Linear Prediction Based Semiblind Channel Estimation for Multiuser OFDM with Insufficient Guard Interval

Parthapratim De, Tsung-Hui Chang, and Chong-Yung Chi

***Abstract*—To meet the demand of high data rate transmissions for multimedia wireless communications, orthogonal frequency division multiplexing (OFDM) systems in conjunction with multiple-input multiple-output (MIMO) signal processing have been considered one of the central techniques in advanced wireless communications. In the paper, two semiblind channel estimation algorithms are proposed for the uplink multiuser OFDM systems with insufficient guard interval, in contrast to sufficient guard interval assumed in most of the prior works. A zero-padding OFDM system, which zero-pads rather than cyclicly prefixing each block, is considered in this paper. By utilizing the relation between the linear prediction error filters (LPEFs) of the received signal with multiple prediction orders and the transmitted data sequence, the first proposed algorithm, namely the multistage LP (MLP) based algorithm, can estimate the MIMO channel coefficients, with only a single pilot OFDM block used. To reduce the sensitivity of the proposed algorithms to the channel order overestimation, it is proposed to implement the LPEFs with a QR-decomposition based approach. This QR-decomposition based approach alternatively computes the LPEFs without direct inversion of the received signal correlation matrix, thus exhibiting robustness against channel order overestimation. Some simulation results are presented to demonstrate the effectiveness and robustness of the proposed algorithms.**

***Index Terms*—Orthogonal frequency-division multiplexing (OFDM), multiple-input multiple-output (MIMO), blind channel estimation, linear prediction (LP), QR decomposition.**

I. INTRODUCTION

MULTIMEDIA applications in wireless communications require very high data rate transmissions. This has emphasized the importance of orthogonal frequency division multiplexing (OFDM) systems as well as multiple-input multiple-output (MIMO) signal processing techniques [1]. Blind and semiblind channel estimation for OFDM [2]–[7] has been of great interest and central importance. This class of channel estimation methods estimates the channel coefficients either using only the received signal or using both the received signal and a small number of pilot signals, thereby achieving higher spectral efficiency (higher data transmission rate).

For MIMO-OFDM or multiuser OFDM systems where usually a large amount of channel coefficients are involved, effective blind and semiblind channel estimation becomes much more challenging and has attracted much attention recently [2]–[5]. Specifically, the semiblind channel estimation methods in [3] and [4] are developed for MIMO-OFDM systems in which the guard interval between successive transmitted data blocks is larger than the length of multipath channels. These methods can estimate the channel coefficients up to a matrix ambiguity without requiring precise knowledge of the channel length. Some other works [6]–[8], [10], however, focus on OFDM systems in which the guard interval is shorter than the length of multipath channels. This might be necessary for spectral efficiency consideration (to achieve higher data rates by shortening cyclic prefix or zero padding length), or because the multipath channel length is underestimated at the transmitter. The associated blind and semiblind channel estimation problems are much more difficult. For example, when the guard interval is less than the channel length, the semiblind subspace method developed in [3] can only estimate the channel coefficients up to a polynomial matrix ambiguity. The identifiability of this subspace method thereby is still in question [3]. Another problem rising in the insufficient guard interval scenario is that the rank of the channel matrix depends on the multipath channel order. Corresponding blind and semiblind channel estimation methods require knowledge of the multipath channel order *a priori*; for example, the blind subspace method in [6] for single-input multiple-output (SIMO) OFDM systems requires the exact knowledge of the channel order.

In the paper, we consider the semiblind channel estimation problem for uplink multiuser OFDM systems with insufficient guard interval. A zero-padding OFDM system, which zero-pads rather than cyclicly prefixing each block, is considered in this paper. A multistage linear prediction (MLP) based semiblind channel estimation method is first developed. We show that the minimum mean squared error (MMSE) linear prediction error filter (LPEF) of the received signal contains the transmitted data sequence up to an ambiguity matrix. By computing these LPEFs of the received signal with different prediction orders, and by utilizing their relation to the transmitted data sequence, we show that a single pilot OFDM block is sufficient to resolve the ambiguity matrix and the MIMO channel coefficients can be estimated using a simple input-output-cross-correlation (IOCC) method. To the best of our knowledge, the proposed approach is the first to successfully achieve this insufficient guard interval semiblind channel estimation by using only one pilot block.

As mentioned earlier that in the insufficient guard interval case, the rank of the MIMO channel matrix depends on the multipath channel order. The existing blind and semiblind channel estimation/data detection methods [6], [7], [11] as well as the proposed MLP based semiblind channel estimation

Manuscript received June 24, 2008; revised April 7, 2009; accepted June 11, 2009. The associate editor coordinating the review of this letter and approving it for publication was R. Mallik.

This work was supported by the National Science Council, R.O.C., under Grants NSC 96-2219-E-007-001 and NSC 96-2219-E-007-004.

The authors are with the Institute of Communications Engineering & Department of Electrical Engineering, National Tsing Hua University, Hsinchu, Taiwan, R.O.C. P. De is the corresponding author (e-mail: pd4267@yahoo.com).

Digital Object Identifier 10.1109/TWC.2009.12.080787

methods, therefore, require the exact knowledge of the channel order. Since most of these approaches involve the pseudo inversion of the received signal correlation matrix whose rank depends on the channel order, overestimation of the channel order would lead to noise enhancement effect due to the inversion of small eigenvalues associated with the noise subspace in the received signal correlation matrix. Thus the performance of the associated channel estimation methods can be seriously degraded. To improve the robustness of the proposed channel estimation methods against the channel order overestimation, we apply the QR-decomposition based LPEF in [9], [12]. This QR-decomposition based approach alternatively computes the LPEFs without inversion of the received signal correlation matrix and thereby is more robust against channel order overestimation. Simulation results are presented to demonstrate the effectiveness and robustness of the proposed channel estimators.

The rest of the paper is organized as follows. The problem statement and the system model are presented in Section II. The proposed MLP based semiblind channel estimation method is presented in Section III. Section IV presents the application of the QR-decomposition based LPEF. Simulation results are provided in Section V. Finally, the conclusions are drawn in Section VI.

II. Problem Statement and System Model

Consider the uplink of a multiuser OFDM system where the base station is equipped with M receive antennas communicating with K single-antenna users over a common frequency band. Assume that $K \leq M$, and that all the users are in perfect synchronization in the uplink transmissions. Let $\mathbf{s}_i^{(k)} = [s_i^{(k)}(N-1), \ldots, s_i^{(k)}(0)]^T$ be the ith transmitted data vector of user k, where N denotes the discrete Fourier transform (DFT) size. The time-domain signals $\mathbf{u}_i^{(k)} := [u_i^{(k)}(N-1), \ldots, u_i^{(k)}(0)]^T$ are obtained by taking the inverse DFT (IDFT) of $\mathbf{s}_i^{(k)}$. In OFDM systems, the ith transmitted OFDM block symbol consists of $\mathbf{u}_i^{(k)}$ padded with a guard interval of Z (≥ 0) zero samples (or preceded with a cyclic prefix). The guard interval enables simple subcarrier-by-subcarrier equalization in the frequency domain at the receiver provided that the guard interval length is larger than or equal to the order of the time-domain channel impulse response [3]. However, if the guard interval is less than the channel order including the case of no guard interval for higher spectral efficiency, inter-block interference (IBI) arises, making the equalization problem much more difficult [10].

