

Maximum-Likelihood Detection of Orthogonal Space-Time Block Coded OFDM in Unknown Block Fading Channels

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Abstract—For orthogonal space-time block coded orthogonal frequency division multiplexing (OSTBC-OFDM) systems, many of the existing blind detection and channel estimation methods rely on the assumption that the channel is static for many OSTBC-OFDM blocks. This paper considers the blind (semiblind) maximum-likelihood (ML) detection problem of OSTBC-OFDM with a single OSTBC-OFDM block only. The merit of such an investigation is the ability to accommodate channels with shorter coherence time. We examine both the implementation and identifiability issues, with a focus on BPSK or QPSK constellations. In the implementation, we propose reduced-complexity detection schemes using subchannel grouping. In the identifiability analysis, we show that the proposed schemes can ensure a probability one identifiability condition using very few number of pilots. For example, the proposed semiblind detection scheme only requires a single pilot code for unique data identification; while the conventional pilot-based channel estimation method requires L pilots where L denotes the channel length. Our simulation results demonstrate that the proposed schemes can provide performance close to that of their nonblind counterparts.

Index Terms—Blind and semiblind detection, maximum likelihood detection, MIMO systems, OFDM, space-time block code.

I. INTRODUCTION

IN frequency selective fading channels, space-time coded orthogonal frequency division multiplexing (OFDM) [1] is a popular approach to providing transmit diversity and coding gains; e.g., space-time trellis coded OFDM [2], [3], and space-time block coded OFDM [4]–[13]. In particular, the combination of orthogonal space-time block codes (OSTBCs) and OFDM, or simply OSTBC-OFDM [5]–[13], has drawn much attention because it attains the maximum transmit diversity and has a simple maximum-likelihood (ML) receiver structure given channel state information (CSI) at the receiver.

Manuscript received March 12, 2007; revised August 27, 2007. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Gerald Matz. This work is supported by the National Science Council, R.O.C., under Grant NSC 95-2219-E-007-004, NSC 95-2219-E-007-001 and NSC 95-2221-E-007-036, and partly by The Chinese University of Hong Kong, under Direct Grant 2050396. Part of the work was presented at the 2007 IEEE International Conference on Acoustics, Speech, and Signal Processing, Honolulu, HI, April 15–20, 2007.

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Digital Object Identifier 10.1109/TSP.2007.909229

Recently, there have been considerable interests in techniques requiring no CSI at the receiver; e.g., blind (semiblind) channel estimation [5]–[7], [11], [12] and the differential schemes [8]–[10]. Many of the existing blind channel estimators, such as those based on second order statistics [5]–[7], assume that the channel is static over many OSTBC-OFDM blocks. For example, the blind subspace-based estimator in [5] uses at least 150 OSTBC-OFDM blocks in the simulations. Thus, these estimators may not be applicable if the channel changes in a block-by-block¹ manner. When the channel can be invariant for at least two blocks, differentially encoded OSTBC-OFDM is a convenient scheme for no-CSI detection. It, however, incurs a 3-dB performance loss in terms of signal-to-noise ratio (SNR).

On the other hand, we have seen significant progress in the blind ML detection techniques for OSTBCs in flat fading channels. In essence, by exploiting the special OSTBC characteristics one can simplify the blind ML receiver realization problem considerably. This has led to various realization methods, such as the low-complexity cyclic ML method [14], [15], the simple norm relaxation method [16], [17], optimal sphere decoding [17], [18], and the efficient quasi-optimal semidefinite relaxation (SDR) method [17], [19]. Extensions to unknown noise covariance and time-selective fading have also been considered in [15] and [18], respectively. These successes have recently motivated investigation of blind ML OSTBC identifiability, a crucial fundamental aspect that determines conditions under which a blind OSTBC scheme can operate properly. A blind ML identifiability analysis with a focus on BPSK/QPSK constellations has been provided in [20]. This work not only proves that there exist OSTBCs having very relaxed identifiability conditions (say, capability of unique code identification with one receive antenna only), it also develops a construction method for such OSTBCs with BPSK/QPSK constellations. Meanwhile, the concurrent works [18], [19] have concentrated on an OSTBC scheme design using dual MPSK constellations, which is also found to attain excellent identifiability. It is worth mentioning that OSTBCs are beneficial not just to the blind ML approach. In the parallel developments of the blind subspace approach, OSTBCs are also found to be a good class of space-time codes bringing about simple estimator structure and attractive identifiability; see [16], [21]–[23] for the details, and see [17] for some discussions comparing the ML and subspace approaches.

The purpose of this paper is to extend the above described blind ML OSTBC technique to the OSTBC-OFDM scenario,

¹In the rest of this paper we assume the tacit understanding that “block” simply stands for “OSTBC-OFDM block”, for the sake of convenience.

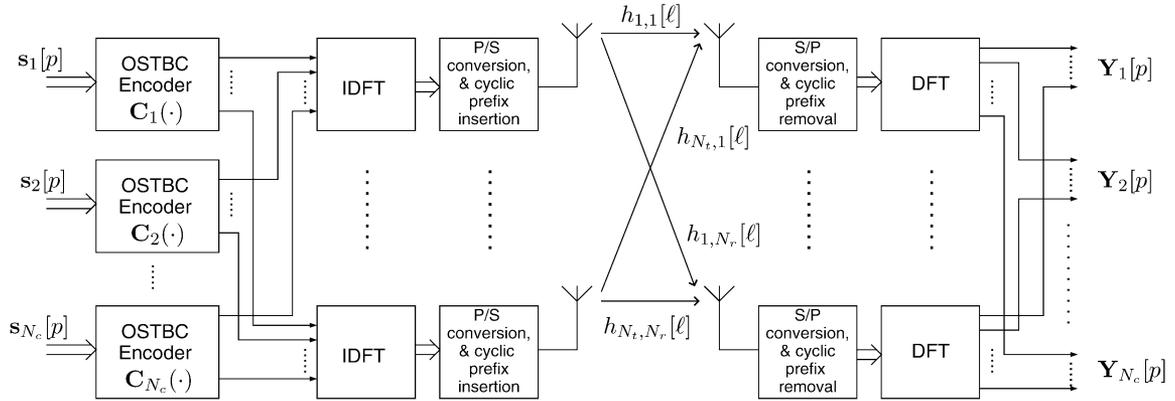


Fig. 1. OSTBC-OFDM system structure diagram.

with an emphasis on BPSK/QPSK constellations. A straightforward approach is to treat the OFDM subchannels as if they were mutually independent flat fading channels. This subchannel-wise approach enables direct application of the previously described flat-fading based blind ML techniques, but it typically works well only when the channel remains static over many (OSTBC-OFDM) blocks. This work follows a different approach that has only been applied to the single-input-multiple-output OFDM scenario so far; see [24] and the references therein, and [25]. The idea is to exploit the intersubchannel relationship, specifically by linking the subchannels through their time-domain characterization. By doing so we establish a subchannel dependent ML approach that can perform well with just one block. The advantages of this approach are the ability to handle block fading channels (i.e., channels that vary from one block to another), and short detection latency which is favorable for delay-constrained applications.

This paper deals with two important issues that were not addressed in the previous studies. First, we consider reduced-complexity implementation by proposing subchannel grouping OSTBC-OFDM (SGOO) blind/semiblind detection schemes. This development is essential because full OSTBC-OFDM (FOO) blind/semiblind ML detection is usually a large scale problem. Specifically the FOO problem size is proportional to the discrete Fourier transform (DFT) size, the latter of which is very large in practice; e.g., 128 for IEEE 802.11a and 2048 for DVB-H². SGOO works by breaking the FOO problem into smaller subproblems, and then by handling each subproblem individually. Further improvement can be obtained by using the low-complexity cyclic ML method to fuse the SGOO solutions to yield a refined solution. We found that this combined method works very well, as the simulation results in Section VI will demonstrate.

Second, we perform a theoretical analysis for blind ML identifiability of OSTBC-OFDM. While an identifiability analysis for OSTBCs in flat fading channels has been given in [20], its implications are not sufficient enough to deal with its OSTBC-OFDM counterpart. We provide a generalization of the existing results, and more importantly we derive new results that connect the blind ML identifiability conditions of OSTBC and OSTBC-OFDM. With these results, we are able to design blind and

semiblind SGOO schemes that guarantee unique identifiability in a probability one sense. The designed schemes require few number of pilots; for example, in our semiblind SGOO scheme, only one pilot OSTBC is needed. This is in sharp contrast to the conventional pilot-aided channel estimation methods [13], [27], in which unique channel identification requires at least L pilot codes where L denotes the channel length.

This paper is organized as follows. Section II reviews the OSTBC-OFDM signal model with an emphasis on BPSK/QPSK constellations. Section III describes the subchannel dependent blind ML detection approach, and the SGOO detection method. The proposed blind and semiblind SGOO schemes are also introduced in that section. Sections IV and V respectively deal with the implementation and identifiability issues, based on a unified framework covering both the SGOO and full OSTBC-OFDM problems. In Section VI, simulation results are presented to demonstrate the performance advantages of the proposed methods. Finally, some conclusions are drawn in Section VII.

II. BACKGROUND

In this review section, we first describe the OSTBC-OFDM system model under consideration in the first subsection. Then, we briefly explain the subchannel-independent blind ML approach and discuss its drawbacks in the second subsection.

A. OSTBC-OFDM System Model

Consider an OSTBC-OFDM system [12], [5] equipped with N_t transmit antennas and N_r receive antennas as illustrated in Fig. 1. As seen in the figure, N_c denotes the discrete Fourier transform (DFT) size of OFDM, or the number of subchannels. Moreover, the length of the employed space-time codes is denoted by T . As a common assumption in space-time-frequency coding, we assume that the channel can at least remain static for T OFDM symbols. Each subchannel, indexed by $n \in \{1, \dots, N_c\}$, has a preassigned OSTBC denoted by a mapping

²DVB-H: Digital video broadcasting-handheld [26].

$$\mathbf{C}_n : \{\pm 1\}^{K_n} \rightarrow \mathbb{C}^{T \times N_t}$$

where K_n is the number of bits per code³. Over a time frame of T OFDM symbols or simply an *OSTBC-OFDM block*, each subchannel will transmit one space-time matrix according to $\mathbf{C}_n(\cdot)$. The model of the resultant received signal can be formulated as

$$\mathbf{Y}_n[p] = \mathbf{C}_n(\mathbf{s}_n[p])\mathbf{H}_n[p] + \mathbf{W}_n[p], \quad (1)$$

where $n = 1, 2, \dots, N_c$, and $p = 1, 2, \dots$ is the OSTBC-OFDM block index. Here, $\mathbf{Y}_n[p] \in \mathbb{C}^{T \times N_r}$ is the received code matrix in the p th block at subchannel n , $\mathbf{s}_n[p] \in \{\pm 1\}^{K_n}$ is the transmitted data vector sequence for subchannel n , $\mathbf{H}_n[p] \in \mathbb{C}^{N_t \times N_r}$ is the MIMO channel frequency response matrix in the p th block at subchannel n , and $\mathbf{W}_n \in \mathbb{C}^{T \times N_r}$ is an AWGN matrix with the average power per entry denoted by σ_w^2 . It should be stressed that the notation $\mathbf{H}_n[p]$ implies a block fading environment⁴ where the channel may change from one block to another.

