|^2 \geq \sigma_{v_{n0}}^2 (\ln \rho^{-1})^{-1} = 1, \quad n = 1, \dots, N, \quad (27)$$

where the equality comes from (25b). Combining constraints (26b) with (26e), we have $\|\mathbf{w}_{n0}^v\|^2 = 1$ for $n = 1, \dots, N$. Similarly, one can rewrite (26c) as

$$\begin{aligned} (\mathbf{w}_{n0}^v)^H \mathbf{A}_\ell \mathbf{w}_{n0}^v &\leq 10 \|\mathbf{w}_{n\ell}^v\|^2 \left(\rho^{-1} \exp\left(\frac{-\sigma_{v_{n\ell}}^2}{\|\mathbf{w}_{n\ell}^v\|^2}\right) - 1 \right) \\ &\leq \max_{\|\mathbf{w}_{n\ell}^v\|^2 \leq 1} 10 \|\mathbf{w}_{n\ell}^v\|^2 \left(\rho^{-1} \exp\left(\frac{-\sigma_{v_{n\ell}}^2}{\|\mathbf{w}_{n\ell}^v\|^2}\right) - 1 \right) = 1, \end{aligned} \quad (28)$$

for $n = 1, \dots, N$, $\ell = 1, 2, 3, 4$, where the 2nd inequality holds with equality if and only if $\|\mathbf{w}_{n\ell}^v\|^2 = 1$, and the last equality comes from (25b). Furthermore,

$$(\mathbf{w}_{n0}^v)^H \mathbf{A}_\ell \mathbf{w}_{n0}^v + (\mathbf{w}_{n0}^v)^H \mathbf{A}_{\ell+1} \mathbf{w}_{n0}^v = 2 \|\mathbf{w}_{n0}^v\|^2 = 2, \quad (29)$$

for $n = 1, \dots, N$, $\ell = 1, 3$. Combining (28) and (29), we obtain

$$(\mathbf{w}_{n0}^v)^H \mathbf{A}_\ell \mathbf{w}_{n0}^v = 1, \text{ and } \|\mathbf{w}_{n\ell}^v\|^2 = 1, \quad (30)$$

for all $n = 1, \dots, N$ and $\ell = 1, 2, 3, 4$. Then, we can derive

$$(\mathbf{w}_{n0}^v)^H (\mathbf{A}_1 - \mathbf{A}_2) \mathbf{w}_{n0}^v = 4 \text{Re} \left\{ ([\mathbf{w}_{n0}^v]_2)^H [\mathbf{w}_{n0}^v]_1 \right\} = 0, \quad (31a)$$

$$(\mathbf{w}_{n0}^v)^H (\mathbf{A}_3 - \mathbf{A}_4) \mathbf{w}_{n0}^v = 4 \text{Im} \left\{ ([\mathbf{w}_{n0}^v]_2)^H [\mathbf{w}_{n0}^v]_1 \right\} = 0, \quad (31b)$$

for all $n = 1, \dots, N$, where $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ denote the real part and the imaginary part of a complex number, and $[\mathbf{w}_{n0}^v]_1$ and $[\mathbf{w}_{n0}^v]_2$ denote the first and second elements of \mathbf{w}_{n0}^v , respectively. Thus, a feasible \mathbf{w}_{n0}^v must satisfy either $[\mathbf{w}_{n0}^v]_1 = 0$ or $[\mathbf{w}_{n0}^v]_2 = 0$. In addition, constraints (26d) and (26e) imply that

$$\begin{aligned} \min_{\|\mathbf{w}_m^c\|^2 \leq 1} \rho e^{\frac{\sigma_{c_m}^2}{\|\mathbf{w}_m^c\|^2}} &\times \prod_{\tau \in \{i,j,k\}} \left(1 + \frac{(\mathbf{w}_{\tau m 0}^v)^H \mathbf{Q}_{v_{\tau m 0}, c_m} \mathbf{w}_{\tau m 0}^v}{\|\mathbf{w}_m^c\|^2} \right) \\ &= \rho e^{\sigma_{c_m}^2} \prod_{\tau \in \{i,j,k\}} \left(1 + (\mathbf{w}_{\tau m 0}^v)^H \mathbf{Q}_{v_{\tau m 0}, c_m} \mathbf{w}_{\tau m 0}^v \right) \\ &\leq 1. \end{aligned} \quad (32)$$

By combining (30), (31), and (32), we come up with the result that the feasible beamformers to problem (26) must satisfy

$$\mathbf{w}_{n0}^v \in \left\{ \bigcup_{\theta \in [0, 2\pi]} [e^{j\theta} \ 0]^T \right\} \cup \left\{ \bigcup_{\theta \in [0, 2\pi]} [0 \ e^{j\theta}]^T \right\}, \forall n, \quad (33a)$$

$$\rho e^{\sigma_{c_m}^2} \prod_{\tau \in \{i,j,k\}} \left(1 + (\mathbf{w}_{\tau m 0}^v)^H \mathbf{Q}_{v_{\tau m 0}, c_m} \mathbf{w}_{\tau m 0}^v \right) \leq 1, \forall m, \quad (33b)$$