Let $h^{(m,k)}(\ell)$, $\ell = 0, 1, \ldots, L$, denote the time-domain channel impulse response from user k to the mth receive antenna, where L stands for the channel order. In the section, we derive the received signal model at the base station with the guard interval length $Z < L \ll N$ (insufficient guard interval case) [3]. The received ith OFDM block including the guard interval of Z zero padded samples at the mth receive antenna is given by

$$y_i^{(m)}(n) = \sum_{k=1}^{K}\sum_{\ell=0}^{L} h^{(m,k)}(\ell)u_i^{(k)}(n-\ell) \underbrace{+\sum_{k=1}^{K}\sum_{\ell=0}^{L-Z-1} h^{(m,k)}(Z+\ell+1)u_{i-1}^{(k)}(N+n-\ell-1) + w_i^{(m)}(n)}_{\text{IBI}}, \tag{1}$$

for $n = 0, \ldots, N+Z-1$, where the second term is due to the IBI and is nonzero only for $n < L-Z$, and $w_i^{(m)}(n)$ is the additive noise. Let us define $\mathbf{y}_i(n) = [y_i^{(1)}(n), y_i^{(2)}(n), \ldots, y_i^{(M)}(n)]^T \in \mathbb{C}^M$, $\mathbf{u}_i(n) = [u_i^{(1)}(n), u_i^{(2)}(n), \ldots, u_i^{(K)}(n)]^T \in \mathbb{C}^K$, $\mathbf{w}_i(n) = [w_i^{(1)}(n), w_i^{(2)}(n), \ldots, w_i^{(M)}(n)]^T \in \mathbb{C}^M$, and define the MIMO channel matrix of the ℓth tap as

$$\mathbf{H}(\ell) = \begin{bmatrix} h^{(1,1)}(\ell) & h^{(1,2)}(\ell) & \cdots & h^{(1,K)}(\ell) \\ h^{(2,1)}(\ell) & h^{(2,2)}(\ell) & \cdots & h^{(2,K)}(\ell) \\ \vdots & \vdots & \ddots & \vdots \\ h^{(M,1)}(\ell) & h^{(M,2)}(\ell) & \cdots & h^{(M,K)}(\ell) \end{bmatrix} \in \mathbb{C}^{M\times K}.$$

It follows from (1) that the $M \times 1$ received signal vector $\mathbf{y}_i(n)$ can be expressed as

$$\mathbf{y}_i(n) = \sum_{\ell=0}^{L} \mathbf{H}(\ell)\mathbf{u}_i(n-\ell) + \sum_{\ell=0}^{L-Z-1} \mathbf{H}(Z+\ell+1)\mathbf{u}_{i-1}(N+n-\ell-1) + \mathbf{w}_i(n). \tag{2}$$

For the signal model in (2), we aim to develop a semiblind channel estimation technique which only requires one OFDM block known to the base station (i.e., a single training (pilot) OFDM block). To this end, three general assumptions are made as follows.

($\mathcal{A}1$) The symbol sequence of each user $s_i^{(k)}(n)$ is temporally white with zero mean and variance σ_s^2, and is statistically uncorrelated with $s_i^{(q)}(n)$ for $k \neq q$.

($\mathcal{A}2$) The noise sequences $w_i^{(m)}(n)$ are stationary, and temporally and spatially white with zero mean and variance σ_w^2.

($\mathcal{A}3$) The symbol sequences $s_i^{(k)}(n)$ are statistically uncorrelated with the noise sequences $w_i^{(m)}(n)$.

Note that because the IDFT matrix is unitary, the assumptions ($\mathcal{A}1$) and ($\mathcal{A}3$) also hold for the time-domain sequences $u_i^{(k)}(n)$. On the other hand, we assume that the channels between the users and the base station are quasi-stationary for P consecutive OFDM blocks. That is, the channel coefficients $\mathbf{H}(\ell)$, $\ell = 0, \ldots, L$, are static from $\mathbf{u}_i(n)$ to $\mathbf{u}_{i+P-1}(n)$ for $n = 0, \ldots, N-1$, and it may vary in the next P blocks. Under this channel assumption, our goal is to estimate the channels coefficients provided that only the block $\mathbf{u}_i(n), n = 0, \ldots, N-1$, is the pilot.

III. Multistage Linear Prediction based Semiblind Channel Estimation

In this section, let us present the proposed MLP based semiblind channel estimator for the MIMO-OFDM model (2). It is to be noted that prior work on linear prediction based SIMO and MIMO channel estimators and equalizers in [11], [14] are for conventional continuous transmission systems. Our contribution in this paper is to use linear prediction for blind channel estimation in a MIMO-OFDM block transmission system (with the guard interval shorter than the length of the multipath channel), which has not been done earlier in literature. It will be seen in this section that the signal model and the channel matrix in a block transmission system are quite different from the case of conventional continuous transmission system.

The relationship between LPEFs of the received signal and the transmitted data sequences is first established in Section III-A. Based on this relation, a semiblind channel estimation method is presented in Sections III-B and III-C.

A. Linear Prediction Error Filter and Its Relationship with the Data Matrix

Here we consider the backward linear prediction of $\mathbf{y}_i(k)$ from the received signal $\{\mathbf{y}_i(N+Z-1),\ldots,\mathbf{y}_i(k+1)\}$ in the noise-free situation (i.e., $\mathbf{w}_i(n) = \mathbf{0}$), for some $k \in \{0,\ldots,N+Z-1\}$. Note that the prediction order is $(N+Z-k-1)$. By stacking $\mathbf{y}_i(n)$ in (2) for $n = k,\ldots,N+Z-1$, let us define

$$\boldsymbol{y}_i^{(k)} = [\mathbf{y}_i^T(N+Z-1),\ldots,\mathbf{y}_i^T(k)]^T = \boldsymbol{\mathcal{H}}^{(k)}\boldsymbol{g}_i^{(k)}, \quad (3)$$

where

$$\begin{aligned}\boldsymbol{g}_i^{(k)} = [&\mathbf{u}_i^T(N-1),\mathbf{u}_i^T(N-2),\ldots,\mathbf{u}_i^T(0),\\ &\mathbf{u}_{i-1}^T(N-1),\ldots,\mathbf{u}_{i-1}^T(N-L+Z+k)]^T\\ &\in \mathbb{C}^{(N+L-Z-k)K}, \quad (4)\end{aligned}$$

and $\boldsymbol{\mathcal{H}}^{(k)}$ is the corresponding channel matrix with the dimension $(N+Z-k)M \times (N+L-Z-k)K$ and is given by equation (5) (shown on the top of next page).

To perform linear prediction, we partition $\boldsymbol{y}_i^{(k)}$ in (3) as

$$\boldsymbol{y}_i^{(k)} \triangleq [(\tilde{\boldsymbol{y}}_i^{(k)})^T, \mathbf{y}_i^T(k)]^T, \quad (6)$$

where $\tilde{\boldsymbol{y}}_i^{(k)} = [\mathbf{y}_i^T(N+Z-1),\ldots,\mathbf{y}_i^T(k+1)]^T$. According to (3), (4) and (5), $\tilde{\boldsymbol{y}}_i^{(k)}$ can be expressed as

$$\tilde{\boldsymbol{y}}_i^{(k)} = \widetilde{\boldsymbol{\mathcal{H}}}^{(k)}\tilde{\boldsymbol{g}}_i^{(k)}, \quad (7)$$

where $\widetilde{\boldsymbol{\mathcal{H}}}^{(k)}$ is the upper-left $(N+Z-k-1)M \times (N+L-Z-k-1)K$ submatrix of $\boldsymbol{\mathcal{H}}^{(k)}$ in (5) (obtained by deleting the last M rows and last K columns of $\boldsymbol{\mathcal{H}}^{(k)}$), and

$$\begin{aligned}\tilde{\boldsymbol{g}}_i^{(k)} = [&\mathbf{u}_i^T(N-1),\ldots,\mathbf{u}_i^T(0),\mathbf{u}_{i-1}^T(N-1),\ldots,\\ &\ldots,\mathbf{u}_{i-1}^T(N-L+Z+k+1)]^T, \quad (8)\end{aligned}$$

(obtained by deleting $\mathbf{u}_{i-1}(N-L+Z+k)$ in (4)). Let $\mathbf{P}_{\mathrm{LP}}^{(k)}$ denote the $M \times (N+Z-k-1)M$ LP matrix. The prediction error vector for (6) is given by

$$\mathbf{d}_i^{(k)} = \mathbf{y}_i(k) - \mathbf{P}_{\mathrm{LP}}^{(k)}\tilde{\boldsymbol{y}}_i^{(k)} = [-\mathbf{P}_{\mathrm{LP}}^{(k)} \mid \mathbf{I}_M]\boldsymbol{y}_i^{(k)}, \quad (9)$$

where $\mathbf{I}_M$ is the $M \times M$ identity matrix. The linear prediction matrix $\mathbf{P}_{\mathrm{LP}}^{(k)}$ can be obtained by the MMSE criterion, i.e., by minimizing the $\mathrm{E}\{\mathrm{trace}\{\mathbf{d}_i^{(k)}(\mathbf{d}_i^{(k)})^H\}\}$. It is easy to show that the optimum $\mathbf{P}_{\mathrm{LP}}^{(k)}$ can be obtained by solving the following linear equation

$$\mathbf{P}_{\mathrm{LP}}^{(k)}E\{\tilde{\boldsymbol{y}}_i^{(k)}(\tilde{\boldsymbol{y}}_i^{(k)})^H\} = \mathrm{E}\{\mathbf{y}_i(k)(\tilde{\boldsymbol{y}}_i^{(k)})^H\}. \quad (10)$$

The LP error vector $\mathbf{d}_i^{(k)}$ in (9) can be shown to be related to the transmitted data sequence $\mathbf{u}_{i-1}(N-L+Z+k)$. To this end, an assumption is made on the rank of $\widetilde{\boldsymbol{\mathcal{H}}}^{(k)}$ in (7):

($\mathcal{A}4$) The MIMO channel matrix $\widetilde{\boldsymbol{\mathcal{H}}}^{(k)}$ has full column rank.