Our focus in this work is on BPSK/QPSK constellations. In this case, each OSTBC function \mathbf{C}_n takes the linear dispersion form [28]

$$\mathbf{C}_n(\mathbf{s}_n[p]) = \sum_{k=1}^{K_n} \mathbf{X}_{n,k} s_{n,k}[p], \quad (2)$$

where $s_{n,k}[p] \in \{\pm 1\}$ is the k th entry of $\mathbf{s}_n[p]$, and $\mathbf{X}_{n,k} \in \mathbb{C}^{T \times N_t}$ are the basis matrices of \mathbf{C}_n . The basis matrices are specially designed such that for any $\mathbf{s}_n[p] \in \{\pm 1\}^{K_n}$,

$$\mathbf{C}_n^H(\mathbf{s}_n[p])\mathbf{C}_n(\mathbf{s}_n[p]) = K_n \mathbf{I}_{N_t}, \quad (3)$$

where \mathbf{I}_{N_t} is the $N_t \times N_t$ identity matrix.

B. Subchannel-Wise Blind ML Detection for Slow Fading Channels

Let us consider a slow fading environment where $\mathbf{H}_n[p]$ is static over P consecutive (OSTBC-OFDM) blocks, say

$$\mathbf{H}_n[1] = \mathbf{H}_n[2] = \dots = \mathbf{H}_n[P] \triangleq \mathbf{H}_n$$

for all subchannels $n = 1, 2, \dots, N_c$. If we treat each \mathbf{H}_n as an independent deterministic unknown, then blind ML detection associated with the observations in (1) for $p = 1, 2, \dots, P$ is given by N_c independent subproblems:

$$\min_{\substack{\mathbf{s}_n[p] \in \{\pm 1\}^{K_n}, p=1,2,\dots,P \\ \mathbf{H}_n \in \mathbb{C}^{N_t \times N_r}}} \sum_{p=1}^P \|\mathbf{Y}_n[p] - \mathbf{C}_n(\mathbf{s}_n[p])\mathbf{H}_n\|_F^2 \quad (4)$$

for $n = 1, 2, \dots, N_c$. The objective of (4) is to find a pair of channel and symbols that gives the least squares approximation error to the observations. In essence, each subproblem in (4) is equivalent to that of blind ML OSTBC detection in flat-

³In the coherent scenario we usually use the same OSTBC encoder for all subchannels. But, in the blind scenario, we shall see that allowing a different OSTBC for each subchannel has some advantage from a blind identifiability standpoint.

⁴More precisely, the block fading assumption necessitates that the channel coherence interval should be longer than $T(N_c + L)T_s$ sec., where L denotes the cyclic prefix length, and T_s is the sampling interval whose unit is second.

fading channels. Hence, the previously developed treatments for the latter [12], [14]–[18], [29] can be directly applied to (4). However, there are several reasons that would render this subchannel-wise blind ML detection approach unsatisfactory:

- i) A moderate to large P (or large data size) is usually required to achieve near coherent performance, from our experience with the flat-fading scenario. This translates into a long channel coherence time which may be violated in certain fast fading environments.
- ii) Each blind detection subproblem in (4) is subject to a sign ambiguity. To fix this problem, we need to place pilot bits or even pilot codes in each subchannel. Each subchannel is unable to take advantage of the pilots in other subchannels.
- iii) The MIMO frequency responses \mathbf{H}_n are actually dependent. They follow a relationship called the *FIR channel parameterization*. Specifically, the (m, i) th entry of \mathbf{H}_n is given by

$$[\mathbf{H}_n]_{m,i} = \frac{1}{\sqrt{N_c}} \sum_{\ell=0}^{L-1} h_{m,i}[\ell] e^{-j2\pi\ell(n-1)/N_c} \quad (5)$$

where $\{h_{m,i}[\ell]\}_{\ell=0}^{L-1}$ is the (finite) impulse response of the channel between the m th transmit antenna and the i th receive antenna, and L is the channel length (or the cyclic prefix length) with $L \ll N_c$ in practice. The subchannel-wise approach, albeit simple, does not exploit the FIR channel relationship.

III. BLIND ML DETECTION IN ONE OSTBC-OFDM BLOCK: AN OVERVIEW

Starting from this section, we concentrate on the subchannel-dependent blind ML approach that enables detection in one block. This section serves as an overview for this approach. The more detailed derivations for implementations and identifiability will be provided in Sections IV and V, respectively. In the first subsection, the problem formulation is presented. In the second subsection, we introduce the subchannel grouping OSTBC-OFDM (SGOO) schemes for complexity reduction.

A. Basic Problem Formulation

Recall the signal model in (1). Our problem is to blindly detect the bit symbols $\{\mathbf{s}_1[p], \dots, \mathbf{s}_{N_c}[p]\}$ from the associated received signal block $\{\mathbf{Y}_1[p], \dots, \mathbf{Y}_{N_c}[p]\}$, thereby enabling detection in one block. Under such circumstances, it is notationally convenient to drop the index p from (1) to form a simplified signal model

$$\mathbf{Y}_n = \mathbf{C}_n(\mathbf{s}_n)\mathbf{H}_n + \mathbf{W}_n, \quad n = 1, 2, \dots, N_c. \quad (6)$$

The key ingredient of the subchannel-dependent approach is to exploit the FIR channel parameterization in (5). Let $\mathbf{f}_n = (1)/(\sqrt{N_c})[1, e^{-j(2\pi)/(N_c(n-1))}, \dots, e^{-j(2\pi)/(N_c(n-1)(L-1)}]^T$ and $\mathbf{h}_{m,i} = [h_{m,i}[0], h_{m,i}[1], \dots, h_{m,i}[L-1]]^T$ be a DFT vector and a time-domain channel vector, respectively. Equation (5) can be reexpressed as

$$[\mathbf{H}_n]_{m,i} = \mathbf{f}_n^T \mathbf{h}_{m,i}. \quad (7)$$

Furthermore, by letting

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_{1,1} & \cdots & \mathbf{h}_{1,N_r} \\ \vdots & \ddots & \vdots \\ \mathbf{h}_{N_t,1} & \cdots & \mathbf{h}_{N_t,N_r} \end{bmatrix} \in \mathbb{C}^{LN_t \times N_r} \quad (8)$$

be the time-domain MIMO channel matrix, each \mathbf{H}_n can be formulated as

$$\mathbf{H}_n = (\mathbf{I}_{N_t} \otimes \mathbf{f}_n^T) \mathbf{H} \quad (9)$$

where \otimes denotes the Kronecker product. Note that if we define

$$\mathbf{G}_n(\mathbf{s}_n) = \mathbf{C}_n(\mathbf{s}_n) (\mathbf{I}_{N_t} \otimes \mathbf{f}_n^T) \quad (10)$$

the model in (6) can be rewritten as

$$\mathbf{Y}_n = \mathbf{G}_n(\mathbf{s}_n) \mathbf{H} + \mathbf{W}_n, \quad n = 1, 2, \dots, N_c. \quad (11)$$

An interesting observation is that from a standpoint of flat-fading based space-time coding, (11) can be viewed as the received signal for a sequence of time-varying space-time block codes \mathbf{G}_n over a flat-fading channel. The blind ML detector for (11) is given by

$$\min_{\substack{\mathbf{s}_n \in \{\pm 1\}^{K_n}, n=1,2,\dots,N_c \\ \mathbf{H} \in \mathbb{C}^{LN_t \times N_r}}} \sum_{n=1}^{N_c} \|\mathbf{Y}_n - \mathbf{G}_n(\mathbf{s}_n) \mathbf{H}\|_F^2. \quad (12)$$

There are two possible approaches for dealing with the above minimization problem. One is to use cyclic minimization [15]. The idea is to cyclically update the channel and symbol estimates, denoted, respectively, by $\hat{\mathbf{H}}$ and $\{\hat{\mathbf{s}}_n\}_{n=1}^{N_c}$, by solving the following two subproblems:

$$\hat{\mathbf{H}} := \arg \min_{\mathbf{H} \in \mathbb{C}^{LN_t \times N_r}} \sum_{n=1}^{N_c} \|\mathbf{Y}_n - \mathbf{G}_n(\hat{\mathbf{s}}_n) \mathbf{H}\|_F^2 \quad (13)$$

$$\{\hat{\mathbf{s}}_n\}_{n=1}^{N_c} := \arg \min_{\substack{\mathbf{s}_n \in \{\pm 1\}^{K_n}, \\ n=1,2,\dots,N_c}} \sum_{n=1}^{N_c} \|\mathbf{Y}_n - \mathbf{G}_n(\mathbf{s}_n) \hat{\mathbf{H}}\|_F^2. \quad (14)$$

The cyclic (or multistage) update continues until some stopping criterion is satisfied; see [15] for the details. It can be shown that (13) is a least-squares (LS) channel estimator fixing $\{\hat{\mathbf{s}}_n\}_{n=1}^{N_c}$, and that (14) is the coherent OSTBC detector given $\hat{\mathbf{H}}$. The two update processes can be shown to be very simple and of low complexity [15]. However, cyclic ML cannot operate properly without reasonable initialization of either $\hat{\mathbf{H}}$ or $\{\hat{\mathbf{s}}_n\}_{n=1}^{N_c}$.

Another approach for handling (12) is based on Boolean quadratic program (BQP) reformulation [17]. To illustrate this, let $\mathbf{s} = [\mathbf{s}_1^T, \dots, \mathbf{s}_{N_c}^T]^T \in \{\pm 1\}^{\bar{K}}$, where $\bar{K} = \sum_{n=1}^{N_c} K_n$ is the total number of bits to be detected. It will be shown in the next section that (12) can be simplified to be the BQP

$$\max_{\mathbf{s} \in \{\pm 1\}^{\bar{K}}} \mathbf{s}^T \mathbf{R} \mathbf{s} \quad (15)$$

for some appropriate $\mathbf{R} \in \mathbb{R}^{\bar{K} \times \bar{K}}$, and that determining the optimal estimate of \mathbf{H} from the solution of (15) is simple. The reformulation in (15) enables us to handle the problem by directly applying a readily available BQP solver; e.g., the quasi-optimal

SDR solver [17], [30] which has a worst-case polynomial-time complexity of $\mathcal{O}(\bar{K}^{3.5})$. This approach also serves as a reliable means of finding good initializations for cyclic ML. We should note that the problem size \bar{K} is proportional to the DFT size N_c . For practical DFT sizes, say, $N_c = 128$ or even $N_c = 2048$, the blind ML BQP is a large scale problem meaning that direct application of BQP solvers would still be computationally too expensive. This inherent difficulty motivates the subchannel grouping method considered in the next subsection.