$$\|\mathbf{w}_{n\ell}^v\|^2 = 1, \quad \forall n, \forall \ell. \quad (33c)$$

By (25d) and constraint (33a), one can see that

$$\begin{aligned} &(\mathbf{w}_{\tau m 0}^v)^H \mathbf{Q}_{v_{\tau m 0}, c_m} \mathbf{w}_{\tau m 0}^v \\ &= \begin{cases} 0, & \text{if } y_{\tau m} = x_{\tau m}, \mathbf{w}_{\tau m 0}^v \in \left\{ \bigcup_{\theta \in [0, 2\pi]} [e^{j\theta} \ 0]^T \right\}, \\ 0, & \text{if } y_{\tau m} = \neg x_{\tau m}, \mathbf{w}_{\tau m 0}^v \in \left\{ \bigcup_{\theta \in [0, 2\pi]} [0 \ e^{j\theta}]^T \right\}, \\ 1/25, & \text{if } y_{\tau m} = x_{\tau m}, \mathbf{w}_{\tau m 0}^v \in \left\{ \bigcup_{\theta \in [0, 2\pi]} [0 \ e^{j\theta}]^T \right\}, \\ 1/25, & \text{if } y_{\tau m} = \neg x_{\tau m}, \mathbf{w}_{\tau m 0}^v \in \left\{ \bigcup_{\theta \in [0, 2\pi]} [e^{j\theta} \ 0]^T \right\} \end{cases}, \end{aligned} \quad (34)$$

where $\tau_m \in \{i_m, j_m, k_m\}, \forall m = 1, \dots, M$. Suppose that $\hat{\mathbf{w}}_{n\ell}^v, \hat{\mathbf{w}}_m^c, n = 1, \dots, N, m = 1, \dots, M, \ell = 0, \dots, 4$, are feasible to (26), and set

$$x_n = \begin{cases} 1, & \text{if } \hat{\mathbf{w}}_{n0}^v \in \left\{ \bigcup_{\theta \in [0, 2\pi]} [e^{j\theta} \ 0]^T \right\}, \\ 0, & \text{if } \hat{\mathbf{w}}_{n0}^v \in \left\{ \bigcup_{\theta \in [0, 2\pi]} [0 \ e^{j\theta}]^T \right\}. \end{cases} \quad (35)$$

Then, we have $(\hat{\mathbf{w}}_{\tau_m 0}^v)^H \mathbf{Q}_{v_{\tau_m 0}, c_m} \hat{\mathbf{w}}_{\tau_m 0}^v = 0$ if and only if $y_{\tau_m} = 1$ according to (34). Furthermore, it can be verified by (25a), (25b) and (34) that condition (33b) is satisfied if and only if $(\mathbf{w}_{\tau_m 0}^v)^H \mathbf{Q}_{v_{\tau_m 0}, c_m} \mathbf{w}_{\tau_m 0}^v = 0$, for some $\tau_m \in \{i_m, j_m, k_m\}, \forall m$, implying that all of the clauses d_1, \dots, d_M are true. Hence, the truth assignment (35) satisfies the given 3-SAT problem instance (24).

On the other hand, suppose that the given 3-SAT problem (24) can be satisfied and $\hat{x}_n \in \{0, 1\}, n = 1, \dots, N$, is a truth assignment such that the clauses d_1, \dots, d_M are true, i.e., constraints (24b) are satisfied. Then, analogous to the above analysis, it can be verified that

$$\begin{aligned} \mathbf{w}_{n0}^v &= \begin{cases} [1 \ 0]^T, & \hat{x}_n = 1, \forall n, \\ [0 \ 1]^T, & \hat{x}_n = 0, \end{cases} \\ \|\mathbf{w}_{n\ell}^v\|^2 &= \|\mathbf{w}_m^c\|^2 = 1, \forall n, \forall m, \forall \ell \neq 0, \end{aligned}$$

is feasible to problem (26). Thus, we have proven the equivalence of the given 3-SAT problem (24) and problem (26), namely, the 3-SAT problem is reducible to problem (20). As a result, determining the feasibility of problem (20) is NP-hard when $N_t \geq 2$, and hence solving problem (19) is also NP-hard according to Lemma 3. ■

Remark 1: The preceding proof of the NP-hardness of the feasibility problem (20) can also be analogously applied to show the NP-hardness of the following CoBF design problem for outage balancing [28]:

$$\min_{\substack{\mathbf{w}_i \in \mathbb{C}^{N_t}, \rho_i \in [0, 1], \\ i=1, \dots, K}} 1/\rho \quad (36a)$$

$$\text{s.t. } \rho_i \exp \left(\frac{(2^{R_i} - 1)\sigma_i^2}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i} \right) \times \prod_{k \neq i} \left(1 + \frac{(2^{R_i} - 1)\mathbf{w}_k^H \mathbf{Q}_{ki} \mathbf{w}_k}{\mathbf{w}_i^H \mathbf{Q}_{ii} \mathbf{w}_i} \right) \leq 1, \quad (36b)$$

$$\|\mathbf{w}_i\|^2 \leq P_i, \quad i = 1, \dots, K, \quad (36c)$$

where $\rho = \min\{\rho_1, \dots, \rho_K\}$ and R_i is the given data rate of the i th user, $i = 1, \dots, K$. Then, following a similar argument as Lemma 3, one can show that problem (36) is solvable (e.g., by bisecting over $\rho \in [0, 1]$) if and only if the feasibility problem (20) (with $\rho_i = \rho, \alpha_i \bar{R} = R_i, i = 1, \dots, K$) is solvable. Therefore, the NP-hardness of problem (20) when $N_t \geq 2$ also implies that problem (36) is NP-hard when $N_t \geq 2$.