For ($\mathcal{A}4$) to be true, the z-domain transfer functions of the M different sub-channels of $\mathbf{H}(\ell)$ must not have any common zeros [11]. Under ($\mathcal{A}4$), the MMSE estimate of $\mathbf{y}_i(k)$ in terms of $\tilde{\boldsymbol{y}}_i^{(k)}$ can be shown to be equal to that in terms of $\tilde{\boldsymbol{g}}_i^{(k)}$ [14]. Denote by $\hat{\mathbf{y}}_i(k|\tilde{\boldsymbol{g}}_i^{(k)})$ the MMSE estimate of $\mathbf{y}_i(k)$ in terms of $\tilde{\boldsymbol{g}}_i^{(k)}$. Then

$$\mathbf{P}_{\mathrm{LP}}^{(k)}\tilde{\boldsymbol{y}}_i^{(k)} = \hat{\mathbf{y}}_i(k|\tilde{\boldsymbol{g}}_i^{(k)}). \quad (11)$$

Recall from (2) that, in the noise-free case

$$\begin{aligned}\mathbf{y}_i(k) = \sum_{\ell=0}^{k}\mathbf{H}(\ell)\mathbf{u}_i(k-\ell) + \sum_{\ell=0}^{L-Z-1}\mathbf{H}(Z+\ell+1)\\ \cdot\mathbf{u}_{i-1}(N+k-\ell-1). \quad (12)\end{aligned}$$

On the other hand, according to (8) and (12), $\hat{\mathbf{y}}_i(k|\tilde{\boldsymbol{g}}_i^{(k)})$ can be obtained as

$$\begin{aligned}\hat{\mathbf{y}}_i(k|\tilde{\boldsymbol{g}}_i^{(k)}) = \sum_{\ell=0}^{k}\mathbf{H}(\ell)\mathbf{u}_i(k-\ell) + \sum_{\ell=0}^{L-Z-2}\mathbf{H}(Z+\ell+1)\\ \cdot\mathbf{u}_{i-1}(N+k-\ell-1). \quad (13)\end{aligned}$$

Therefore, by (9), (11), (12) and (13), we have

$$\mathbf{d}_i^{(k)} = \mathbf{y}_i(k) - \hat{\mathbf{y}}_i(k|\tilde{\boldsymbol{y}}_i^{(k)}) = \mathbf{H}(L)\mathbf{u}_{i-1}(N-L+Z+k). \quad (14)$$

The form of equation (14), for the noisy signal case, has been discussed in literature. The multistep linear prediction error filter (PEF), in the noiseless case, perfectly reduces the number of transmitted symbols affecting any received symbol. However, it can be shown that the additive noise introduces an unwanted component comprising of residual ISI and colored noise at the output of PEF.

B. Multistage Linear Prediction

Equation (14) implies that the prediction error vector $\mathbf{d}_{i+1}^{(k)}$ contains the transmitted signal $\mathbf{u}_i(N-L+Z+k)$ up to the ambiguity matrix $\mathbf{H}(L)$. By varying the index k from 0 to $L-Z-1$, one can obtain equations similar to (14) for signals $\mathbf{u}_i(N-L+Z), \mathbf{u}_i(N-L+Z+1),\ldots,\mathbf{u}_i(N-1)$. In order to obtain equations corresponding to (14) for $\mathbf{u}_i(0),\mathbf{u}_i(1),\ldots,$ we consider the backward linear prediction of $\mathbf{y}_i(L+q)$ from the received signal $\{\mathbf{y}_i(N+Z-1),\ldots,\mathbf{y}_i(L+q+1)\}$ in the noise-free situation, for some $q \in \{0,\ldots,N+Z-L-1\}$. The procedure is basically identical to that in Section III-A. We stack $\mathbf{y}_i(n)$ for $n = L+q,\ldots,N+Z-1$, and define

$$\boldsymbol{\mathcal{H}}^{(k)} = \left[\begin{array}{ccccc|ccc} \mathbf{H}(Z) & \cdots & \mathbf{H}(L) & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{H}(0) & \mathbf{H}(1) & \dots & \mathbf{H}(L) & \ddots & \ddots & \ddots & \dots \\ \mathbf{0} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \mathbf{H}(0) & \dots & \mathbf{H}(L-Z) & \mathbf{0} & \dots & \dots \\ \hline \vdots & \ddots & \ddots & \ddots & \mathbf{H}(L-Z-1) & \mathbf{H}(L) & \mathbf{0} & \dots \\ \vdots & \ddots & \ddots & \ddots & \vdots & \vdots & \ddots & \ddots \\ \vdots & \dots & \dots & \mathbf{0} & \mathbf{H}(0) & \mathbf{H}(Z+1) & \dots & \mathbf{H}(L) \end{array}\right]. \tag{5}$$

$$\bar{\boldsymbol{y}}_i^{(q)} = [\mathbf{y}_i^T(N+Z-1), \dots, \mathbf{y}_i^T(L+q)]^T = \overline{\boldsymbol{\mathcal{H}}}^{(q)} \bar{\boldsymbol{g}}_i^{(q)}, \tag{15}$$

where

$$\bar{\boldsymbol{g}}_i^{(q)} = [\mathbf{u}_i^T(N-1), \mathbf{u}_i^T(N-2), \dots, \mathbf{u}_i^T(q)]^T \in \mathbb{C}^{(N-q)K},$$

and

$$\overline{\boldsymbol{\mathcal{H}}}^{(q)} = \begin{bmatrix} \mathbf{H}(Z) & \cdots & \mathbf{H}(L) & \mathbf{0} & \cdots \\ \vdots & \ddots & \ddots & \ddots & \ddots \\ \mathbf{H}(0) & \mathbf{H}(1) & \dots & \mathbf{H}(L) & \ddots \\ \mathbf{0} & \ddots & \ddots & \ddots & \ddots \\ \vdots & \ddots & \mathbf{H}(0) & \dots & \mathbf{H}(L) \end{bmatrix} \in \mathbb{C}^{(N+Z-L-q)M \times (N-q)K}. \tag{16}$$

It should be noted that there is no IBI involved in $\bar{\boldsymbol{y}}_i^{(q)}$. Again, $\bar{\boldsymbol{y}}_i^{(q)}$ is partitioned into

$$\bar{\boldsymbol{y}}_i^{(q)} = [(\tilde{\bar{\boldsymbol{y}}}_i^{(q)})^T, \mathbf{y}_i^T(L+q)]^T, \tag{17}$$

where $\tilde{\bar{\boldsymbol{y}}}_i^{(q)} = [\mathbf{y}_i^T(N+Z-1), \dots, \mathbf{y}_i^T(L+q+1)]^T$. The vector $\tilde{\bar{\boldsymbol{y}}}_i^{(q)}$ can be expressed as

$$\tilde{\bar{\boldsymbol{y}}}_i^{(q)} = \tilde{\bar{\boldsymbol{\mathcal{H}}}}^{(q)} \tilde{\bar{\boldsymbol{g}}}_i^{(q)}, \tag{18}$$

where $\tilde{\bar{\boldsymbol{\mathcal{H}}}}^{(q)}$ is the upper-left $(N+Z-L-q-1)M \times (N-q-1)K$ submatrix of $\bar{\boldsymbol{\mathcal{H}}}^{(k)}$, and $\tilde{\bar{\boldsymbol{g}}}_i^{(q)} = [\mathbf{u}_i^T(N-1), \dots, \mathbf{u}_i^T(q+1)]^T$.