B. Subchannel Grouping OSTBC-OFDM

When dealing with a large scale problem, we would often consider decoupling the original problem into smaller subproblems for complexity reduction. This is the idea behind *subchannel grouping* (SG). Essentially, we group the N_c subchannels into a number of subsets, and then apply blind ML detection to each subset individually. Suppose that we have P groups, and let $\mathcal{S}_p \subset \{1, 2, \dots, N_c\}$ be the subchannel index set for the p th group. Fixing group p , we have a dimension reduced signal model

$$\mathbf{Y}_n = \mathbf{G}_n(\mathbf{s}_n) \mathbf{H} + \mathbf{W}_n, \quad n \in \mathcal{S}_p \quad (16)$$

and a group-wise blind ML problem

$$\min_{\substack{\mathbf{s}_n \in \{\pm 1\}^{K_n}, n \in \mathcal{S}_p \\ \mathbf{H} \in \mathbb{C}^{LN_t \times N_r}}} \sum_{n \in \mathcal{S}_p} \|\mathbf{Y}_n - \mathbf{G}_n(\mathbf{s}_n) \mathbf{H}\|_F^2. \quad (17)$$

We call (17) the SG OSTBC-OFDM (SGOO) problem. To distinguish SGOO from the complete OSTBC-OFDM problem in (12), we call (12) the full OSTBC-OFDM (FOO) problem.

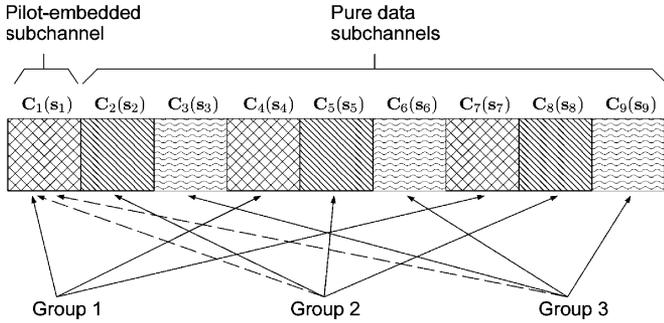
In designing a blind or semiblind SGOO scheme, there are three factors to consider: The SG assignment, the placement of pilots, and the choosing of the codes $\{\mathbf{C}_n\}_{n=1}^{N_c}$. A carelessly designed SGOO scheme may have poor data identifiability, meaning that the scheme would not operate properly even in the absence of noise. Here we describe two SGOO schemes that will be theoretically proven to exhibit good identifiability (in Section V):

Semiblind SGOO Scheme: In this scheme, subchannel 1 is assigned for pilot transmission (or \mathbf{s}_1 is assumed to be known at the receiver). The SG assignment is depicted in Fig. 2 and is given by

$$\mathcal{S}_1 = \left\{ 1 + m \frac{N_c}{M} \mid m = 0, 1, \dots, M-1 \right\} \quad (18a)$$

$$\mathcal{S}_p = \{1\} \cup \left\{ p + m \frac{N_c}{M} \mid m = 0, 1, \dots, M-1 \right\}, \quad p = 2, \dots, P \quad (18b)$$

where $M = N_c/P$. It is assumed that $M > L$. This SG assignment is similar to that for coherent and differential space-time-frequency coding [4], [9], [10], as well as for training-based channel estimation [27], [31], [32] but there is some subtle difference. We let all the groups access subchannel 1, and thus each SGOO problem is able to use the pilots to fix the sign ambiguity effect. As for code selection, we can simply employ the same


 Fig. 2. Proposed SG scheme for $N_c = 9$ and $P = 3$.

OSTBC in every subchannel; i.e., $\mathbf{C}_1(\cdot) = \dots = \mathbf{C}_{N_c}(\cdot) = \mathbf{C}(\cdot)$.

Blind SGOO Odd-Even Scheme: In a blind scheme, only one pilot bit is used. We employ the same SG assignment as in (18), and assign the pilot bit to the 1st bit of the symbol of subchannel 1, i.e., $s_{1,1}$. The blind SGOO design presents a more difficult challenge. For example, unlike the semiblind SGOO scheme described above, a blind SGOO scheme with universal OSTBC for all subchannels does not necessarily result in good identifiability. In fact, simulation results in Section VI will show that such a blind scheme could exhibit poor error performance. To guarantee good identifiability in the blind case, we employ the *odd-even* OSTBC arrangement, first introduced in [20] for flat-fading blind ML OSTBC detection. The arrangement is as follows: Subchannels 2 to N_c (the data subchannels) use a universal OSTBC, denoted by $\mathbf{C}_e(\cdot)$, while subchannel 1 (the pilot-embedded subchannel) uses a different OSTBC, denoted by $\mathbf{C}_o(\cdot)$; that is, $\mathbf{C}_1(\cdot) = \mathbf{C}_o(\cdot)$ and $\mathbf{C}_2(\cdot) = \dots = \mathbf{C}_{N_c}(\cdot) = \mathbf{C}_e(\cdot)$. Let

$$\mathbf{C}_e(\mathbf{s}) = \sum_{k=1}^K \mathbf{X}_k s_k$$

and assume that K is even. Being the code function for carrying most information symbols, \mathbf{C}_e would be chosen to be a maximal-rate BPSK/QPSK OSTBC [33], which often has an even K . The “odd” OSTBC function \mathbf{C}_o is constructed from \mathbf{C}_e by taking out one bit:

$$\mathbf{C}_o(\mathbf{s}) = \sum_{k=1}^{K-1} \mathbf{X}_k s_k.$$

Some discussions are now in order:

- i) Once we solve all SGOO problems (say, in a quasi-optimal fashion by SDR), we can enhance the quality of the obtained solution by applying the cyclic ML refinement mentioned in Section III-A. Our experience with simulations is that this combined method works very well, and this will be illustrated in Section VI.
- ii) If the above proposed semiblind scheme and blind odd-even scheme are applied to the flat fading scenario [specifically by modifying $\mathbf{G}_n(\mathbf{s}_n) = \mathbf{C}_n(\mathbf{s}_n)$ which removes the subcarriers], the existing analysis results [20] will be sufficient in showing that these two schemes

achieve good identifiability conditions. Such desirable conditions do not directly carry over into the SGOO scenario, however. The SGOO identifiability analysis has an intricate relationship with the assigned SG pattern, as we will see in Section V. But, in summary, it will be proven that the SG assignment used in the two schemes is sufficient in leading to good identifiability conditions.

IV. BLIND ML RECEIVER REALIZATION USING A UNIFIED TREATMENT

In this section, we present the detailed derivations of the blind ML BQP reformulation, for both SGOO and FOO. To facilitate the development, it is desirable to establish a unified OSTBC-OFDM (UOO) formulation for the two problems. Consider the following generalized signal model:

$$\mathbf{Y}_m = \mathbf{G}_m(\mathbf{s}_m)\mathbf{H} + \mathbf{W}_m, \quad m = 1, 2, \dots, M \quad (19)$$

$$\mathbf{G}_m(\mathbf{s}_m) = \mathbf{C}_m(\mathbf{s}_m) \left(\mathbf{I}_{N_t} \otimes \mathbf{f}_m^T \right). \quad (20)$$

Here, $\mathbf{C}_m : \{\pm 1\}^{\kappa_m} \rightarrow \mathbb{C}^{T \times N_t}$ is a κ_m -bit OSTBC function which admits a linear dispersion expression

$$\mathbf{C}_m(\mathbf{s}_m) = \sum_{k=1}^{\kappa_m} \mathbf{X}_{m,k} s_{m,k} \quad (21)$$

where $\mathbf{s}_m \in \{\pm 1\}^{\kappa_m}$ is the bit vector transmitted by $\mathbf{C}_m(\cdot)$, $\mathbf{f}_m = [1, z_m^{-1}, \dots, z_m^{-(L-1)}]^T$ where $z_m \in \mathbb{C}$, \mathbf{H} is the deterministically unknown channel, and \mathbf{W}_m is AWGN. This UOO formulation is generalized in the sense that we only assume z_m to be distinct; i.e., $z_m \neq z_n$ for any $m \neq n$. We see that (19) represents FOO in (11) if $M = N_c$, $z_m = e^{j(2\pi)/(N_c)(m-1)}$, $\mathbf{C}_m(\cdot) = \mathbf{C}_m(\cdot)$, $\mathbf{Y}_m = \sqrt{N_c} \mathbf{Y}_m$, $\mathbf{W}_m = \sqrt{N_c} \mathbf{W}_m$ and so on. Similarly, an equivalence can be established for each SGOO problem in (16).

Let us define $\mathcal{Y} = [\mathbf{Y}_1^T, \dots, \mathbf{Y}_M^T]^T$, $\mathcal{W} = [\mathbf{W}_1^T, \dots, \mathbf{W}_M^T]^T$, $\mathbf{s} = [\mathbf{s}_1^T, \dots, \mathbf{s}_M^T]^T$, and

$$\mathcal{G}(\mathbf{s}) = [\mathbf{g}_1^T(\mathbf{s}_1), \dots, \mathbf{g}_M^T(\mathbf{s}_M)]^T. \quad (22)$$