V. CONCLUSION

In this paper, we have studied the complexity status of the outage constrained SRM and MMF CoBF problems. In

particular, we have established the NP-hardness of the two problems by showing that they are at least as difficult as the Max-Cut problem and the 3-SAT problem, respectively. Besides, a subclass, i.e., the SISO case, of the outage constrained MMF problem is identified polynomial-time solvable. Since the MMF CoBF problem is known polynomial-time solvable under perfect CSI, our result implies that the outage constrained CoBF design problems are indeed more challenging. Motivated by our complexity analysis results, efficient algorithms for obtaining high-quality approximate solutions to the outage constrained CoBF problems are further pursued in the companion paper [31].

APPENDIX A PROOF OF LEMMA 1

Notice that the function $\Psi_i(x, \{p_k\}_{k \neq i})$ in (10) is continuously differentiable with respect to $(x, \{p_k\}_{k \neq i})$, and is strictly increasing in x . By the implicit function theorem [35], there exists a unique continuously differentiable function $\zeta_i(\{p_k\}_{k \neq i}) > 0$ satisfying

$$\Psi_i(\zeta_i(\{p_k\}_{k \neq i}), \{p_k\}_{k \neq i}) = 1, \forall \{p_k\}_{k \neq i}.$$

Therefore, we can equivalently express the rate outage constraint (7b) as

$$\frac{2^{R_i} - 1}{Q_{ii} p_i} \leq \zeta_i(\{p_k\}_{k \neq i}) \Leftrightarrow R_i \leq \log_2(1 + \zeta_i(\{p_k\}_{k \neq i}) Q_{ii} p_i),$$

for $i = 1, \dots, K$. Moreover, the objective function of problem (7) is nondecreasing with respect to R_1, \dots, R_K , respectively. So, without loss of optimality, we can assume that equality holds. Therefore, problem (7) is equivalent to problem (9). ■

APPENDIX B PROOF OF LEMMA 2

Suppose that $\{\hat{p}_{ia}^v, \hat{p}_{jk}^e\}_{v_{ia} \in \mathcal{U}_v, e_{jk} \in \mathcal{U}_e}$ is an optimal solution of problem (13). It is easy to see from (13a) that $\hat{p}_{ij}^e = 0.7$ for all $e_{ij} \in \mathcal{U}_e$, since user e_{ij} does not interfere with any other user.

To complete the proof, we first show that $\hat{p}_{ia}^v \in \{0, 1\}$ for all $v_{ia} \in \mathcal{U}_v$. For notational simplicity, let us focus on proving the case of $a = 0$. For any given $i \in \mathcal{V}$, let us assume that $\{\hat{p}_{jb}^v\}_{(j,b) \neq (i,0)}$ are known and fixed. In this case, it is clear that \hat{p}_{i0}^v must also be optimal to the following problem:

$$\begin{aligned} \max_p F(p) &\triangleq \log_2(1 + p \zeta^v(\hat{p}_{i1}^v)) + \log_2(1 + \hat{p}_{i1}^v \zeta^v(p)) \\ &\quad + \sum_{\{j|e_{ij} \in \mathcal{U}_e\}} \alpha_{e_{ij}} \log_2(1 + 0.7 \zeta^e(p, \hat{p}_{j1}^v)) \end{aligned} \quad (A.1a)$$

$$\text{s.t. } 0 \leq p \leq 1, \quad (A.1b)$$

which is obtained from (13) by fixing $p_{jb}^v = \hat{p}_{jb}^v$ for all $(j, b) \neq (i, 0)$ and excluding the terms irrelevant to user v_{i0} . Hence, we focus on showing that the optimal solution to (A.1) is either

$p = 0$ or $p = 1$. To this end, we need the following lemma, which is proved in Appendix C.

Lemma 4: *The objective function of problem (A.1), i.e., $F(p) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, is a differentiable quasiconvex function. Furthermore, there exists at most one $p \geq 0$ such that $dF(p)/dp = 0$.*

Suppose that $p^* \in (0, 1)$ is optimal to (A.1), which implies $dF(p^*)/dp = 0$. Then, by Lemma 4, we have $dF(p)/dp < 0$ for all $p \in [0, p^*)$ and $dF(p)/dp > 0$ for all $p \in (p^*, +\infty)$, which contradicts the fact that p^* maximizes $F(p)$ over the interval $0 \leq p \leq 1$, implying that the optimal solution to problem (A.1) is either $p = 0$ or $p = 1$. Hence, we have proved $\hat{p}_{i0}^v \in \{0, 1\}$. Similarly, we can show that $\hat{p}_{i1}^v \in \{0, 1\}$. As a result, the optimal solution of problem (13) must satisfy

$$\begin{bmatrix} \hat{p}_{i0}^v \\ \hat{p}_{i1}^v \end{bmatrix} \in \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}, i = 1, \dots, |\mathcal{V}|,$$

and $\hat{p}_{jk}^e = 0.7$ for all $e_{jk} \in \mathcal{U}_e$.