Denote by $\bar{\mathbf{P}}_{\mathrm{LP}}^{(q)}$ the $M \times (N+Z-L-q-1)M$ matrix for the MMSE predictor of $\mathbf{y}_i(L+q)$ from the received signal $\{\mathbf{y}_i(N+Z-1), \dots, \mathbf{y}_i(L+q+1)\}$. Then $\bar{\mathbf{P}}_{\mathrm{LP}}^{(q)}$ satisfies

$$\bar{\mathbf{P}}_{\mathrm{LP}}^{(q)} \, \mathrm{E}\{\tilde{\bar{\boldsymbol{y}}}_i^{(q)} (\tilde{\bar{\boldsymbol{y}}}_i^{(q)})^H\} = \mathrm{E}\{\mathbf{y}_i(L+q) (\tilde{\bar{\boldsymbol{y}}}_i^{(q)})^H\}, \tag{19}$$

and the corresponding prediction error vector is given by $\bar{\mathbf{d}}_i^{(q)} = [-\bar{\mathbf{P}}_{\mathrm{LP}}^{(q)} \;\; | \;\; \mathbf{I}_M] \bar{\boldsymbol{y}}_i^{(q)}$. To show that the above $\bar{\mathbf{d}}_i^{(q)}$ contains the transmitted data $\mathbf{u}_i(q)$ (up to an ambiguity matrix), as in $(\mathcal{A}4)$ in Section III-A, we need the assumption that the channel matrix $\tilde{\bar{\boldsymbol{\mathcal{H}}}}^{(q)}$ in (18) has full column rank. This condition holds only if the matrix $\tilde{\bar{\boldsymbol{\mathcal{H}}}}^{(q)}$ is a tall matrix. That is, $(N+Z-L-q-1)M \geq (N-q-1)K$, or equivalently

$$q \leq k_{max} \triangleq \frac{(N+Z-L)M - NK}{(M-K)}. \tag{20}$$

By following the same derivations as in (11), (12), (13) and (14) under the assumption that $\tilde{\bar{\boldsymbol{\mathcal{H}}}}^{(q)}$ in (18) has full column rank for $q = 1, \dots, k_{max}$, one can obtain

$$\bar{\mathbf{d}}_i^{(q)} = [-\bar{\mathbf{P}}_{\mathrm{LP}}^{(q)} \;\; | \;\; \mathbf{I}_M] \bar{\boldsymbol{y}}_i^{(q)} = \mathbf{H}(L)\mathbf{u}_i(q), \tag{21}$$

for $q = 1, \dots, k_{max}$. As presented in the next subsection, (14) and (21) can be utilized to develop a semiblind channel estimation method for the MIMO-OFDM model (2).

C. Semiblind Channel Estimation

Recall from Section II that we have assumed that the channel coefficients $\mathbf{H}(\ell)$, $\ell = 0, \dots, L$, are fixed during the transmissions of $\mathbf{u}_i(n)$, $\mathbf{u}_{i+1}(n), \dots, \mathbf{u}_{i+P-1}(n)$ for $n = 0, \dots, N-1$ (P consecutive OFDM blocks). By assuming that the ith OFDM block data $\mathbf{s}_i(n)$, $n = 1, \dots, N$, are pilots (thus $\mathbf{u}_i(n)$, $n = 1, \dots, N$ are known to the base station), let us show in the subsection that (14) and (21) can be exploited for estimation of $\mathbf{H}(\ell)$ for $\ell = 0, \dots, L$.

To this end, let us define

$$\begin{aligned} \mathbf{P}_u &= [\mathbf{u}_i(0), \mathbf{u}_i(1), \dots, \mathbf{u}_i(k_{max}), \mathbf{u}_i(N-L+Z), \\ &\qquad \mathbf{u}_i(N-L+Z+1), \dots, \mathbf{u}_i(N-1)], \\ \mathbf{D}_i &= [\bar{\mathbf{d}}_i^{(0)}, \bar{\mathbf{d}}_i^{(1)}, \dots, \bar{\mathbf{d}}_i^{(k_{max})}, \mathbf{d}_{i+1}^{(0)}, \mathbf{d}_{i+1}^{(1)}, \dots, \mathbf{d}_{i+1}^{(L-Z-1)}]. \end{aligned} \tag{22}$$

From (14) and (21), one has

$$\mathbf{D}_i = \mathbf{H}(L)\mathbf{P}_u. \tag{23}$$

Without loss of generality, we assume that the pilot matrix $\mathbf{P}_u$ has full row rank and that $K \leq L-Z-1+k_{max}$. Because $\mathbf{P}_u$ is known to the base station, the ambiguity matrix $\mathbf{H}(L)$ can be estimated by

$$\hat{\mathbf{H}}(L) = \mathbf{D}_i \mathbf{P}_u^{\dagger}, \tag{24}$$

where $\mathbf{P}_u^\dagger$ denotes the matrix pseudo inverse of $\mathbf{P}_u$. On the other hand, since (14) for $k=0$ implies

$$\begin{aligned}[\mathbf{d}_{i+1}^{(0)},\ldots,\mathbf{d}_{i+P-1}^{(0)}] &= [-\mathbf{P}_{\mathrm{LP}}^{(0)} \mid \mathbf{I}_M]\,[\boldsymbol{y}_{i+1}^{(0)},\ldots,\boldsymbol{y}_{i+P-1}^{(0)}] \\ &= \mathbf{H}(L)[\mathbf{u}_i(N-L+Z),\ldots,\mathbf{u}_{i+P-2}(N-L+Z)],\end{aligned} \tag{25}$$

with the use of $\hat{\mathbf{H}}(L)$, one can estimate the data sequence $\{\mathbf{u}_i(N-L+Z),\ldots,\mathbf{u}_{i+P-2}(N-L+Z)\}$ by

$$\hat{\mathbf{u}}_j(N-L+Z) = (\hat{\mathbf{H}}(L))^\dagger \mathbf{d}_{j+1}^{(0)}, \tag{26}$$

for $j=i,\ldots,i+P-2$. Finally, according to (2), the estimated sequence $\{\hat{\mathbf{u}}_i(N-L+Z),\ldots,\hat{\mathbf{u}}_{i+P-2}(N-L+Z)\}$ can be utilized for estimation of $\mathbf{H}(\ell)$ via the input-output-cross-correlation (IOCC) method. That is, for $\ell=0,1,\ldots,L$,

$$\begin{aligned}\hat{\mathbf{H}}(\ell) &= \mathrm{E}[\mathbf{y}_i(N-L+Z+\ell)\hat{\mathbf{u}}_i^H(N-L+Z)] \\ &\approx \frac{1}{(P-1)}\sum_{j=i}^{i+P-2}\mathbf{y}_j(N-L+Z+\ell)\hat{\mathbf{u}}_j^H(N-L+Z),\end{aligned} \tag{27}$$

where the ensemble average has been replaced by the time average. We summarize in Table I the above MLP based semiblind channel estimation algorithm in terms of time average estimate.

IV. QR Decomposition Based LP Error Filter

As presented in Section III, the LP estimators $\mathbf{P}_{\mathrm{LP}}^{(k)}$ and $\bar{\mathbf{P}}_{\mathrm{LP}}^{(q)}$ are obtained by solving the linear equations given by (10) and (19), respectively. Specifically, if the channel matrix $\tilde{\boldsymbol{\mathcal{H}}}^{(k)}$ in (7) has full column rank, then under Assumption ($\mathcal{A}1$), $\mathrm{E}\{\tilde{\boldsymbol{y}}^{(k)}(\tilde{\boldsymbol{y}}^{(k)})^H\} = \tilde{\boldsymbol{\mathcal{H}}}^{(k)}(\tilde{\boldsymbol{\mathcal{H}}}^{(k)})^H$ would have rank of $(N+L-Z-k-1)K$. Then the linear system in (10) is rank deficient, and the (minimum norm) solution of (10) is given by

$$\mathbf{P}_{\mathrm{LP}}^{(k)} = \mathrm{E}\{\mathbf{y}_i(k)(\tilde{\boldsymbol{y}}_i^{(k)})^H\}(\mathrm{E}\{\tilde{\boldsymbol{y}}_i^{(k)}(\tilde{\boldsymbol{y}}_i^{(k)})^H\})^\dagger. \tag{28}$$

Because the matrix pseudo inverse of $\mathrm{E}\{\tilde{\boldsymbol{y}}^{(k)}(\tilde{\boldsymbol{y}}^{(k)})^H\}$ requires knowledge of its rank, which however depends on the channel order L, an accurate estimate of the channel order is thus necessary before the matrix pseudo inversion. If the channel order is overestimated, the noise subspace dimension in the correlation matrix $\mathrm{E}\{\tilde{\boldsymbol{y}}^{(k)}(\tilde{\boldsymbol{y}}^{(k)})^H\}$ is then underestimated, and consequently some of the small noise subspace eigenvalues are erroneously classified in the signal subspace and would be amplified in the matrix pseudo inversion [13]. This noise enhancement effect would significantly degrade the performance of the proposed semiblind channel estimator. To avoid this problem, in this section we present an alternative approach to fulfill the LPEFs in (9) and (21) based on QR-decomposition. It is worthwhile to mention that this channel order overestimation problem never occurs for OFDM systems with sufficient guard interval, as reported in [3] and [5].