Equation (19) can be rewritten as

$$\mathcal{Y} = \mathcal{G}(\mathbf{s})\mathbf{H} + \mathcal{W} \quad (23)$$

and its respective blind ML problem is

$$\min_{\mathbf{s} \in \{\pm 1\}^{\bar{\kappa}}} \left\{ \min_{\mathbf{H} \in \mathbb{C}^{L \times N_t \times N_r}} \|\mathcal{Y} - \mathcal{G}(\mathbf{s})\mathbf{H}\|_F^2 \right\} \quad (24)$$

where $\bar{\kappa} = \sum_{m=1}^M \kappa_m$ is the total number of bits. Our first step is to investigate the inner minimization of (24). The inner minimization is an LS problem given \mathbf{s} , which has a unique solution

$$\hat{\mathbf{H}}(\mathbf{s}) = [\mathcal{G}^H(\mathbf{s})\mathcal{G}(\mathbf{s})]^{-1} \mathcal{G}^H(\mathbf{s})\mathcal{Y} \quad (25)$$

if and only if $\mathcal{G}(\mathbf{s})$ is of full column rank. Let us study conditions for $\mathcal{G}(\mathbf{s})$ to have full column rank. Let

$$\mathcal{F} = \begin{bmatrix} \mathbf{f}_1^T \\ \vdots \\ \mathbf{f}_M^T \end{bmatrix} = \begin{bmatrix} 1 & z_1^{-1} & \cdots & z_1^{-(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & z_M^{-1} & \cdots & z_M^{-(L-1)} \end{bmatrix} \in \mathbb{C}^{M \times L} \quad (26)$$

which is a Vandermonde matrix. By noting the following expression of $\mathcal{G}(\mathbf{s})$:

$$\mathcal{G}(\mathbf{s}) = \begin{bmatrix} \mathbf{C}_1(\mathbf{s}_1) & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{C}_M(\mathbf{s}_M) \end{bmatrix} \begin{bmatrix} \mathbf{I}_{N_t} \otimes (\mathbf{e}_1^T \mathcal{F}) \\ \vdots \\ \mathbf{I}_{N_t} \otimes (\mathbf{e}_M^T \mathcal{F}) \end{bmatrix}$$

where $\mathbf{e}_m \in \mathbb{R}^M$ is the unit vector with the m th entry equal to 1, $\mathcal{G}(\mathbf{s})$ can be rewritten as

$$\mathcal{G}(\mathbf{s}) = \mathcal{D}\mathcal{C}(\mathbf{s})\mathbf{\Pi}(\mathbf{I}_{N_t} \otimes \mathcal{F}) \quad (27)$$

where $\mathcal{D}\mathcal{C} = \text{blkdiag}[\mathbf{C}_1(\mathbf{s}_1), \dots, \mathbf{C}_M(\mathbf{s}_M)] \in \mathbb{C}^{MT \times MN_t}$ (a block diagonal matrix), and $\mathbf{\Pi} \in \mathbb{C}^{MN_t \times MN_t}$ is a permutation matrix given by

$$\mathbf{\Pi} = [\mathbf{I}_{N_t} \otimes \mathbf{e}_1, \dots, \mathbf{I}_{N_t} \otimes \mathbf{e}_M]^T.$$

It can be shown that, for any $\mathbf{A} \in \mathbb{C}^{M \times M}$, the following commutativity property holds:

$$\mathbf{\Pi}^T(\mathbf{A} \otimes \mathbf{I}_{N_t})\mathbf{\Pi} = \mathbf{I}_{N_t} \otimes \mathbf{A}. \quad (28)$$

Using (27) and (28), one can verify that

$$\mathcal{G}^H(\mathbf{s})\mathcal{G}(\mathbf{s}) = \mathbf{I}_{N_t} \otimes (\mathcal{F}^H \mathbf{D}_\kappa \mathcal{F}) \quad (29)$$

where $\mathbf{D}_\kappa = \text{diag}(\kappa_1, \dots, \kappa_M)$. Using (29) and some standard matrix results, we show that

$$\text{rank}\{\mathcal{G}(\mathbf{s})\} = \text{rank}\{\mathcal{G}^H(\mathbf{s})\mathcal{G}(\mathbf{s})\} = N_t \min\{M, L\}. \quad (30)$$

We, therefore, conclude from (30) that $\mathcal{G}(\mathbf{s})$ has full column rank if and only if $M \geq L$.

The second step of the BQP reformulation is to substitute the inner minimization solution in (25) into the blind ML problem in (24). The resulting problem is given by

$$\min_{\mathbf{s} \in \{\pm 1\}^\kappa} \|\mathcal{Y} - \mathbf{P}_{\mathcal{G}(\mathbf{s})}\mathcal{Y}\|_F^2 \quad (31)$$

where $\mathbf{P}_{\mathcal{G}(\mathbf{s})} = \mathcal{G}(\mathbf{s})[\mathcal{G}^H(\mathbf{s})\mathcal{G}(\mathbf{s})]^{-1}\mathcal{G}^H(\mathbf{s})$ is the orthogonal projector of $\mathcal{G}(\mathbf{s})$. Equation (31) can be simplified to a BQP, by substituting (29) and (21) into (31). The development is conceptually identical to that in flat-fading blind ML OSTBC detection [17], [18], though the derivations in this case appear to be notationally more involved. Hence, for brevity, the result is given without proof as follows:

Proposition 1: Suppose that $M \geq L$. For the blind ML problem in (24), an optimal symbol solution can be obtained by solving the BQP

$$\max_{\mathbf{s} \in \{\pm 1\}^\kappa} \sum_{m=1}^M \sum_{n=1}^M \mathbf{s}_m^T \mathbf{R}_{m,n} \mathbf{s}_n = \max_{\mathbf{s} \in \{\pm 1\}^\kappa} \mathbf{s}^T \mathbf{R} \mathbf{s} \quad (32)$$

where $\mathbf{R}_{m,n} \in \mathbb{R}^{\kappa_m \times \kappa_n}$ has its (k, ℓ) entry given by

$$[\mathbf{R}_{m,n}]_{k,\ell} = \text{Re} \{ \text{Tr} \{ \gamma_{m,n} \mathcal{Y}_m^H \mathcal{X}_{m,k} \mathcal{X}_{n,\ell}^H \mathcal{Y}_n \} \} \quad (33)$$

$$\gamma_{m,n} = \mathbf{e}_m^T \mathcal{F} (\mathcal{F}^H \mathbf{D}_\kappa \mathcal{F})^{-1} \mathcal{F}^H \mathbf{e}_n, \quad (34)$$

and

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_{1,1} & \cdots & \mathbf{R}_{1,M} \\ \vdots & \ddots & \vdots \\ \mathbf{R}_{M,1} & \cdots & \mathbf{R}_{M,M} \end{bmatrix}. \quad (35)$$

The associated optimal channel solution is obtained by substituting the optimal solution of (32) into (25).

As mentioned earlier, the BQP problem in (32) can be effectively handled by readily available algorithms; e.g., the SDR algorithm which yields a complexity of $\mathcal{O}(\bar{\kappa}^{3.5})$. Readers are referred to [17] for detailed descriptions of SDR and the other available BQP solvers.

The above developed blind ML framework can be easily extended to the semiblind case. In this paper, we are interested in using only 1 pilot code. Without loss of generality, assume that \mathbf{s}_1 is known. Let $\mathbf{s}_d = [\mathbf{s}_2^T, \dots, \mathbf{s}_M^T]^T \in \{\pm 1\}^{\kappa - \kappa_1}$ be the unknown data vector. The semiblind ML problem is given by

$$\min_{\substack{\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_d^T]^T \\ \mathbf{s}_d \in \{\pm 1\}^{\kappa - \kappa_1}}} \left\{ \min_{\mathcal{H} \in \mathbb{C}^{LN_t \times N_r}} \|\mathcal{Y} - \mathcal{G}(\mathbf{s})\mathcal{H}\|_F^2 \right\}. \quad (36)$$

Like Proposition 1, problem (36) can be reformulated as a BQP. That reformulation shares the same idea as that in the flat-fading scenario [17], and its details are omitted here for conciseness.

We notice that the blind ML BQP in (32) has at least two solutions: if \mathbf{s}^* is a solution of (32) then $-\mathbf{s}^*$ is also a solution of (32). This sign ambiguity may be fixed by assigning one bit as the pilot. As for the semiblind ML problem in (36), there should be no such problem. However, it can be shown that

Lemma 1: *The solution to the blind ML problem in (24) is unique up to a sign only if $M > L$. The solution to the semiblind ML problem in (36) is unique only if $M > L$.*

The proof of this lemma is given in Appendix I, where we show that the blind and semiblind ML problems may give multiple solutions if $M \leq L$.

V. BLIND ML IDENTIFIABILITY ANALYSIS

Following the development in the previous section, this section considers blind ML identifiability analysis under the UOO framework. Through the process we will see that the semiblind and blind SGOO schemes proposed in Section III-B, as special cases of UOO, have a rather relaxed probability one identifiability condition. In the first subsection, we briefly review and generalize some existing identifiability results for OSTBCs in flat fading channels [20]. Then, in the second subsection, the

relationship of the existing results and the UOO identifiability conditions is explored.

A. Review and Generalization of Some Existing Results

A key result that will be used in this paper is *probability one blind/semiblind identifiability*, which was developed for OSTBCs [20]. The essence of the result may be well described by considering a noise-free generic MIMO problem

$$\mathbf{Y} = \mathbf{G}\mathbf{H}, \quad \mathbf{G} \in \mathcal{G} \tag{37}$$

where \mathbf{G} is redefined as a transmitted code matrix drawn from a (finite) codeword set $\mathcal{G} \subset \mathbb{C}^{T \times N_t}$, and again $\mathbf{Y} \in \mathbb{C}^{T \times N_r}$ and $\mathbf{H} \in \mathbb{C}^{N_t \times N_r}$ are the received signal matrix and MIMO channel of the problem, respectively. Our treatment is general in the sense that \mathcal{G} is not restricted to any particular class of schemes. Hence, it may be applied not only to OSTBC, but also to UOO as well as other space-time-frequency and space-time coding schemes. For the UOO framework in (23), the blind problem is equivalent to that in (37) with a codeword set $\mathcal{G} = \{\mathcal{G}(\mathbf{s}) \in \mathbb{C}^{MT \times LN_t} | \mathbf{s} \in \{\pm 1\}^{\bar{K}}\}$ where $\mathcal{G}(\cdot)$ is given in (27). As for the semiblind problem, we have $\mathcal{G} = \{\mathcal{G}([\mathbf{s}_1^T \mathbf{s}_d^T]^T) \in \mathbb{C}^{MT \times LN_t} | \mathbf{s}_d \in \{\pm 1\}^{\bar{K}-\kappa_1}\}$ where \mathbf{s}_1 is fixed.

Consider applying blind ML detection to (37), in the same way as before. To uniquely determine the true \mathbf{G} from \mathbf{Y} , it is essential that the following ambiguity situation does not hold

$$\mathbf{G}\mathbf{H} = \mathbf{G}'\mathbf{H}' \tag{38}$$

for any $\mathbf{G}' \in \mathcal{G} \setminus \{\mathbf{G}\}$ and $\mathbf{H}' \in \mathbb{C}^{N_t \times N_r}$. Simply speaking, \mathbf{G} is said to be unique identifiable if (38) cannot be satisfied. Consider the following definition.

Definition 1: A codeword set \mathcal{G} is said to be *pairwise non-transformable (PNT)* if, for any two distinct codewords $\mathbf{G}, \mathbf{G}' \in \mathcal{G}$, there does not exist a matrix $\mathbf{U} \in \mathbb{C}^{N_t \times N_t}$ such that

$$\mathbf{G}'\mathbf{U} = \mathbf{G}. \tag{39}$$

Moreover, \mathcal{G} is said to be *PNT up to a sign (PNT- ± 1)* if (39) does not hold for any $\mathbf{G} \in \mathcal{G}$ and $\mathbf{G}' \in \mathcal{G} \setminus \{\pm \mathbf{G}\}$.