What remains to be proved is that $[\hat{p}_{i0}^v \ \hat{p}_{i1}^v]^T = [0 \ 0]^T$ or $[\hat{p}_{i0}^v \ \hat{p}_{i1}^v]^T = [1 \ 1]^T$ for some $i \in \mathcal{V}$ is strictly suboptimal to problem (13). Let \hat{R} denote the objective value of problem (13) achieved by $\{\hat{p}_{ia}^v, \hat{p}_{jk}^e\}_{v_{ia} \in \mathcal{U}_v, e_{jk} \in \mathcal{U}_e}$, and define

$$\begin{aligned} \mathcal{I}_0 &= \left\{ i \in \mathcal{V} \mid [\hat{p}_{i0}^v \ \hat{p}_{i1}^v]^T = [0 \ 0]^T \right\}, \\ \mathcal{I}_1 &= \left\{ i \in \mathcal{V} \mid [\hat{p}_{i0}^v \ \hat{p}_{i1}^v]^T = [1 \ 1]^T \right\}. \end{aligned}$$

We can bound \hat{R} from below and above as follows:

$$\begin{aligned} \hat{R} &\geq \sum_{v_{ia} \in \mathcal{U}_v} \log_2(1 + \hat{p}_{ia}^v \zeta^v(\hat{p}_{ia}^v)) \\ &= \sum_{i=1}^{|\mathcal{V}|} (\log_2(1 + \hat{p}_{i0}^v \zeta^v(\hat{p}_{i0}^v)) + \log_2(1 + \hat{p}_{i1}^v \zeta^v(\hat{p}_{i1}^v))) \\ &= (|\mathcal{V}| - |\mathcal{I}_0| - |\mathcal{I}_1|) \times \log_2(1 + 1 \cdot \zeta^v(0)) \\ &\quad + 2|\mathcal{I}_1| \log_2(1 + 1 \cdot \zeta^v(1)) \\ &= (|\mathcal{V}| - |\mathcal{I}_0| - |\mathcal{I}_1|) \times 0.5973 + 2|\mathcal{I}_1| \times 0.0671 \\ &= |\mathcal{V}| \times 0.5973 - (|\mathcal{I}_0| \times 0.5973 + |\mathcal{I}_1| \times 0.4631), \end{aligned} \tag{A.2a}$$

$$\begin{aligned} \hat{R} &\leq \sum_{v_{ia} \in \mathcal{U}_v} \log_2(1 + \hat{p}_{ia}^v \zeta^v(\hat{p}_{ia}^v)) \\ &\quad + \sum_{e_{jk} \in \mathcal{U}_e} \alpha_{e_{jk}} \log_2(1 + 0.7 \cdot \zeta^e(0, 0)) \text{ (by Lemma 5)} \\ &= |\mathcal{V}| \times 0.5973 - (|\mathcal{I}_0| \times 0.5973 + |\mathcal{I}_1| \times 0.4631) \\ &\quad + \log_2(1 + 0.7 \cdot \zeta^e(0, 0)) \text{ (by (12b))} \\ &= |\mathcal{V}| \times 0.5973 \\ &\quad - (|\mathcal{I}_0| \times 0.5973 + |\mathcal{I}_1| \times 0.4631 - 0.4426), \end{aligned} \tag{A.2b}$$

where the above numerical values are computed from the parameters set in (12). From (A.2a), one can see that \hat{R} is no less than $|\mathcal{V}| \times 0.5973$ when $|\mathcal{I}_0| = |\mathcal{I}_1| = 0$. However, from (A.2b), it is clear that \hat{R} is smaller than $|\mathcal{V}| \times 0.5973$, when either $|\mathcal{I}_0| \neq 0$ or $|\mathcal{I}_1| \neq 0$. Thus, the optimal power pattern $[\hat{p}_{i0}^v \ \hat{p}_{i1}^v]^T$ must be either $[0 \ 1]^T$ or $[1 \ 0]^T$. This completes the proof of Lemma 2. ■

APPENDIX C

PROOF OF LEMMA 4

Let $f(p)$ denote the derivative of $F(p)$ with respect to p . It can be obtained as follows.

$$\begin{aligned} f(p) \ln 2 &= \frac{dF(p)}{dp} \cdot \ln 2 \\ &= \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p\zeta^v(\hat{p}_{i1}^v)} + \frac{\hat{p}_{i1}^v}{1 + \hat{p}_{i1}^v \zeta^v(p)} \frac{d\zeta^v(p)}{dp} \\ &\quad + \sum_{\{j|e_{ij} \in \mathcal{U}_e\}} \frac{0.7\alpha_{e_{ij}}}{1 + 0.7\zeta^e(p, \hat{p}_{j1}^v)} \frac{d\zeta^e(p, \hat{p}_{j1}^v)}{dp} \end{aligned}$$

By applying implicit function theorem [35] to (14), we can obtain closed-form expressions for the derivatives of $\zeta^v(p)$ and $\zeta^e(p, \hat{p}_{j1}^v)$, which are given in (A.3a) and (A.3b). Hence, $f(p) \ln 2$ can be expressed in closed-form as (A.3c), shown at the bottom of the page.