Let us illustrate the proposed QR-decomposition method by considering the LP problem in (10) for $k=0$. The extension of this idea to other values of k and to the LP problem in (19) is straightforward and thus is omitted here. Since, in practice, the ensemble average is replaced by the time average, by (3) we define the received data matrix

$$\begin{aligned}\mathbf{Y}_i &\triangleq [\boldsymbol{y}_i,\boldsymbol{y}_{i+1},\ldots,\boldsymbol{y}_{i+P-1}] \\ &\triangleq \boldsymbol{\mathcal{H}}\mathbf{G}_i = \boldsymbol{\mathcal{H}}[\boldsymbol{g}_i,\boldsymbol{g}_{i+1},\ldots,\boldsymbol{g}_{i+P-1}] \in \mathbb{C}^{(N+Z)M\times P}\end{aligned} \tag{29}$$

where $\boldsymbol{y}_i = \boldsymbol{y}_i^{(0)}$, $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{H}}^{(0)}$ and $\boldsymbol{g}_i = \boldsymbol{g}_i^{(0)}$, in which we have dropped the superscript for notational simplicity. Similarly, the superscripts in (7) and (9) are also removed in this section. According to (6), $\mathbf{Y}_i$ in (29) can be partitioned as

$$\mathbf{Y}_i := \begin{bmatrix}\widetilde{\mathbf{Y}}_i \\ \mathbf{B}_i\end{bmatrix} = \begin{bmatrix}\tilde{\boldsymbol{y}}_i & \tilde{\boldsymbol{y}}_{i+1} & \cdots & \tilde{\boldsymbol{y}}_{i+P-1} \\ \mathbf{y}_i(0) & \mathbf{y}_{i+1}(0) & \cdots & \mathbf{y}_{i+P-1}(0)\end{bmatrix} \tag{30}$$

where

$$\widetilde{\mathbf{Y}}_i = [\tilde{\boldsymbol{y}}_i,\tilde{\boldsymbol{y}}_{i+1},\ldots,\tilde{\boldsymbol{y}}_{i+P-1}] \in \mathbb{C}^{(N+Z-1)M\times P}, \tag{31}$$

$$\mathbf{B}_i = [\mathbf{y}_i(0),\mathbf{y}_{i+1}(0),\ldots,\mathbf{y}_{i+P-1}(0)] \in \mathbb{C}^{M\times P}. \tag{32}$$

Then the time average counterpart of (10) can be written as

$$\hat{\mathbf{P}}_{\mathrm{LP}}(\widetilde{\mathbf{Y}}_i\widetilde{\mathbf{Y}}_i^H) = \mathbf{B}_i\widetilde{\mathbf{Y}}_i^H, \tag{33}$$

and its associated solution is given by

$$\hat{\mathbf{P}}_{\mathrm{LP}} = \mathbf{B}_i\widetilde{\mathbf{Y}}_i^H(\widetilde{\mathbf{Y}}_i\widetilde{\mathbf{Y}}_i^H)^\dagger. \tag{34}$$

Note that we have used $\hat{\mathbf{P}}_{\mathrm{LP}}$ to represent the time average estimate of $\mathbf{P}_{\mathrm{LP}}$. Substituting (34) into (9) gives rise to the optimum error matrix

$$\begin{aligned}\mathbf{E}_i &\triangleq [-\hat{\mathbf{P}}_{\mathrm{LP}} \mid \mathbf{I}_M]\mathbf{Y}_i \\ &= \mathbf{B}_i - \mathbf{B}_i\widetilde{\mathbf{Y}}_i^H(\widetilde{\mathbf{Y}}_i\widetilde{\mathbf{Y}}_i^H)^\dagger\widetilde{\mathbf{Y}}_i.\end{aligned} \tag{35}$$

Since $\widetilde{\mathbf{Y}}_i\widetilde{\mathbf{Y}}_i^H \approx P(\tilde{\boldsymbol{\mathcal{H}}}\tilde{\boldsymbol{\mathcal{H}}}^H)$ when P is large, it can be seen from (34) and (35) that the rank of $\tilde{\boldsymbol{\mathcal{H}}}$ (or the channel order L) needs to be exactly known at the base station for computing the pseudo inversion of $\widetilde{\mathbf{Y}}_i\widetilde{\mathbf{Y}}_i^H$ [13]. This direct matrix pseudo inversion can be avoided by applying the QR-decomposition based LPEF which is an extension of the author's work reported in [12]. To illustrate this, let us consider the QR-decomposition of matrix $\widetilde{\mathbf{Y}}_i^H$ [12]

$$\widetilde{\mathbf{Y}}_i^H\mathbf{\Pi} = \mathbf{Q}\begin{bmatrix}\mathbf{R}_1 & \mathbf{R}_2 \\ \mathbf{0}_{P-r,r} & \mathbf{0}_{P-r,(N+Z-1)M-r}\end{bmatrix}, \tag{36}$$

where $r = \mathrm{rank}(\tilde{\boldsymbol{\mathcal{H}}}) = (N+L-Z-1)K$, $\mathbf{R}_1 \in \mathbb{R}^{r\times r}$ is a non-singular upper triangular matrix, $\mathbf{R}_2 \in \mathbb{R}^{r\times((N+Z-1)M-r)}$, $\mathbf{Q} \in \mathbb{C}^{P\times P}$ is a unitary matrix, $\mathbf{\Pi} \in \mathbb{R}^{(N+Z-1)M\times(N+Z-1)M}$ is a permutation matrix making the diagonal elements of $\mathbf{R}_1$ have magnitudes in descending order, and $\mathbf{0}_{P-r,r}$ stands for the $(P-r)\times r$ zero matrix. By substituting (36) into (33), one can show that by extending the results in [12]

$$[-\hat{\mathbf{P}}_{\mathrm{LP}} \mid \mathbf{I}_M]\mathbf{Y}_i[\mathbf{Q}]_{1:r} = \mathbf{0}_{M,r}, \tag{37}$$

where $[\mathbf{Q}]_{1:r} \in \mathbb{C}^{P\times r}$ consists of the first r columns of $\mathbf{Q}$. On the other hand, using (30) and (36) it can be shown that by extending the results in [12]

$$[-\hat{\mathbf{P}}_{\mathrm{LP}} \mid \mathbf{I}_M]\mathbf{Y}_i[\mathbf{Q}]_{r+1:P} = \mathbf{B}_i[\mathbf{Q}]_{r+1:P}. \tag{38}$$