It can be verified from (38) and (39) that the PNT and PNT- ± 1 conditions are necessary for unique code identifiability and unique code identifiability up to a sign, respectively. PNT and PNT- ± 1 are also sufficient identifiability conditions, in a probability one sense. This is described in the following theorem:

Theorem 1: Assume that \mathbf{H} is Gaussian distributed, and that at least one column of \mathbf{H} has a positive definite covariance matrix. Then, for the blind ML detection of (37):

- i) the code matrix \mathbf{G} is uniquely identifiable with probability one if \mathcal{G} is PNT; and
- ii) the code matrix \mathbf{G} is uniquely identifiable up to a sign with probability one if \mathcal{G} is PNT- ± 1 .

Note that the i.i.d. Rayleigh fading channels satisfy the channel assumption in Theorem 1. The proof is presented in Appendix II. The idea behind the proof is to show that the ambiguity in (38) happens with probability zero, under the premises in Theorem 1.

We should emphasize that Definition 1 and Theorem 1 are generalization of the probability one identifiability result in [20],

which was only for orthogonal codes. In [20], we have the following definition.

Definition 2: Let $\mathbf{C} : \{\pm 1\}^K \rightarrow \mathbb{C}^{T \times N_t}$ be an BPSK or QPSK OSTBC function, and $\mathcal{G} = \{\mathbf{C}(\mathbf{s}) | \mathbf{s} \in \{\pm 1\}^K\}$. If \mathcal{G} is PNT- ± 1 , then $\mathbf{C}(\cdot)$ is said to be *strictly nonrotatable*.

The use of the term “nonrotatable” was due to the observation that \mathbf{U} in (39) must be a rotation matrix if (39) is to be satisfied by the OSTBC. An important question is where to find a strictly nonrotatable OSTBC. This aspect has been studied in [20], and simply speaking not all the existing OSTBCs have the strictly nonrotatable property. But there exists a simple way of converting a (BPSK/QPSK) OSTBC to a strictly nonrotatable OSTBC⁵.

Lemma 2 (Ma [20]): Given a BPSK/QPSK OSTBC $\mathbf{C}_e(\mathbf{s}) = \sum_{k=1}^K \mathbf{X}_k s_k$, where K is even, construct

$$\mathbf{C}_o(\mathbf{s}) = \sum_{k=1}^{K-1} \mathbf{X}_k s_k. \tag{40}$$

The following concatenated code is strictly nonrotatable:

$$\mathbf{C}(\mathbf{s}) = [\mathbf{C}_o^T(\mathbf{s}_1) \mathbf{C}_e^T(\mathbf{s}_2) \dots \mathbf{C}_e^T(\mathbf{s}_M)]^T \tag{41}$$

where $\mathbf{s} = [\mathbf{s}_1^T \mathbf{s}_2^T \dots \mathbf{s}_M^T]^T$, $\mathbf{s}_1 \in \{\pm 1\}^{K-1}$, and $\mathbf{s}_m \in \{\pm 1\}^K$ for $m = 2, \dots, M$.

B. Identifiability of OSTBC-OFDM

We now focus on the symbol identifiability of UOO under the following assumptions:

- A1)** $M > L$ (this is a necessary identifiability condition; cf., Lemma 1).
- A2)** \mathcal{H} is Gaussian distributed and at least one column of \mathcal{H} has a positive definite covariance matrix.

Our aim is to derive conditions under which the super-code \mathcal{G} in (27) has the PNT/PNT- ± 1 property for the semiblind/blind case, thereby achieving the probability-1 identifiability stated in Theorem 1. To do this, let

$$\mathcal{C}(\mathbf{s}) = [\mathcal{C}_1^T(\mathbf{s}_1), \dots, \mathcal{C}_M^T(\mathbf{s}_M)]^T \tag{42}$$

be a concatenation of the OSTBCs in UOO. We consider the following condition:

C1) Let $\mathbf{P} = \mathcal{F}(\mathcal{F}^H \mathcal{F})^{-1} \mathcal{F}^H \in \mathbb{C}^{M \times M}$. For any $m, n \in \mathcal{I}_M$ with $m \neq n$, where $\mathcal{I}_M = \{1, 2, \dots, M\}$, there exists an length- I index sequence $\{\varpi_1, \varpi_2, \dots, \varpi_I\} \subseteq \mathcal{I}_M$ such that $\varpi_1 = m, \varpi_I = n$, and the $I - 1$ elements (ϖ_i, ϖ_{i+1}) in \mathbf{P} are nonzero, i.e.,

$$P_{\varpi_i, \varpi_{i+1}} \neq 0, \quad i = 1, 2, \dots, I - 1. \tag{43}$$

We have the following lemma for **C1**).

Lemma 3: Under **C1**), \mathcal{G} is PNT if and only if \mathcal{C} is PNT. Moreover, under **C1**) \mathcal{G} is PNT- ± 1 if and only if \mathcal{C} is strictly nonrotatable.

The proof is provided in Appendix III. Lemma 3 has profound implication: If we can guarantee that **C1**) holds, then the study

⁵The procedure was proposed to construct the so-called nonintersecting subspace OSTBCs. This class of codes is a subset of the strictly nonrotatable code class, which has added advantage in the flat fading scenario (see [20] for the details).

of identifiability of UOO reduces to that of its OSTBCs. Now, a key question is whether the subcarrier sets of FOO and SGOO satisfy **C1**). Fortunately, the answer is yes, through careful investigation. Consider the following two lemmas which are relevant to FOO and SGOO.

Lemma 4: *Suppose that*

$$\{z_1, z_2, \dots, z_M\} = \left\{1, e^{j2\pi/M}, \dots, e^{j2\pi(M-1)/M}\right\}.$$

Then, the corresponding \mathcal{F} satisfies **C1**).

Lemma 5: *Suppose that we expand the problem size of UOO from M to $M + 1$, and that*

$$\begin{aligned} \{z_1, z_2, z_3, \dots, z_{M+1}\} \\ = \left\{1, e^{j\alpha}, e^{j\alpha+j2\pi/M}, \dots, e^{j\alpha+j2\pi(M-1)/M}\right\} \end{aligned}$$

for some $\alpha \in \mathbb{R}$. Then, the corresponding \mathcal{F} satisfies **C1**).

The proofs of the two lemmas are presented in Appendices IV and V, respectively. We note that Lemma 4 is applicable to the FOO problem and the SGOO problem associated with \mathcal{S}_1 [see (18a)], and that Lemma 5 is applicable to the SGOO problems associated with \mathcal{S}_p for $p = 2, \dots, M$ [see (18b)]. Hence, we conclude the following.

Remark 1: The FOO problem satisfies **C1**). All SGOO problems satisfy **C1**).

Now, the remaining problem in our identifiability analysis is to examine the PNT/PNT ± 1 condition of \mathcal{C} in (42).

Semiblind Detection Case: The pairwise transformation identity for \mathcal{C} , given in (39) in Definition 1, can be explicitly expressed as

$$\mathbf{C}_m(\mathbf{s}'_m)\mathbf{U} = \mathbf{C}_m(\mathbf{s}_m), \quad m = 1, 2, \dots, M \quad (44)$$

where $\mathbf{s}, \mathbf{s}' \in \{\pm 1\}^{\bar{k}}$ are distinct. Since the OSTBCs have full column rank and subchannel 1 contains only the pilot; i.e., $\mathbf{s}'_1 = \mathbf{s}_1$, from (44) we must have $\mathbf{U} = \mathbf{I}_{N_t}$. Thus, (44) can be all satisfied only when $\mathbf{s}_m = \mathbf{s}'_m$ for all m . In other words, by Definition 1, the codewords of \mathcal{C} have to be PNT in the semiblind case. Hence, by Lemma 3 and Theorem 1, we obtain the following theorem.

Theorem 2: *Consider the semiblind ML detection of the UOO problem [in (36)]. Under **C1**), the data vector \mathbf{s}_d is uniquely identifiable with probability one.*

Blind Detection Case: Let us focus on the odd-even scheme described in Lemma 2, the same arrangement used in the blind SGOO odd-even scheme proposed in Section III-B. Specifically, given an OSTBC $\mathbf{C}_e(\cdot)$ with even number of bits K , choose

$$\mathbf{C}_1(\cdot) = \mathbf{C}_o(\cdot), \quad \mathbf{C}_m(\cdot) = \mathbf{C}_e(\cdot), \quad m = 2, 3, \dots, M \quad (45)$$

where $\mathbf{C}_o(\cdot)$ is the “odd” counterpart of $\mathbf{C}_e(\cdot)$, defined in the same way as (40). Since Lemma 2 indicates that the resultant \mathcal{C} is strictly nonrotatable (or PNT ± 1), we have the following theorem.

Theorem 3: *Consider the blind ML detection of UOO [in (24)], and suppose that the odd-even arrangement in (45) is employed. Under **C1**), the data vector \mathbf{s} is uniquely identifiable up to a sign with probability one.*

Now we are ready to consider the identifiability of SGOO, as a special case of UOO. Due to Remark 1 and Theorems 2 and 3, the following important conclusion is reached.

Corollary 1: *In the semiblind SGOO scheme in Section III-B, the data symbols are uniquely identifiable with probability one. In the blind SGOO odd-even scheme in Section III-B, the data symbols are uniquely identifiable up to a sign with probability one. The same identifiability holds for its FOO counterpart.*

VI. SIMULATION RESULTS

This section presents three simulation examples to justify the efficacy of the proposed blind/semiblind ML methods. Either the QPSK Alamouti code ($T = 2, N_t = 2, K = 4$) [34] or the QPSK 4×3 OSTBC code ((120) of [33]) ($T = 4, N_t = 3, K = 6$) was used. The DFT size N_c was 256 for all the examples. Since the respective FOO problems are large scale optimization problems meaning that they cannot be handled efficiently, we consider the SGOO schemes only⁶. The blind or semiblind ML SGOO BQP was handled by the SDR algorithm [17]. The obtained SGOO solutions were then refined by a one-cycle cyclic ML procedure; the relevant equations are given in (13) and (14). We compare the proposed schemes to the coherent ML detector (which has perfect CSI) and the pilot-based LS channel estimator [13], [27], [35]. Assume that the LS method employed N_{LS} pilot codes, where $N_{LS} \geq L$ and divides N_c . Following [31], the pilot placement of LS method is given by

$$\mathcal{S} = \left\{1, 1 + \frac{N_c}{N_{LS}}, 1 + \frac{N_c}{N_{LS}} \cdot 2, \dots, 1 + \frac{N_c}{N_{LS}} \cdot (N_{LS} - 1)\right\}.$$

If not mentioned specifically, we set $N_{LS} = L$. The differential OSTBC-OFDM scheme [8] was also compared, which was the one by applying the differential OSTBC scheme [36] to each subchannel. In the simulations, the coefficients of \mathcal{H} are zero-mean i.i.d. complex Gaussian distributed with variance equal to 1, and change from one OSTBC-OFDM block to next. The SNR per subchannel is defined as

$$\begin{aligned} \text{SNR} &= \frac{E \{ \|\mathcal{D}_{\mathcal{C}}(\mathbf{s})\mathbf{\Pi}(\mathbf{I}_{N_t} \otimes \mathbf{F})\|_F^2 \}}{TN_c\sigma_w^2} \\ &= \frac{N_t \text{Tr}(\mathbf{F}^H \mathcal{D}_K \mathbf{F})}{TN_c\sigma_w^2} \end{aligned}$$

where $\mathcal{D}_{\mathcal{C}}(\mathbf{s}) = \text{blkdiag}[\mathbf{C}_1(\mathbf{s}_1), \dots, \mathbf{C}_{N_c}(\mathbf{s}_{N_c})]$, $\mathbf{F} = [\mathbf{f}_1, \dots, \mathbf{f}_{N_c}]^T$ is the $N_c \times L$ DFT submatrix and $\mathcal{D}_K = \text{diag}\{K_1, \dots, K_{N_c}\}$. The detector performance was evaluated in terms of average symbol error rate (SER), and there were 10 000 trials performed in each simulation example.