Our goal is to show that, for any $\bar{P} \in [0, 1]$, it holds true that

$$f(p) > 0 \text{ for all } p > \bar{P} \text{ if } f(\bar{P}) \geq 0, \tag{A.4a}$$

$$f(p) < 0 \text{ for all } p < \bar{P} \text{ if } f(\bar{P}) \leq 0. \tag{A.4b}$$

Since $f(p)$ is a continuous function, conditions in (A.4) imply that either of the following two statements must be true.

$$\frac{d\zeta^v(p)}{dp} = -\frac{\zeta^v(p)}{\sigma^2 + \sigma^2 p \zeta^v(p) + p} \tag{A.3a}$$

$$\frac{d\zeta^e(p, \hat{p}_{j1}^v)}{dp} = -\frac{\zeta^e(p, \hat{p}_{j1}^v) (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))}{(1 + p\zeta^e(p, \hat{p}_{j1}^v)) [\hat{p}_{j1}^v + \sigma^2 (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))] + p (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))} \tag{A.3b}$$

$$\begin{aligned} f(p) \ln 2 &= \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p\zeta^v(\hat{p}_{i1}^v)} - \frac{\hat{p}_{i1}^v \zeta^v(p)}{1 + \hat{p}_{i1}^v \zeta^v(p)} \frac{1}{\sigma^2 + \sigma^2 p \zeta^v(p) + p} \\ &\quad - \sum_{\{j|e_{ij} \in \mathcal{U}_e\}} \frac{\alpha_{e_{ij}} \cdot 0.7 \zeta^e(p, \hat{p}_{j1}^v)}{1 + 0.7 \zeta^e(p, \hat{p}_{j1}^v)} \frac{1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v)}{(1 + p\zeta^e(p, \hat{p}_{j1}^v)) [\hat{p}_{j1}^v + \sigma^2 (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))] + p (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))} \end{aligned} \tag{A.3c}$$

- 1) There exists $0 \leq \bar{P} \leq 1$ such that $f(\bar{P}) = 0$, which, according to (A.4a) and (A.4b), implies that $f(p) > 0$ for all $p > \bar{P}$, $f(p) < 0$ for all $p < \bar{P}$.
- 2) Either $f(p) > 0$ for all $0 \leq p \leq 1$, or $f(p) < 0$ for all $0 \leq p \leq 1$, i.e., the function $F(p)$ is either strictly increasing or strictly decreasing.

That is, $F(p)$ is a quasiconvex function, and there exists at most one $p \geq 0$ such that $dF(p)/dp = 0$.

Note that the two conditions in (A.4) are actually equivalent. Hence, it suffices to prove (A.4a). To this end, let us introduce the following lemma, which is proved in Appendix D.

Lemma 5: *Given $\bar{p} \geq 0$ fixed, the functions $\zeta^v(p)$, which satisfies (14a), and $\zeta^e(p, \bar{p}) = \zeta^e(\bar{p}, p)$, which satisfies (14b), are strictly decreasing for $p \geq 0$, while the functions $p\zeta^v(p)$ and $p\zeta^e(p, \bar{p})$ are strictly increasing for $p \geq 0$.*

Given any $\bar{P} \geq 0$ and based on Lemma 5, we can derive a lower bound $g(p|\bar{P})$ of $f(p) \ln 2$ for all $p \geq \bar{P}$, which is given in (A.5), shown at the bottom of the page. To obtain the first inequality in (A.5), one can observe that $\frac{\hat{p}_{i1}^v \zeta^v(p)}{1 + \hat{p}_{i1}^v \zeta^v(p)}$ and $\frac{1}{\sigma^2 + \sigma^2 p \zeta^v(p) + p}$ in the second term are increased due to the fact that $\zeta^v(p) \leq \zeta^v(\bar{P})$, $\frac{x}{1+x}$ is nondecreasing in x for any $x \geq 0$ and $p\zeta^v(p) \geq \bar{P}\zeta^v(\bar{P})$; similar reasons also apply to the summation term since $\zeta^e(p, \hat{p}_{j1}^v) \leq \zeta^e(\bar{P}, \hat{p}_{j1}^v)$ and $p\zeta^e(p, \hat{p}_{j1}^v) \geq \bar{P}\zeta^e(\bar{P}, \hat{p}_{j1}^v)$ for all $p \geq \bar{P}$. The second inequality in (A.5) is obtained by the monotonicity of $\frac{x}{1+x}$ and $\zeta^e(p, \hat{p}_{j1}^v)$. Note that the monotonic properties of $\zeta^v(p)$, $\zeta^e(p, \hat{p}_{j1}^v)$, $p\zeta^v(p)$, $p\zeta^e(p, \hat{p}_{j1}^v)$ and $\frac{x}{1+x}$ are strict, so the two inequalities in (A.5) hold with equality if and only if $p = \bar{P}$; that is,

$$\begin{cases} f(p) \ln 2 > g(p|\bar{P}), & p > \bar{P}, \\ f(p) \ln 2 = g(p|\bar{P}), & p = \bar{P}. \end{cases} \quad (\text{A.6})$$