Combining (37) and (38) gives rise to

$$\begin{aligned}[-\hat{\mathbf{P}}_{\mathrm{LP}} \mid \mathbf{I}_M]\mathbf{Y}_i &= [\mathbf{0}_{M,r},\ \mathbf{B}_i[\mathbf{Q}]_{r+1:P}]\mathbf{Q}^H \\ &= \mathbf{B}_i[\mathbf{Q}]_{r+1:P}[\mathbf{Q}]_{r+1:P}^H, \end{aligned} \quad (39)$$

which therefore provides an alternative representation of (35) without pseudo inversion of $\widetilde{\mathbf{Y}}_i\widetilde{\mathbf{Y}}_i^H$. Derivation of equations (37) and (38) are not provided in this paper due to space limitations. The advantage of the QR-decomposition based method may be analyzed as follows. Suppose that the rank of $\tilde{\boldsymbol{\mathcal{H}}}$ is overestimated as $r' = (N + L' - Z - 1)K$ where $L' = L + \triangle L$ and $\triangle L \geq 0$. By (39), the LP error matrix by QR-decomposition becomes

$$[-\hat{\mathbf{P}}_{\mathrm{LP}} \mid \mathbf{I}_M]\mathbf{Y}_i = \mathbf{B}_i[\mathbf{Q}]_{r'+1:P}[\mathbf{Q}]_{r'+1:P}^H, \quad (40)$$

where the noise subspace vectors $[\mathbf{Q}]_{r+1:r'}$ are lost and they would degrade the performance of channel estimation algorithm. However, compared to the direct pseudo inversion of $\widetilde{\mathbf{Y}}_i\widetilde{\mathbf{Y}}_i^H$ used in (35) and in most of conventional LP based MIMO blind equalizers [5], [11], the QR-decomposition based method in (39) shows much better robustness against channel order overestimation, as demonstrated in our simulation results later. To emphasize the proposed MLP based channel estimation algorithm using the QR-decomposition based LPEFs, we refer to it as the "MLP-QR" based method in the paper. The importance of developing a blind channel estimation algorithm robust to channel order overestimation has been highlighted in [13] and one such algorithm has been developed in [15] for a single-input multiple-output (SIMO) continuous transmission system. Our QR decomposition based algorithm is completely different from that algorithm in [15] and is developed for a MIMO-OFDM and block transmission system. Our algorithm, being based on QR decomposition, is computationally more efficient than the algorithm in [15] which is based on singular value decomposition (SVD). This robust algorithm is an important contribution of this paper.

V. Simulation Results

In the section, some simulation results are presented to evaluate the performance of the proposed semiblind channel estimation methods for MIMO-OFDM with insufficient guard interval $(0 < Z < L)$. A two-user $(K = 2)$ uplink OFDM system with the number of subcarriers equal to 16 $(N = 16)$ was considered. The number of receive antennas was 6 $(M = 6)$. The channel coherence time was assumed to be 600 OFDM blocks $(P = 600)$. The data signals $s_i^{(k)}(n)$, $k = 1, 2$, were BPSK modulated. Each simulation result was obtained by averaging over 125 trials. For each coherence interval (trial), independent and identically distributed complex Gaussian channel coefficients with zero mean and unit variance were generated. The receiver signal-to-noise ratio (SNR) was defined as

$$\mathrm{SNR} = \frac{\mathrm{E}(||\mathbf{y}_i(n) - \mathbf{w}_i(n)||^2)}{\mathrm{E}(||\mathbf{w}_i(n)||^2)},$$

and the performance of the channel estimator was measured with the normalized MSE (NMSE)

$$\mathrm{NMSE} = \frac{1}{125}\sum_{p=1}^{125}\left\{\frac{\sum_{\ell=0}^{L}||\mathbf{H}^{(p)}(\ell) - \hat{\mathbf{H}}^{(p)}(\ell)||_F^2}{\sum_{\ell=0}^{L}||\mathbf{H}^{(p)}(\ell)||_F^2}\right\}, \quad (41)$$

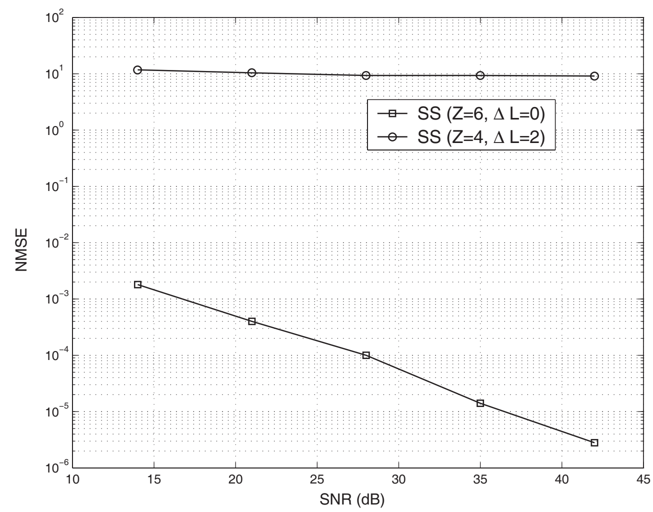

Fig. 1. Performance simulation results (NMSE) for $L = 6$, $M = 6$. Subspace method of [3]. Plots for $Z = 6$ (sufficient guard interval) with exact channel order and for $Z = 4$ (insufficient guard interval) with channel order overestimation by 2.

where $||\mathbf{H}^{(p)}(\ell)||_F$ denotes the Frobenious norm of $\mathbf{H}^{(p)}(\ell)$, and $\mathbf{H}^{(p)}(\ell)$ is the MIMO channel coefficient matrix for tap l in the pth trial. Recall that the proposed MLP based semiblind channel estimation method obtains the estimates of $\mathbf{u}_i(N-L+Z), \mathbf{u}_{i+1}(N-L+Z), \ldots, \mathbf{u}_{i+P-2}(N-L+Z)$ (see (26)) followed by the estimates of the channel coefficients by the IOCC method (27). By supposing that the data sequence of $\mathbf{u}_i(N-L+Z), \mathbf{u}_{i+1}(N-L+Z), \ldots, \mathbf{u}_{i+P-2}(N-L+Z)$ is perfectly known to the base station, the channel coefficients can be estimated by (27), yielding a NMSE which provides a lower bound to that of the proposed methods. Since there are no existing algorithms which can perform semiblind channel estimation for the considered scenario, we compared the proposed methods with this "perfect method", and referred to the associated NMSE curves as "Lower Bound" in the simulation results to be presented below.

First, the time domain subspace based blind channel estimator [3] is simulated. Its performance for sufficient guard interval $(Z = 6)$ with exact channel order and also its performance for insufficient guard interval $(Z = 4)$ with channel order overestimation by 2 $(\triangle L = 2)$ are shown in Figure 1. The figure shows that while the subspace method of [3] performs very well in the sufficient guard interval case, its performance is inferior when there is channel order overestimation and the guard interval is of insufficient length. It is in this scenario that multistage linear prediction based channel estimators proposed in this paper perform well and have an advantage over the subspace based method of [3].

A. Performance of MLP based method with exact knowledge of channel order

In this subsection, the performance of the proposed MLP based semiblind channel estimator is investigated for the insufficient guard interval scenario, provided that the base station perfectly knows the channel order. Figure 2 shows the results (NMSE v.s. SNR) of the MLP-QR based algorithm for $L = 4$, and $Z = 2$ and $Z = 3$, respectively. One

TABLE I
SUMMARY OF PROPOSED MLP BASED SEMIBLIND CHANNEL ESTIMATION ALGORITHM

Given the received signals $\{\mathbf{y}_i(n), \mathbf{y}_{i+1}(n), \ldots, \mathbf{y}_{i+P-1}(n)\}$ for $n = 0, \ldots, N+Z-1$, the pilot block $\mathbf{u}_i(n)$ for $n = 0, \ldots, N-1$, the channel order L, and the guard interval length Z.

Step 1. For $k = 0, 1, \ldots, L-Z-1$ and $j = i+1, \ldots, i+P-1$, let $\tilde{\boldsymbol{y}}_j^{(k)} = [\mathbf{y}_j^T(N+Z-1), \ldots, \mathbf{y}_j^T(k+1)]^T$ and $\boldsymbol{y}_j^{(k)} = [\tilde{\boldsymbol{y}}_j^{(k)T}, \mathbf{y}_j^T(k)]^T$. For each k,
i) compute

$$\mathbf{R}_{\mathbf{y}\tilde{\boldsymbol{y}}}^{(k)} = \frac{1}{P}\sum_{j=i+1}^{i+P-1} \mathbf{y}_j(k)(\tilde{\boldsymbol{y}}_j^{(k)})^H, \ \mathbf{R}_{\tilde{\boldsymbol{y}}\tilde{\boldsymbol{y}}}^{(k)} = \frac{1}{P}\sum_{j=i+1}^{i+P-1} \tilde{\boldsymbol{y}}_j^{(k)}(\tilde{\boldsymbol{y}}_j^{(k)})^H,$$

and the singular value decomposition (SVD) of $\mathbf{R}_{\tilde{\boldsymbol{y}}\tilde{\boldsymbol{y}}}^{(k)} = \mathbf{USV}^H$;
ii) invert the largest $(N+L-Z-k-1)K$ diagonal values of $\mathbf{S}$ and set the other diagonal values of $\mathbf{S}$ to zero to generate $\mathbf{S}^\dagger$; and
iii) according to (10), compute the LP matrix by

$$\hat{\mathbf{P}}_{\rm LP}^{(k)} = \mathbf{R}_{\mathbf{y}\tilde{\boldsymbol{y}}}^{(k)}\mathbf{VS}^\dagger\mathbf{U}^H,$$

and the prediction error vector $\hat{\mathbf{d}}_{i+1}^{(k)} = -\hat{\mathbf{P}}_{\rm LP}^{(k)}\tilde{\boldsymbol{y}}_{i+1}^{(k)} + \mathbf{y}_{i+1}(k)$.