Simulation Example 1: Fig. 3 illustrates the results for the QPSK 4×3 OSTBC. In Fig. 3(a), we show the BER performance of the proposed SGOO schemes with and without the cyclic ML solution refinement. In the legend, “CML” stands for cyclic ML, and “Odd-Even” refers to the blind SGOO odd-even scheme. One can see that, for both the blind and semiblind SGOO schemes, the cyclic ML solution refinement procedure enhances the performance of SGOO quite significantly. This

⁶Readers who are interested in the FOO performance are referred to [35], where two simulation examples for FOO with smaller problem size were provided.

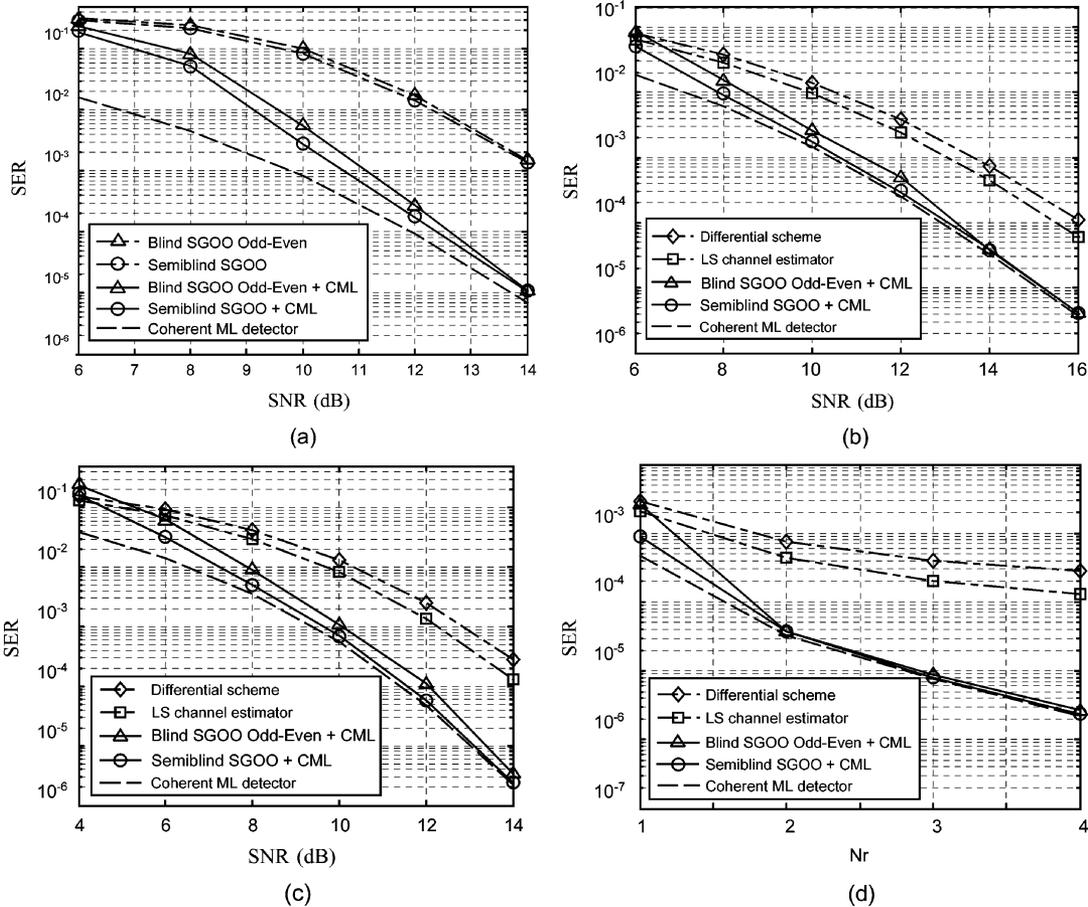


Fig. 3. Performance (SER) of the proposed schemes for QPSK 4×3 OSTBC, $P = 16$, (a) $L = 12$ and $N_r = 3$, (b) $L = 8$ and $N_r = 2$, (c) $L = 8$ and $N_r = 4$, (d) $L = 8$ and SNR = 14 dB.

empirical finding implies that the SGOO solutions may provide sufficiently good initialization for cyclic ML to arrive at a near-optimal FOO solution.

Fig. 3(b), (c), and (d) compares the performance of the proposed method to that of the pilot-based LS method and the differential scheme, under various conditions. We see that both the semiblind and blind SGOO schemes outperform the LS method and the differential scheme. One can also observe that the performance of the two proposed schemes is close to that of the coherent ML detector.

Simulation Example 2: This example aims to illustrate the performance differences of identifiable and nonidentifiable blind schemes. The QPSK Alamouti code is employed. It is known that the Alamouti code is not identifiable in the flat fading context, without using the odd-even arrangement [20] or other methods [16], [18], [19]. The results are plotted in Fig. 4. In the legend, “Blind SGOO” stands for the direct application of the Alamouti code to SGOO (i.e., all subchannels employ the QPSK Alamouti code), while “Blind SGOO Odd-Even” is the proposed SGOO odd-even scheme. We should recall that “Blind SGOO Odd-Even” works by removing only one bit symbol from “Blind SGOO.” The figure indicates that the Alamouti code is still nonidentifiable in the OSTBC-OFDM context, and that the odd-even arrangement is successful in turning the nonidentifiable blind SGOO scheme to an identifiable one. As

a reference, we also show the SER of the semiblind SGOO scheme in Fig. 4. One can see that the semiblind SGOO scheme achieves near-optimal performance, once again.

Simulation Example 3: In this example, we compare the performance of the proposed ML detector and the Swindlehurst-Leus subspace detector [37]. The Swindlehurst-Leus subspace detector was not developed for the block-fading OSTBC-OFDM scenario, but we found that the detector is, in essence, applicable to that case. However, this method works only when some restrictive assumptions are satisfied. For example, it requires the channel matrix \mathcal{H} to have full row rank, which translates into the necessity of $N_r \geq LN_t$. This requirement is impractical even when the channel length is moderate. In contrast with the subspace method, the proposed ML method does not suffer from this limitation, as we have proven in Section V. It has also been verified from Fig. 3(d) that the proposed schemes work well even when $N_r = 2$ (while $LN_t = 24$). For fair comparison, the subspace method was applied to the semiblind SGOO scheme by replacing the SGOO ML detector with a Swindlehurst-Leus subspace counterpart. The obtained solutions were also refined by the cyclic ML method. Fig. 5 presents the performance comparison for the semiblind case for QPSK Alamouti code with $L = 3$, $P = 8$ and $N_r = 6$. One can see from this figure that the proposed ML method significantly outperforms the subspace method.

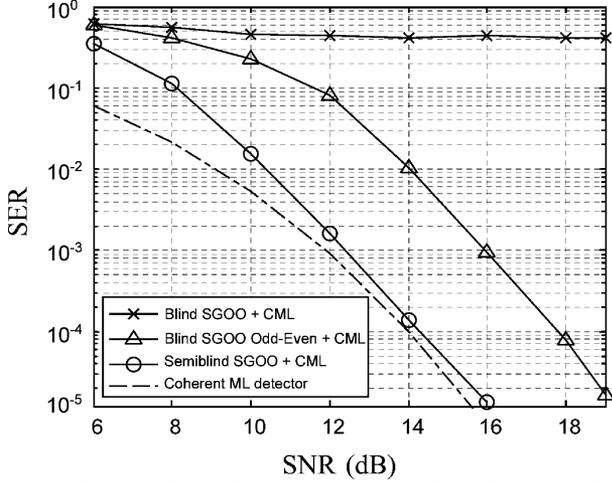


Fig. 4. Performance (SER) of the proposed schemes for QPSK Alamouti code, $P = 8$, $L = 8$ and $N_r = 4$.

VII. CONCLUSION AND DISCUSSION

In this paper, we have developed a blind ML OSTBC-OFDM framework that covers both the practical implementation and theoretical identifiability issues. The proposed framework features blind detection in one OSTBC-OFDM block, a characteristic that is not present in most existing blind methods. We have proposed subchannel grouping OSTBC-OFDM (SGOO) detection schemes that aim to overcome the large scale optimization problem inherent in full OSTBC-OFDM (FOO) detection, thereby enabling realizable implementations in practical OFDM applications. Our analysis has shown that both the SGOO and FOO schemes guarantee unique symbol identifiability in a probability one sense. Using simulations, we have demonstrated that the SGOO schemes, when coupled with the cyclic ML method, can outperform the pilot-based LS method and the differential scheme. In fact, the simulation results indicated that the proposed detectors can exhibit near-coherent performance.

Although our focus has been on BPSK/QPSK constellations, the results can be extended to general MPSK constellations. The implementation in this extension may be handled effectively by incorporating the readily available MPSK quadratic programming methods [18], [19], [38]. As for the identifiability, it is not difficult to see that our semibind identifiability result, which assumes constant modulus OSTBCs only, is perfectly applicable to the MPSK case. For a similar reason, it is likely that the application of the dual MPSK constellation schemes [18], [19] to OSTBC-OFDM should result in the same blind identifiability condition as the odd-even BPSK/QPSK scheme proposed here. These directions provide an interesting avenue for future research.