Next, we show that $g(p|\bar{P}) \geq 0$ for all $p \geq \bar{P}$ if $f(\bar{P}) \geq 0$, which, together with (A.6) then implies (A.4a). By defining

$$\begin{aligned} a_0 &= \sigma^2 + \sigma^2 \bar{P} \zeta^v(\bar{P}) \geq 0, \\ b_0 &= \hat{p}_{i1}^v \zeta^v(\bar{P}) \geq 0, \\ a_j &= (1 + \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v)) [\hat{p}_{j1}^v + \sigma^2 (1 + \hat{p}_{j1}^v \zeta^e(\bar{P}, \hat{p}_{j1}^v))] \\ &\quad + \hat{p}_{j1}^v \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v) \geq 0, \\ b_j &= 0.7 \zeta^e(\bar{P}, \hat{p}_{j1}^v) \geq 0, \\ c_j &= b_j \cdot (1 + \hat{p}_{j1}^v \zeta^e(\bar{P}, \hat{p}_{j1}^v)) \geq 0, \forall j \in \{k | e_{ik} \in \mathcal{U}_e\}, \end{aligned}$$

which are independent of p , one can respectively express $g_0(p|\bar{P})$ and $g_j(p|\bar{P})$ as

$$g_0(p|\bar{P}) = \frac{b_0}{(1 + b_0)(a_0 + p)}, \quad (\text{A.7a})$$

$$g_j(p|\bar{P}) = \frac{\alpha_{e_{ij}} c_j}{(1 + b_j)(a_j + p)}, \forall j \in \{k | e_{ik} \in \mathcal{U}_e\}, \quad (\text{A.7b})$$

Suppose that there exists $0 \leq \bar{P} \leq 1$ satisfying $f(\bar{P}) \geq 0$. Then, by (A.6), we have $f(\bar{P}) \ln 2 = g(\bar{P}|\bar{P}) = \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + \bar{P} \zeta^v(\hat{p}_{i1}^v)} - g_0(\bar{P}|\bar{P}) - \sum_{\{j | e_{ij} \in \mathcal{U}_e\}} g_j(\bar{P}|\bar{P}) \geq 0$. Since $\frac{\zeta^v(\hat{p}_{i1}^v)}{1 + \bar{P} \zeta^v(\hat{p}_{i1}^v)} > 0$, $g_0(\bar{P}|\bar{P}) \geq 0$ and $g_j(\bar{P}|\bar{P}) > 0$ for all $j \in \{k | e_{ik} \in \mathcal{U}_e\}$, we can further infer from $g(\bar{P}|\bar{P}) \geq 0$ that, when $p = \bar{P}$,

$$\begin{aligned} & \beta_0 \cdot \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p \zeta^v(\hat{p}_{i1}^v)} - g_0(p|\bar{P}) \\ &= \frac{[\zeta^v(\hat{p}_{i1}^v) a_0 \beta_0 (1 + b_0) - b_0]}{(1 + \zeta^v(\hat{p}_{i1}^v) p) (1 + b_0)(a_0 + p)} \\ &\quad + \frac{[\beta_0 (1 + b_0) - b_0] \cdot \zeta^v(\hat{p}_{i1}^v) p}{(1 + \zeta^v(\hat{p}_{i1}^v) p) (1 + b_0)(a_0 + p)} \\ &\geq 0, \end{aligned} \quad (\text{A.8a})$$

$$\begin{aligned} f(p) \ln 2 &= \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p \zeta^v(\hat{p}_{i1}^v)} - \frac{\hat{p}_{i1}^v \zeta^v(p)}{1 + \hat{p}_{i1}^v \zeta^v(p)} \frac{1}{\sigma^2 + \sigma^2 p \zeta^v(p) + p} \\ &\quad - \sum_{\{j | e_{ij} \in \mathcal{U}_e\}} \frac{\alpha_{e_{ij}} \cdot 0.7 \zeta^e(p, \hat{p}_{j1}^v)}{1 + 0.7 \zeta^e(p, \hat{p}_{j1}^v)} \frac{1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v)}{(1 + p \zeta^e(p, \hat{p}_{j1}^v)) [\hat{p}_{j1}^v + \sigma^2 (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))] + p (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))} \\ &\geq \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p \zeta^v(\hat{p}_{i1}^v)} - \frac{\hat{p}_{i1}^v \zeta^v(\bar{P})}{1 + \hat{p}_{i1}^v \zeta^v(\bar{P})} \frac{1}{\sigma^2 + \sigma^2 \bar{P} \zeta^v(\bar{P}) + p} \\ &\quad - \sum_{\{j | e_{ij} \in \mathcal{U}_e\}} \frac{\alpha_{e_{ij}} \cdot 0.7 \zeta^e(\bar{P}, \hat{p}_{j1}^v)}{1 + 0.7 \zeta^e(\bar{P}, \hat{p}_{j1}^v)} \frac{1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v)}{(1 + \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v)) [\hat{p}_{j1}^v + \sigma^2 (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v))] + \hat{p}_{j1}^v \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v) + p} \\ &\geq \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p \zeta^v(\hat{p}_{i1}^v)} - \frac{\hat{p}_{i1}^v \zeta^v(\bar{P})}{1 + \hat{p}_{i1}^v \zeta^v(\bar{P})} \frac{1}{\sigma^2 + \sigma^2 \bar{P} \zeta^v(\bar{P}) + p} \\ &\quad - \sum_{\{j | e_{ij} \in \mathcal{U}_e\}} \frac{\alpha_{e_{ij}} \cdot 0.7 \zeta^e(\bar{P}, \hat{p}_{j1}^v)}{1 + 0.7 \zeta^e(\bar{P}, \hat{p}_{j1}^v)} \frac{1 + \hat{p}_{j1}^v \zeta^e(\bar{P}, \hat{p}_{j1}^v)}{(1 + \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v)) [\hat{p}_{j1}^v + \sigma^2 (1 + \hat{p}_{j1}^v \zeta^e(\bar{P}, \hat{p}_{j1}^v))] + \hat{p}_{j1}^v \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v) + p} \\ &= \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p \zeta^v(\hat{p}_{i1}^v)} - g_0(p|\bar{P}) - \sum_{\{j | e_{ij} \in \mathcal{U}_e\}} g_j(p|\bar{P}) \triangleq g(p|\bar{P}), \forall p \geq \bar{P} \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned}
& \beta_j \cdot \frac{\zeta^v(\hat{p}_{i1}^v)}{1 + p\zeta^v(\hat{p}_{i1}^v)} - g_j(p|\bar{P}) \\
&= \frac{[\zeta^v(\hat{p}_{i1}^v) a_j \beta_j (1 + b_j) - \alpha_{e_{ij}} c_j]}{(1 + \zeta^v(\hat{p}_{i1}^v) p) (1 + b_j) (a_j + p)} \\
&+ \frac{[\beta_j (1 + b_j) - \alpha_{e_{ij}} c_j] \cdot \zeta^v(\hat{p}_{i1}^v) p}{(1 + \zeta^v(\hat{p}_{i1}^v) p) (1 + b_j) (a_j + p)} \\
&\geq 0, \forall j \in \{k | e_{ik} \in \mathcal{U}_e\}, \tag{A.8b}
\end{aligned}$$