Step 2. Compute $k_{max} = \frac{(N+Z-L)M-NK}{(M-K)}$.
For $q = 0, 1, \ldots, k_{max}$ and $j = i, \ldots, i+P-1$, let $\tilde{\bar{\boldsymbol{y}}}_j^{(q)} = [\mathbf{y}_j^T(N+Z-1), \ldots, \mathbf{y}_j^T(L+q+1)]^T$ and $\bar{\boldsymbol{y}}_j^{(q)} = [(\tilde{\bar{\boldsymbol{y}}}_j^{(q)})^T, \mathbf{y}_j^T(L+q)]^T$. For each q,
i) compute

$$\mathbf{R}_{\mathbf{y}\tilde{\bar{\boldsymbol{y}}}}^{(q)} = \frac{1}{P}\sum_{j=i}^{i+P-1} \mathbf{y}_j(L+q)(\tilde{\bar{\boldsymbol{y}}}_j^{(q)})^H, \ \mathbf{R}_{\tilde{\bar{\boldsymbol{y}}}\tilde{\bar{\boldsymbol{y}}}}^{(q)} = \frac{1}{P}\sum_{j=i}^{i+P-1} \tilde{\bar{\boldsymbol{y}}}_j^{(q)}(\tilde{\bar{\boldsymbol{y}}}_j^{(q)})^H,$$

and the SVD of $\mathbf{R}_{\tilde{\bar{\boldsymbol{y}}}\tilde{\bar{\boldsymbol{y}}}}^{(q)} = \bar{\mathbf{U}}\bar{\mathbf{S}}\bar{\mathbf{V}}^H$;
ii) invert the largest $(N-q-1)K$ diagonal values of $\bar{\mathbf{S}}$ and set the other diagonal values to zero to generate $\bar{\mathbf{S}}^\dagger$; and
iii) according to (19), compute the LP matrix by

$$\hat{\bar{\mathbf{P}}}_{\rm LP}^{(q)} = \mathbf{R}_{\mathbf{y}\tilde{\bar{\boldsymbol{y}}}}^{(q)}\bar{\mathbf{V}}\bar{\mathbf{S}}^\dagger\bar{\mathbf{U}}^H,$$

and the prediction error vector $\hat{\bar{\mathbf{d}}}_i^{(q)} = [-\hat{\bar{\mathbf{P}}}_{\rm LP}^{(q)} \mid \mathbf{I}_M]\bar{\boldsymbol{y}}_i^{(q)}$.

Step 3. Construct from the pilot block $\mathbf{u}_i(n)$

$$\begin{aligned}\mathbf{P}_u = &[\mathbf{u}_i(0), \mathbf{u}_i(1), \ldots, \mathbf{u}_i(k_{max}), \mathbf{u}_i(N-L+Z), \\ &\mathbf{u}_i(N-L+Z+1), \ldots, \mathbf{u}_i(N-1)],\end{aligned}$$

and let $\mathbf{D}_i = [\bar{\mathbf{d}}_i^{(0)}, \bar{\mathbf{d}}_i^{(1)}, \ldots, \bar{\mathbf{d}}_i^{(k_{max})}, \mathbf{d}_{i+1}^{(0)}, \mathbf{d}_{i+1}^{(1)}, \ldots, \mathbf{d}_{i+1}^{(L-Z-1)}]$.
Estimate the ambiguity matrix $\hat{\mathbf{H}}(L) = \mathbf{D}_i\mathbf{P}_u^\dagger$.

Step 4. Estimate the data sequence

$$\hat{\mathbf{u}}_j(N-L+Z) = (\hat{\mathbf{H}}(L))^\dagger[-\hat{\mathbf{P}}_{\rm LP}^{(0)} \mid \mathbf{I}_M]\boldsymbol{y}_{j+1}^{(0)}$$

for $j = i, \ldots, i+P-2$.
Estimate $\mathbf{H}(\ell)$, $\ell = 0, \ldots, L$ via IOCC

$$\hat{\mathbf{H}}(\ell) = \frac{1}{(P-1)}\sum_{j=i}^{i+P-2} \mathbf{y}_j(N-L+Z+\ell)\hat{\mathbf{u}}_j^H(N-L+Z),$$

can observe from this figure that the NMSE performance of the proposed MLP-QR based algorithm for $Z = 3$ is better than that of $Z = 2$. Moreover, for $Z = 3$, the performance difference between the proposed method and the lower bound curve is small for SNR ≥ 35 dB. Figure 3 shows consistent performance results for $L = 6$, and $Z = 4$ and $Z = 5$, respectively. It can be seen, from Figure 3, that for SNR≥ 35 dB, the performance of the proposed method is very close to the lower bound curves, demonstrating the efficacy of the proposed MLP based method.

B. Performance under channel order overestimation

As discussed in Section IV, if the channel order is overestimated at the base station, directly computing the LPEF by (35) in the proposed semiblind channel estimation methods would seriously degrade estimation performance; whereas this effect can be alleviated by using the QR-decomposition based method. In the subsection, let us validate this by simulation results. Figure 4 displays the performance results for $L = 4$ and $Z = 2$ under channel order overestimation of 5 and 8 ($\triangle L = 5$, $\triangle L = 8$), respectively. The curves "MLP" denote the MLP based method in Table I which uses the direct pseudo inversion as in (35) to implement the LPEFs. The curve of performance lower bound was also displayed in the figure. It can be seen from this figure that for $\triangle L = 0$ (perfect knowledge of channel order) both MLP-QR and MLP based algorithms almost have the same NMSE performance. However, for the cases of $\triangle L > 0$, one can see that the MLP-QR performs much better than the MLP and exhibits small performance loss compared to that of $\triangle L = 0$. Consistent simulation results for $L = 4$ and $Z = 3$ can be observed in Figure 5. Simulation results with 32 sub-carriers and 4 receive antennas as in [3], 3 users, $Z = 4$, $L = 6$ are shown in Figure 6. The user data signals are QPSK modulated. The channel is a Rayleigh fading channel.

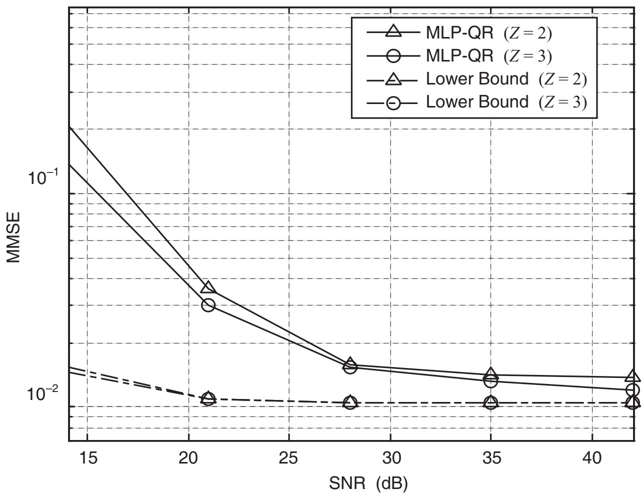

Fig. 2. Performance simulation results (NMSE) for $L = 4$, $M = 6$ (insufficient guard interval).

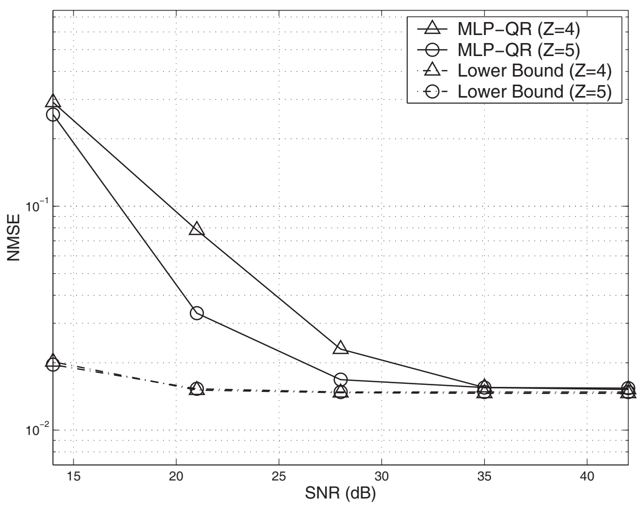

Fig. 3. Performance simulation results (NMSE) for $L = 6$, $M = 6$ (insufficient guard interval).