APPENDIX I PROOF OF LEMMA 1

For the blind ML problem in (24), the blind channel estimate is unique only if $\mathcal{G}(\mathbf{s})$ is of full column rank. We have shown in (30) that $\mathcal{G}(\mathbf{s})$ has full column rank if and only if $M \geq L$. Now, let us consider the case where $M = L$. Since \mathcal{F} is invertible in this case, (34) can be reduced to

$$\begin{aligned} \gamma_{m,n} &= \mathbf{e}_m^T \mathcal{F} \mathcal{F}^{-1} \mathbf{D}_\kappa^{-1} \mathcal{F}^{-H} \mathcal{F}^H \mathbf{e}_n \\ &= \mathbf{e}_m^T \mathbf{D}_\kappa^{-1} \mathbf{e}_n \end{aligned} \quad (\text{A.1})$$

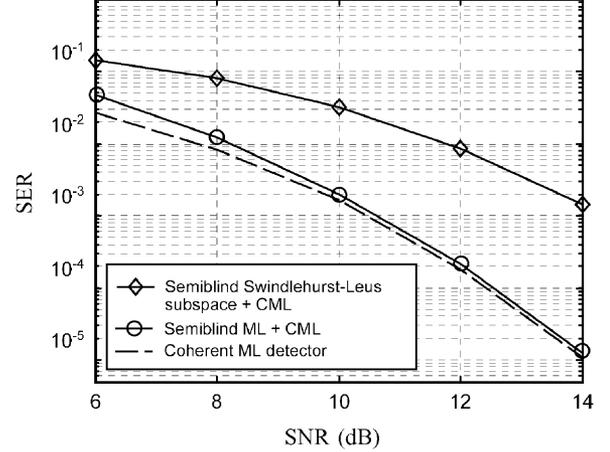


Fig. 5. Performance (SER) comparison to the Swindlehurst-Leus subspace method for Alamouti code, $L = 3$, $P = 8$ and $N_r = 6$.

i.e., $\gamma_{m,n} = 0 \quad \forall m \neq n$. Substituting this result into (33), we obtain $\mathbf{R}_{m,n} = \mathbf{0} \quad \forall m \neq n$. Subsequently, the blind BQP in (32) reduces to M independent subproblems:

$$\max_{\mathbf{s}_m \in \{\pm 1\}^{\kappa_m}} \mathbf{s}_m^T \mathbf{R}_{m,m} \mathbf{s}_m \quad (\text{A.2})$$

for $m = 1, 2, \dots, M$. Each subproblem in (A.2) is subject to a sign ambiguity of its own. Hence, if $\{\mathbf{s}_1^*, \dots, \mathbf{s}_M^*\}$ is a solution to (A.2) then any $\{\pm \mathbf{s}_1^*, \dots, \pm \mathbf{s}_M^*\}$ also serves as a solution to (A.2). Similarly, we can find the same problem in the semibind case. ■

APPENDIX II PROOF OF THEOREM 1

Suppose that (38) holds, and that there is no \mathbf{U} satisfying (39). Let \mathbf{G}'^\dagger denote the pseudoinverse of \mathbf{G}' . Premultiplying $\mathbf{G}'\mathbf{G}'^\dagger$ on the both sides of (38) results in

$$\mathbf{G}'\mathbf{U}\mathbf{H} = \mathbf{G}'\mathbf{H}' \quad (\text{A.3})$$

where we denote $\mathbf{U} = \mathbf{G}'^\dagger \mathbf{G}$, and we have used the basic property $\mathbf{G}'\mathbf{G}'^\dagger \mathbf{G}' = \mathbf{G}'$ at the right-hand side (RHS). Substituting (A.3) into (38), we obtain

$$(\mathbf{G} - \mathbf{G}'\mathbf{U})\mathbf{H} = \mathbf{0}. \quad (\text{A.4})$$

Let $\Phi = \mathbf{G} - \mathbf{G}'\mathbf{U}$, which must not equal $\mathbf{0}$. The probability that (A.4) holds is given by

$$\begin{aligned} \Pr\{\Phi\mathbf{H} = \mathbf{0}\} &= \Pr\left\{\bigcap_{i=1}^{N_r} \Phi\mathbf{h}_i = \mathbf{0}\right\} \\ &\leq \Pr\{\Phi\mathbf{h}_i = \mathbf{0}\} \end{aligned} \quad (\text{A.5})$$

for all $i = 1, 2, \dots, N_r$. Here \mathbf{h}_i denotes the i th column of \mathbf{H} . Without loss of generality, suppose that \mathbf{h}_1 is Gaussian distributed with a positive definite covariance matrix. Then one can show that $\Pr\{\Phi\mathbf{h}_1 = \mathbf{0}\}$ is of measure zero [39]. Hence, we have $\Pr\{\Phi\mathbf{H} = \mathbf{0}\} = 0$, which is equivalent to saying that (38)

holds with probability zero. It also follows that Theorem 1 is true. \blacksquare

APPENDIX III PROOF OF LEMMA 3

Assume that **C1** holds. Proving Lemma 3 is equivalent to proving the following alternative statement: Given $\mathbf{s}, \mathbf{s}' \in \{\pm 1\}^{\bar{\kappa}}$, there exists a matrix $\mathbf{U} \in \mathbb{C}^{LN_t \times LN_t}$ such that

$$\mathcal{G}_m(\mathbf{s}'_m)\mathbf{U} = \mathcal{G}_m(\mathbf{s}_m), \quad m = 1, 2, \dots, M \quad (\text{A.6})$$

if and only if there exists a matrix $\mathbf{Q} \in \mathbb{C}^{N_t \times N_t}$ such that

$$\mathcal{C}_m(\mathbf{s}'_m)\mathbf{Q} = \mathcal{C}_m(\mathbf{s}_m), \quad m = 1, 2, \dots, M. \quad (\text{A.7})$$

The sufficiency of the statement in (A.6)–(A.7) is straightforward. Suppose that (A.7) holds, and let $\mathbf{U} = \mathbf{Q} \otimes \mathbf{I}_L$. By recalling that $\mathcal{G}_m(\mathbf{s}_m) = \mathcal{C}_m(\mathbf{s}_m)(\mathbf{I}_{N_t} \otimes \mathbf{f}_m^T)$, we obtain

$$\mathcal{G}_m(\mathbf{s}'_m)\mathbf{U} = \mathcal{C}_m(\mathbf{s}'_m)(\mathbf{Q} \otimes \mathbf{f}_m^T). \quad (\text{A.8})$$

Since $\mathbf{Q} \otimes \mathbf{f}_m^T = (\mathbf{Q} \otimes \mathbf{1})(\mathbf{I}_{N_t} \otimes \mathbf{f}_m^T) = \mathbf{Q}(\mathbf{I}_{N_t} \otimes \mathbf{f}_m^T)$, the RHS of (A.8) can be reduced to that of (A.6).

To prove necessity, suppose that (A.6) holds. Equation (A.6) can be rewritten as $\mathcal{G}(\mathbf{s}')\mathbf{U} = \mathcal{G}(\mathbf{s})$, which can be further expanded as

$$D_{\mathcal{C}}(\mathbf{s}')\mathbf{\Pi}(\mathbf{I}_{N_t} \otimes \mathcal{F})\mathbf{U} = D_{\mathcal{C}}(\mathbf{s})\mathbf{\Pi}(\mathbf{I}_{N_t} \otimes \mathcal{F}). \quad (\text{A.9})$$

Let the thin singular value decomposition of \mathcal{F} be $\mathcal{F} = \mathbf{V}\mathbf{\Sigma}\mathbf{W}^H$, where $\mathbf{V} \in \mathbb{C}^{M \times L}$ is semiunitary, $\mathbf{\Sigma} \in \mathbb{C}^{L \times L}$ is diagonal and invertible, and $\mathbf{W} \in \mathbb{C}^{L \times L}$ is unitary. Postmultiplying both sides of (A.9) by $\mathbf{I}_{N_t} \otimes \mathbf{W}\mathbf{\Sigma}^{-1}$ yields

$$D_{\mathcal{C}}(\mathbf{s}')\mathbf{\Pi}(\mathbf{I}_{N_t} \otimes \mathbf{V})\tilde{\mathbf{U}} = D_{\mathcal{C}}(\mathbf{s})\mathbf{\Pi}(\mathbf{I}_{N_t} \otimes \mathbf{V}), \quad (\text{A.10})$$

$$\tilde{\mathbf{U}} = (\mathbf{I}_{N_t} \otimes \mathbf{\Sigma}\mathbf{W}^H)\mathbf{U}(\mathbf{I}_{N_t} \otimes \mathbf{W}\mathbf{\Sigma}^{-1}). \quad (\text{A.11})$$

We first show that $\tilde{\mathbf{U}}$ is unitary, a property that will prove useful later. Let

$$\tilde{\mathcal{G}}(\mathbf{s}) = \left(D_{\kappa}^{-1/2} \otimes \mathbf{I}_T \right) D_{\mathcal{C}}(\mathbf{s}')\mathbf{\Pi}(\mathbf{I}_{N_t} \otimes \mathbf{V}). \quad (\text{A.12})$$

It can be shown that $\tilde{\mathcal{G}}(\mathbf{s})$ is semiunitary for any $\mathbf{s} \in \{\pm 1\}^{\bar{\kappa}}$. Equation (A.10) implies that

$$\tilde{\mathcal{G}}(\mathbf{s}')\tilde{\mathbf{U}} = \tilde{\mathcal{G}}(\mathbf{s}). \quad (\text{A.13})$$

Since $\tilde{\mathcal{G}}(\mathbf{s})$ and $\tilde{\mathcal{G}}(\mathbf{s}')$ are semiunitary, $\tilde{\mathbf{U}}$ has to be unitary in order to satisfy (A.13).

Second, we show that if $P_{m,n} \neq 0$ for some (m, n) , then there exists a matrix \mathbf{Q} such that $\mathcal{C}_m(\mathbf{s}'_m)\mathbf{Q} = \mathcal{C}_m(\mathbf{s}_m)$ and

$\mathcal{C}_n(\mathbf{s}'_n)\mathbf{Q} = \mathcal{C}_n(\mathbf{s}_n)$. Combining this result with **C1** will lead to (A.7), the final result. From (A.10), the following two equations are obtained:

$$\mathcal{C}_m(\mathbf{s}'_m)(\mathbf{I}_{N_t} \otimes \mathbf{e}_m^T \mathbf{V})\tilde{\mathbf{U}} = \mathcal{C}_m(\mathbf{s}_m)(\mathbf{I}_{N_t} \otimes \mathbf{e}_m^T \mathbf{V}) \quad (\text{A.14})$$

$$\mathcal{C}_n(\mathbf{s}'_n)(\mathbf{I}_{N_t} \otimes \mathbf{e}_n^T \mathbf{V})\tilde{\mathbf{U}} = \mathcal{C}_n(\mathbf{s}_n)(\mathbf{I}_{N_t} \otimes \mathbf{e}_n^T \mathbf{V}). \quad (\text{A.15})$$