where

$$\begin{aligned}
\beta_0 &= \frac{g_0(\bar{P}|\bar{P})}{g_0(\bar{P}|\bar{P}) + \sum_{\{k|e_{ik} \in \mathcal{U}_e\}} g_k(\bar{P}|\bar{P})}, \\
\beta_j &= \frac{g_j(\bar{P}|\bar{P})}{g_0(\bar{P}|\bar{P}) + \sum_{\{k|e_{ik} \in \mathcal{U}_e\}} g_k(\bar{P}|\bar{P})}, \quad \forall j \in \{k | e_{ik} \in \mathcal{U}_e\},
\end{aligned}$$

and $\beta_0 + \sum_{\{j|e_{ij} \in \mathcal{U}_e\}} \beta_j = 1$. Next, we aim to show that $[\beta_0(1 + b_0) - b_0] \geq 0$ and $[\beta_j(1 + b_j) - \alpha_{e_{ij}} c_j] \geq 0$, which, when applied to (A.8), imply that, $\frac{\beta_0 \zeta^v(\hat{p}_{i1}^v)}{1 + p\zeta^v(\hat{p}_{i1}^v)} - g_0(p|\bar{P}) \geq 0$ and each $\frac{\beta_j \zeta^v(\hat{p}_{i1}^v)}{1 + p\zeta^v(\hat{p}_{i1}^v)} - g_j(p|\bar{P}) \geq 0$ for all $p \geq \bar{P}$. To this end, consider the following inequalities

$$\rho e^{\sigma^2 \zeta^v} (1 + p\zeta^v) \geq \rho(1 + \sigma^2 \zeta^v)(1 + p\zeta^v), \tag{A.9a}$$

$$\rho e^{\sigma^2 \zeta^e} (1 + p\zeta^e) (1 + \hat{p}_{j1}^v \zeta^e) \geq \rho(1 + \sigma^2 \zeta^e)(1 + p\zeta^e), \tag{A.9b}$$

for all $\zeta^v \geq 0$, $\zeta^e \geq 0$, $p \geq 0$ and $\hat{p}_{j1}^v \geq 0$, where the inequalities are owing to $e^x \geq 1 + x$ for all x . Since $\rho = 0.95 < 1$, for any $p \geq 0$, there must exist $\bar{\zeta}^v(p) = \bar{\zeta}^e(p) > 0$ such that

$$\rho(1 + \sigma^2 \bar{\zeta}^v(p))(1 + p\bar{\zeta}^v(p)) = \rho(1 + \sigma^2 \bar{\zeta}^e(p))(1 + p\bar{\zeta}^e(p)) = 1.$$

Besides, by (14a), one has

$$\begin{aligned}
& \rho e^{\sigma^2 \zeta^v(p)} (1 + p\zeta^v(p)) = 1, \\
& \rho e^{\sigma^2 \zeta^e(p, \hat{p}_{j1}^v)} (1 + p\zeta^e(p, \hat{p}_{j1}^v)) (1 + \hat{p}_{j1}^v \zeta^e(p, \hat{p}_{j1}^v)) = 1.
\end{aligned}$$