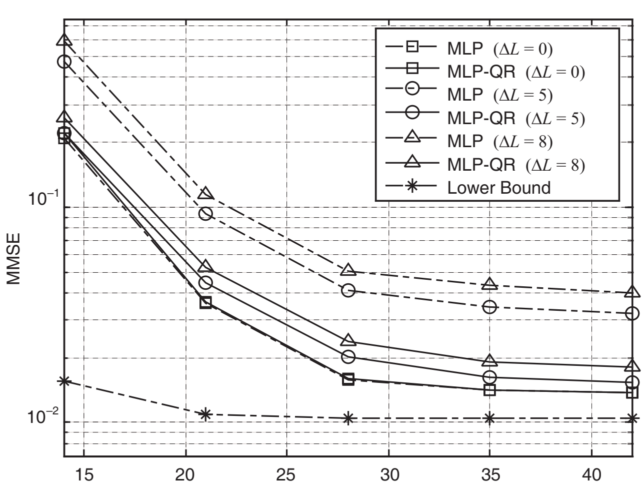

Fig. 4. Performance comparison results (NMSE) for $L = 4$, $M = 6$ and $Z = 2$ (insufficient guard interval).

VI. Conclusions

In the paper, we have presented a MLP based semiblind channel estimation algorithm for multiuser OFDM systems

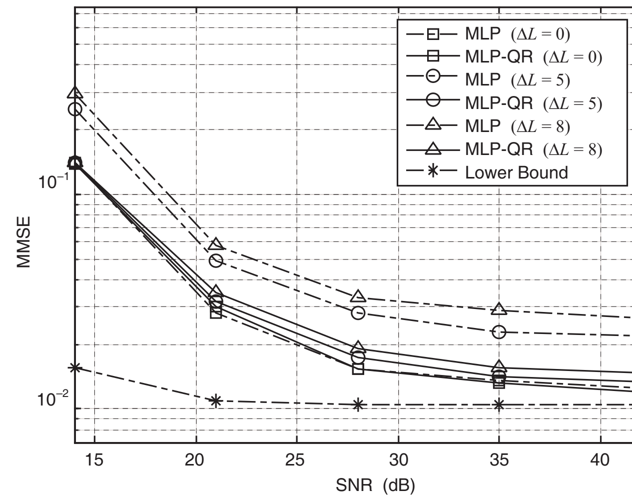

Fig. 5. Performance comparison results (NMSE) for $L = 4$, $M = 6$ and $Z = 3$ (insufficient guard interval).

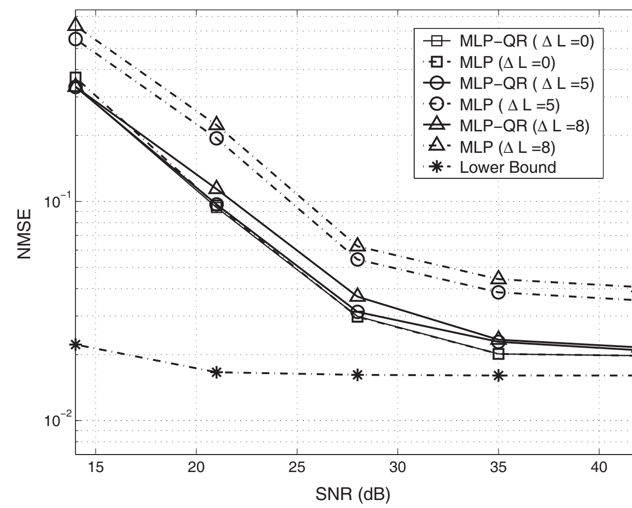

Fig. 6. Performance comparison results (NMSE) for $L = 6$, $M = 6$ and $Z = 4$ (insufficient guard interval), $N = 32$ subcarriers, $K = 3$ users, QPSK modulation.

with insufficient guard interval. We have shown that, by utilizing the relation between the LPEFs of the received signal with multiple prediction orders and the transmitted data sequence, the MIMO channel coefficients can be estimated without ambiguity by the proposed MLP based algorithm using a single pilot OFDM block only. To alleviate the sensitivity of the proposed algorithms to the channel order overestimation, we have also proposed to implement the associated LPEFs in a QR-decomposition based method. Our simulation results have shown that the proposed MLP based algorithm performs well in the insufficient guard interval situation. The QR-decomposition based LPEFs exhibit superior robustness against channel order overestimation.

References

[1] D. Gesbert, M. Shafi, D.-S. Shiu, P. J. Smith, and A. Naguib, "From theory to practice: an overview of MIMO space-time coded wireless systems," *IEEE J. Select. Areas Commun.*, vol. 21, no. 2, pp. 281-302, Apr. 2003.

[2] H. Fu, P. H.-W. Fung, and S. Sun, “Semi-blind channel estimation for MIMO-OFDM,” in *Proc. IEEE PIMRC,* vol. 3, Barcelona, Spain, Sept. 2005, pp. 1850-1854.

[3] Y. Zeng and T-S Ng, “A semi-blind channel estimation method for multiuser multiantenna OFDM systems,” *IEEE Trans. Signal Processing,* vol. 52, no. 5, pp. 1419-1429, May 2004.

[4] S. Yatawatta and A. P. Petropulu, “Blind channel estimation in MIMO OFDM systems with multiuser interference,” *IEEE Trans. Signal Processing,* vol. 54, no. 3, pp. 1054-1068, Mar. 2006.

[5] F. Wan, W. Zhu, and M. N. S. Swamy, “Linear prediction based semi-blind channel estimation for MIMO-OFDM system,” in *Proc. IEEE ISCAS*, New Orleans, USA, May 2007, pp. 3239-3242.

[6] S. Roy and C. Li, “A subspace based blind channel estimation method for OFDM systems without cyclic prefix,” *IEEE Trans. Wireless Commun.,* vol. 1, no. 4, pp. 572-579, Oct. 2002.

[7] G. Leus and M. Moonen, “Per-tone equalization for MIMO-OFDM systems,” *IEEE Trans. Signal Processing*, vol. 51, no. 11, pp. 2965-2975, Nov. 2003.

[8] M. Toeltsch and A. F. Molisch, “Efficient OFDM transmission without cyclic prefix over frequency-selective channels,” in *Proc. IEEE PIMRC*, London, UK, Sept. 2000, pp. 1363-1367.

[9] P. De and B. Shah, “Robust QR-decomposition based blind equalizers,” in *Proc. IEEE Radio and Wireless Symposium*, Long Beach, USA, Jan. 2007, pp. 217-220.

[10] A. F. Molisch, M. Toeltsch, and S. Vermani, “Iterative methods for cancellation of inter-carrier interference in OFDM systems,” *IEEE Trans. Veh. Technol.*, vol. 56, no. 4, pp. 2158-2167, July 2007.

[11] J. Tugnait and B. Huang, “Multistep linear predictors based blind identification and equalization of multiple input multiple output channels,” *IEEE Trans. Signal Processing*, vol. 48, no. 1, pp. 26-38, Jan. 2000.

[12] P. De, “Combined linear prediction and subspace based blind equalizers,” in *Proc.IEEE ISCAS*, New Orleans, USA, May 2007, pp. 3479-3482.

[13] A. P. Liavas, P. A. Regalia, and J.-P. Delmas, “On the robustness of the linear prediction method for blind channel identification with respect to effective channel undermodeling/overmodeling,” *IEEE Trans. Signal Processing*, vol. 48, no. 5, pp. 1477-1481, May 2000.

[14] C. B. Papadias and D. T. M. Slock, “Fractionally spaced equalization of linear polyphase channels and related blind techniques based on multichannel linear prediction,” *IEEE Trans. Signal Processing*, vol. 47, no. 3, pp. 641-654, Mar. 1999.

[15] H. Gazzah, P. Regalia, J. Delmas, and K. Abed-Meraim, “A blind multichannel identification algorithm robust to order overestimation,” *IEEE Trans. Signal Processing,* vol. 50, no. 6, pp. 1449-1458, June 2002.