We notice that $P_{m,n} = \mathbf{e}_m^T \mathcal{F}(\mathcal{F}^H \mathcal{F})^{-1} \mathcal{F}^H \mathbf{e}_n = \mathbf{e}_m^T \mathbf{V}\mathbf{V}^H \mathbf{e}_n$. By using the property $(\mathbf{I}_{N_t} \otimes \mathbf{e}_m^T \mathbf{V})(\mathbf{I}_{N_t} \otimes \mathbf{e}_n^T \mathbf{V})^H = P_{m,n} \mathbf{I}_{N_t}$ and the unitarity of $\tilde{\mathbf{U}}$, we find that

$$\begin{aligned} & \mathcal{C}_m(\mathbf{s}_m)\mathcal{C}_n^H(\mathbf{s}_n) \\ &= \frac{1}{P_{m,n}} \mathcal{C}_m(\mathbf{s}'_m)(\mathbf{I}_{N_t} \otimes \mathbf{e}_m^T \mathbf{V}) \\ & \quad \times \tilde{\mathbf{U}}\tilde{\mathbf{U}}^H (\mathbf{I}_{N_t} \otimes \mathbf{e}_n^T \mathbf{V})^H \mathcal{C}_n^H(\mathbf{s}'_n) \\ &= \frac{1}{P_{m,n}} \mathcal{C}_m(\mathbf{s}'_m)(\mathbf{I}_{N_t} \otimes \mathbf{e}_m^T \mathbf{V}) \\ & \quad \times (\mathbf{I}_{N_t} \otimes \mathbf{e}_n^T \mathbf{V})^H \mathcal{C}_n^H(\mathbf{s}'_n) \\ &= \mathcal{C}_m(\mathbf{s}'_m)\mathcal{C}_n^H(\mathbf{s}'_n). \end{aligned} \quad (\text{A.16})$$

Postmultiplying both sides of (A.16) by $\mathcal{C}_n(\mathbf{s}_n)$ results in

$$\mathcal{C}_m(\mathbf{s}'_m)\mathbf{Q}_m = \mathcal{C}_m(\mathbf{s}_m), \quad (\text{A.17})$$

where $\mathbf{Q}_m = (1)/(\kappa_n)\mathcal{C}_n^H(\mathbf{s}'_n)\mathcal{C}_n(\mathbf{s}_n)$. Similarly, premultiplying (A.16) by $\mathcal{C}_m^H(\mathbf{s}_m)$ leads to

$$\mathcal{C}_n(\mathbf{s}'_n)\mathbf{Q}_n = \mathcal{C}_n(\mathbf{s}_n) \quad (\text{A.18})$$

where $\mathbf{Q}_n = (1)/(\kappa_m)\mathcal{C}_m^H(\mathbf{s}'_m)\mathcal{C}_m(\mathbf{s}_m)$. By premultiplying (A.18) by $\mathcal{C}_n^H(\mathbf{s}'_n)$, we achieve

$$\mathbf{Q}_n = \frac{1}{\kappa_n} \mathcal{C}_n^H(\mathbf{s}'_n)\mathcal{C}_n(\mathbf{s}_n). \quad (\text{A.19})$$

Hence, we obtain

$$\mathbf{Q} \equiv \mathbf{Q}_n = \mathbf{Q}_m. \quad (\text{A.20})$$

By **C1** and by induction, we conclude that (A.17), (A.18), and (A.20) holds for any (m, n) with $m \neq n$ and $P_{m,n} \neq 0$. \blacksquare

APPENDIX IV PROOF OF LEMMA 4

The corresponding \mathcal{F} satisfies $\mathcal{F}^H \mathcal{F} = M\mathbf{I}_L$. Subsequently

$$\begin{aligned} P_{m,m+1} &= M\mathbf{e}_m^T \mathcal{F}\mathcal{F}^H \mathbf{e}_{m+1} = M \sum_{\ell=0}^{L-1} (z_m z_{m+1}^*)^{-\ell} \\ &= M \sum_{\ell=0}^{L-1} e^{-j\frac{2\pi}{M}\ell} \neq 0 \end{aligned}$$

since $M > L$. It follows that for any (m, n) with $n > m$, we have a sequence $\{\varpi_1, \varpi_2, \dots, \varpi_I\} = \{m, m+1, \dots, n\}$ satisfying (43). ■

APPENDIX V
PROOF OF LEMMA 5

Without loss of generality, reorder $\{z_m\}$ such that $z_m = e^{j\alpha + j(2\pi)/(M)(m-1)}$ for $m = 1, 2, \dots, M$, and $z_{M+1} = 1$. Its Vandermonde matrix \mathcal{F} can be expressed as

$$\mathcal{F} = \begin{bmatrix} \mathbf{\Upsilon} \\ \boldsymbol{\nu}^H \end{bmatrix} \quad (\text{A.21})$$

where

$$\mathbf{\Upsilon} = \begin{bmatrix} 1 & z_1^{-1} & \dots & z_1^{-(L-1)} \\ \vdots & \vdots & & \vdots \\ 1 & z_M^{-1} & \dots & z_M^{-(L-1)} \end{bmatrix} \quad (\text{A.22})$$

and $\boldsymbol{\nu}^H = [1, z_{M+1}^{-1}, \dots, z_{M+1}^{-(L-1)}]$. The above expression leads to a partitioned form for $\mathbf{P} = \mathcal{F}(\mathcal{F}^H \mathcal{F})^{-1} \mathcal{F}^H \in \mathbb{C}^{(M+1) \times (M+1)}$

$$\begin{aligned} \mathbf{P} &= \begin{bmatrix} \mathbf{\Upsilon}(\mathcal{F}^H \mathcal{F})^{-1} \mathbf{\Upsilon}^H & \mathbf{\Upsilon}(\mathcal{F}^H \mathcal{F})^{-1} \boldsymbol{\nu} \\ \boldsymbol{\nu}^H (\mathcal{F}^H \mathcal{F})^{-1} \mathbf{\Upsilon}^H & \boldsymbol{\nu}^H (\mathcal{F}^H \mathcal{F})^{-1} \boldsymbol{\nu} \end{bmatrix} \\ &\triangleq \begin{bmatrix} \mathbf{A} & \mathbf{b} \\ \mathbf{b}^H & c \end{bmatrix}. \end{aligned} \quad (\text{A.23})$$

Let us consider the closed form of $(\mathcal{F}^H \mathcal{F})^{-1} = (\mathbf{\Upsilon}^H \mathbf{\Upsilon} + \boldsymbol{\nu} \boldsymbol{\nu}^H)^{-1}$. It can be shown, in the same way as the proof of Lemma 4, that $\mathbf{\Upsilon}^H \mathbf{\Upsilon} = M \mathbf{I}_L$. Using the matrix inversion lemma, we get

$$(\mathcal{F}^H \mathcal{F})^{-1} = \frac{1}{M} \left(\mathbf{I}_L - \frac{1}{L+M} \boldsymbol{\nu} \boldsymbol{\nu}^H \right). \quad (\text{A.24})$$

Substituting (A.24) into the submatrices in (A.23), we show that \mathbf{b} and \mathbf{A} have simplified forms

$$\mathbf{b} = \frac{1}{M} \left(1 - \frac{L}{L+M} \right) \mathbf{\Upsilon} \boldsymbol{\nu}, \quad (\text{A.25})$$

$$\mathbf{A} = \frac{1}{M} \left(\mathbf{\Upsilon} \mathbf{\Upsilon}^H - \frac{1}{L+M} (\mathbf{\Upsilon} \boldsymbol{\nu})(\mathbf{\Upsilon} \boldsymbol{\nu})^H \right). \quad (\text{A.26})$$

Recall that our aim is to show **C1)**, which says that for any $m, n \in \mathcal{I}_{M+1}$ with $m \neq n$ (Note that we have extended M to $M+1$ in this lemma), there exists a sequence $\{\varpi_1, \varpi_2, \dots, \varpi_I\} \subseteq \mathcal{I}_{M+1}$ such that $\varpi_1 = m, \varpi_I = n$, and $P_{\varpi_i, \varpi_{i+1}} \neq 0$ for $i = 1, 2, \dots, I-1$. To prove this, let \mathcal{I} be the set

$$\mathcal{I} = \{i | b_i \neq 0, i \in \mathcal{I}_M\} \quad (\text{A.27})$$

(where b_i denotes the i th element of \mathbf{b}) and $\bar{\mathcal{I}} = \mathcal{I}_M \setminus \mathcal{I}$. The set \mathcal{I} must be nonempty, since $\mathbf{\Upsilon}$ has full column rank implying that $\mathbf{\Upsilon} \boldsymbol{\nu} \neq \mathbf{0}$. Moreover, it can be shown that $[\mathbf{\Upsilon} \mathbf{\Upsilon}^H]_{m, m+1} \neq 0$ for $m = 1, 2, \dots, M-1$, a familiar result presented in Lemma 4. Applying the above results to (A.25)–(A.26), we locate some nonzero elements of \mathbf{P} that are sufficient for this proof:

$$P_{m, M+1} \neq 0, P_{M+1, m} \neq 0, \quad \forall m \in \mathcal{I}, \quad (\text{A.28})$$

$$P_{m, m+1} \neq 0, \quad \forall m \in \bar{\mathcal{I}}. \quad (\text{A.29})$$

Our investigation is divided into two cases as follows.

Case A: Suppose that $1 \leq m < n \leq M$: If $m \in \mathcal{I}$ and $n \in \mathcal{I}$, then a feasible hopping sequence $\{\varpi_1, \dots, \varpi_I\}$ is simply $\{m, M+1, n\}$ due to (A.28). If $m \in \mathcal{I}$ and $n \in \bar{\mathcal{I}}$, we can find a hopping sequence in the following way. Let p be a number such that $m \leq p < n, p \in \mathcal{I}$, and $i \in \bar{\mathcal{I}}$ for all $i = p+1, p+2, \dots, n$. By inspection, such a p always exists. Using (A.28)–(A.29), we obtain a feasible hopping sequence $\{m, M+1, p, p+1, p+2, \dots, n\}$. Using the same idea, we can show that hopping sequences exist for the remaining subcases, namely the subcase $m \in \bar{\mathcal{I}}, n \in \mathcal{I}$, and the subcase $m \in \bar{\mathcal{I}}, n \in \bar{\mathcal{I}}$.

Case B: Suppose that $1 \leq m \leq M, n = M+1$: If $m \in \mathcal{I}$ then the hopping sequence is simply $\{m, M+1\}$. If $m \in \bar{\mathcal{I}}$, then either one of the following possibilities must hold. In the first possibility, there exists a number q such that $m < q \leq M, q \in \mathcal{I}$, and $i \in \bar{\mathcal{I}}$ for all $i = m, m+1, \dots, q-1$. The corresponding hopping sequence is $\{m, m+1, \dots, q-1, q, M+1\}$. In the second possibility, there exists a number p such that $1 \leq p < m, p \in \mathcal{I}$, and $i \in \bar{\mathcal{I}}$ for all $i = p+1, p+2, \dots, m$. The corresponding hopping sequence is $\{M+1, p, p+1, p+2, \dots, m\}$.

Combining the results in Cases A and B, we conclude that **C1)** holds. ■

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