Thus, it follows from (A.9a) that $\bar{\zeta}^v(p) \geq \zeta^v(p)$ for all $p \geq 0$. Therefore, for $0 \leq \bar{P} \leq 1$, we have

$$\begin{aligned}
& \zeta^v(\hat{p}_{i1}^v) a_0 \\
&= \zeta^v(\hat{p}_{i1}^v) \sigma^2 (1 + \bar{P} \zeta^v(\bar{P})) \\
&\leq \zeta^v(0) \sigma^2 (1 + 1 \cdot \zeta^v(1)) \\
&\leq \ln \rho^{-1} \times (1 + \bar{\zeta}^v(1)) \\
&= \ln \rho^{-1} \times \left(1 - \frac{1 + \sigma^2}{2\sigma^2} + \sqrt{\frac{1}{\rho\sigma^2} + \frac{(1 + \sigma^2)^2}{4\sigma^4} - \frac{1}{\sigma^2}} \right) \\
&\approx 0.0537 < 1, \tag{A.10}
\end{aligned}$$

where the first inequality comes from Lemma 5, $\zeta^v(0) = \frac{\ln(1/\rho)}{\sigma^2}$ is obtained from (14a), and the approximate value 0.0537 is obtained by using $\rho = 0.95$ and $\sigma^2 = 0.1$ in (12a). By applying (A.10) to (A.8a), we conclude that

$$\beta_0(1 + b_0) - b_0 \geq 0; \tag{A.11}$$

otherwise (A.8a) does not hold true. Similarly, we have $\bar{\zeta}^e(p) \geq \zeta^e(p, \hat{p}_{j1}^v)$, for all $p \geq 0$, $\hat{p}_{j1}^v \geq 0$,

$$\begin{aligned}
& \zeta^v(\hat{p}_{i1}^v) a_j \\
&= \zeta^v(\hat{p}_{i1}^v) [(1 + \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v)) [\hat{p}_{j1}^v + \sigma^2(1 + \hat{p}_{j1}^v \zeta^e(\bar{P}, \hat{p}_{j1}^v))] \\
&\quad + \hat{p}_{j1}^v \bar{P} \zeta^e(\bar{P}, \hat{p}_{j1}^v)] \\
&\leq \zeta^v(0) [(1 + \zeta^e(1, \hat{p}_{j1}^v)) [1 + \sigma^2(1 + \zeta^e(\bar{P}, 1))] + \zeta^e(1, \hat{p}_{j1}^v)] \\
&\leq \frac{1}{\sigma^2} \ln \rho^{-1} [(1 + \bar{\zeta}^e(1)) [1 + \sigma^2(1 + \bar{\zeta}^e(1))] + \bar{\zeta}^e(1)] \\
&\approx 0.6181 < 1, \tag{A.12}
\end{aligned}$$

and thus

$$\beta_j(1 + b_j) - \alpha_{e_{ij}} c_j \geq 0, \forall j \in \{k | e_{ik} \in \mathcal{U}_e\}, \tag{A.13}$$

as inferred from (A.8b). By (A.8), (A.11) and (A.13), we have

$$\frac{\beta_j \zeta^v(\hat{p}_{i1}^v)}{1 + p\zeta^v(\hat{p}_{i1}^v)} - g_j(p|\bar{P}) \geq 0, \text{ for } p \geq \bar{P}, j \in \{0\} \cup \{k | e_{ik} \in \mathcal{U}_e\}.$$

By summing up the above inequalities, we have $g(p|\bar{P}) \geq 0$ for all $p \geq \bar{P}$. Hence, we have proved (A.4a), and the proof of Lemma 4 is complete. ■

APPENDIX D PROOF OF LEMMA 5

Suppose that $0 \leq p_1 < p_2$. Then, we can obtain by (14a) that

$$\begin{aligned}
1 &= 0.95 e^{\zeta^v(p_2) \sigma^2} (1 + \zeta^v(p_2) p_2) \\
&= 0.95 e^{\zeta^v(p_1) \sigma^2} (1 + \zeta^v(p_1) p_1) \\
&< 0.95 e^{\zeta^v(p_1) \sigma^2} (1 + \zeta^v(p_1) p_2).
\end{aligned}$$

Note that the above inequality is strict since $\zeta^v(p) > 0$ for any $p \geq 0$. Because the function $\rho e^{\zeta^v(p) \sigma^2} (1 + \zeta^v(p) p)$ is strictly increasing in $\zeta^v(p)$, the above inequality implies that $\zeta^v(p_2) < \zeta^v(p_1)$, i.e., the function $\zeta^v(p)$ is a strictly decreasing function for $p \geq 0$. Accordingly, we can further obtain

$$\begin{aligned}
1 &= 0.95 e^{\zeta^v(p_2) \sigma^2} (1 + \zeta^v(p_2) p_2) \\
&= 0.95 e^{\zeta^v(p_1) \sigma^2} (1 + \zeta^v(p_1) p_1) \\
&> 0.95 e^{\zeta^v(p_2) \sigma^2} (1 + \zeta^v(p_1) p_1),
\end{aligned}$$

which implies $\zeta^v(p_2) p_2 > \zeta^v(p_1) p_1$, namely, $p\zeta^v(p)$ is strictly increasing for $p \geq 0$. The remaining statements about $\zeta^e(p, \bar{p})$ and $p\zeta^e(p, \bar{p})$ in Lemma 5 can be proved similarly. ■

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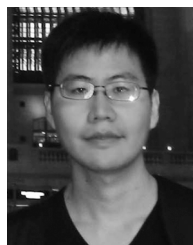
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